In Defence of Factions

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We model faction formation in a world where party politicians' objective is the development of an informed program of governance. Politicians' preferences reflect their own views and their information that, when aggregated via intra-party deliberations, influences the party manifesto. By joining a faction a politician increases the influence of its leader on the manifesto, but foregoes his individual bargaining power. For broad model specifications we find that a faction formation process allows power to be transferred to moderate politicians. This facilitates information sharing, so increasing the capacity of the party to attain its objective. These positive welfare effects may hold even when factionalism restricts intra-party dialogue, and hold a fortiori when information is freely exchanged across factions. We conclude that the existence of ideological factions may benefit a party: it provides a means to tie uninformed or extremist politicians to more moderate and informed faction leaders.

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INTRODUCTION

Analysis of political parties often makes use of the "unitary actor" assumption: the party is a single player with a common and known objective. While, arguably, this assumption is useful when considering certain party roles, such as bargaining over government formation, it is less so when considering others. In many democracies, for example, parties have a rich tradition of manifesto writing. In the Westminster system, which tends to produce single party majorities, a manifesto establishes a direct mandate for governance linking pre-election promises to policies implemented post-election. Where coalition governance is the norm, there is a close link between policies proposed in the manifesto and final outcomes (Back et al., 2011). Indeed it has been argued (Budge (1987), p.15), that manifestos "form genuine statements of preference rather than mere bargaining counters in negotiations." Manifesto commitments are enforced, albeit imperfectly, via party discipline which itself relates to institutional features (such as the confidence procedure and party whips).

The extant literature suggests two factors that are important in determining a party's program. The first are its internal factions. The second is inherent uncertainty as to which proposals allow the party to achieve its objectives. Budge et al. (2011) analyze party manifestos in twenty democracies in the post war period and find that "parties are ideologically based and pursue their own policies, that they are internally factionalized and operating under high levels of uncertainty." In order to understand the policies proposed by a party we therefore need to account for the balance of power between its factions and the mechanisms deployed to reduce uncertainty in manifesto writing. Here, we develop a model in which the manifesto is the outcome of bargaining between factions and where politicians preferences over manifesto commitments are formed through intra-party deliberations.

The critical and novel feature of our model is that stable party factions emerge endogenously, due to strategic choices made by politicians who consider the effect of joining one faction (or another) or of remaining unaligned. It is logical that, if party factions influence policy, then politicians will anticipate this effect when joining. Of course politicians join factions for many reasons, amongst them the prospect of material reward. While such motives have been studied formally (see Persico et al. (2011)), to our knowledge there is no model of factions forming between politicians whose motives are, at least in part, ideological and who care about the policies proposed in the party manifesto. Without a model it is hard to evaluate the tradeoffs faced by such politicians when joining a faction and, moreover, the normative consequences of factionalism.

Our normative approach to the impact of factions, from the perspective of intra-party deliberation, is warranted. Theoretical argument and careful empirical evidence supports the view

that intra-party debate can benefit a party. The argument goes back to Burke (1851) who, in his essay "Of Parties in General," defined a party as "a body of men united for promoting by their joint endeavors the national interest." His view was that the combined wisdom and judgement of politicians formed the basis of a clear mandate for governance and that intra-party deliberation enhanced this prospect. Empirical evidence comes from Wantchekon (2003) and Fujiwara and Wantchekon (2013): the former distinguishes the impact of appeals based on manifesto promises of public goods provision from those offering material favors in exchange for votes; the latter shows that pooling information via communication between politicians and experts leads to more informed and effective party programmes.

From this perspective, factionalism might be seen as detrimental. White and Ypi (2011) refer back to Burke in distinguishing a party that justifies its programme in the "name of the whole" from a faction that "exhibits no concern to justify its program to the community in toto." Indeed, and although factions are seen as a "fact of life within parties" there is a common perception that they, nevertheless, exacerbate non-cooperative behaviour and so are antithetical to achievement of common goals. Boucek (2009) provides a review of the field and notes that negative perceptions of factionalism originated with Hume (1877: Essay VIII) and are still predominant. She, however, offers a different perspective, conjecturing that "by facilitating the aggregate capacity of parties while preserving subgroup identities, a factional structure may be instrumental in promoting intra-party cooperation and in building integrated parties." An example is provided by McAllister (1991) whose in-depth study of the Australian Labour Party concludes that "modern factions are open, policy-oriented groups whose activities are generally seen as accommodative rather than disruptive."

We can abjudicate between these perspectives as the presumed negative consequences of factionalism are related to three facts relevant to our model. First, when joining a faction, a politician relinquishes his independence and increases the power of a faction leader. Second, in influencing the programme, such leaders consider only their own payoff and not what is best for the party "in toto." Third, factionalism may restrict intra-party deliberation and hence communication of policy relevant information to decision-makers; so we compare an "open" world where deliberations take place between politicians independently of their factional identity, to a "closed" one where communication between politicians belonging to different factions is stifled.

As a benchmark, motivated by the normative critiques of factionalism above, we analyze a party in which power is shared equally between politicians of different viewpoints who deliberate openly over outcomes. Our results challenge the negative perception of factions and, indeed, are in line with Boucek's claim that factions "facilitate the aggregate capacity

of parties." Here, party welfare depends, in part, upon the amount of policy-relevant information pooled via intra-party deliberations, precisely the mechanism identified empirically by Fujiwara and Wantchekon (2013) amongst others. Surprisingly, we find that parties can maximize the potential impact of such deliberations by allowing, rather than prohibiting, faction formation. Moreover, when led by politicians who are moderate with respect to the spectrum of opinions, factions are strictly welfare enhancing. These positive welfare effects may hold even when factionalism restricts intra-party dialogue. And they hold a fortiori when information is freely exchanged across factions.

The key intuition and novel insight that emerges from our analysis is that factions provide a mechanism by which power can be transferred to better informed politicians. We provide an organizational rationale for factions that we label "tying in the extremists": factions tie politicians with the most extreme views to more moderate faction leaders who are better placed to take party decisions due to others' willingness to communicate (truthfully) with them. Conversely we provide conditions under which factions are detrimental to party welfare: this can only occur when their leaders hold extreme views relative to other members of the faction.

Our notion of tying in the extremists resonates with detailed empirical study of intra-party politics. Notably, Muller-Rommel (1982) analyses the German Social Democrats between the years 1969-80 during which, under party leader Willy Brandt, they embarked on a strategy of "openness" toward informally organized groups amongst its leftist activist base (the so called JUSOS). Muller-Rommel (1982) highlights (p.263) that the integration of these radical groups into the party mainstream was achieved by increasing influence of existing organized factions,[3] thus he finds (p.266) that party factions were a "stabilising element in the structural development of the party." His argument corresponds with our (more general) finding that factions (can) improve the efficiency of parties by allowing for better exchange of policy relevant information via intra-party deliberation.[4] He concludes (p.263) that "well functioning internal factional communication structures are a necessary condition for successfully enhancing the ability of informal groups to act. The efficiency of factional activity increases with the frequency of communication between (if possible) many faction members." Indeed our theoretical model shows that efficiency is enhanced when communication is "open" between party factions.

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[3]He describes the formation and development of the main intraparty groupings–to the left the *Frankfurter Kreiss* (or Circle), on the center-left the *Godesberger Kreiss*, on the centre-right the *Tubinger Kreiss*, and to the right the *Kurt-Schumacher Kreiss*. For each faction, Muller Rommel identifies the membership of the party's Bundestag *fraktion* (caucus), its internal factions, and their main programmatic goals. Of particular interest was the development of a leftist faction, that became known as the *Leverkusener Kreiss*, following the 1969 election. It grew to approximately one third, numbering between forty and sixty deputies, and played a pivotal role in the inclusion of leftist groups and politicians.

[4]The information gathering exercises and informal internal communication structures of the different factions are described in great detail. For example, we learn that the Leverkussen Circle met regularly with policy experts and practitioners and that faction members held office on the 16th floor of the Bundestag and met regularly for dinner, and always on Tuesdays and Thursdays, to deliberate over policy (p.153-154).

1. Intra-party bargaining and deliberation

Before providing an overview of our main findings and more detailed analysis, we pause in order to justify the key assumptions that underpin our model. The first and central aspect of our model is that manifesto outcomes reflect weighted intra-party bargaining between factions. The second is that intra-party deliberations are used to alleviate the uncertainty faced by a party when drafting their policy proposals.

1.1. **Intra-party bargaining.** We view a faction as a resource that allows its leader influence over policy. Our view thus accords with that of Laswell (1931) whose definition in the *Encyclopedia of Social Sciences* states: "the term faction is commonly used to designate any constituent group of a larger unit which works for the advancement of particular persons or policies."

We study the influence of ideological party factions on its manifesto and so set aside the view that factions form to pursue particular material interests (as noted already, the endogenous formation of such "factions of interest" has already been studied). We view the manifesto as an outcome of bargaining between party faction leaders. More specifically, in our model the weight given to a faction leader's views is increasing in the size of his faction. Correspondingly, when joining a faction, a politician increases the influence of its leader on final policy. We discuss the general claim and the specific assumption in turn, and with reference to the empirical literature on political parties.

The existence of ideological factions has been noted in several empirical studies. Rose (1964) identified the organizing principle of a faction's ideology– shared values around which faction members ultimately cohere and coordinate activity. Analyzing over 100 parties on several dimensions, Janda (1980) found that the most common basis of factionalism was ideological and that factions typically also had distinct leaders. Moreover, recent text analysis using state of the art estimation and measurement techniques on party documents and speeches provides strong evidence of ideological intra-party groupings (see Laver and Giannetti (2004), Konig (2006), and Bernauer and Brauninger (2008) amongst others).[5]

Our claim that the manifesto reflects the outcomes of bargaining between factions is of course a generalisation. As stated by Lo et al. (2014): "there are few common procedures that govern how parties arrive at the final draft. At one extreme, a small committee composed of the party leadership may write the manifesto. At the other extreme, a party conference may discuss various proposals and vote on an issue-by-issue basis on amendments." Put otherwise, there are a large number of extensive form games that could be analyzed to represent the specific

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[5]A common distinction is made between ideological factions of "principle" and factions of "interest" (see Bettcher (2005). Things may not be so clear cut. Boucek (2012) finds that faction motivations are themselves endogenous to national and party level institutions.

protocols adopted by different parties. While our reduced-form approach can be justified due to the feasibility constraint imposed by analyzing such a broad class of games, a stronger defence is built on empirical grounds: our approach is consistent with broad findings in the empirical literature showing that a party manifesto (or changes to it over time) reflect the weights of different party factions.

The earliest claims that party manifestos could be related to the bargaining power of factions can be traced back to Harmel et al. (1995). They understood manifesto (as well as other party) change as due to alternation in a party's "dominant faction."[6] For example, they argue that replacement of the "left" with the "new socialist right" as the party's dominant faction led to the adoption of the Bad Godesberg programme by the German Social Democratic Party in 1958. Budge et al. (2011) analyze the time series of party positions measured by manifesto documents and find that "factional struggles power the policy alternation pattern observed on the left-right continuum." Similarly, Lo et al. (2014) view a manifesto as a resolution mechanism for two factions (or, equivalently, two groups of voters), differentiated according to their ideology and competing for control.

Our more specific claim, that outcomes correspond to weighted bargaining, can also be justified. As noted by Levy (2004), even "the mechanisms by which Western European parties reach internal compromise in their manifestos mimic some form of a weighted average of the ideal policies of their factions; the parties delegates vote on the policy principals in an annual conference whereas the balance of power between factions, that is, the proportion of votes that each faction receives, is translated into policy recommendations. These policy recommendations are then incorporated into the party's election manifesto."[7] Indeed, the statistical evidence of Lo et al. (2014) is consistent with the conjecture that, on average, the manifesto content is a weighting of the ideological differences between different factions.

A recent and specific case study is by Ceron (2012) who provides strong evidence in support of the assumption that "factions negotiate over party positions according to the bargaining power of each subgroup." He uses data from party congresses in Italy to measure faction preferences and size and relates this to policy positions of Italian parties. His empirical finding is that "party position will be closely related to the weighted mean of the factions' positions. Factions then affect the party proportionally to their strength." These findings are perhaps surprising since on many dimensions, e.g. candidate selection, Italian parties are quite centralised (see Galasso and Nannicini (2011)). Importantly for our purposes, Ceron

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[6]While the authors do not provide an exact account of their coding, the discussion makes clear that changes in a party's dominant faction reflects the relative size of factions as well as to their leaders.

