The Three A's of Government Formation: Appointment, Allocation, and Assignment

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Abstract. How does the Prime Minister organize her government so that she can implement her policy agenda? In our model, a Prime Minister appoints individuals to her cabinet, allocates their portfolios, and assigns their policy tasks - that is, she decides the relevant jurisdiction of departments and the type of proposals a minister can make. Upon appointment ministers obtain expertise on policies specific to their jurisdiction and strategically communicate this information to the Prime Minister before a policy is implemented. Assignment allows the Prime Minister to implement her agenda even when she is constrained to appoint ministers whose policy preferences are far from her own. A Prime Minister weakly prefers a diverse cabinet. In equilibrium, the Prime Minister is indifferent between delegating policy or implementing policy herself.

1. Introduction

The standard view of relations in parliamentary democracy, certainly under the Westminster model, is of a dominant Prime Minister whose power is nevertheless constrained by cabinet government. But what are the sources of the Prime Minister's influence? How effective are the instruments at her disposal in allowing her to implement her policy agenda? And does a diverse cabinet act as an effective constraint on the exercise of Prime Ministerial power? In this paper we develop a formal model that provides answers to these questions and that builds on key structural features of parliamentary government, which are most developed in the Westminster system in which the Prime Minister is usually leader of a majority party. Our focus is on the way that a Prime Minister organizes her government in such a system. We highlight three instruments at her disposal: (i) the *appointment* of her ministers - the Prime Minister chooses who will serve under her and who will remain on the back-benches;

(ii) the *allocation of portfolios*- the Prime Minister decides which ministers will run which government departments; and (iii) the *assignment* of a ministers responsibilities within her department. With reference to the latter we suppose that the Prime Minister can actively define her minister's brief: the set of issues over which the minister can decide (which we define as his jurisdiction); and the type of proposals that a minister can make.

The appointment of ministers is the most well understood instrument at the Prime Minister's disposal, and has been analyzed by Huber and Martinez-Gallardo (2008), Dewan and Myatt (2007), and Thies (2001), amongst others. The allocation of ministerial portfolios is central to the seminal models by Laver and Shepsle (1996) and Austen-Smith and Banks (1990). The key insight of the portfolio allocation models is that, once we consider the parliamentary procedures by which the rights to make and implement policy on specific issues are allocated, we can find equilibria of a well defined government formation game in multiple policy dimensions. These models treat a minister's responsibilities as fixed: upon being appointed (and allocated a portfolio) he implements the party's preferred policy on a set of issues aligned on a single policy dimension over which he has jurisdiction. We start from the premiss that defining ministerial responsibilities is a critical element of a Prime Minister's strategic plan and should be considered alongside the other instruments at her disposal.

Following standard models of delegation we assume that the Prime Minister cannot formulate policy on all dimensions. The consequent division of labor involves strategic appointment of cabinet members, allocation of ministerial portfolios, and assignment of ministerial responsibilities. We explore how a Prime Minister organizes her government in a multidimensional policy space and show that the existence of a such a policy space creates strategic opportunities for the Prime Minister to define ministerial responsibilities in a way that aligns ministers' incentives with her own. Surprisingly, this holds even when she is constrained to appoint ministers with ideal policies far from her own preferred ones.

We develop this insight in a model with imperfect information: ministers have known policy biases but have expertise on policies under their jurisdiction. Specifically ministers have private information which they can reveal truthfully or otherwise to the Prime Minister. We show that when the Prime Minister assigns responsibilities optimally (from her perspective)

then she is able to obtain the same outcomes as if she were perfectly informed and implemented policy directly. In fact, we show that when the Prime Minister optimally chooses ministerial responsibilities then she is strictly indifferent between implementing policy herself or delegating to her ministers. This is true even when her cabinet is ideologically diverse; in fact we show that a Prime Minister (weakly) prefers diversity.

The paper is organized as follows. We first (section 2) offer a discussion of the related literature before (section 3) providing some empirical motivation for our model. We introduce our modeling framework in section 4. In section 5, we analyze a two minister model that allows us to explore the relative effectiveness of the three instruments that the Prime Minister has at her disposal and to establish some baseline results. In the remainder of the paper we provide a more general analysis, first exploring the process of appointment, allocation and assignment in large cabinets (section 6), and then taking account of the size of adjustments required to status quo jurisdictions (section 7). In section 8 we explore the robustness of our findings in different institutional environments: we consider that the Prime Minister is elected by some subsection of party members; consider different ways of conceptualizing the cabinet; allow for collusion between ministers; and finally consider a world where ministers not only have policy biases but also hold expertise on issues (that may be correlated with these biases). Section 9 considers the novel empirical implications of our analysis and section 10 concludes. All proofs are contained in the appendix.

2. Related literature

Our model relates to a growing literature that uses principal agent theory to understand the multiple relations in parliamentary democracies (Strøm, Muller, and Bergstrom, 2003; Thies, 2001; Martin and Vanberg, 2004). Our analysis of aggregation of dispersed information by cabinet members is circumscribed in the cheap talk literature (Crawford and Sobel, 1982) that analyzes strategic communication by an agent to a principal who implements policy. Whilst this literature has been used to explore information transmission in Congress (Gilligan and Krehbiel, 1987; Baron, 2000; Patty, 2009), to our knowledge our work is the first to apply this machinery to parliamentary democracies. In the classic model of committee organization in the U.S. House (Gilligan and Krehbiel, 1987), the parent body chooses the procedural

rules that, in turn, provide incentives for committee members to acquire information in a unidimensional policy space. In our model of Westminster government, a Prime Minister assigns a minister's task by defining his jurisdiction and responsibilities. Ministers acquire information (costlessly) on policy issues over which they have jurisdiction, and so, and in contrast to the classic model, expertise is endogenously assigned by the Prime Minister.

The fact that departmental jurisdictions overlap, means that a multi-dimensional model is appropriate to our setting. Our framework and initial results (propositions 1 and 2) applies the multi-dimensional cheap talk model introduced by Battaglini (2002) and highlights the importance of these results toward understanding government organization. Whereas Battaglini shows that in a multi-dimensional cheap talk setting there is a fully revealing truth-telling equilibrium where jurisdictions are orthogonal to the biases of senders, we show that in our world this is true of any truth-telling equilibrium. Using this framework our results can clearly be distinguished from those of the unidimensional model that has been used to analyze Congress: whereas in that model information transmission improves when the (median) preference of the committee and the parent body are not far apart, in our model diversity does not undermine the power of a Prime Minister and can sometimes benefit her.

The analysis of jurisdictional assignments as equilibrium phenomena has been studied previously by Ting (2002). He analyzes optimal jurisdictional assignments in the presence of moral hazard when the legislature controls the agency budget and contractual rewards and shows conditions under which a legislature would wish to consolidate bureaucratic tasks in a single agency. Indridason and Kam (2008) also look at moral hazard in a situation where the budget resources a minister can claim for a given portfolio may be transferred to a rival minister following a reshuffle. In contrast to these models asymmetric information due to expertise. A similarity arises between the work of Indridason and Kam and ours in that the Prime Minister can exploit differences between her ministers and align their incentives with her own.

Finally, a critical question that we address is the optimal assignments when the Prime Minister is constrained to making only small adjustments to ministerial responsibilities. The political situation we analyze is a special case of the assignment problem studied in mathematics (Burkhard, Dell'Amico, and Martello, 2009).

3. Changing Ministerial Assignments

Our key contribution is in exploring the ways that a Prime Minister adapts the machinery of government as part of her cabinet management strategy. A recent report by the Institute of Government into the changing structure of Whitehall states that the ability of the Prime Minister to rearrange departments serves as "such an important tool that only one new Prime Minister since 1950 has chosen not to reconfigure departments in some way after assuming the leadership."[1] Such reconfiguration typically takes one of two forms: new departments are created from a rearrangement of existing departmental responsibilities; or the department structure remains the same but ministerial responsibilities are reassigned across departments.

An example of the first comes with the creation of the Department of Energy and Climate Change in October 2008 taking over *energy* functions from Department for Business, Enterprise and Regulatory Reform and taking over *climate change* functions from the Department for Environment, Food and Rural Affairs. A further example concerns the recent overhaul of the Home Office, that included the setting up of a new Ministry of Justice and a new Office for Security and Counter-Terrorism within the Home Office. A still recent example is the setting up of the Department for International Development in 1997 following a transfer of responsibilities from the Foreign and Commonwealth Office.