[7]In so far as such formal mechanisms are binding. Michels (1915/1958) classic analysis of the German Socialist Party, showed that although policymaking was governed by constitutional procedures these were ignored by the party oligarchy. McKenzie (1955) revealed few differences in practice between procedures adopted by the British Labour and Conservative parties despite the constitutional status of the party conference in the latter.

finds that allowing for the identity of the median (politician or faction) in the party does not improve the fit of his statistical model when accounting for the bargaining weights of the party factions. This finding corresponds with our predictions (proposition 4) below.

Of course there are mitigating factors. For example, Ceron shows that, in Italy, the relationship between the proportional strength of factions and policy position is stronger when leaders are elected by party committee. This suggests that our assumptions are weaker when leaders have a degree of autonomy from the parliamentary party. Such autonomy might arise when a leader's power stems from her direct relationship with voters or extra-parliamentary groups. Nevertheless, even in such less likely cases, the qualitative evidence suggests that, at times, our weighted bargaining assumption may hold. Beer (1965), for example, studies the British Labour Party in the 1950's and 60's. It was part of a two-party system whereby its manifesto provided a direct mandate for governance. Under its constitution, the party's National Executive Committee in charge of writing the manifesto was to implement the wishes of the party conference. Beer shows (chapter 8) that the party manifesto was neither controlled by a centralised leadership or a majority in party conference. Instead, he argues that the party was divided "vertically between two sets of followers and leaders" (p.231). Each faction "had an ideology and each sought to commit Labour to its ideological position." While the revisionists behind Hugh Gaitskell had the "balance of advantage in program-making" they were unable always to impose their views on the fundamentalist faction lead by Ernest Bevan. The latter could count on a "solid core of about a quarter" of the parliamentary party which included Bevan's "able lieutenants," key members of the National Executive Council who directly influenced the manifesto.[8]

1.2. **Intra-party deliberations.** We allow our politicians a rich set of motivations: they have (idiosyncratic) ideological concerns but also share a common desire that the party develop an informed program of governance. As we will describe in more detail in the following section, this statement (and so our model) is consistent with different interpretations. A leading interpretation is that politicians desire a program that is informed as to the wishes of the party's target voters. A different interpretation draws upon Burke's view of programmatic political parties cited earlier. According to this view the party program reflects politicians' views about the national interest. Whichever interpretation of politicians motives we adopt,

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[8]The British Labour Party provides several episodes consistent with our claim that factions tie extremists to more moderate and better informed leaders. One such example follows its defeat to the Conservatives in the 1987 general election. The party's Policy Review involved a series of working groups each run by a Shadow Minister from the Shadow Cabinet (elected by MPs and broadly representative of the party's different ideological factions). A key division was that between Modernisers and Traditionalists, the former providing the origins of what was to become New Labour. Although the party was centralised under its centre-right leader Neil Kinnock and his Private Office, on key policy areas it was important to keep the left on board. A key element of reform was the policy review for Employment, Training, and Labour Law drafted by Michael Meacher, Shadow Secretary for Employment, and a traditionalist who could deliver the support of the left. (Source, private conversation with David Soskice, January 2015).

there is inherent uncertainty as to which policies should be pursued toward such aims. Thus, insofar as a politician (or perhaps the party leader) wishes to appeal to voters (or to the national interest) she alone does not know which policies best serve that purpose.

This uncertainty faced by parties in the manifesto writing process has been the subject of several recent empirical papers. These relate the time series of party manifesto observations (analyzed on the left-right dimension) to inductive and myopic decision rules that a vote-maximizing party may adopt from one election to another: Somer-Topcu (2009) explores policy adjustments as a response to electoral losses in the previous election; while Adams and Somer-Topcu (2009) consider policy change in a current election as related to moves made by other parties in previous ones.[9]

While commonly observed and relatively easy to measure factors, such as previous policies and election results, are easily incorporated in our model (a common prior will suffice), our focus instead is on intra-party deliberations that form part of the manifesto writing process. That intra-party deliberations take place is self-evident: at any party gathering, politicians will exchange views. Indeed, deliberation is a main mechanism deployed by parties to reduce uncertainty and is widely used by different parties. Consider, for example, the two main British political parties who, as we write, are finalising their manifestos for the upcoming 2015 election. Central to the manifesto preparations of the Labour Party is its Policy Review led by key members of the party's Shadow Cabinet, each with a different policy brief. It is, as described in the documentation of the party, "a comprehensive process of discussion looking at every aspect of Labour policy in order to support the development of our manifesto for 2015." Deliberations are also central to the Conservative manifesto writing process via the Policy Forum that produces discussion briefs on manifesto issues-for example, most recently, housing and planning - that are then submitted to the party's Central Office.

The existence of such deliberations does not, of course, ensure their relevance to the manifesto writing process. Such deliberative processes might be adopted for other purposes. For example they can provide coordination around a leader's views.[10] However, as noted by Landa and Meirowitz (2009)," if political debate is to serve a purpose beyond allowing politicians to simply coordinate on a particular policy choice—if the purpose is to meaningfully change the preferences of politicians over outcomes—then the information held by participants about policy choices cannot be common knowledge: some information must be private." Consequently, we start from the premise that, although no politician is perfectly informed about the wishes of the party's target voters, or the best policies to pursue from a national interest perspective, each has some private information that is relevant to choices a party makes.

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[9]More generally, Callander (2011) has analyzed a dynamic theoretical model in which the search for "good policies" is related to observed policy outcomes that correspond to a Brownian motion.

[10]See Dewan and Myatt (2007), for a formal analysis.

Such information may be obtained from constituency surgeries, interactions with research staff, or other sources. It can be exchanged in informal discussions or in formal consultation and intra-party deliberations such as those mentioned above. And it can affect the views of those (faction leaders) who determine the party policy.

As customary in the formal-theoretic literature, we identify debate with verbal (cheap talk) communication between privately-informed participants seeking to arrive at a collective choice.[11] Specifically, we assume that each politician observes a private but imperfect signal. Using the multi-player communication model by Galeotti et al. (2013), we study the strategic communication of such information to party decision-makers who, in our model, are faction leaders. This setup proves useful when analyzing manifestos as a collection of policy decisions to be made, where politicians have private information that is relevant to each of these decisions, and where players preferences are affected by communication between them. Indeed as the model can be used to describe any directed network of communication, it covers the types of discussions that politicians might have in the deliberative forums mentioned above. Our analysis of intraparty deliberation provides a framework for exploring the normative consequences of factionalism. As stated by Landa and Meirowitz: "Unless we understand the conditions under which the incentives in deliberative environments encourage agents to be sincere or fully revealing, as opposed to insincere or withholding information, we cannot hope to offer a coherent (stable) normative argument for institutional design."

2. Our Contribution to the Related Literature

We contribute to a literature on endogenous parties stimulated by Krehbiel (1993), who raised the possibility that effects attributed to party organization may be due to similarity of preferences, that influenced a large body of work: Patty (2008) formalizes the notion of a "cartel party" and shows the effect of the size of the majority on its ability to discipline its members; Eguia (2011a,b) looks at the role of parties in enabling blocs of politicians to achieve mutually beneficial outcomes that could not be obtained were each legislator to act in isolation. Morelli (2004) analyzes the effect of the electoral system on party formation; whereas Snyder and Ting (2002) explore the effect of party labels. Levy (2004) explores party formation using the citizen-candidate model and shows that parties are "effective"– they influence outcomes independently–only when the policy space is multidimensional. In our paper, a party is given by the set of players (politicians) and its internal factions form endogenously. These partitions are effective, in a sense similar to Levy, when parties are sufficiently heterogenous.

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[11]See Minozzi et al. (2013) for a dissenting voice arguing that such a formulation of political debate and the deliberative process omits key, philosophical reasons for deliberation, such as a desire on the part of participants to publicly articulate reasons for their desired outcomes.

We share a focus on the intra-party determinants of the party platform with Caillaud and Tirole (2002, 1999) who also analyze the allocation of control rights on the quality of the party platform. In their model the platform has an ideological and a valence component (a left policy, for example, can be of high or low quality). Ideological dissonance between the party leadership and its rank and file provides incentives for the former to invest in the quality of the platform design. The authors contrast platform design in an authoritarian party (where control is exercised by the leadership) and a democratic one run by the rank and file. Castanheira et al. (2010) analyze platform design in a similar set up with two party competition that allows them to explore rent-seeking behavior given the equilibrium degree of intra-party competition.

In our model, quality depends upon the pooling of policy relevant information via intra-party deliberation rather than on investments made by party leaders. Our theoretical investigation into strategic aspects of intra-party deliberation follows Galeotti et al. (2013) whose framework for the study of multi-player communication provides tractability and substantive new political insights as witnessed by a host of recent papers: Patty and Penn (2014) study information transmission in small networks of decision makers; Patty (2013) determines the optimal exclusion and inclusion policies to maximize information sharing among cabinet members; Gailmard and Patty (2009) study transparency and optimal delegation by a principal to informed agents; Dewan et al. (2015) investigate the optimal assignment of decision-making power in the executive of a parliamentary democracy; and Penn (2015) studies the formation of stable aggregations of different units within an association.

While these papers build on the same methodology, each studies distinct political problems. At the purely theoretical level, the closest papers to ours are the latter two. Nevertheless, there are significant differences making our analysis both novel and challenging. Specifically, unlike Penn (2015) who holds the allocation of authority as exogenous, here the allocation is endogenously transferred to the leaders of factions that form in equilibrium. And unlike Dewan et al. (2015), who study the optimal allocation of authority within a group of politicians, we solve the significantly more complex problem of determining the equilibrium outcome of voluntary transfer of authority.

We believe our model is the first that explores the endogenous formation of ideological factions of "principle". However, a recent paper by Persico et al. (2011) looks at the formation of factions of "interest". In their model factional allegiance allows politicians to advance their careers and deliver pork to their constituents. A faction is a "team of politicians who are mutually dependent on each other for career advancement." Despite this critical difference, the two models have a common component in that both build on an intuitive assumption that factions become stronger– they have more bargaining power–the larger they are; in their

model a candidate's ability to obtain pork for his district depends upon the size and composition of the faction to which he belongs; in ours, a politician anticipates that joining a faction strengthens the position of its leader and influence on policy.

3. MODEL

In this section we describe our model of endogenous faction formation and political debate within the party. We define a factional structure as a pair consisting of a partition of the party into factions, together with a leader for each faction. The influence of a faction leader on the party manifesto increases in the size of his faction. Intra party debate takes place after the faction building stage, before the party manifesto is produced, and so is anticipated by politicians. We study factional structures that satisfy a specific stability/equilibrium requirement, motivated by the idea that factional structures should not be upset by unilateral deviations. The factional structure is stable if there is an equilibrium (within a class with desirable properties to be specified) in which each politician chooses to remain in his faction. Importantly, our model's restrictions are minimal. We do not commit to any specific game form that describes which politicians become faction leaders and how factions form. Thus, our results may be applied to political environments characterized by different mechanisms.

3.1. **Party Factions and the Manifesto.** We let $\mathcal{I} = \{1, ..., I\}$ be the set of prominent, ideologically differentiated politicians in a party. A factional structure within $\mathcal{I}$ is defined as a partition $\mathcal{C} = \{F_1, ..., F_N\}$ of $\mathcal{I}$, together with a function $\mathcal{L}$ assigning a leader $l \in F$ for each faction F. We let $\mathcal{L}^{-1}(l) \equiv \mathcal{F}(l)$ denote the faction of leader l.

To determine the stability of each factional structure $(\mathcal{C}, \mathcal{L})$ we introduce the following political game. Initially, the party is factionalized according to the structure $(\mathcal{C}, \mathcal{L})$. In the first stage of the game, each player i chooses whether to stay in his own faction, move to a different faction, or act independently. When joining a faction F led by leader l, a politician i increases the influence of l in the party, measured by the influence weight $\pi_l \in (0, 1)$, and we normalize the sum of the influence weights across leaders to one. For simplicity, the implications of a leader l's influence on the party manifesto are summarized by a single strategic variable $\hat{y}_l$ that lies on the left-right spectrum $\mathbb{R}$. We assume that only faction leaders exert any influence on the party manifesto and that if a faction leader moves to a different faction he transfers his influence to the leader of that faction. We denote the factional structure that arises after this first stage of the game as $(\mathcal{C}', \mathcal{L}')$, to distinguish it from the initial structure $(\mathcal{C}, \mathcal{L})$.

Each politician i would like that all leaders' choices $\hat{y}_l$ to be as close as possible to his ideal point on $\mathbb{R}$. The ideal point is parameterized as $\theta + b_i$, where θ is an unknown state of the

world, uniformly distributed on $[0, 1]$. The state θ represents the common, but uncertain component of the preferences of politicians in $\mathcal{I}$. Each one of them would like the 'best decisions' to be made as far as θ is concerned. We are agnostic about the precise definition of "best" as different parties may have different objectives. Interpretations include best from the voters' perceived perspective, so that politicians share a common aim of election, or best in the sense of policies that are in voters' interest. The relevant point is that, whatever politicians' objectives, they are uncertain as to which policies best serve their intentions. As we shall see, this implies that it is beneficial to all politicians in $\mathcal{I}$ that as much information as possible on θ is conveyed to faction leaders through political debate. For each individual politician i, the individual bias b_i represents the extent and direction of i's bliss point deviation from θ. This may reflect i's own personal views and/or those of i's constituents.

The focus of our model is the impact of factions on political debate within parties. In order to study intra-party communication, we adopt the following functional form for each politician i's terminal payoff:

$$u_i(\hat{\mathbf{y}}, \theta, \pi) = - \sum_{F \in \mathcal{C}'} \pi_{\mathcal{L}'(F)} \left(\hat{y}_{\mathcal{L}'(F)} - \theta - b_i \right)^2,$$

where we set $\pi_{\mathcal{L}'(F)} = |F|/I$, the fraction of politicians in I that belong to F, and we also define $u_i(\mathcal{L}'(F), \hat{y}_{\mathcal{L}'(F)}, \theta) = - \left(\hat{y}_{\mathcal{L}'(F)} - \theta - b_i \right)^2$, for future reference. As shown in Galeotti et al. (2013), this utility specification yields a simple and tractable equilibrium solution, within the communication game that we describe below.

3.2. **Intra-party Debate.** We stipulate that each politician i has some private information on θ. This, as already noted, is a necessary feature of a model of political debate. Here, each politician's information may come from his own research or information gleaned from interest groups, think tanks, private experts and other interested parties. We represent i's private information as a signal s_i that follows a simple statistical law: conditional on the state θ, s_i takes the value equal to one with probability θ and to zero with probability $1 - \theta$.

Before faction leaders exert their influence on the manifesto, politicians can convey their views and information to them. Communication takes a very simple form. Each politician i may send (possibly differentiated) message $\hat{m}_{ij} \in \{0, 1\}$ to any other politician j. A pure communication strategy of player i is a function m_{ij} of s_i. Each communication strategy m_{ij} from i to j may either be truthful, in that a politician reveals his signal to j so that $m_{ij}(s_i) = s_i$ for $s_i \in \{0, 1\}$, or not: in the latter case, $m_{ij}(s_i)$ does not depend on s_i. For any communication strategy profile $\mathbf{m}$, we let each politician j's information $d_j(\mathbf{m})$ be the number of binary signals known by player j after communication takes place, and note that $d_j(\mathbf{m})$ equals the

number of players i whose strategy m_{ij} is truthful plus one. Our model allows for communication with multiple politicians, but we restrict to simultaneous messaging.[12]

As noted in our introduction, the existence of factions may exacerbate noncooperative forms of intra-party behaviour. We study the (possibly) negative consequences of factions in prohibiting intra-party communication, analyzing communication under two different social norms. Under *open communication*, all politicians can communicate to any other politician. Communication is *closed* if politicians can communicate only within their faction–i.e. players are restricted to sending at most one truthful message.

After debate takes place, by sequential rationality, each leader l makes choice $\hat{y}_l$ to maximize his expected utility $E[u_l(\hat{\mathbf{y}}, \theta, \pi)|s_l, \hat{\mathbf{m}}_{-l,l}]$, where the expectation is taken with respect to θ, and the pair $(s_l, \hat{\mathbf{m}}_{-l,l})$ represents l's information (specifically, l's signal and the messages received by other politicians). We denote the associated policy strategy as $y_l : \{0,1\}^{\mathcal{I}} \to \Re$.

3.3. **Equilibrium Concept and Selection.** Our solution concept is pure-strategy Perfect Bayesian Equilibrium. There may be multiple equilibria $(\mathbf{m}, \mathbf{y})$ in the communication games taking place after factions are established. For example, the strategy profile where no player is truthful and no information is aggregated in the party is always an equilibrium. Politicians' welfare, however, increases when information is aggregated: faction leaders are better informed, so the quality of the party programme is enhanced. Correspondingly we adopt the standard equilibrium selection criterion: in the political game, we select equilibria $(\mathcal{C}', \mathcal{L}'; \mathbf{m}, \mathbf{y})$ that maximize each politician's welfare and information aggregation on the equilibrium path. Specifically, we focus on equilibria $(\mathcal{C}', \mathcal{L}'; \mathbf{m}, \mathbf{y})$ such that $d_l(\mathbf{m})$ is as large as possible, for all faction leaders l. As we shall see, our simple information and signal structure means that, along the equilibrium path, beliefs are straightforward to define using Bayes' Rule.

In order to construct out-of equilibrium beliefs we need to consider deviations in the stage in which players choose their factional allegiance. If a player j deviates from the initial factional structures $(\mathcal{C}, \mathcal{L})$ –by choosing to join a faction different from his initial one, or by acting independently and retaining power over y_j– then we restrict beliefs at any subsequent information set as follows. We posit that beliefs are consistent with play of the non-informative strategy m_{ij} for all players $i \neq j$, so that $d_j(\mathbf{m}) = 1$; whereas $d_l(\mathbf{m})$ is still as large as possible, for all leaders $l \in \mathcal{L}'$. An interpretation of this restriction is that any player j who deviates from his initial factional allocation is punished by "ostracism" —i.e., nobody in the party is willing to talk to him anymore, whereas faction leaders are still optimally informed within the party.[13]

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[12]We restrict to multi-player simultaneous cheap talk as multi-player multi-round cheap talk is very involved. For an analysis of a dynamic setting and a single informed sender, see Ivanov (2013).

[13]Cutting down all communication to a politician who deviates from his initial factional allocation provides strong incentives against deviation. Note, however, the existence of a more potent punishment for j's deviation: he would

Within the equilibrium class defined above, we select the equilibria $(\mathcal{C}', \mathcal{L}'; \mathbf{m}, \mathbf{y})$ that maximize welfare within the party. Our notion of welfare is ex-ante utilitarian. We let $\mathcal{U}_i(j, \mathbf{m}, y_j) = -E[(y_j(s_j, \hat{\mathbf{m}}_{-j,j}) - \theta - b_i)^2]$ be the equilibrium utility of politician i from the influence of politician j over the party programme in equilibrium $(\mathcal{C}', \mathcal{L}'; \mathbf{m}, \mathbf{y})$. So, the Utilitarian party welfare $\mathcal{W}(\mathbf{m}, \mathbf{y}; \mathcal{C}', \mathcal{L}')$ is the sum of the utilities $\mathcal{U}_i(\mathbf{m}, \mathbf{y}; \mathcal{C}', \mathcal{L}') = \sum_{F \in \mathcal{C}'} \pi_{\mathcal{L}'(F)} \mathcal{U}_i(\mathcal{L}'(F), \mathbf{m}, y_{\mathcal{L}'(F)})$ across the politicians $i \in \mathcal{I}$. However, some of our results hold also if considering all Pareto optimal equilibria, instead of only ex-ante Utilitarian optimal ones.

3.4. **Our Faction Stability Criterion.** We define a factional structure $(\mathcal{C}, \mathcal{L})$ *stable,* if the game with initial structure $(\mathcal{C}, \mathcal{L})$ has an equilibrium $(\mathcal{C}', \mathcal{L}'; \mathbf{m}, \mathbf{y})$ among those selected above, in which each politician (including leaders) confirms allegiance to his initial faction, i.e., $\mathcal{C}' = \mathcal{C}$, and $\mathcal{L}' = \mathcal{L}$. In line with the empirical literature that points to ideologically cohesive groupings, as mentioned earlier, we restrict to ideologically connected factions: If politicians i and j belong to faction F, then so does any politician k whose bias b_k is between b_i and b_j.

Our core results compare these party structures with those that would exist if factions did not form. The benchmark of a *non-factional* party $\mathcal{P} = (\{\{i\} : i \in \mathcal{I}\}, \mathcal{I})$ is defined as the one in which factions are prohibited and so each politician i decides on y_i but all politicians can communicate freely with each other. In the statement of our results we make use of the fact that $\mathcal{P}$ could arise (possibly) as a stable factional structure, or it could be imposed were factionalism to be prohibited.[14]

Our model restrictions are minimal and so its results may be applied to political environments characterized by different mechanisms determining how factions form and who leads them. For example, a simple game is one in which there are no factions at the beginning of the game, and politicians first choose whether to try and form a faction becoming potential leaders; then the political game described above is played. Another possibility is to presume that the party is already factionalized at the beginning of the game, that different faction members challenge each other for leadership, and that then faction members may choose to change allegiance depending on the outcome of the power struggle for faction leadership.[15]

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be worse off if all players consequently communicated noninformatively with all faction leaders. Although more potent, we consider it unrealistic to impose a complete breakdown of intra-party communication and, happily, our results do not require us to make this restriction.

[14] Our benchmark is purposively normative, motivated by critiques of factionalism mentioned earlier. An alternative approach is to consider a party that first elects a leader who determines the policy position. In ongoing work (citation omitted) we show that majority rule can then lead to inefficiency relative to a weighted utilitarian benchmark.

[15] Less obviously, our model also covers a game form in which politicians delegate authority to each other in subsequent stages. In that game, a player endowed with authority from another politician may further transmit this authority to a third politician. Potentially, a network of delegated authority arises in equilibrium with a structure more complex than the partitional factional structures we consider here. Omitted results, available upon request, show that all equilibrium networks are equivalent to a partitional structure.

4. Equilibrium Communication and Faction Formation

In any equilibrium of our game, a politician anticipates the effect of his faction allegiance on final party policy. Consequently, he is willing to be in a faction only if he expects that the leader will be more informed than he is in equilibrium. As customary, we solve our game backwards and here we use results by Galeotti et al. (2013) to determine the equilibria of the intra-party deliberation stage before then deriving the stable factional structures $(\mathcal{C}, \mathcal{L})$.

4.1. **Equilibrium Communication.**

The solution of the communication game is described as follows. Given communication equilibrium strategies $\mathbf{m}$, a politician j's choice is $y_j = E\left[\theta | s_j, \hat{\mathbf{m}}_{-j,j}\right] + b_j$.[16] In words, the preferred choice of each politician j equals the expected value of the common state θ given his equilibrium information, plus his ideological bias b_j. Further, in anticipation of the final decisions, the profile of communication strategies $\mathbf{m}$ is an equilibrium if and only if, whenever a politician i is truthful to a politician j,

$$|b_i - b_j| \leq \frac{1}{2\left[d_j(\mathbf{m}) + 2\right]}.$$

The possibility for truthful communication from i to j is independent of whether i is truthful or not to any other politician; and it becomes less likely with (i) an increase in the bias difference $|b_i - b_j|$ and (ii) an increase in the information held by j in equilibrium. Intuitively, ideological differences can prohibit the effective aggregation of information.

On the basis of this finding, we can define the maximum possible information d_j held by politician j in the intra-party communication stage as the integer d that solves the equation $n_j\left([2(d+2)]^{-1}\right) = d$, where the function $n_j : \mathbb{R}_+ \to \mathbb{N}$ is defined as $n_j(b) = |\{i \in \mathcal{I} : |b_i - b_l| \leq b\}|$ —for any set A, the notation $|A|$ denotes the cardinality of A. We note that, by construction, the maximum information d_j coincides with $d_j(\mathcal{P})$, the equilibrium information held by each politician j in the non-factionalized party.

For any initial factional structure $(\mathcal{C}, \mathcal{L})$, the equilibrium information $d_j(\mathcal{C}, \mathcal{L})$ of politician j depends on whether j is a faction leader or not, and on whether communication is open or closed. For each leader $l \in \mathcal{L}$, under open communication, $d_l(\mathcal{C}, \mathcal{L}) = d_l$, the leader is as informed as possible in equilibrium. Under closed communication, instead $d_l(\mathcal{C}, \mathcal{L}) = \min\{d_l, |F(l)|\}$: the leader is as informed as possible, compatible with the constraint that only members of his own faction communicate with him.