Major organizational changes are typically taken without a major change to the departmental structure and involve a simple transfer of policy responsibilities. The history of the Home Office is particularly interesting with a plethora of policy functions being transferred and withdrawn from its jurisdiction over the course of its history. At different points in time it has been responsible for sport, broadcasting, and adoption and child care which it received following a transfer of responsibilities from the Ministry of Health in 1947 with the same policy powers being transferred back to the Department of Health and Social Security in 1971 (the DHSS was itself, a department that merged functions previously held by other departments). More recently in 2007 criminal justice, prisons, probation and legal affairs were transferred to the Ministry of Justice while counter-terrorism strategy was brought

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[1]White and Dunleavy (2010)

under the Home Office's remit. These are a few of more than forty changes made in the twentieth century.[2]

Such changes, which amount to a major reorganization of the policy responsibilities and functions of British government, fall under the Royal Prerogative which is exercised by the Prime Minister.[3] No primary legislation is required and changes are made in the absence of an objection from either House.[4] Indeed a recent report of the House of Commons Public Administration Select Committee, stated that "it is anomalous that it is so procedurally straightforward for the Prime Minister to reorganize the Civil Service by amending the functions of the ministers it serves, when reorganizing other public services may often involve statutory consultation, parliamentary approval, or even primary legislation."[5]

An important and previously uninvestigated consequence of these organizational changes is that there is considerable overlap in the jurisdictions of different departments. To provide some examples from the current assignment of Whitehall departments: policies on Health, whilst part of the domain of the Ministry for Health, are also dealt with in the Department of Education; policies on terrorism, counter-terrorism, and security, come under the purview of the Foreign and Commonwealth Office and the Home Office; International Relations falls under the jurisdiction of the Secretary of State for Defence and the Minister for Environment, Food and Rural Affairs.

Our notion of assignment takes account of this jurisdictional overlap but goes further. A key aspect of our set up is that a Prime Minister can limit the types of proposals that a minister can make. Ministers must make policy proposals, within the area of their jurisdiction, that accord with guidelines set down by the Prime Minister. A leading example is

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[2]A full record of all such changes to British government from 1964-1992 gathered by Iain McLean is held at http://www.nuffield.ox.ac.uk/politics/whitehall/; Chester and Wilson (1968) look at changes from 1960-83.

[3]Relatedly, in the German Federal Republic the reassignment of policy competencies ultimately falls under the *Richtlinienkompetenz* of the Chancellor according to Article 65 of the Basic Law.

[4]In response to the question made on February 6th 2006 by Lord Stoddart of Swindon, who asked Her Majesty's Government "whether they will issue a Green Paper on the proposed reorganisation and splitting of the Home Office, and allow for a period of public debate and consultation and the issuing of a White Paper before any Bill to implement such reorganization is presented to Parliament", the Minister of State for the Home Office, Baroness Scotland of Asthal, replied unequivocally: "questions of changes to the machinery of government are decided by the Prime Minister." By contrast, it was Congress that set up the Homeland Security Department in response to the Sept. 11, 2001, terrorist attacks.

[5]Seventh Report of Sessions 2006-2007 of the House of Commons Public Administration Select Committee: Machinery of Government Changes.

where the Prime Minister provides direction over spending priorities. A stark and currently very relevant case is where the Prime Minister wishes to ring-fence or prioritize certain areas of spending over others. Such ringfencing protects budgets according to policy priorities identified by the Prime Minister. For example, in a recent announcement David Cameron committed to increase the Foreign Aid budget, administered by the Department for International Development, by 0.7 per cent of gross national income by 2013 despite overall public spending cuts totalling 83 billion pounds. Other examples include the budget for the Department of Education which is due to rise year on year, though capital spending has been slashed by 60%. These examples are indicative of the ways in which the Prime Minister can limit the type of proposals that a minister brings to cabinet and introduces to the floor of the House of Commons.

Existing formal models of Parliamentary Democracy have thus far treated the jurisdictions of departments as fixed and so have not shed light on this aspect of governmental organization. Neither have existing models taken account of the considerable overlap in jurisdictions that is found between Whitehall departments. And we know little about how the Prime Minister's ability to set policy guidelines affects the policies that are implemented. What explains why Prime Ministers have adapted the structure of Whitehall departments and why jurisdictional overlap is so common? Is there a strategic rationale for the underlying structure to collective decision making in Westminster Democracies? And how does control over ministerial assignments fit alongside other mechanisms by which the Prime Minister affects policy outcomes such as ministerial appointments? In the following section we describe a model that allows us to explore these questions.

4. A Formal Analysis of the Westminster model

We develop a formal model that explores a situation in which an executive leader organizes her cabinet. Her organizational strategy has three elements: she selects the personnel who will serve in her cabinet; she allocates their portfolios; and she then assigns ministerial responsibilities in a way in which we make precise below. The strategic actors in our model are then a Prime Minister and her ministers who are motivated by policy. Outcomes are

determined by strategic interaction between ministers who report to the Prime Minister on their assignment; and the Prime Minister who organizes her cabinet and implements policy.

4.1. **Payoffs.** We initially restrict to a government that consists of two ministerial posts, though we later extend our analysis to a larger cabinet. The preferences of ministers are defined over policy outcomes $x \in \mathbb{R}^2$ and are single peaked and quadratic with bliss points at m_i and $i \in \{1,2\}$ so that payoffs are defined as $u_i(x) = -\sum_{n=1}^{2}(x^n - m_i^n)^2$.[6]We write the Prime Minister's ideal point as pm^* and, for notational simplicity, normalize so that it is located at the origin $pm^* = (0,0)$. This implies that the ministers' bias with respect to the Prime Minister is $(m_i - pm^*) = m_i$.

4.2. **The Prime Minister's Instruments.** The Prime Minister must appoint ministers, allocate a portfolio to each, and then assign their responsibilities. The first element of assignment is the set of policies over which the minister reports to cabinet. This is what we term a ministers "jurisdiction". For two policy dimensions it is possible that a Prime Minister would assign all issues related to dimension X to the portfolio of one minister, and the other Y to the other minister. As we have seen, however, there may be a degree of overlap in the jurisdictions of different departments. So we allow for the fact that a Prime Minister might give some influence over issues on X and Y to both ministers.

The second element of a ministers assignment is the type of proposal that a minister can make within his jurisdiction. As we have seen, although ministers are responsible for their own departments, the Prime Minister sets important policy guidelines that constrains the type of proposals ministers can make. To illustrate consider figure 1 that depicts a two dimensional policy space involving policy X and policy Y. We indicate by m_1 a minister's policy bias relative to that of the Prime Minister (recall that the Prime Minister's ideal policy lies at the origin and so in this example the minister's bliss point is to the north-east of that point). If the minister had influence on both policies, then he would wish to implement a policy that is greater than pm^* on both policy dimensions. Consider, for example a situation where the minister could decide how much money should be allocated to policies X and Y. The minister would like to spend more on both relative to the Prime Minister. If, however,

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[6]This is without loss of generality. More specifically, we require that utility functions are continuous, quasi-concave, and differentiable.
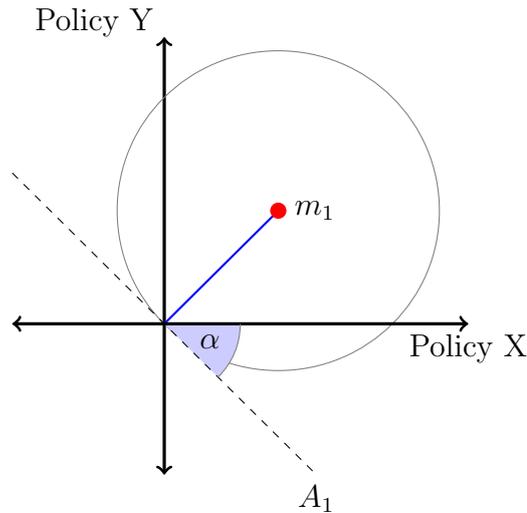
FIGURE 1. Defining a Minister's Assignment

the minister were forced to choose a level of spending along the line given by A_1 then her preferred allocation on that line is the same as that of the Prime Minister: her indifference curve is tangent to A_1 at the origin.[7]

An implication of this example, that we explore more fully, is that by carefully defining ministerial responsibilities a Prime Minister may be able to align the incentives of her ministers with her own. A critical aspect is that jurisdictional overlap creates a multidimensional problem that provides the Prime Minister greater strategic leeway. To return to our spending example, if the Prime Minister were only able to define her minister's responsibilities along a single dimension, then she could constrain his proposals in a very stark fashion; by for example setting a maximum (or minimum) level of spending. In multiple dimensions, however, the Prime Minister can define the ministerial brief in a way that provides her with strategic advantage.[8]

4.3. **The Prime Minister and her Cabinet.** To complete our formal setting we now describe more fully the relationship between the Prime Minister and her team of ministers, the timing of our game, what ministers know, and how they know it.