The information $d_j(\mathcal{C}, \mathcal{L})$ of any politician j, influences the expected utility accrued by any player i anticipating the influence exerted by j on the manifesto. But player i's expected

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[16]Politician j may exert influence even if he is not a leader in $\mathcal{L}$, as he may choose to withdraw allegiance to his faction, in the first stage of the game, and act independently in the party.

utility is also influenced by the ideological distance between player i and player j. Specifically, the expected utility accrued by any player i for politician j's influence is:

$$U_i\left(j, \mathcal{C}, \mathcal{L}\right) = -\frac{1}{6\left(d_j\left(\mathcal{C}, \mathcal{L}\right) + 2\right)} - \left(b_i - b_j\right)^2. \tag{1}$$

The expected utility of player i decreases in the ideological difference with politician j and increases in the equilibrium information $d_j\left(\mathcal{C}, \mathcal{L}\right)$ held by j.

4.2. **Stable Factional Structures.** We now turn to the novel aspects of our enquiry, analyzing a model of faction stability built on the notion of voluntaryism: each politician i freely chooses his factional allegiance, anticipating the process of intra-party communication and the final party program. In so doing he maximizes his own expected utility. Expression (1) allows us to find a simple characterization of stable factional structures, which we report in the following Lemma 1 (the proof is omitted as it is immediate). To simplify the exposition of this result and subsequent ones, we henceforth restrict attention to equilibria of the communication game in which politicians babble to every politician j who is not a faction leader, so that $d_j\left(\mathcal{C}, \mathcal{L}\right) = 1$.[17]

Lemma 1. *Every stable factional structure $(\mathcal{C}, \mathcal{L})$ is such that for any faction F and politicians $i \in F$ and $j \in \mathcal{I}$,*

$$\frac{1}{6\left(d_{\mathcal{L}(F)}\left(\mathcal{C}, \mathcal{L}\right) + 2\right)} + \left(b_i - b_{\mathcal{L}(F)}\right)^2 \leq \frac{1}{6\left(d_j\left(\mathcal{C}, \mathcal{L}\right) + 2\right)} + \left(b_i - b_j\right)^2. \tag{2}$$

The result is intuitive. The factional structure $(\mathcal{C}, \mathcal{L})$ is stable to unilateral deviations if each politician $i \in F$ prefers that his faction leader $\mathcal{L}(F)$ has influence over the manifesto, rather than maintaining his independence or throwing his weight behind a different faction leader. By construction, this choice is checked under the stipulation that leaders are as informed as possible, compatible with equilibrium of the intra-party communication game, whereas all other politicians do not receive any information.[18]

As our model is one of cheap talk, we do not assume that a faction leader is informed by all members of her faction. Indeed it may be that a politician will join a faction while being unable to communicate truthfully with its leader.

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[17]Non-truthful communication with politicians other than faction leaders can be sustained in equilibrium, since decisions $\hat{y}_j$ made by politicians $j \neq i$ that do not lead a faction do not affect i's payoff. Consequently, the restriction that $d_j\left(\mathcal{C}, \mathcal{L}\right) = 1$ is with no loss of generality.

[18]We surmise that Lemma 1 and all subsequent results hold also when changing the order of moves in the political game, so that players first communicate and then form factions. In fact, the choice of politicians of whether to confirm their allegiance to a faction leader or not in expectation that he will be informed through private communication is equivalent to the choice of whether to confirm allegiance or not after private communication has taken place, as politicians cannot observe the messages sent to the faction leaders. A difference in these games is that the stable factional structure could no longer be enforced through "ostracism".

Lemma 2. *Politician i does not communicate with his faction leader $\mathcal{L}(F)$ when the ideological difference between i and $\mathcal{L}(F)$ is neither so small that i can always communicate with his leader, nor so large that he does not join her faction. Specifically, it is required that*

$$\frac{1}{2\left[d_{\mathcal{L}(F)}\left(\mathcal{C},\mathcal{L}\right)+2\right]} < \left|b_i - b_{\mathcal{L}(F)}\right| \leq \sqrt{\frac{1}{18} - \frac{1}{6\left[d_{\mathcal{L}(F)}\left(\mathcal{C},\mathcal{L}\right)+2\right]}}. \tag{3}$$

For there to exist a range of $\left|b_i - b_{\mathcal{L}(F)}\right|$ such that this condition can be satisfied, it must be that the leader is sufficiently informed: $d_{\mathcal{L}(F)}\left(\mathcal{C},\mathcal{L}\right) \geq 3$.

This result is intuitive. If the ideological difference between the politician and the faction leader is small then he will communicate information truthfully to her. If, on the other hand, it is too large, then he will not join her faction. For intermediate values, the politician is willing to join the faction, as long as its leader is sufficiently informed. But at the same time, he is unable to truthfully communicate with the leader, in equilibrium. Why? Although he is happy to throw his weight behind the faction leader, the politician still wishes to influence her views. But because he is not ideologically close enough to reveal his information truthfully, in equilibrium, any such advise would be ignored.[19] Realistically, then, some faction members will be closer to their leader than others: they will be able to communicate information and so affect the faction leaders views while other members will not.

5. PARTY WELFARE

Using equality (1) allows us to express party welfare as a simple function of the factional structure $(\mathcal{C},\mathcal{L})$:

$$\mathcal{W}\left(\mathcal{C},\mathcal{L}\right) = -\sum_{i\in\mathcal{I}}\sum_{F\in\mathcal{C}}\pi_{\mathcal{L}(F)}\left[\frac{1}{6\left(d_{\mathcal{L}(F)}\left(\mathcal{C},\mathcal{L}\right)+2\right)} + \left(b_i - b_{\mathcal{L}(F)}\right)^2\right]. \tag{4}$$

The party values information, i.e. high $d_{\mathcal{L}(F)}\left(\mathcal{C},\mathcal{L}\right)$, and moderation, i.e. $b_{\mathcal{L}(F)}$ close to the party average $\sum_{i\in\mathcal{I}}b_i/I$ with faction leaders $\mathcal{L}(F)$. As implied by Proposition 1 of Dewan et al. (2015), the optimal allocation of bargaining authority empowers a single leader who, amongst the set of politicians, provides an optimal mix of moderation and informed policy. Central to our analysis, however, is that party politicians cannot be coerced into granting all power to a benevolent party dictator. Instead the organizational principle of faction formation is voluntary transfer of authority to faction leaders. This raises the possibility that a factionalized party may provide a second-best alternative to one that is centralized under a welfare-maximizing leader.

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[19]Instead, if the leader $L(F)$ is not well informed ($d_{\mathcal{L}(F)}\left(\mathcal{C},\mathcal{L}\right) < 3$), then i joins her faction only if he is ideologically so close that he can also communicate to her truthfully, in equilibrium.

In order to pursue this line of enquiry we require a benchmark for comparison. As noted in our introductory remarks there are, we believe, three principal reasons for the negative connotations of factionalism: that party politicians relinquish their independence when joining a faction; that when doing so they consider only their own payoff; and that factionalism can be inimical to intra-party cooperation. In this respect our model offers a natural benchmark, namely the policy programme that would prevail if factions were prohibited from forming, defined earlier as $\mathcal{P} = (\{\{i\} : i \in \mathcal{I}\}, \mathcal{I})$, under open communication. Thus, we contrast a world where a politician exerts influence over policy in accord with his own judgement, with one where he has the opportunity of joining a faction and so increasing the power of its leader. To make the comparison interesting, we focus on the case in which the faction formation process does not achieve the first best of a unified party behind a single leader.

5.1. **Factionalism under Open Communication in a Canonical Model.** We consider welfare implications of the faction formation process under open communication, within a broad class of ideology distributions that impose some structure on the primitives of our model. We assume that there are an odd number of politicians, denote the median politician by m, and assume that the ideology distribution is *symmetric* around m and *single peaked* at m, in the sense that politicians are more ideologically clustered as they get closer to m. Formally, we define the ideology distribution b as single peaked and symmetric at m, when for any $i = 1, ..., m-1$, $b_{i+1} - b_i > 0$ weakly decreases in i, and $b_{i+1} - b_i = b_{n-i+1} - b_{n-i}$.[20] This focus is based on the supposition that the ideology distribution is often concentrated closer to the party median, with fewer politicians found at the ideological extremes of the party.

For this canonical case, we can prove that factionalism is beneficial to the party under open communication, that it can never be detrimental to attaining party goals, and that factions should not be prohibited from forming. In doing so we uncover a novel rationale for factions.

Proposition 1. *Suppose that the ideology distribution* b *is single-peaked and symmetric, that communication is open across factions, and that a single party-wide faction is not a stable structure. Then, it can never benefit any politician in the party to prohibit factionalism: if the optimal stable factional structure* $(\mathcal{C}, \mathcal{L})$ *does not coincide with the non-factional party* $\mathcal{P}$, *then it strictly improves the welfare of every single politician.*

The proof of Proposition 1 is involved and so relegated to the Appendix. The key fact that leads to the result is that, when the ideology distribution b is single-peaked and symmetric, the maximal amount of information d_j obtained with intra-party communication weakly increases as politician j becomes more moderate. As a consequence it can never be the case

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[20]The definition of symmetric and single-peaked ideology for an even number of politicians is analogous but slightly more involved. All results hold with even n also.

(by equation 3) that a moderate politician joins a faction led by a more extreme politician; as moderate faction leaders are more informed, extreme politicians transfer power to them. Further, because the ideology distribution b is symmetric, for every transfer of power from a leftist politician to a more moderate leader there is a corresponding and symmetric transfer from a right-wing player to a more moderate and informed leader. Each player benefits from these symmetric transfers, regardless of their ideology. From expression (1), the ideological loss of each politician i due to j's decisions is a convex function of the ideological distance between i and j. So even an extremist gains more from a transfer of power to a moderate leader on the opposite side of the ideological spectrum, than he loses from a symmetric transfer of power away from his ideological bliss point.

Our result uncovers a role for party factions. They provide a mechanism for tying extreme politicians into partitions led by more moderate and informed leaders. Proposition 1 shows that, within a canonical model specification, their formation always benefits the party. Later, we develop this insight further, fully characterizing the optimal factional structure for a sub-case of this environment. Before doing so, and having shown the benefits of faction formation, we explore conditions under which factions may or may not have negative welfare effects, in a much simpler model specification. We also ask whether our results change when considering closed communication.

5.2. **Factionalism in a Simple Model with Three Ideological Tendencies.** In our introduction, we noted that factions are commonly perceived as detrimental to the achievement of common party goals. We have shown that this need not be so. Indeed, party objectives can sometimes be better achieved when factionalism is allowed than when it is prohibited. To uncover a negative role for factions, we must then move beyond the canonical class of single-peaked and symmetric ideology distributions for which, as we have seen, factions involve empowering relatively moderate and better informed faction leaders.

We first note that factions can be detrimental, relative to a benchmark where they are prohibited, only when (i) a party without factions is optimal and (ii) this outcome can only be achieved by prohibiting them. Put otherwise, it must be that a non factional party is optimal, but that it is not a stable structure when politicians can voluntarily join factions. We demonstrate that this occurs only when factions form around leaders whose views are extreme relative to those of their members. When factionalism increases the power of these extreme-ideology politicians it can be detrimental to the party as a whole.

We illustrate these findings in a simple environment, where the party is formed of three distinct ideological groups: left, center and right. There are n_L politicians with ideology $-b_L$, n_C politicians with ideology zero, and n_R politicians with ideology b_R. Following Rose

(1964), we refer to these leftist, centrist and right-wing groups as ideological *tendencies*. Our distinction between a faction and a tendency closely mimics that of Rose (1964) who described the former as "self-consciously organized as a body, with a measure of discipline and cohesion", whereas the latter is instead "a stable set of attitudes, rather than a stable set of politicians."(Empirical evidence for this distinction from the British House of Commons, using proximity scaling of divisions, is provided by Wood and Jacoby (1984).) We thus study how a party of underlying tendencies relates to a stable factional structure: as we shall see, intuitively, stable factions form when the ideological differences between tendencies is not too large. We explore the welfare consequences of these intra-party partitions. While we do not impose any restrictions on n_L, n_C and n_R, we have in mind an application to sufficiently large parties where the number of informed politicians is non-negligible for each tendency.[21]

There are three possibilities with respect to the stable factional structures. The first is that the programme is decided in its entirety by the leader of a party-wide faction. When the party is unified in such a faction with a welfare-maximizing leader, then welfare is higher than in a non-factional party as the first best is achieved. Another possibility is that the party is not factionalized. Welfare is of course equivalent whether factions are allowed or prohibited. Of more interest is the third and remaining possibility: two tendencies are joined in a cohesive faction, thus leaving the remaining tendency isolated.

In dealing with this case, and without loss of generality, we take the right-wing tendency to be isolated. Still there are two possibilities. The leader of the faction may be of the left or the center: we refer to the former as a left-center faction and the latter as a center-left faction. In exploring welfare implications it is immediate that a center-left faction benefits the party always: leftist politicians must benefit, as joining a faction led by a centrist leader is a voluntary choice; centrist politicians benefit *a fortiori* as faction formation increases the power of a centrist leader; and right-wing politicians benefit as a center-left faction reduces influence of the left. This thus reinforces our earlier message that factionalism is beneficial when extreme politicians are tied into factions led by moderates.

To see why factionalism can be detrimental we consider the case where the welfare maximizing equilibrium involves a left-center faction. Centrists must benefit as faction formation is voluntary, as do leftists *a fortiori*; however, increasing the influence of a leftist leader may be so detrimental to right-wing politicians as to induce an overall welfare loss for the party.

Proposition 2. *Suppose that there are three ideological tendencies: left, center, and right. If the optimal stable party structure under open communication involves a center-left (or center-right) faction, then factionalism is beneficial to the party. If the optimal stable party structure*

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[21]We note that when the ideological difference between two tendencies is not too large, then a leader of the resulting faction that forms between them is informed by *all* members of her faction.

under open communication is such that a left-center or right-center faction forms, then factionalism may be detrimental to the party.

In a three-tendency model, our result highlights that a necessary though not sufficient condition for factionalism to be detrimental is that the faction leader is extreme. Investigating further, we present details of an example for particular parameter values.

Example 1. *Consider a stable left-center faction: Let $n_C = 3$, $n_R = 3$ and set n_L sufficiently large so that the faction leader is from the left (in, fact, $n_L \geq 5$ will suffice). Suppose b_L and b_R are sufficiently large that L and R tendency politicians cannot communicate to C tendency politicians, nor viceversa: specifically, set $b_L > 1/12$, and $b_R > 1/12$. At the same time, take b_L sufficiently small that C tendency politicians are willing to be in the faction led by a leftist leader: specifically, set $b_L \leq \sqrt{\frac{1}{30} - \frac{1}{6n_L+12}}$, noting that the right hand side of this inequality is larger than $1/12$ because $n_L \geq 5$.*

In order to compare welfare in the party with a left-center faction to that of a party in which factionalism is prohibited, we specialize the example to $n_L = 6$ and $b_L = 1/10$. Straightforward algebra reveals the following: when $b_R > 1/4$, factionalism is detrimental to the party, whereas if $1/12 < b_R < 1/4$, then factionalism is beneficial to the party.

Our example demonstrates that factionalism may be detrimental to the party when the tendency of right-wing politicians is fairly extreme. In this case, they are severely hurt by the influence of the leftist leader of a left-center faction and this more than cancels the gains of other politicians from faction formation. Beyond this somewhat pathological circumstance, factions strictly benefit the party and so should never be prohibited.

The conditions we identified for factions to be detrimental to the party appear stringent. This suggests the beneficial consequences of allowing politicians to freely join factions and thereby increase the influence of their leaders. As argued earlier, however, factionalism may damage parties by exacerbating noncooperative intra-party behaviour. To explore this issue we analyze a closed world where intra-party dialogue is restricted. Specifically we assume that communication can occur only within factions.

Our analysis reveals, surprisingly, that whether communication is open or closed is *immaterial* as long as the number of informed politicians in any tendency is not too small (specifically, it is sufficient to set $n_L \geq 3$, $n_C \geq 3$ and $n_R \geq 3$). This is because there is no ideology configuration b such that a politician j would be willing to communicate truthfully with faction leader l if allowed to do so, yet be unwilling to be in l's faction. In fact, both the decision to communicate truthfully and to join the faction depend on the leader's information d_l and on the ideology difference $|b_i - b_l|$. However, the willingness to be truthful decreases in d_l,

whereas the willingness to be in the faction increases in d_l. When $n_L \geq 3$, $n_C \geq 3$ and $n_R \geq 3$, any leader l has information d_l no smaller than 3 (at a minimum, he can gather truthful messages of the politicians of his same tendency). For $d_l \geq 3$, politician i cannot be willing to communicate truthfully to l, without being simultaneously willing to stay in l's faction.

Proposition 3. *When there are three tendencies such that $n_L \geq 3$, $n_C \geq 3$, and $n_R \geq 3$, the restrictions that factions may impose on intra-party communication are irrelevant to the welfare consequences of factionalism. Factionalism with closed communication within factions benefits the party unless it is divided into factions with non-centrist leaders.*

Somewhat contrary to conventional wisdom, the overall conclusion of our welfare analysis is that, by and large, factions based on ideology are beneficial to the party. This insight stems from two aspects of factionalism that we believe have previously been overlooked. Firstly, factions form via a process of voluntary transfer of power over the party decisions. Secondly, factions are by definition part of a larger body that has a common objective. Our specific message is that factions provide a mechanism for preventing politicians with relatively extreme views, or relatively poor information, from acting autonomously in the party.

6. Characterization of Optimal Party Factions

We have established that factionalism is beneficial to a party under a broad set of circumstances. Correspondingly, we have shown that a negative association between ideological factions and welfare can be established only under quite restrictive conditions. Moreover we have uncovered an intriguing rationale for factions in tying extreme and poorly informed politicians into partitions led by moderate and well informed leaders. Here we develop this insight further, undertaking the full characterization of optimal party factions within a simple environment that is a subcase of the canonical model studied earlier.

6.1. **Open Communication in a Model with Equidistant Ideological Neighbours.** We study a party with an odd number of politicians, $I \geq 5$ where the ideological distance between each politician and his closest ideological neighbor is fixed and equal to β. So we assume that $b_i = i\beta$ for $i = 1, ...I$. Considering politicians i and j, we let $i - j$ denote the number of "ideological positions" separating i from j. Further, we assume throughout that $K = I$: each politician is initially endowed with authority over a single dimension.

Using this example we derive an iterative construction of the optimal stable factional structure, based on the following intuitive ideas. In the optimal structure, politicians should join a faction led by a more moderate politician whenever possible. Politicians are constrained: as the ideological distance β increases, they will be unable in equilibrium to join factions

led by politicians that are too ideologically distant. We show that the optimal structure is determined by the requirement that the most extreme politicians join factions led by as moderate a politician as possible, thus reinforcing the imperative of tying in the extremists. In fact we show that this imperative dominates all other requirements. In particular, selecting the stable faction that ensures that extremists are tied to leaders with intermediate ideology requires that such leaders do not join factions led by even more moderate politicians, even though there are stable factions in which they do.

The details of our analysis are somewhat intricate and so we begin our exposition with the informal description of a simple example.

Example 2. *Open Communication in a Party with 7 Politicians. We determine the optimal stable factional structure that maximizes party welfare for the case of $I = 7$ politicians and open communication with equidistant bias $b_i = i\beta$ for $i = 1, ...I$.*

Dispensing with much detail, we focus on the provision of a clear exposition of our insights.[22] Depending on the parameter value β, the optimal stable structure may take different forms. We distinguish between three mutually exclusive and exhaustive cases defined by the degree of ideological distance between a politician and his nearest ideological neighbor.

Small Distance. We first suppose that the ideological distance between a politician and his closest ideological neighbor is small, specifically, $\beta \leq \frac{1}{45}\sqrt{5}$. We can then show that the party should be united under the leadership of the median politician $m = 4$. First, we note that, in this case, the analysis in section 4 implies that the maximum equilibrium information of the median player 4, defined as d_4, is at least 3. Moreover, we note that, by inequality (2), all politicians are willing to join a faction led by player 4 when $d_4 \geq 3$ and $\beta \leq \frac{1}{45}\sqrt{5}$. Uniting the party under the most moderate politician yields the optimal structure, as it minimizes the aggregate ideological loss.

Intermediate Distance. Now suppose that the ideological distance between a politician and his closest ideological neighbor is not small, nor it is too large. Specifically, suppose that $\frac{1}{45}\sqrt{5} < \beta \leq \frac{1}{30}\sqrt{5}$. Then we show that in the optimal stable structure, the party is divided into 3 factions: $\{1, 2, 3\}$, $\{4\}$ and $\{5, 6, 7\}$ whose leaders are 3, 4, and 5 respectively.

We begin as before by calculating the maximum equilibrium information d_j of any player j. Doing so, and because $\frac{1}{45}\sqrt{5} < \beta \leq \frac{1}{30}\sqrt{5}$, we obtain that $d_j = 3$. Further, because $\frac{1}{45}\sqrt{5} < \beta$, the most extreme politicians 1 and 7 are no longer willing to join a faction led by the median player 4. However, it is still possible that they join a faction led by a more moderate politician and in so doing that they improve party welfare. Inequality (2) implies that any politician i would join a faction led by a more moderate politician j who is at most 2 ideological positions

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[22]The precise analysis is presented in an online appendix.

away from i: so player 1 is willing to join a faction led by either 2 or 3. If the faction is led by 3 then, since it must be ideologically connected, 2 must join it also. So faction $\{1, 2, 3\}$ forms. If 1 decides to act independently, however, then 2 can join a faction led by 4 in which case faction $\{2, 3, 4\}$ forms. It can be shown that the former configuration is more advantageous. Because of the concavity in the politicians' payoff functions, the party's (collective) first order concern is to avoid extreme political stands. This can be achieved by tying the extremists into factions led by more moderate leaders. Extending this reasoning, it can be shown that the optimal stable factional structure comprises 2 factions: $\{1, 2, 3\}$, and $\{5, 6, 7\}$, whose leaders are 3, and 5 respectively.[23] This configuration leaves the most moderate politician 4 isolated.

Now suppose that the ideological distance between a politician and his closest ideological neighbor is larger than in the previous example, but not too large. Specifically suppose that $\frac{1}{30}\sqrt{5} < \beta \leq \frac{1}{12}\sqrt{2}$. Following the same steps as in the previous case, we find that politicians are only willing to stay in factions led by their immediate ideological neighbors. The party imperative– avoiding extreme ideological stands– requires that politicians 1 and 7 form a faction led by their respective neighbors 2 and 6, so that the optimal factional structure is $\{1, 2\}$ and $\{6, 7\}$. As a consequence, the remaining players 3 and 5 then form a faction $\{3, 4, 5\}$ led by 4.

Large Distance. Finally, we consider a highly polarized party, specifically $\beta > \frac{1}{12}\sqrt{2}$. Then, politicians are unwilling to join factions even when they are led by their most immediate neighbor. Hence, the only possible stable structure is one where each politician acts independently, i.e. no factions are formed.

We now derive a general characterization, building on two central points arising from the example above.

First, we have seen that, in order to improve party welfare, politicians should join factions led by more moderate politicians. When the ideological distance is not too small, extreme politicians are unwilling to join a faction led by the median politician, but every politician is willing to join a faction led by a more moderate politician who is ideologically separated by, at most, a specific number of ideological positions. Intuitively, the number of ideological positions is a decreasing function of the ideological distance β. For example, we have seen that for $\frac{1}{30}\sqrt{5} < \beta \leq \frac{1}{12}\sqrt{2}$, politicians are willing to transfer influence only to immediate ideological neighbors, whereas for $\frac{1}{45}\sqrt{5} < \beta \leq \frac{1}{30}\sqrt{5}$, they are also willing to transfer influence to politicians that differ by two ideological positions.

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[23]These factional structure are stable because leaders are more informed than more extreme politicians, and do not have an incentive to transfer influence to them. There exist suboptimal stable factional structures, in which moderate politicians transfer influence to more extremist leaders.

Second, we have seen that the optimal organization of the party when β is intermediate involves factions that include extreme politicians and the *most* moderate politicians possible. This holds regardless of the reduced capability of more moderate politicians to transfer influence. In our seven player example, we have seen that if 1 is willing to be in a faction led by 2 but not by 3, then the faction $\{1, 2\}$ is part of the optimal factional structure even though 2 would willing to be in a faction led by 3. Making (already) moderate politicians adopt (more) moderate stances is less important for party welfare than tying extreme politicians to more moderate leaders.