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[7]The analysis extends to any continuous multidimensional policy space where preferences are strictly convex. Consider for example the case where the minister is, relative to the Prime Minister, in favor of stricter regulation of environmental standards, via a carbon tax for example, and greater provision for nuclear power. He might then be required to couple any proposed plan to increase nuclear capacity, via state support for capital investment in nuclear plants, with a reduction in carbon taxes.

[8]We are saying nothing new here. This insight was developed in the seminal paper by Austen-Smith (1993).

4.3.1. *Information.* The source of a minister's influence in our model is an informational asymmetry. We assume that there are underlying social, economic and political fundamentals that are (initially) not directly observed by either the Prime Minister or her ministers. We capture these fundamentals via the vector $\theta \in \Theta = \mathbb{R}^2$. Once a minister has been allocated a department and its jurisdiction is assigned, however, he acquires all information relevant to policy-making in that jurisdiction. An implication of our assumption is that in assigning responsibilities the Prime Minister decides which of her minister will become informed and on which set of issues. Having defined a minister's jurisdiction the Prime Minister completes the assignment by specifying the minister's task: the minister, upon receiving the policy-relevant information in their jurisdiction, must report a single dimensional variable to the PM, $s_i : \Theta \to \mathbb{R}$. An assignment, however, restricts ministers to reporting on one of two directions $A_i \in \mathbb{R}^2$, $i = \{1, 2\}$ that span the whole policy space (i.e. $A_1 \neq \lambda A_2, \forall \lambda \in \mathbb{R}$).

4.3.2. *Timing.* In the first stage of our model the Prime Minister appoints her ministers, allocates their portfolios and assigns their responsibilities. In the second stage ministers bring a proposal to the Prime Minister that conforms with the responsibilities assigned to them. Finally, and upon receiving the minister's proposal the Prime Minister implements policy which we denote as $y \in \mathbb{R}^2$. Her chosen policy may depend on the declarations made by ministers so that $y((s_1(\theta), s_2(\theta)))$. We write the policy outcomes as x, which depends on both the chosen policy y and the underlying fundamentals θ. Specifically the final outcome satisfies $x = y + \theta$. The policy that the Prime Minister chooses depends upon her beliefs about these fundamentals given the declarations of her ministers. We write the posterior belief of the PM on the possible states of the world as $\mu : \mathbb{R} \times \mathbb{R} \to P(\Theta)$.

4.4. **Solution Concept.** Our solution concept is Perfect Bayesian Equilibrium. Loosely speaking, this requires that the actions of the Prime Minister and her ministers be sequentially rational given their beliefs, that their beliefs be consistent with rational play and Bayes rule along the equilibrium path, and with some beliefs off the equilibrium path. We focus attention on the existence of a *truthful and fully revealing equilibrium*: ministers adopt a truthful strategy, that is they report the true state of the world within their ministerial responsibility and, in combining her ministers' joint declarations, the Prime Minister learns

all there is to know about the true state of the world. In game theoretic terms a fully revealing equilibrium is defined by posterior beliefs cumulated on the true state of the world, $\mu(s_1(\theta), s_2(\theta))(\theta) = 1$. Of course an equilibrium may not be truthful and yet still be fully revealing. This occurs, for example, if the ministers systematically misreport: it may be common knowledge that a minister exaggerates by adding a bias to his report and that the Prime Minister takes this into account by discounting such reports. Applying reasoning akin to the revelation principle, it can be shown that there is no loss of generality in restricting attention to fully revealing equilibria that are at once truthful and involve degenerate out of equilibrium beliefs.[9] The intuition behind this result is that cabinet members only care about final outcomes: when a fully revealing equilibrium exists they may as well report truthfully (they need never construct complicated out-of-equilibrium beliefs, as the preferred policy of the Prime Minister is an equilibrium outcome in either case).

5. Three A's in a Two Member Cabinet

In this section we establish our core results in a simple two member cabinet. We develop our results by looking at two distinct and extreme cases. In the first, assignments are given by the policy axis and the only instrument the Prime Minister can use in order to implement her policy agenda is to choose which individuals sit in her cabinet and to allocate a portfolio to each. In the second case, we assume the Prime Minister has no discretion over who to appoint but is able to allocate portfolios and assign ministerial responsibilities.

5.1. **Appointing Ministers.** We begin our analysis by focussing on the simplest scenario in which departmental jurisdictions are fixed, but the Prime Minister has complete discretion to appoint her ministers. To make things interesting we make a slight restriction on the type of ministers that are available assuming that there will always be some ideological conflict. That is, a Prime Minister is unable to appoint ministers who share her ideal point.

As a toy example that helps illustrate some of the main ideas and introduces our basic notation, consider a situation where the ideal points satisfy $pm^* = (0,0)$, $m_1 = (0,1)$ and $m_2 = (1,0)$ respectively (the first coordinate represents the ideal point on policy X, and the second coordinate the ideal point on policy Y). In this situation the Prime Minister has

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[9]See Lemma 1 of Battaglini (2002).

perfectly aligned preferences with m_1 on policy X, (both the Prime Minister and her minister would like the policy outcome to be as close as possible to the origin) whilst, analogously, on policy Y, the Prime Minister and m_2 are perfectly aligned.

Under these circumstances, and given any underlying value of θ, the Prime Minister can strategically appoint her ministers so that ministers reveal their information truthfully. Consider a situation where m_1 is appointed to the department that has full jurisdiction over policy X, so that his assignment is $A_1 = (1,0)$, or any proportional vector to this one, and agent 2 has full jurisdiction on the second policy dimension so that he reports on $A_2 = (0,1)$. Now imagine that m_2 truthfully reveals her information to the Prime Minister (so that $s_2(\theta) = \theta_2$). Armed with this information the Prime Minister will implement a policy ($y_2 = -\theta_2$) such that $x_2 = 0$. In order to compute Minister 1's best response to truthful revelation by minister 2 we can draw the indifference curve tangent to his assignment: the point of tangency coincides with the ideal point of the Prime Minister. Thus minister 1 delivers a report that yields a policy outcome $x_1 = 0$. In a truthful equilibrium, the minister reports $s_1(\theta) = \theta_1$ and the Prime Minister implements $y_1 = -s_1(\theta)$. In this example the Prime Minster elicits all the information relevant to full implementation of her agenda: $s_i(\theta) = \theta_i$ for m_i is a best response for minister i when the other minister reveals her information truthfully. It remains to be shown that this is in fact part of a (perfect Bayesian) equilibrium.

Proposition 1. *When ministerial responsibilities coincide with policy dimensions and each minister's assignment is orthogonal to his bias then there is an equilibrium where each minister reports truthfully the state of the world given his assignment. The Prime Minister believes these statements, and the policy that is implemented yields an outcome coinciding with the Prime Ministers ideal point.*

This first result shows that, without specifying the agents' messages nor the response of the Prime Minister to these messages, there is an equilibrium in which the preferred policy of the Prime Minister is implemented.[10] One such equilibrium is both *truthful and fully revealing.*[11]

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[10]As usual in these environments there are many more equilibria (e.g. a babbling equilibrium where no information is revealed and the messages are never used by the Prime Minister).

[11]As noted earlier, there may be other fully revealing equilibria (not truthful) where ministers systematically overstate the true state of the world ($s_i(\theta) = \theta_i + \delta$), but where the Prime Minister, taking such behavior

5.2. **Allocating Portfolios and Assigning Ministerial Responsibilities.** The results of the previous section reveal conditions under which cabinet selection acts as an instrument allowing the Prime Minister to implement her agenda. Of course, there are limitations to the use of this tool. A Prime Minister may be forced to select on some individual trait other than the political preferences of her appointee - for example, talent, experience, or following in the party - and may find that those best able to serve under her do not share her political opinions.[12] This might suggest then that the Prime Minister's position is weakened. Indeed the view that a Prime Minister's power is both limited and contextual due to restrictions on her ability to appoint is commonly held.

This viewpoint overlooks the fact that the Prime Minister has other instruments at her disposal, and that their use may also influence the policies that are implemented. In particular she can decide which policies form part of a minister's jurisdiction and the type of proposals a minister can make. In the previous section we looked at a situation where assignments are aligned with the policy axis and so a department is the only one with jurisdiction in a given policy area. Although analytically convenient, as we have seen, usually there is some overlap in the jurisdictions of different departments. Figure 1 depicts a situation where the minister's policy bias relative to that of the Prime Minister is given by m_1 and the assignment A_1 involves both policy X and policy Y. Recall that the Prime Minister's ideal policy lies at the origin and so in this example the minister's bliss point is to the north-east of that point. Ideally, from his perspective, the minister would recommend a policy that is greater than pm^* on both policy dimensions. Suppose however, that the minister' assignment is given by A_1: then he must acknowledge a trade-off in which he can increase X only by decreasing Y. A natural way to think of this trade-off regards policies over distribution, where X and Y involve spending on particular policies and/or targeted distribution to specific constituencies. For example, a report on A_1 to the northwest of the origin ties an increase in spending

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into account, implements the policy $(y(s) = -(s_1 - \delta, s_2 - \delta))$. When the Prime Minister is able to appoint a minister whose bias is orthogonal to the jurisdiction, then she can obtain full information and so implement her preferred policy.