Our findings suggest an iterative procedure to calculate the welfare-maximizing factional structure $(\mathcal{C}, \mathcal{L})$. Initially, as a function of the ideological difference β, we calculate the maximum number q of ideological positions across which a politician is willing to transfer influence. Then, we build the optimal party organization. We make extreme politicians 1 and I be led by politicians that are q ideological positions more moderate, (i.e. $q + 1$ and $I - q$ respectively). Since factions must be ideologically connected, this implies that all politicians between 1 and q (respectively, between $I - q$ and I) also join the faction led by $q + 1$ (respectively, $I - q$). If the remaining politicians $q + 2, ..., I - q - 1$ are now willing to be led by the median $m = (I + 1)/2$, then they should be united in a single faction under her. Otherwise, the iterative procedure continues by building two factions $\{q + 2, q + 3, ..., 2q + 1\}$ and $\{I - 2q, I - 2q + 1, ..., I - q - 1\}$, and then considering the remaining politicians $2q + 2, ..., I - 2q - 1$. This procedure continues until the remaining politicians are united under a faction led by the median.

Proposition 4 below confirms that the conjectured iterative procedure delivers the optimal factional structure, determines q precisely as a function of β, and in so doing calculates the equilibrium information d_l of all faction leaders l. Moreover, it derives the number G of factions in the optimal stable structure and their size. In the statement below, for any real number x, the notation $\lfloor x \rfloor$ denotes the largest integer smaller than x, whereas the notation $\lceil x \rceil$ denotes the smallest integer larger than x.

Proposition 4. *Suppose that there are an odd number $I \geq 5$ of politicians, and that ideological neighbors are at fixed ideological distance β. The optimal stable factional structure is characterized as follows: (i) The leader of each faction is always its most moderate politician; (ii) Letting $g(d) = (2 \cdot \lceil (d-1)/2 \rceil \cdot (d+2))^{-1}$ for any integer d, and letting d^* be the integer d such that $g(d+1) < \beta \leq g(d)$, each leader l's equilibrium information equals $d(\beta) = \min\{I, d^*\}$, (iii) Letting $q(\beta) = \left\lfloor \frac{1}{\beta} \sqrt{\frac{d(\beta)-1}{18(d(\beta)+2)}} \right\rfloor$, there are $G = 2 \cdot \left\lfloor \frac{I-1}{2(q(\beta)+1)} \right\rfloor + 1$ factions, symmetrically arranged around the median politician $m = (I+1)/2$; all factions are of size $q(\beta) + 1$, but the one containing the median politician m, which is of size $M = I - (G-1)(q(\beta)+1)$.*

A consequence of our result is that, in the optimal stable party faction, the median politician $m = (I + 1)/2$ never joins the faction of a different leader: he is the most moderate politician, hence his decisions minimize the aggregate ideological loss of the party. Further, he is never less informed than any other politician. Combining these two considerations delivers the implication that it is never beneficial for him to join the faction of another.

Before concluding our analysis of the optimal partition of parties into factions with open communication, we highlight one further substantive implication of our findings. We note that an external observer cannot always infer party policy from a simple summary statistic such as the median preference. Indeed, and as a consequence of our rationale for factionalism, we observe that when the ideological distance between ideological neighboring politicians is neither too large nor too small, then the optimal party structure comprises the factions $\{1, 2, 3\}$, $\{4\}$ and $\{5, 6, 7\}$ for $\beta \leq \frac{1}{30}\sqrt{5}$, whereas for $\beta > \frac{1}{30}\sqrt{5}$, it comprises the factions $\{1, 2\}$, $\{3, 4, 5\}$ and $\{6, 7\}$. Hence, the programme will differ according to β, although the ideology of the median politician 4 remains unchanged (when renormalizing the ideologies, setting $b_i = (i - 4)\beta$, for $i = 1, ..., 7$). Whilst intuitively one might think that knowledge of the party median provides a strong indicator of the policies a party will pursue, our analysis suggests that this is the case only for small ideological differences within the party. Otherwise a detailed analysis of the stable party factions is required. So we conclude that factions matter for collective choice.

6.2. **Closed Communication with Equidistant Ideological Neighbors.** We conclude this section by addressing the case of closed communication. To do so, we revisit Example 2, with 7 politicians and equidistant ideological neighbors.

Example 3. *A Party with 7 Politicians and Closed Communication. We study the optimal stable factional structure for the case of $I = 7$ politicians with equidistant bias $b_i = i\beta$ for $i = 1, ...I$, who can communicate only with members of their faction.*

Small Distance. When $\beta \leq \frac{1}{45}\sqrt{5}$, as in the case of open communication, the party should be united under the leadership of the median politician $m = 4$. Indeed, the constraint that communication occurs only among the same faction members does not play any role if the party is united.

Intermediate Distance. In the range $\frac{1}{45}\sqrt{5} < \beta \leq \frac{1}{30}\sqrt{5}$, there are two optimal configurations: the one with factions $\{1, 2, 3, 4\}$ and $\{5, 6, 7\}$, and the one with factions $\{1, 2, 3\}$ and $\{4, 5, 6, 7\}$. In both configurations, the faction leaders are 3 and 5 respectively. To appreciate the difference with the case of open communication, recall that there the optimal configuration is $\mathcal{C} = \{\{1, 2, 3\}, \{4\}, \{5, 6, 7\}\}$, with leaders 3, 4, and 5. The reason why $(\mathcal{C}, \mathcal{L})$ does not maximize welfare with closed communication is because it requires the median politician,

player 4, to be isolated. But being isolated, player 4 can count only on his own information, and this makes his decision very imprecise. As a result, 4 is now willing to transfer authority to player 3 or 5, and this improves party welfare.

In the range $\frac{1}{30}\sqrt{5} < \beta \leq \frac{1}{12}\sqrt{2}$, the optimal configuration is composed of three factions $\{1, 2\}$, $\{3, 4, 5\}$ and $\{6, 7\}$, with leaders 2, 4 and 6 respectively, as in the case of open communication.

Large Distance. When $\beta > \frac{1}{12}\sqrt{2}$, then as with open communication, the only possibility is that no factions form. Communication between politicians could not take place even with open communication. *A fortiori*, it cannot occur when communication is constrained to happen only within factions. As a result, all politicians prefer to act independently.[24]

The significance of contrasting our 7 player examples under closed and open communication is in highlighting that the provisions of the party programme need not be uniquely identified by the ideological distribution of politicians. Here, we show that the social norms that determine how communication takes place within a party play a role in determining the optimal stable factions, and hence the details of the final party programme.

The prohibition of communication may reduce the number of factions in the optimal structure. Ostensibly, the party will be less divided. Closed communication makes division more costly and hence, in the optimal stable structure, the party needs to be more unified.

While this might suggest that closed communication may benefit the party, this is not so. This is because, in contrast to open communication, power may be transferred to relatively extreme leaders. (Unlike in example 2, here it is optimal that politician 4 joins the faction led by either politician 3 or 5 as otherwise he would be uninformed). Moreover, such leaders are (weakly) less informed than a leader would be in the case of open communication. Closed communication is thus always (weakly) detrimental to the party, and to each single individual politician, relative to open communication. The intuition then is simple: closed communication implies that power may be transferred to relatively extreme faction leaders who are weakly less informed. Closed communication is detrimental as it constrains the amount of information conveyed to faction leaders in equilibrium, though, by expression (1), each politician is better off if leaders are as informed as possible.

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[24]The ranges for the different optimal party configurations are the same as under open communication because, at the relevant thresholds, the number of politicians in each optimal faction under closed communication is at least as large as d_l, the maximum amount of information of each leader l in the communication game. So, the constraint of closed communication does not bind. For example, at the boundary $\beta = \frac{1}{45}\sqrt{5}$, the party is united and hence the closed communication constraint is irrelevant, and at the boundary $\beta = \frac{1}{30}\sqrt{5}$, the optimal factions are composed of at least 3 politicians, and the maximum amount of information d_l is also 3.

7. CONCLUDING COMMENTS

Although factions are ubiquitous, they are commonly perceived as playing a negative role in political life. Nevertheless, several empirical studies have suggested that factions can play a role in integrating extreme ideological streams of a party. We provide a formal defence of factions consistent with this more positive view. Our key research findings can be summarized succinctly: factions play a role in tying extreme party politicians to more moderate faction leaders, who are better informed and so better placed to make decisions on behalf of a party. This leads to a defence of factions: they enhance the quality of party programmes, by keeping the actions of the most extreme and less informed politicians in check, under guidance of more informed and moderate ones.

We conclude, then, by highlighting possible extensions, applications, and central lessons of our analysis.

How might our insights extend to a world with political competition? We were motivated in part by recent research (Fujiwara and Wantchekon (2013)) showing that intra-party deliberation enhances the quality of programmatic platforms and so provides electoral benefits. Our results reveal that factions can facilitate information sharing via intra-party deliberation and suggests that they may also provide electoral benefits.[25] So a reasonable conjecture, albeit one left to further research, is that our central insights extend to a competitive environment.

A limiting factor with respect to the generality of our conclusions is that we assume a party whose politicians share a common objective. We can be agnostic as to what the common objective is–the central point is that whatever politicians' objectives, and insofar as the party programme can be used to attain them, party politicians are uncertain as to which policies best serve their intentions. We do assume, however, that party politicians are alike with respect to the weight they place on the common objectives and their idiosyncratic (ideological) concerns. Nevertheless, we can confirm that in at least one obvious and realistic case, where politicians with the most extreme ideological preferences care less about the common objective, all of our results hold.

While there are limitations, it is also likely that our insights travel beyond political parties. Many organizations have a common objective to reach informed decisions, consist of a diverse body of opinion, and maintain organized internal groups who seek to control decision making. Consider, as one example, the case of an academic department that has a number of faculty positions to be filled, would like the hires that are made to be of a quality that enhances the reputation of the department, and where information about candidates is dispersed amongst

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[25]Muller-Rommel (1982) extends his argument about the integrating role of factions in the German SPD, cited earlier, by stating (page 267) that "intraparty factionalism neither had a negative effect on electoral results, nor was serious consideration given to a the party splitting."

faculty members who also have idiosyncratic biases. Our insights extend to such an environment: starting from a point where all or some have a say in the hiring process, it would be beneficial for groups to form behind distinct leaders of factions who make final decisions.

Our defence of factions provides lessons to be learned: restrictions on information sharing between factions can be detrimental. This suggests that an optimal party design involves factions that tie extreme members to moderate leaders, so facilitating better information-sharing, alongside institutions that enable inter-factional dialogue.

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APPENDIX

Proof of Lemma 2. The condition for i to be in the faction led by $\mathcal{L}(F)$ is

$$\frac{1}{6\left(d_{\mathcal{L}(F)}\left(\mathcal{C},\mathcal{L}\right)+2\right)} + \left(b_i - b_{\mathcal{L}(F)}\right)^2 \leq \frac{1}{6(1+2)}. \tag{5}$$

Player i does not communicate with player j when $|b_i - b_j| > \frac{1}{2(d_j+2)}$. Therefore player i does not communicate with his faction leader $\mathcal{L}(F)$ when the latter condition holds together with condition (5) for $j = \mathcal{L}(F)$. Rearranging these conditions, we obtain the inequality chain given in 3. For this to hold, it must be that $\left(\frac{1}{2\left[d_{\mathcal{L}(F)}(\mathcal{C},\mathcal{L})+2\right]}\right)^2 < \frac{1}{18} - \frac{1}{6\left[d_{\mathcal{L}(F)}(\mathcal{C},\mathcal{L})+2\right]}$, or that $\frac{1}{2\left[d_{\mathcal{L}(F)}(\mathcal{C},\mathcal{L})+2\right]} < \frac{1}{6}(\sqrt{3}-1)$, or that $d_{\mathcal{L}(F)}\left(\mathcal{C},\mathcal{L}\right) > \frac{3}{\sqrt{3}-1} - 2$, so that $d_{\mathcal{L}(F)}\left(\mathcal{C},\mathcal{L}\right) \geq 3$.