[12]An alternative explanation is that the Prime Minister prefers to have her opponents in the cabinet. John Major was unwilling to sack three staunch opponents of his policies arguing that "we don't want another three bastards out there. What's Lyndon Johnson's maxim?..." Johnson had famously declined to sack FBI director Edgar Hoover on the basis that "it's probably better to have him inside the tent pissing out than outside pissing in".

on policy Y to a decrease in spending on policy X, relative to the Prime Minster's ideal point; the opposite spending pattern is implied by a report to the southeast of the origin. Thus, although relative to the Prime Minister, the minister would like to spend more on both policies (constituencies), he is forced into a trade-off between the two. The Prime Minister has the power to determine this trade-off by defining his minister's brief. In our model this is achieved by determining the slope of A_1. She will do so optimally given her knowledge of the minister's bias. We are now ready to show that in any fully revealing equilibrium (in which the Prime Minister implements her preferred policy) each minister's assignment needs to be orthogonal to his own bias.

Lemma 1. *Fix the ideal points of ministers and allow the Prime Minister to choose their assignment. In a fully revealing equilibrium, a ministers assignment is orthogonal to his bias and not affected by the ideal point of the remaining cabinet ministers. Multiplying a minister's bias by a constant does not change his assignment.*

The implication of this result is that, when the Prime Minister is constrained to appoint a minister with a given and known bias then she can always define a minister's responsibilities assign in such a way that he reports truthfully. This result is not affected by the size of the minister's bias relative to the Prime Minister's ideal point, nor the distribution of ideal points in the cabinet. The following proposition shows that a set of orthogonal assignments form part of a fully-revealing perfect-Bayesian equilibrium.

Proposition 2. *Assume that the biases of the ministers with respect to the the ideal point of the Prime Minister are linearly independent. Allow the Prime Minister to choose ministerial assignments. She then elicits full information from her ministers and so can fully implement her policy agenda.*

A technicality in the wording of the proposition deserves further attention: the biases of the ministers with respect to the ideal point of the Prime Minister need to be *linearly independent*. By Lemma 1 we know that in a fully revealing equilibrium each assignment needs to be orthogonal to its minister's bias. If biases are linearly dependant, assignments

would coincide and the Prime Minister would no longer be able to infer the true state of the world in our two-dimensional policy space. Instead, linear independence ensures that both assignments span the whole policy space and the Prime Minister is then able to illicit all information she needs to implement her preferred policy.

According to one prominent and widely held view, the Prime Minister's control over policy is limited by the need to include ministers who (i) do not share her policy preferences and (ii) are either too senior, talented, or well supported in the party, to be overlooked.[13] Our analysis suggests that these are necessary though not sufficient conditions. According to our view the Prime Minister is constrained, that is she is unable to fully implement her agenda, only when each of the following conditions hold with respect to a particular minister: the Prime Minister is forced to appoint the minister even though the ministers preferences are not aligned with her own; the minister has veto power over his/her appointment to a particular ministry; the minister has veto-power over any changes to his assignment; in such circumstances, the minister's report cannot yield implementation of the Prime Minister's preferred policy. In all other situations, even when the Prime Minister is constrained to appoint ministers whose preferred policies are very far from her own, she nevertheless is able to implement her desired policies. Moreover, and perhaps surprisingly, this does not depend on the assumption that the Prime Minister implements policy as we show in the following corollary to proposition 2.

Corollary to Proposition 2: *When the configuration of tasks is optimally designed from the Prime Minister's perspective, the Prime Minister is strictly indifferent between choosing policy herself or fully delegating the task to her minister.*

The implication of this result is stark. As long as the Prime Minister has full control over assignment, her influence is undiminished even when she allows policy to be chosen and implemented by her ministers. Thus what Lupia (2003) has referred to as the "perils of delegation" in parliamentary democracies are avoided so long as the Prime Minister has full control over the allocation of ministerial tasks. When policy decisions are delegated to

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[13]King (1994) argued that a few "big beasts of the jungle" maintained such stature as to be able to impose their views on policy outcomes in their departments (see also Laver and Shepsle (2000)).

ministers they implement precisely those policies the Prime Minister would implement in the event that she had full and perfect information.

As we noted earlier our model is related to the classic application of the cheap talk literature to Congress (Gilligan and Krehbiel, 1987). There it is well known that the legislative body can extract more information when committee members have biases that are small relative to the Floor median. Do the insights from the Congressional literature on information transmission carry over in our multidimensional setting more suited to Westminster? Here we observe that our results do not hinge on the distance of ministers ideal points from the preferred outcomes of the Prime Minister. Given any bias, however big, the Prime Minister can design a ministerial brief such that the minister will report truthfully and given the optimal assignments to all cabinet members the Prime Minister extracts all relevant information. In fact the Prime Minister is never worse off when the cabinet is more diverse.

6. Three A's in a Large Cabinet

In the remainder of the paper we consider how robust our findings are to different assumptions about the institutional environment. Thus far our analysis has relied on a two-member cabinet. It is natural to consider the implications of analyzing a fully-fledged cabinet consisting of an arbitrary number of ministers deciding over multiple policy issues and so extend our model to consider a multi-member cabinet with $n > 2$ distinct policy issues related to the same number of government departments.

Proposition 3. *Consider a situation where the number of policy issues decided by the government is $n > 2$ so that the cabinet consists of n ministers with fixed ideal points. Allowing the Prime Minister to choose the assignment of each of her ministers, and assuming that the biases of at least two ministers with respect to the ideal point of the Prime Minister are linearly independent, the Prime Minister elicits full information from her agents and so can fully implement her policy agenda.*

Perhaps surprisingly, when moving to the general case we can use less restrictive assumptions about the ideal points of the ministers. In particular, for $n > 2$, we only need two ministers'
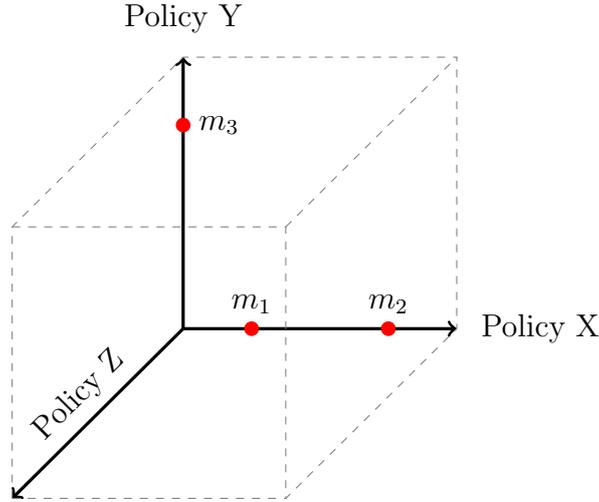
FIGURE 2. Portfolio Allocation when $n = 3$

ideal points to be linearly independent (i.e. the orthogonal hyperplanes to their biases span the whole policy space). This can be shown with a three minister example as illustrated in figure 2. Here note that the ideal points of m_1 and m_2 are linearly dependent: *both* ministers agree with the Prime Minister on two of the three policy dimensions. Following our earlier logic, it is straightforward to see that the Prime Minister can elicit all of the relevant information when assigning jurisdiction over policy X to m_3, jurisdiction over policy Y to m_1, and jurisdiction over policy Z to m_2; each minister needs to report on the assignment formed by the policy axis. This is not the only way in which the Prime Minister can organize her government and be in a position to implement her agenda: she could, for instance, obtain the same outcome by switching the jurisdictions of m_1 and m_2. In fact she could give overlapping jurisdiction on policy Y and Z as long as these were linearly independent. We note that in such a situation both minister acquire information on policies Y and Z but not on X, whereas minister 3 only acquires information on policy X. This shows that in dimensions $n > 2$ a minister's expertise can be limited to policies over which he has jurisdiction. Thus the Prime Minister, as part of the assignment process, endogenously determines the expertise of her ministers.

7. ADJUSTING ASSIGNMENTS

A key innovation in our analysis is that we consider the assignment of ministerial departments as being part of a Prime Minister's strategic plan. As we have seen, when the Prime Minister
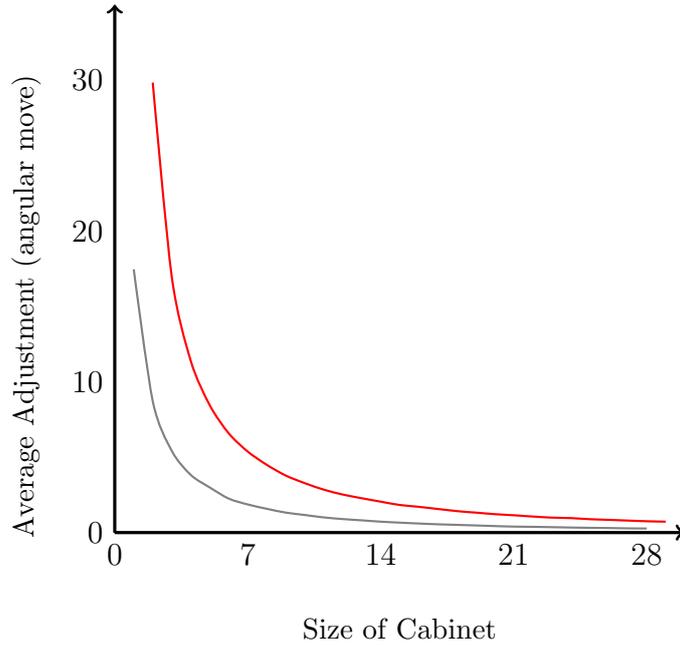
FIGURE 3. Average Adjustment of Reorganizing Assignments
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can create departments with overlapping jurisdictions and optimally assign tasks then she can obtain her desired outcomes even when constrained with regard to the appointments she makes to cabinet. Most models of government formation do not consider this aspect. In the portfolio allocation models, for example, each minister's jurisdiction involves a single policy dimension. An interesting theoretical question then is the extent that the optimal assignment from the Prime Minister's perspective deviates from a status quo where ministers' assignments are given by the policy axis as in figure 1. Our focus is in the angular move α, defined as the necessary adjustment making a minister's assignment orthogonal to his bias, when starting from this status quo. Our interest in this parameter can be motivated from the following thought experiment. Imagine that ministers enter government with the expectation that they will be given full control over a well defined portfolio with jurisdiction over a given policy area (i.e the policy axis X in figure 1) How much must the Prime Minister adjust this portfolio to correct for the bias of her minister and in order to align his incentives with her own. We know that, in any fully revealing equilibrium, the Prime Minister will adjust until A_i is orthogonal to m_i (as illustrated in figure 1 for $i = 1$ and $A = 1$). Thus, for example, when $m_i = (1, 1)$, the move is $45°$.

To explore our question we ask how does α respond to the size of government.[15] The political implications of increasing the size of the government are the following: on the one hand, an increase in the size of government implies an increase in the number of possible assignments and this is associated with an increase in the complexity of the Prime Minister's assignment problem; on the other, an increase in the size of government increases the possibility, that for a given set of ministers, and on any particular assignment, the Prime Minister can find a minister with an ideal point that is orthogonal to the status quo assignments in which case he need make no adjustments to that portfolio. We look at the extreme case where the Prime Minister has no appointment power, thus we treat each minister's ideal point as a random draw from a known distribution. We assume that the size of the adjustment(the angle α)that produces an assignment orthogonal to a minister's bias is independently drawn from a distribution F with density f. We call AA_n as the average size of adjustment when assigning n ministers. We analyze how this average adjustment responds to increasing the size of government (i.e. as n grows large).

Proposition 4. *When there is a strictly positive probability that the adjustments required to existing portfolios are arbitrarily small (i.e. $F(\alpha) > 0$ for all $\alpha > 0$), the average adjustment of the optimal assignment tends to zero as the number of policy dimensions increases ($\lim_{n\to\infty} AA_n = 0$).*

In the limit, as the size of the government grows large, the necessary adjustments from the status quo provisions in order for the Prime Minister to implement her agenda go to zero. Pushing further we analyze the same parameter in commonly sized cabinets (i.e. n just below 30). An immediate concern is how fast the average adjustment AA_n converges to zero and whether, in commonly sized cabinets these are also negligible. We analyze this question numerically by assuming that there are n ministers whose bias with respect to the Prime Minister on each policy dimension is drawn from a normal distribution with zero mean and unit variance. As for our our earlier results, status quo jurisdictions are the coordinate axis. We compute the minimum angular move so that each assignment is orthogonal to the bias

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[15]Recall that we need as many ministers as policy dimensions so that in our analysis an increase in the number of policy dimensions is equivalent to an increase in the size of government.

of each minister. For each of the 500 simulations we run, we find the optimal assignment that minimizes the size of the required adjustment. We compute the average adjustment among all our simulations. Full details of our algorithm can be found in the appendix, here we concentrate instead on our substantive results represented by figure 3.

The upper (red) line in figure 3 shows the rate of convergence when the Prime Minister has no freedom to appoint who sits in her cabinet, but can make assignments in such a way that ministers will report truthfully. We see that, as the number of jurisdictions increases, the average adjustments to the status quo fall rapidly toward zero.[16] When $n = 2$ the (average) necessary angular move away from the status quo jurisdictions is $\alpha = 29.832°$. This falls to $\alpha = 0.722°$ as the government size increases to $n = 29$.[17] The lower (gray) line in figure 3 shows a situation where the Prime Minister has some discretion over appointments and has twice as many ministerial options as jurisdictions. As might be expected, convergence is much faster as under these circumstances the Prime Minister can seat in her cabinet those ministers with the smallest biases. These graphs provide an indication as to how the instruments at the Prime Minister's disposal, namely her ability to appoint ministers, allocate portfolios, and assign tasks, interact to allow her to implement her agenda. Moreover they show that for these particular parameters, the adjustments required to the status quo assignments (i.e those that coincide with the policy axis) in a reasonably sized government are negligible.

8. Changing the Institutional Environment

Our results are based upon a somewhat stylized and reduced form view of the Westminster Model. Here we consider how robust are our findings to the addition of some institutional detail. First we consider what would happen if the Prime Minister were to be held accountable via an electoral mechanism, that is she could be replaced by some section of the polity. Next we consider alternative views of the cabinet: in one, ministers deliberate over outcomes; in another they are able to collude before providing their reports to the Prime

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[16]Here we depict the average results from 500 simulations.

[17]In all our numerical simulations, total costs also go to zero.

Minister. Finally we consider a world where ministers are distinguished by their expertise as well as by their policy preferences.

8.1. **Choosing the Prime Minister.** Thus far we have not considered any mechanism by which the Prime Minister can be held to account for the organization of her government and the decisions that are reached given her organizational strategy. In some sense we might view the Prime Minister as the agent of her party who is endowed with the powers to appoint ministers, allocate them to portfolios, and assign tasks. The question then arises: which agent would the party choose? Our key results reveal that when the Prime Minister has control over assignments, even though she may be constrained with regard to appointments, she can implement policies as if she were fully informed on the relevant issues. An implication of these findings is that it straightforward to extend our analysis to a world in which the Prime Minister were selected by the party of some subset of its membership. For example we could extend our analysis to a world where there was an open entry contest to become the Prime Minister, in which eligible citizens (however defined) cast their votes between competing candidates. A straightforward application of the citizen-candidate framework (Besley and Coate, 1997), yields the following: when voters anticipate the final implemented policies, there is an equilibrium in which at least one candidate stands for the office of Prime Minister; the equilibrium is Pareto efficient; if there exists an individual amongst the polity whose ideal point is a Condorcet winner then that citizen is elected unopposed.

8.2. **Cabinet Deliberations.** One objection to our analysis, with its focus on allocation and assignment, is that a Prime Minister might not need such powerful instruments at her disposal in order to implement her agenda. Instead she could use cabinet deliberations as a mechanism for learning the true state of the world before making policy decisions. One way she could do this is to have ministers report everything they know rather than limiting the type of proposals that they can make. Then any discrepancy between their reports would immediately reveal that at least one minister did not report truthfully. In such a world, Ambrus and Takahashi (2008) show that there is a fully revealing equilibrium, so long as off the equilibrium path actions can be appropriately punished. Applied to our world the intuition is straightforward: when the Prime Minister can commit to implementing

a policy commonly disliked by her ministers in the event their statements do not match, then ministers' willingness to conceal information is avoided. This logic yields an outcome - full implementation of the Prime Minister's agenda- that is observationally equivalent to ours. However that model, unlike ours, can not account for changes to ministerial assignments: in particular it provides no insight as to why the assignment of departments has changed.[18]

8.3. **Collusion Between Ministers.** A final theoretical concern is collusion-proofness. Asking ministers to report on the whole dimensionality introduces scope for collusion between ministers. They may jointly agree to report in a way that yields a final outcome closer to their bliss point. It is straightforward to show that the equilibrium constructed here is collusion-proof in the sense that, although Pareto improvements from the ministers' perspective are always possible, they do not form part of an equilibrium in the absence of a commitment mechanism. It remains an open question whether ministers are able to forge an agreement (an enforceable side-contract *a la* Laffont and Martimort (1997)) that avoids unilateral deviations.