Proof of Proposition 1. Suppose that the ideology distribution is symmetric and single-peaked and that the non-factional party $\mathcal{P} = (\{\{i\} : i \in \mathcal{I}\}, \mathcal{I})$ is not the optimal stable factional structure, or else the comparison is trivial. If the non-factional party is a stable factional structure, we have the result immediately. So, suppose that $\mathcal{P}$ is not a stable factional structure. By construction, in $\mathcal{P}$ all politicians i are as informed as possible: $d_i(\mathcal{P})$ is the integer d solving the equation $n_i\left([2(d+2)]^{-1}\right) = d$. So, for $\mathcal{P}$ not to be a stable factional structure, there must be some politician i prefers to be in a faction led by a different politician j, rather than acting autonomously, i.e.,

$$\frac{1}{6\left(d_i\left(\mathcal{P}\right)+2\right)} > \frac{1}{6\left(d_j\left(\mathcal{P}\right)+2\right)} + \left(b_i - b_j\right)^2. \tag{6}$$

Hence, it must be the case that $d_j(\mathcal{P}) > d_i(\mathcal{P})$. Next, observe that for any $j = 1,..m$, $d_j(\mathcal{P})$ weakly increases in j, because the ideology distribution b is single-peaked and symmetric (and symmetrically, it weakly decreases in j, for $j = m, ..., I$). In fact, note that because for any $j = 1,..m-1$, $b_{j+1} - b_j$ weakly decreases in j, it follows that $n_j(b) = |\{i \in \mathcal{I} : |b_i - b_j| \leq b\}|$ weakly increases in j, and hence the integer d such that $n_j\left([2(d+2)]^{-1}\right) = d$ weakly increases in j. Hence, for any politician i and j satisfying inequality (6), it must be that j is closer to m than i: all politicians not acting independently in the party must join factions led by more moderate politicians. By symmetry of the ideology distribution b, for any i willing to be in a faction led by j, it must also be the case that $n - i + 1$ is willing to be in a faction led by $n - j + 1$; and it must also be that case that $d_j(\mathcal{P}) = d_{n-j+1}(\mathcal{P})$ and $d_i(\mathcal{P}) = d_{n-i+1}(\mathcal{P})$. By inspection of expression (1), we see that the utility of every single politician i' is improved by these politicians joining factions led by more moderate leaders. This is because, first, leaders j and $n - j + 1$ are more informed than politicians i and $n - i + 1$. Second, even when player i' suffers an ideological loss by i joining the faction led by j (which happens if $i' < i < j$ or if $j < i < i'$), because of the convexity of the ideological loss $(b_{i'} - b_{j'})^2$, player i' ideologically gains more from politician $n - i + 1$ joining the faction led by $n - j + 1$, than he loses from i joining the faction led by j.

Consider now the factional structure $(\mathcal{C}, \mathcal{L})$ iteratively constructed as follow. Let F_0 be the largest set of ideologically-adjacent politicians i such that $U_i(m, \mathcal{P}) \geq U_i(i, \mathcal{P})$, i.e., that they weakly prefer to join a faction led by m rather than acting autonomously in the party. Note that F_0 is non-empty as it contains m. Let l_{-1} be the largest index i smaller than m such that i does not belong to F_0; and l_1 be the smallest index i larger than m such that i does not belong to F_0. Let F_{-1} be the largest set of ideologically-adjacent politicians i not in F_0 such that $U_i(l_{-1}, \mathcal{P}) \geq U_i(i, \mathcal{P})$; and analogously define F_1. For any $k \geq 2$, let l_{-k} be the largest index i smaller than m such that i does not belong to F_{k-1} and F_{-k} be the largest set of ideologically-adjacent politicians i not in F_k such that $U_i(l_{-k}, \mathcal{P}) \geq U_i(i, \mathcal{P})$; defining l_k and F_k analogously. Iterate the procedure until a partitional structure $(\mathcal{C}, \mathcal{L})$ is obtained.

Now, we prove that the obtained factional structure $(\mathcal{C}, \mathcal{L})$ is stable and provides welfare larger than $\mathcal{P}$ so concluding the proof. To see that $(\mathcal{C}, \mathcal{L})$ is a stable factional structure, we first note that for any $l \in \mathcal{L}$, $U_i(l, \mathcal{C}, \mathcal{L}) = U_i(l, \mathcal{P})$ Further, for any $F \in \mathcal{C}$ and any $i \in F$, $U_i(l(F), \mathcal{C}, \mathcal{L}) = U_i(l(F), \mathcal{P}) \geq U_i(i, \mathcal{P}) \geq U_i(i, \mathcal{C}, \mathcal{L})$; by construction of $(\mathcal{C}, \mathcal{L})$ and because $d_i(\mathcal{C}, \mathcal{L}) \leq d_i(\mathcal{P})$ by definition. As far as the comparison between $U_i(l(F), \mathcal{C}, \mathcal{L})$ and $U_i(l(F'), \mathcal{C}, \mathcal{L})$ for any $F' \neq F$ is concerned, we proceed as follows. If the leader $l(F')$ is further away from m than $l(F)$, then it immediately follows from the previous step that $U_i(i, \mathcal{P}) \geq U_i(l(F'), \mathcal{P}) = U_i(l(F'), \mathcal{C}, \mathcal{L})$. Because by construction $U_i(l(F), \mathcal{C}, \mathcal{L}) = U_i(l(F), \mathcal{P}) > U_i(i, \mathcal{P})$, it must be that $U_i(l(F), \mathcal{C}, \mathcal{L}) > U_i(l(F'), \mathcal{C}, \mathcal{L})$. Finally, suppose that $l(F')$ is closer to m than $l(F)$. By construction, $U_{l(F)}(l(F), \mathcal{C}, \mathcal{L}) = U_{l(F)}(l(F), \mathcal{P}) > U_{l(F)}(l(F'), \mathcal{P}) = U_{l(F)}(l(F'), \mathcal{C}, \mathcal{L})$. Hence, because $l(F)$ is between i and $l(F')$, it follows that $U_i(l(F), \mathcal{C}, \mathcal{L}) > U_i(l(F'), \mathcal{C}, \mathcal{L})$, by convexity of the ideological loss $(b_i - b_l)^2$ in expression (1). To conclude that $(\mathcal{C}, \mathcal{L})$ gives a strictly larger welfare than $\mathcal{P}$, we note that (i) $(\mathcal{C}, \mathcal{L})$ cannot coincide with $\mathcal{P}$, or else $\mathcal{P}$ would be a stable factional structure; and (ii) $(\mathcal{C}, \mathcal{L})$ requires that some players i join factions led by more moderate and more informed leaders, which benefits all players, as shown earlier.

Proof of Proposition 2. We first consider the case in which the leader is a centrist in the center-left faction. Let W^L, W^C, W^R and $\bar{W}^L$, $\bar{W}^C$, $\bar{W}^R$ be the aggregate welfare of politicians of tendency L, C and R under factionalization and non-factionalization respectively. Politicians of tendency R and C strictly benefit from politicians of tendency L joining the faction led by the centrist leader, so that $W^C > \bar{W}^C$ and $W^R > \bar{W}^R$. If it were the case, by contradiction, that $W^L + W^C + W^R < \bar{W}^L + \bar{W}^C + \bar{W}^R$, then it would be the case that $\bar{W}^L > W^L$. Thus, non-factionalization would be a stable factional structure, and, indeed, the welfare maximizing one; but this would contradict the premise.

Suppose that the leader of the left-center faction is the leftist politician. We show with a parameterized example that factionalization may or may not be detrimental to the party. Let $n_C = 3$, $n_R = 3$ and set n_L sufficiently large so that a leftist leader is able to attract C tendency politicians (in, fact, $n_L \geq 5$ will suffice). We construct the example so that b_L and b_R are sufficiently large that L and R tendency politicians cannot communicate to C tendency

politicians, nor viceversa: Specifically, $b_L > \frac{1}{2(n_C+1+2)} = \frac{1}{2(3+1+2)} = 1/12$, and $b_R > 1/12$. At the same time, we want b_L sufficiently small that C tendency politicians are willing to join the faction led by the leftist leader: Specifically, $\frac{1}{6(n_C+2)} = \frac{1}{6(3+2)} \geq \frac{1}{6(n_L+2)} + b_L^2$, i.e., $b_L \leq \sqrt{\frac{1}{30} - \frac{1}{6n_L+12}}$, and the latter is larger than $1/12$ because $n_L \geq 5$. We now compare welfare in the non-factionalized party, where each politician makes his own decision,

$$-n_L \left(\frac{n_L}{6\left(n_L+2\right)} + \frac{n_C}{6\left(n_C+2\right)} + n_C b_L^2 + \frac{n_R}{6\left(n_R+2\right)} + n_R(b_L+b_R)^2 \right)$$

$$-n_C \left(\frac{n_L}{6\left(n_L+2\right)} + n_L b_L^2 + \frac{n_C}{6\left(n_C+2\right)} + \frac{n_R}{6\left(n_R+2\right)} + n_R b_R^2 \right)$$

$$-n_R \left(\frac{n_L}{6\left(n_L+2\right)} + n_L(b_L+b_R)^2 + \frac{n_C}{6\left(n_C+2\right)} + n_C b_R^2 + \frac{n_R}{6\left(n_R+2\right)} \right)$$

with the welfare of the left-center party:

$$-n_L \left(\frac{n_L+n_C}{6\left(n_L+2\right)} + \frac{n_R}{6\left(n_R+2\right)} + n_R(b_L+b_R)^2 \right)$$

$$-n_C \left(\frac{n_L+n_C}{6\left(n_L+2\right)} + (n_L+n_C)b_L^2 + \frac{n_R}{6\left(n_R+2\right)} + n_R b_R^2 \right)$$

$$-n_R \left(\frac{n_L+n_C}{6\left(n_L+2\right)} + (n_L+n_C)(b_L+b_R)^2 + \frac{n_R}{6\left(n_R+2\right)} \right)$$

Specializing the example to $n_L = 6$ and $b_L = 1/10$ for brevity, the difference between the first and the second expression is $\frac{9}{5}b_R - \frac{9}{20}$. Hence, whenever $b_R > 1/4$, factionalism is detrimental to the party, whereas if $1/12 < b_R < 1/4$, then it is beneficial.

Proof of Proposition 3. By our previous analysis, a politician i is willing to communicate truthfully to a politician j in equilibrium only if $|b_i - b_j| \leq [2(d_j\left(\mathcal{C},\mathcal{L}\right)+2)]^{-1}$, whereas he is willing to join the faction led by j instead of acting autonomously if $[6\left(d_i\left(\mathcal{C},\mathcal{L}\right)+2\right)]^{-1} \geq [6\left(d_j\left(\mathcal{C},\mathcal{L}\right)+2\right)]^{-1} + (b_i-b_j)^2$. Because $d_i\left(\mathcal{C},\mathcal{L}\right) = 1$, rearranging the second inequality and simplifying, we obtain: $|b_i - b_j| \leq \sqrt{[6\left(1+2\right)]^{-1} - [6\left(d_j\left(\mathcal{C},\mathcal{L}\right)+2\right)]^{-1}}$. The function $(2(d+2))^{-1}$ decreases in d, whereas the function $\sqrt{\left(6\left(1+2\right)\right)^{-1} - \left(6\left(d+2\right)\right)^{-1}}$ increases in d, and they cross at $d = 2.0981$. Hence, for $d_j\left(\mathcal{C},\mathcal{L}\right) \geq n_j = 3$, $\sqrt{\left(6\left(1+2\right)\right)^{-1} - \left(6\left(d_j\left(\mathcal{C},\mathcal{L}\right)+2\right)\right)^{-1}} > [2(d_j\left(\mathcal{C},\mathcal{L}\right)+2)]^{-1}$, so if a politician i is willing to communicate truthfully to a politician j, then he is also willing to join a faction led by j.

Proof of Proposition 4. We begin by establishing a few intermediate results. The first explores the structure of any player j's maximal information d_j.

Lemma A.2. *The median politician m's maximal equilibrium information equals $d_m = \min\{I, d^*\}$ whenever $g(d^*+1) < \beta \leq g(d^*)$, where $g(d) = (2 \cdot \lceil (d-1)/2 \rceil \cdot (d+2))^{-1}$ for all integer d. Further, letting $\delta = \lfloor (d_m-1)/2 \rfloor$, player j's maximal equilibrium information $d_j = d_m$ whenever $j = \delta+1, ..., n-\delta$; for all other politicians j, it is the case that $d_j \leq d_m$.*

Proof. We know that i communicates with m if and only if $|b_i - b_m| \leq (2\left(d_m+2\right))^{-1}$. Because the median politician m has exactly $\lfloor (d_m-1)/2 \rfloor$ politicians willing to communicate on one side and $\lceil (d_m-1)/2 \rceil$ on the other, it follows that the condition simplifies to:

$\beta \leq (2 \cdot \lceil (d_m - 1)/2 \rceil \cdot (d_m + 2))^{-1}$, for the player who is ideologically most distant from m. The first result then immediately follows. To prove the second result, note that a left-wing politician that has at least $\lfloor (d_m - 1)/2 \rfloor$ politicians to his left can elicit as much information as the median politician. Due to the equidistant bias structure, no politician can elicit more.