8.4. **Expertise.** A Prime Minister may consider motives other than the desire to counter ministers biases when organizing her government. In particular, she may wish to account for a minister's expertise when allocating his portfolio. Until now we have considered only a world where all ministers are experts (they are perfectly informed) on the issues over which they have jurisdiction. Here we consider a world where this is no longer so for all ministers: some ministers are better informed on some issues than are others. The question we ask is whether the Prime Minister can organize her cabinet so that (in equilibrium) she obtains truthful reports from her ministers.

When potential cabinet members have a known bias on a policy where their expertise lies, the Prime Minister needs to trade-off receiving a biased report from an an expert, or a noisy report (on an assignment orthogonal to the sender's bias) from a relatively less well informed

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[18]A further argument in favor of our equilibrium is more theoretical. Suppose that each minister observes θ_i with some noise. As all reports contain error the Prime Minister is then unable to draw comparisons between them (and so equilibria of the sort discussed above will fall away). However, as long as such noise is of a particular type, equilibria of the sort we have constructed in Proposition 2 remain. Battaglini (2004) provides a proof for this claim; Levy and Razin (2007) show the limitations of the fully revealing equilibrium when the signals received on each dimension are not independent.

minister.[19] In this circumstance the Prime Minister will generally be unable to aggregate information efficiently. To illustrate this point we develop a numerical example where the Prime Minister delegates the implementation of policy to two ministers on A_i.[20]

We consider a situation with a two dimensional policy space where two ministers have biases $(m_1, 0)$ and $(0, m_2)$. The true state of the world is unknown but its prior distribution is common knowledge, $\theta_i \sim N(0, \sigma^2)$ for $i = 1, 2$ and θ_1 and θ_2 are independent. Minister 1 perfectly observes the true state of the world in the first dimension but privately observes a noisy signal on the second dimension: $s_1 \sim N(\theta_2, \tau^2)$. By contrast, minister 2 can perfectly observe the second dimension but privately observes a noisy signal in the first one: $s_2 \sim N(\theta_1, \tau^2)$ (both signals are independent). Upon receiving the relevant information, ministers implement a policy constrained by their ministerial brief: $(A_1, 1)$ for minister 1 and $(1, A_2)$ for minister 2. In other words, minister i needs to decide a real value r_i and the overall implemented policy is equal to $r_1 \cdot (A_1, 1) + r_2 \cdot (1, A_2)$.

In the appendix we provide step by step calculations providing equilibrium declarations of ministers given any pair of assignments. We then look at the optimal assignment by the Prime Minister when taking into account the ministers' best responses. In order to illustrate our point we give particular values to the biases and the variance of the signals (we consider the case where biases and variances of posterior beliefs on the true state of the world are all equal to one, $m_1 = m_2 = 1$ and $\text{var}(\theta_i \mid s_j) = 1$ for $i \neq j$). We find that the (unique) optimal assignment is not orthogonal to the ministers' biases. This result shows that the Prime Minister will prefer extracting better information by utilizing a minister's expertise even when this implies that the assignments are not orthogonal to the ministers' biases. This of course implies that ministers will no longer report truthfully (the ministers' indifference curves are no longer tangent to their jurisdictions at the origin -the preferred policy of the Prime Minister) and so the final policy implemented will not be the Prime Minister's ideal

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[19]When bias and expertise are negatively (and perfectly) correlated, the previous analysis applies: by assigning an orthogonal assignment to a minister's bias, the Prime Minister not only avoids the bias in her report but assignments coincide with the dimension where ministers are experts.

[20]Adding expertise to our primitive model with strategic information transmission in a multidimensional model (instead of the Prime Minister delegating the implementation of policies) is not trivial. The problem lies in characterizing equilibrium reports when jurisdictions are not orthogonal: using a similar rational to the one in Crawford and Sobel (1982), we know that the message space will be partitioned; characterizing those partitions in a multidimensional space remains an open question in the theoretical literature.

one. Analyzing the way in which the Prime Minister modifies assignments according to the relative expertise of her ministers and their biases constitutes ongoing work.

9. Empirical Consequences and Implications of our Model

Before concluding we provide a brief overview of our results and discuss their empirical relevance. We have studied an abstract model of government formation that highlights the importance of jurisdictional assignments as part of a Prime Minister's organizational strategy. An immediate empirical payoff of our model is that we can use the fully revealing equilibrium described in Proposition 2 to derive a number of comparative static results that show how jurisdictions change according to differences in the composition of cabinet (according to members ideal points). It is straightforward to show that, when the Prime Minister is constrained to changing assignments, then any exogenous shift in a minister's bias will lead to a change in his (orthogonal) assignment. It is difficult to construct an empirical test of these claims. In particular, although some steps have been taken to put together data that details how jurisdictions have altered in the British case, we as yet have no independent measures of ministerial ideal points. Since ministers are bound by the convention of collective responsibility, and in any event partisan votes are enforced by government whips, we cannot obtain measures of variance in ministers ideal points that would allow us to test our ideas.[21] We note of course that any model (or claim or conjecture) about how assignments varied with the ideal points of ministers, would face the same common problem: we can not always obtain exogenous variation on the variables of interest; and, as famously pointed out by Krehbiel (1993), this is in particular the case where the variable of interest concerns the preferences of political actors.

Of course, it would be desirable to directly test some of the implications of our model. It is possible to generate (uncontaminated) data from a suitable experimental setting. Here we discuss ongoing work that provides the first test of our theory. In that work we analyze whether a subject acting as Prime Minister can offset the bias of a subject acting as minister

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[21]Some recent studies have used MPs positions on nonwhipped votes to test party strength and to provide an indication of the average dispersion of ideal points amongst groups of MPs (Dewan and Spirling, 2010). Such data is not helpful, however, since we require a measure of the distance between the Prime Minister and her Minister on those policies over which she has jurisdiction; the number of unwhipped votes in that category are likely to be small and the sample unrepresentative.

by allowing the latter to choose (implement a policy) on a particular set of outcomes. We simplify our model and focus on the relationship between the Prime Minister and a single minister (by assuming a nonstrategic second minister). We allow the subject acting as Prime Minister to choose among a menu of ministerial briefs (each corresponding to a different trade-off). Knowing the true state of the world, the subject acting as a minister then chooses a final allocation from the menu assigned to him. This simple experimental design allows us to analyze whether assignments are made optimally and the "minister" responds truthfully. More elaborate experiments should introduce the strategic interaction that arises between two ministers and on aspects such as collusion or expertise.

Given the problems of measurement and identification that arise when exploring the phenomenon at hand, our formal model does proves useful and non-obvious insights into the consequences of the organizational powers of the Prime Minister which we elaborate upon here. The first key insight - one that we believe has been overlooked by analysts who have focussed entirely on the composition of cabinets and turnover of ministerial positions - is that the allocation of tasks is a powerful tool for the chief of the executive to have at her disposal. Contrary to conventional wisdom we show that the Prime Minister's ability to implement her desired policies is not necessarily diminished when she is unable to fully determine who sits in her cabinet; when she is able to assign ministerial jurisdictions and responsibilities then she can still implement her agenda. Would the Prime Minister nevertheless prefer a cabinet in which the views of her colleagues are aligned with her own? Our model shows that, whilst this view may be correct in a unidimensional world, where the ministers acted as either veto players or agents of the Prime Minister, such an inference is incorrect in a multidimensional setting in which ministers are the providers of information.

A further empirical insight stems from our analysis of relations between the Prime Minister and her ministers. One of our key results is that when the Prime Minister designs her government optimally, then she is strictly indifferent between implementing policy herself or having her minister do so. The conventional view of British politics is that ministers are not autonomous: Prime Ministers regularly intervene in the running of departments and there is no constitutional enshrined protection for ministerial autonomy as provided under German Basic Law, for example. Our analysis allows us to explore a relevant counterfactual:

what would happen if ministers were free to implement their desired policy, as the principal of ministerial autonomy implies, though are constrained to act only on those policies over which they are granted jurisdiction and according to the responsibilities defined by the Prime Minister? We show then that the same policy outcomes prevail irrespective of whether the Prime Minister or her minister implements policy.

10. Conclusion

Although the analysis of strategic information transmission has been central to the formal literature on Congressional committees and their relationship to the U.S. House of Representatives, these models have not, until now, been applied in a parliamentary setting. We analyze how ministerial appointments, responsibilities, and the structure of government departments, can be adapted strategically by a Prime Minister who aggregates information provided by (biased) ministers who are experts in their brief. In assigning a ministers' responsibilities optimally, the Prime Minister is able to extract all of the relevant information so that she can implement her preferred policies. In fact, once she has organized her government optimally, the Prime Minister is indifferent between implementing policy herself or delegating this task to her ministers. Perhaps surprisingly this result does not depend on the extent of ministers' biases relative to the Prime Minister. We also provide an example where ministers have exogenously given expertise that is correlated with their biases and show that then the Prime Minister is unable to implement her desired policies.

We end by suggesting other avenues for future research on this topic. Although our model captures some of the critical features of Westminster democracy, we use simplifying assumptions. In our basic model civil servants are both perfectly informed and perfect agents of ministers, and so we abstract from the agency problem between a Prime Minister and her ministers and the incentives required for a bureaucrat to obtain information (Bawn, 1995). However our model does extend to the basic agency problem between a minister and her civil servants. Indeed the permanency of officials in most parliamentary systems means that the assignment of tasks is likely to be used by a ministerial principle to extract information from her agents. Whilst our model provides useful insights to Westminster democracy, it

abstracts from issues of party competition and coalition formation and the basic framework developed here could be extended to include variations on this institutional architecture.

11. Appendix

Proof of Proposition 1. Assignments are the coordinate axis ($A_1 = (1,0)$ and $A_2 = (0,1)$) and the Prime Minister chooses two ministers with preferences $m_1 = (0, m_1^2)$ and $m_2 = (m_2^1, 0)$. We can show that there exists a truthful fully revealing equilibrium where a minister with preferences m_1 (m_2) has jurisdiction on the first (second) policy dimension. This equilibrium has the following features: each minister reports the true state of the world given their ministerial brief $s_i(\theta) = \theta_i$, $i = 1, 2$; the Prime Minister follows the ministers' advice in implementing policy so that $y(s_1(\theta), s_2(\theta)) = -(A_1 \cdot s_1(\theta) + A_2 \cdot s_2(\theta))$ and, finally, the Prime Minister's beliefs accumulate all mass on the jointly reported state of the world, $\mu(s_1(\theta), s_2(\theta))(\theta) = 1$.

We show that these strategies and beliefs constitute an equilibrium. The interim utility of minister i is $u_i(y(s_1(\theta), s_2(\theta))) = -(-s_1(\theta) + \theta_1 - m_i^1)^2 - (-s_2(\theta) + \theta_2 - m_i^2)^2$. Ministers want to send the report that maximizes their interim utility. Taking as given the behavior of minister 2, the optimal behavior of minister 1 is to report $s_1(\theta) = \theta_1$ (i.e. the prescribed behavior in a truthful and fully revealing equilibrium). Minister 2's optimal behavior can analogously be proved. Beliefs are consistent with equilibrium behavior as they accumulate all mass on the true state of the world. Finally, the Prime Minister's behavior is optimal given these beliefs: the Prime Minister implements a policy, $y = -\theta$, that yields his preferred policy outcome $x = (0,0)$, and so has no incentive to deviate. $\square$

Proof of Lemma 1. Given minister j's report, minister i's optimal report should lead to pm^* when the equilibrium is fully revealing. As can be observed in figure 1, this can only happen when minister i's indifference curve over pm^*, is tangent to the set of policies from which he can choose, i.e. his own assignment. Indifference curves of quadratic utilities are circles, and any tangent line to a circle is orthogonal to the radius of the circle on the tangency point. This implies that the direction of the assignment is orthogonal to the bias between the minister and the Prime Minister, $(m_i - pm^*) \cdot A_i = 0$. It follows that an assignment

is orthogonal to a minister's bias and thus is invariant with respect to changes in another minister's bias; and, multiplying a minister's bias by a constant does not change the optimal assignment as the orthogonal direction remains unchanged. □

Proof of Proposition 2. We show that when the Prime Minister chooses orthogonal assignment to the ministers' biases, agents report the true state of the world in the new coordinate system induced by their assignments. The Prime Minister then holds beliefs that allow the implementation of her preferred policy. Consider two arbitrary bliss points $m_1, m_2 \in \mathbb{R}^2$. Lemma 1 implies that, without loss of generality, we can restrict attention to bliss points of the form: $m_1 = (m_1^1, 1)$ and $m_2 = (m_2^1, 1)$. Two orthogonal assignments read as $A_1 = (-1, m_1^1)$ and $A_2 = (-1, m_2^1)$. It is useful to first express the true state of the world in the new set of coordinates given by A_1 and A_2.[22]

A necessary and sufficient condition for this set of equations to have a solution is that the biases need to be linearly independent. By solving these two equations we can describe the truth-telling equilibrium strategies and beliefs. In this equilibrium each minister reports the coordinate of the true state of the world in their assignment:

$$s_1(\theta) = \frac{\theta_2 + \theta_1 m_2^1}{m_1^1 - m_2^1} \quad \text{and} \quad s_2(\theta) = \frac{-\theta_2 - \theta_1 m_1^1}{m_1^1 - m_2^1}.$$

In a truthful and fully revealing equilibrium, the Prime Minister follows her ministers' advice and so implements policy following the expression for $y(s_1(\theta), s_2(\theta))$ in the proof of Proposition 1, and, as in that earlier result, her beliefs put all weight on the jointly reported state of the world. In order to prove the optimality of the prescribed strategies we compute the interim utility of minister 1 given minister 2's behavior:

$$u_1(y(s_1, s_2(\theta))) = -(\underbrace{s_1 + s_2(\theta) + \theta_1 - m_1^1}_{x_1})^2 - (\underbrace{-s_1 m_1^1 - s_2(\theta) \cdot m_2^1 + \theta_2}_{x_2} - 1)^2$$

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[22]We need to find x and y to solve $x \cdot A_1 + y \cdot A_2 = \theta$. That is, we need to solve the following system of equations $-x - y = \theta_1$ and $x m_1^1 + y m_2^1 = \theta_2$.

which we use to find the optimal response of the first minister by computing the first order condition of the optimization problem is. Its solution yields

$$s_1 \left(1 + \left(m_1^1\right)^2\right) = \frac{1}{\left(m_1^1 - m_2^1\right)} \left(1 + \left(m_1^1\right)^2\right) \theta_2 + \left(\left(m_1^1\right)^2 + 1\right) \theta_1 m_2^1$$

which is precisely the behavior prescribed above. Minister 2's optimal behavior is analogously proved. Beliefs are consistent with equilibrium behavior as they accumulate all mass on the true state of the world. Finally, the Prime Minister's behavior is optimal given beliefs: the Prime Minister implements the policy $y = -\theta$ that yields her preferred policy outcome $x = (0, 0)$, thus has no incentive to deviate. $\square$

Note (1) on Proof to Proposition 2: While assignments do not depend on any other minister's bias, equilibrium declarations do depend on the other minister's assignments. Assignments will generally not be given by the policy axis thus the new coordinates of the state of the world (i.e. the ministers declarations) depend on both ministerial briefs. In equilibrium then, a minister takes into account the (equilibrium) messages reported by other ministers. Aggregation of truthful messages yields the expected final policy.

Note (2) on Proof to Proposition 2: By looking at the objective function of minister 1 (analogously minister 2), we can see that in a fully revealing equilibrium his best response depends linearly on the action of the other minister which implies that there can be (at most) one equilibrium. In other words, when the Prime Minister optimally assigns ministerial tasks, the minister cannot collude on an equilibrium different than the fully revealing one.

Proof of Proposition 3. The rationale of lemma 1 still applies when n is larger than 2: in a fully revealing equilibrium assignments need to be orthogonal to the biases. However, whereas when $n = 2$ the orthogonal direction is uniquely determined, when $n > 2$ this is no longer the case as there are now $n - 1$ orthogonal directions. This allows the Prime Minister to elicit full information when (at least) two ministers have linearly independent biases. To see this, consider the most extreme case where $n - 1$ ministers have exactly the same bias and the nth minister has a bias that is linearly independent to the rest. We can choose $n - 1$ assignments in the orthogonal hyper-plane of the $n - 1$ ministers and an orthogonal direction for the nth minister that is linearly independent to the previous hyper-plane. This

set of assignments is orthogonal to each minister's biases and spans the whole n-dimensional policy space. Once we have determined this set of orthogonal jurisdictions we can follow the proof of proposition 2 and rewrite any point in the Euclidean space using the new set of coordinates. The coordinate in the direction of each assignment is precisely the declaration of the minister that has been given such ministerial brief. After receiving all ministers' declarations, the posterior beliefs of the Prime Minister are concentrated on the true state of the world and so she implements a policy that yields her preferred policy outcome. $\square$

Proof of Proposition 4. Frenk, van Houweninge, and Rinnoy Kan (1987) show that the asymptotic behavior of the expected average cost in a random assignment problem is determined by that of the smallest order statistic when the costs are i.i.d. and their distribution function F satisfies the following two *near zero conditions*: (1) $\lim_{n\to\infty} F^{-1}\left(\frac{1}{n}\right) = 0$; and (2) F is of *positive decrease* at 0 (i.e. $\exists a \in (0,1) : \lim_{x\downarrow 0} \frac{F(x)}{F(ax)} > 1$. This implies that when both these conditions are satisfied the average cost of the random assignment problem tends to zero. We can apply this rationale to the average adjustment the Prime Minister needs to incur in the optimal assignment of tasks.