The second Lemma explicitly calculates the maximal number of ideological positions q between a politician and his faction leader with maximal information d.

Lemma A.3. *Politician i prefers to join a faction led by $q' + i$ if such politician has information d, instead of acting autonomously if and only if*

$$q' \leq q \equiv \left\lfloor \frac{1}{\beta} \sqrt{\frac{d-1}{18(d+2)}} \right\rfloor.$$

Proof. The proof immediately follows from inequality (2), once noted that $(b_{q'+i} - b_i)^2 = (q\beta)^2$ and rearranging the expression in that earlier result.

We proceed with the proof of Proposition 4 by showing that every politician i is willing to join the faction led by one of the maximally informed politicians $j = \delta + 1, ..., n - \delta$. Evidently, the worst case scenario is given by the most extreme politicians 1, and I. Simplifying the condition that guarantees the politician 1 is willing to join a faction let by $\delta + 1$, we obtain:

$$\delta\beta = \lfloor (d_m - 1)/2 \rfloor \beta \leq \sqrt{\frac{d_m(\beta) - 1}{18(d_m(\beta) + 2)}}.$$

Because d_m is a step function strictly decreasing in β, it is sufficient to check the condition at the sequence $\{\beta_d\}_{d=1}$ of the maximal β associated to each d_m. Since $\beta_d = (2 \cdot \lceil (d-1)/2 \rceil \cdot (d+2))^{-1}$, the condition then becomes

$$\frac{\lfloor (d_m - 1)/2 \rfloor}{2 \lceil (d_m - 1)/2 \rceil (d_m + 2)} \leq \sqrt{\frac{d_m - 1}{18(d_m + 2)}},$$

which always holds. In concluding that each politician i improves welfare by joining a faction led by a more moderate politician, with information d_m, and no more than q ideological positions away from i, we restrict the set of possible optimal stable factional structures. In the optimal one, each faction has cardinality no larger than $q + 1$, its leader is the most moderate politician, and all faction leaders have the same information, d_m. Continuing, note that the problem is symmetric around m. Hence, finding the optimal stable factional structure is equivalent to finding the optimal partition of the set $\{1, 2, ..., m\}$, among the partitions with elements of cardinality no larger than $q + 1$.

Next, observe that the party gain when politicians join factions led by moderates diminishes with the moderation of the politician joining the faction. Formally suppose that politician j

joins a faction led by $j + k$, where $j + k \leq m$. The per player party welfare increment is:

$$
\begin{aligned}
\Delta \mathcal{W}_{j,k} &= -\sum_{i \in \mathcal{I}} \frac{(b_i - b_{j+k})^2}{I} - \frac{1}{6[d_{j+k} + 2]} + \sum_{i \in \mathcal{I}} \frac{(b_i - b_j)^2}{I} + \frac{1}{6[d_j + 2]} \\
&= -\sum_{i \in \mathcal{I}} \frac{(b_i - b_M + b_M - b_{j+k})^2}{I} + \sum_{i \in \mathcal{I}} \frac{(b_i - b_M + b_M - b_j)^2}{I} - \frac{1}{6[d + 2]} + \frac{1}{6[d + 2]} \\
&= -\sum_{i \in \mathcal{I}} \frac{\left[(b_i - b_M)^2 + (b_M - b_{j+k})^2\right]}{I} + \sum_{i \in \mathcal{I}} \frac{\left[(b_i - b_M)^2 + (b_M - b_j)^2\right]}{I} \\
&= -(b_M - b_{j+k})^2 + (b_M - b_j)^2 = (M - j)^2 \beta^2 - (M - j - k)^2 \beta^2
\end{aligned}
$$

which is decreasing in j, because $\frac{\partial}{\partial j}\left((M - j)^2 \beta^2 - (M - j - k)^2 \beta^2\right) = -2\beta^2 k < 0$.

To complete the proof, suppose that we start from the non-factionalized party $\mathcal{P}$, restrict attention to $\{1, 2, ..., m\}$, the half-left set of politicians, and we construct the optimal stable structure by making politicians join factions one by one. If any of the politicians $k = 1, 2, ..q$ joins the faction led by $q + 1$, there is a larger welfare gain than if any politician $j + k$ joins a faction led by $q + j + 1$. Hence, the first element of the optimal stable partition on $\{1, 2, ..., m\}$ must be $\{1, 2, ..., q + 1\}$. Iterating the argument, we obtain that the optimal stable partition on $\{1, 2, ..., m\}$ corresponds to the factional structure reported in the hypothesis.

ANALYTICAL DERIVATIONS NOT SUBMITTED FOR PUBLICATION

Formal analysis of example 1. As shown in the main text, depending on the parameter value β, the optimal stable structure may take different forms. The exposition below fills in the details.

We first show that, when $\beta \leq \frac{1}{45}\sqrt{5}$, the party is united under the leadership of the median politician $m = 4$.

First, note that the maximum equilibrium information d_4 of player 4 is at least 3 for $\beta \leq 1/10$. In fact, for d_4 to be equal to 3, it must be that one neighbor on each side of 4 is willing to communicate to him. For this to be so, it must be that $\beta \leq [2(3+2)]^{-1} = 1/10$. Because $\frac{1}{45}\sqrt{5} < 1/10$, it follows that when $\beta \leq \frac{1}{45}\sqrt{5}$, the median politician's maximum equilibrium information d_4 is at least 3.

Second, note that for the party to be united under 4, and because of inequality 2, it must be that:
$$\frac{1}{6\left(d_i\left(\mathcal{C},\mathcal{L}\right)+2\right)} \geq \frac{1}{6\left(d_4\left(\mathcal{C},\mathcal{L}\right)+2\right)} + \left(b_i - b_4\right)^2,$$
with $d_i\left(\mathcal{C},\mathcal{L}\right) = 1$ for all $i \neq 4$, and $d_4\left(\mathcal{C},\mathcal{L}\right) \geq 3$. Evidently, the worse case scenario is found when considering the most extreme politicians $i = 1, 7$ and $d_4\left(\mathcal{C},\mathcal{L}\right) = 3$. When setting $i = 1$ or $i = 7$, we have that $b_i - b_4 = 3\beta$, so that simplifying the above expression we obtain the condition $\beta \leq \frac{1}{45}\sqrt{5}$.

For $\frac{1}{45}\sqrt{5} < \beta \leq \frac{1}{30}\sqrt{5}$, we show that the party is divided in 3 factions: $\{1,2,3\}$, $\{4\}$, and $\{5,6,7\}$, whose leaders are respectively 3, 4 and 5.

Because $\frac{1}{30}\sqrt{5} < 1/10$, again, it is the case that the maximum equilibrium information d_4 of player 4 is at least 3. But now, it cannot be larger than 3 either. In fact, this would require that at least 2 politicians on one side of 4 are truthful to him in equilibrium. This would require that $2\beta \leq [2(d_4+2)]^{-1}$, where d_4 would be at least 4. In sum, if the maximum equilibrium information d_4 of player 4 were larger than 3, it would be necessary that $\beta \leq [2 \cdot 2(4+2)]^{-1} = 1/24$. But because $1/24 < \frac{1}{45}\sqrt{5}$, this possibility is ruled out by $\beta > \frac{1}{45}\sqrt{5}$.

Having concluded that the maximum equilibrium information d_4 of player 4 is exactly 3 it is immediate that this is also true for any other player $j = 2, 3, 4, 5, 6$. Each one of these players, in fact, has at least two neighbors on the same side and one on the opposite side.

Because $\beta > \frac{1}{45}\sqrt{5}$, uniting the party under politician 4 is not an option anymore. It is possible, however, that any politician i joins a faction led by another politician $j = 2, 3, 4, 5, 6$ who is at most 2 ideological steps away from i. This is because $[6\left(d_i\left(\mathcal{C},\mathcal{L}\right)+2\right)]^{-1} \geq [6\left(d_j\left(\mathcal{C},\mathcal{L}\right)+2\right)]^{-1} + (2\beta)^2$, when $d_i\left(\mathcal{C},\mathcal{L}\right) = 1$ and $d_j\left(\mathcal{C},\mathcal{L}\right) = 3$, by using inequality 2.

Now note that party welfare maximization requires that politicians join factions led by more moderate politicians. This observation rules out many factional structures as optimal, though

allows several possibilities including, for example, $\mathcal{C}_1 = \{\{1,2,3\},\{4\},\{5,6,7\}\}$ with leaders $\mathcal{L}_1 = \{3,4,5\}$, or $\mathcal{C}_2 = \{\{1,2\},\{3,4,5\},\{6,7\}\}$ with leaders $\mathcal{L}_2 = \{2,4,6\}$.

To see that $(\mathcal{C}_1, \mathcal{L}_1)$ provides a higher party welfare than $(\mathcal{C}_2, \mathcal{L}_2)$, we need only compare the ideological losses, because the equilibrium information of all leaders equals 3 in both factional configurations. Under $(\mathcal{C}_1, \mathcal{L}_1)$, politicians 3 and 5 lead factions of 3 politicians each. Under $(\mathcal{C}_2, \mathcal{L}_2)$, politicians 2 and 6 lead factions of 2 politicians each, and 4 leads a faction of 3 politicians. Hence the net difference between $(\mathcal{C}_1, \mathcal{L}_1)$ and $(\mathcal{C}_2, \mathcal{L}_2)$ is that the influence weight $2/7$ is taken from the intermediate ideology politicians 3 and 5 each, half of it $(1/7)$ is given to median player 4 and half to a more extremist politician, respectively 2 and 6. Because players' utilities are quadratic losses, they are concave functions, and thus spreading influence reduces the aggregate players' utilities. In fact, the aggregate ideological loss induced by $(\mathcal{C}_1, \mathcal{L}_1)$ is:

$$3\sum_{i=1}^{7}(i\beta - 3\beta)^2 + \sum_{i=1}^{7}(i\beta - 4\beta)^2 + 3\sum_{i=1}^{7}(i\beta - 5\beta)^2 = 238\beta^2,$$

which is smaller than the ideological loss induced by $(\mathcal{C}_2, \mathcal{L}_2)$,

$$2\sum_{i=1}^{7}(i\beta - 2\beta)^2 + 3\sum_{i=1}^{7}(i\beta - 4\beta)^2 + 2\sum_{i=1}^{7}(i\beta - 6\beta)^2 = 308\beta^2.$$

The same argument based on the concavity of the player's utilities can be repeated for all possible stable factional structures, to establish that the structures $\mathcal{C}_1 = \{\{1,2,3\},\{4\},\{5,6,7\}\}$ with leaders 3, 4, and 5 are optimal.

For $\frac{1}{30}\sqrt{5} < \beta \leq \frac{1}{12}\sqrt{2}$, we now show that the optimal factional structure comprises the three factions $\{1,2\}$, $\{3,4,5\}$, and $\{6,7\}$, with leaders 2, 4 and 6 respectively.

In fact, because $\beta > \frac{1}{30}\sqrt{5}$, players are not willing to join factions led by politicians who are 2 ideological steps away. But because $\beta \leq \frac{1}{12}\sqrt{2}$, we now establish that they are willing to join factions led by immediate ideological neighbors. In fact, as seen before, for $\beta \leq 1/10$, the maximum equilibrium information is $d_j = 3$. In this case, the condition required by inequality 2 for politicians to join the faction led by an immediate neighbor simplifies to $\beta \leq \frac{1}{15}\sqrt{5}$ which is larger than $1/10$ and hence does not impose any restriction. For $1/10 < \beta \leq 1/8$, the possible leaders have information $d_j = 2$. Hence, the condition required by inequality 2 for politicians to join a faction led by an immediate neighbor simplifies to $\beta \leq \frac{1}{12}\sqrt{2}$, which is smaller than $1/8$, and hence is the binding condition.

As in the previous case, although (as established) the optimal stable structure requires politicians to join factions led by their moderate neighbors, there are several possible structures. These include $\mathcal{C}_2 = \{\{1,2\},\{3,4,5\},\{6,7\}\}$ with leaders $\mathcal{L}_2 = \{2,4,6\}$. In order to conclude that, indeed, this is the optimal stable structures, we proceed as in case 3, using arguments based on the concavity of the player's utilities.

Finally, for $\beta > \frac{1}{12}\sqrt{2}$, the only possibility is that the party is non-factionalized. In fact, in this parameter range, the maximum equilibrium information of any possible leader j is $d_j = 2$. At the same time, politicians are not willing to join factions led by even immediate neighbors with such equilibrium information.