When there is a strictly positive probability that the required adjustment is arbitrarily small, $F(x) > 0$ for all $x > 0$ and the distribution is atomless in 0 we have that $\lim_{n\to\infty} F^{-1}\left(\frac{1}{n}\right) = 0$. Having a strictly positive probability of arbitrarily small realizations of the random variable implies that there exists $r > 0$ (possibly very small) where $F'(x) = f(x)$ is bounded and $f(x) > 0$ for all $x \in (0, r)$. Thus, $\lim_{x\to 0} f(x) > 0$ or the limit of the kth derivative when x tends to zero should be different from 0 for a finite k. In either case, by applying *Hôpital's rule* (possibly repeatedly) we see that condition (2) is satisfied. Given that both conditions are satisfied we know that the average adjustment goes to zero as we increase the number of policy dimensions.

When there is a positive mass on 0, we can apply the results above simply by considering the adjustments to be $\tilde{x} = (x + \varepsilon)$ where x is distributed according to F and ε is uniformly distributed in $[0, r]$, where $r > 0$ is arbitrarily small. The previous argument now applies and we know that the the average adjustment of the optimal assignment when adjustments are augmented by ε can only be higher. $\square$

11.1. **Algorithm for Numerical Simulations:** In our simulation we assume there are n citizens whose bias with respect to the Prime Minister on each policy dimension is drawn from a normal distribution with zero mean and unit variance. Status quo assignments are the coordinate axis (i.e. $A_i = (0, ..., 0, 1, 0, ...0)$). The minimum adjustment is the angular move that is required to make assignments A_i orthogonal to the bias of citizen j as $\alpha(m_j, A_i)$. We compute α using the definition of the scalar product between two vectors, i.e. $\alpha(m_j, A_i) = 90° - \arccos(m_j^i / ||m_j||_2)$.

Once we compute the minimum adjustment that is needed to make each germane assignment orthogonal to a citizen bias, we can apply the Hungarian algorithm (see Kuhn (1955)) to find the optimal assignment, i.e. the one that minimizes the average adjustment. We iterate our simulations 500 times and report the average results in figure 3 (the gray line in this figure corresponds to a situation with n policy dimensions and $2n$ citizens).

11.2. **A Model with Expertise.** There are two ministers with biases $(m_1, 0)$ and $(0, m_2)$. The prior distribution of the true state of the world is $\theta_i \sim N(0, \sigma^2)$ for $i = 1, 2$ and θ_1 and θ_2 are independent. Minister 1 perfectly observes the true state of the world in the first policy dimension but privately observes a noisy signal in the second one: $s_1 \sim N(\theta_2, \tau^2)$. Minister 2 perfectly observes the second policy dimension but privately observes a noisy signal in the first one: $s_2 \sim N(\theta_1, \tau^2)$ (both signals are independent). Upon receiving the relevant information, ministers need to implement a policy following their ministerial brief: $(A_1, 1)$ for minister 1 and $(1, A_2)$ for minister 2. In other words, minister i needs to decide a real value r_i and the overall implemented policy is equal to $r_1 \cdot (A_1, 1) + r_2 \cdot (1, A_2)$.[23]

The Prime Minister assigns tasks in order to maximize his utility while taking into account that ministers choose a policy that most benefits themselves. His maximization program

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[23]Our setting is equivalent to the case where the Prime Minister can commit to using ministers' reports in a predetermined way.

reads as follows:

$$\max_{A_1,A_2} -E\left((r_1 \cdot A_1 + r_2 + \theta_1)^2 + (r_1 + r_2 \cdot A_2 + \theta_2)^2\right)$$

$$\text{subject to} \begin{cases} r_1 \in \arg\max_{r_1} -E\left((r_1 \cdot A_1 + r_2 + \theta_1 - m_1)^2 + (r_1 + r_2 \cdot A_2 + \theta_2)^2\right) \\ r_2 \in \arg\max_{r_2} -E\left((r_1 \cdot A_1 + r_2 + \theta_1)^2 + (r_1 + r_2 \cdot A_2 + \theta_2 - m_2)^2\right) \end{cases}$$

After observing signal s_i, minister i's expected value on the true state of the world in the second policy dimension θ_j is $E(\theta_j \mid s_i) = \frac{s_i \sigma^2}{\tau^2 + \sigma^2}$ (DeGroot, 1970). By taking first order conditions we find the optimal decision of minister i:

$$r_i = \frac{1}{1 + A_i^2}\left((m_i - \theta_i)A_i - E(\theta_j \mid s_i) - E(r_j)(A_1 + A_2)\right).$$

The equilibrium of the simultaneous game played by the ministers requires the computation of minister i's expected value of minister j's implemented policy: $E(r_i \mid s_j, \theta_j)$ for $i \neq j$. The equilibrium given the Prime Minister's assignments is:

$$\begin{cases} r_1 = \frac{\alpha - \theta_1 A_2}{A_1 A_2 - 1} + \frac{A_1 m_1\left(1 + A_2^2\right) - A_2 m_2 (A_1 + A_2)}{(A_1 A_2 - 1)^2}, \alpha = \frac{s_1 \sigma^2}{\tau^2 + \sigma^2} \\ r_2 = \frac{\beta - \theta_2 A_1}{A_1 A_2 - 1} + \frac{A_2 m_2\left(1 + A_1^2\right) - A_1 m_1 (A_1 + A_2)}{(A_1 A_2 - 1)^2}, \beta = \frac{s_2 \sigma^2}{\tau^2 + \sigma^2} \end{cases}$$

The first term in the policy implemented by each minister captures the truthful (but noisy) aspect of his action. The second term captures the drift included in their implemented policy; this drift exists as long as jurisdictions are not orthogonal to biases. The Prime Minister objection function can now be rewritten as:

$$-E\left((r_1 \cdot A_1 + r_2 + \theta_1)^2 + (r_1 + r_2 \cdot A_2 + \theta_2)^2\right) =$$

$$= \frac{-1}{(A_1 A_2 - 1)^2}\left((1 + A_1^2)\text{Var}(X) + (1 + A_2^2)\text{Var}(Y)+\right.$$

$$\left. + A_1^2 A_2^2 (m_1^2 + m_2^2) + A_1^2 m_1^2 + A_2^2 m_2^2 - 2A_1 A_2 m_1 m_2 (A_1 + A_2)\right)$$

where $X = \alpha - \theta_2$ and $Y = \beta - \theta_1$.[24] This expression captures the trade-offs the Prime Minister faces in the presence of expertise. When ministerial briefs are orthogonal to the biases ($A_1 = A_2 = 0$) the Prime Minister utility is equal to $(-\text{Var}(X) - \text{Var}(Y))$; indeed, there is no bias in the report but the implementation of policy is noisy. Instead, when the

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[24] $E(X) = E(Y) = 0, \text{Var}(X) = \text{Var}(Y) = \frac{\tau^2 \sigma^2}{\tau^2 + \sigma^2}$, and $\text{Cov}(X,Y) = 0$.

Prime Minister's assignments coincide with the dimensions where each minister has expertise $(A_1, A_2 \to \infty)$ her utility is $(-m_1^2 - m_2^2)$; she avoids the noise in the perception of the true state of the world but implementation of policy is biased towards the ministers preferred policies. When $\mathrm{Var}(X) = \mathrm{Var}(Y) = m_1 = m_2 = 1$, the objective function of the Prime Minister is $\left(-2 - 2\frac{A_1+A_2}{(A_1 A_2 - 1)^2}\left(A_1 + A_2 - A_1 A_2\right)\right)$. By analyzing this function we find that it is minimized at $A_1 = A_2 = x$ where x is the unique real root of $(x^3 - 4x^2 + 3x - 4)$ –the value is approximately 3.4675. In this case, the objective function takes a value -1.4192. Instead, when the Prime Minister assigns orthogonal jurisdictions that coincide with the policy axis the Prime Minister's utility is equal to -2 (so, moving assignments away from the policy axis implies an improvement of about 30%).

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