JAE Journal of Agricultural Economics

Journal of Agricultural Economics, Vol. 63, No. 1, 2012, 25–55
doi: 10.1111/j.1477-9552.2011.00319.x

On the Role of Risk Versus Economies of Scope in Farm Diversification With an Application to Ethiopian Farms

Jean-Paul Chavas and Salvatore Di Falco[1]

(Original submitted March 2010, Revision received June 2011, accepted July 2011.)

Abstract

This article investigates the economics of farm diversification. The analysis assesses economies of diversification using a certainty equivalent measure. It identifies two components: one associated with expected income, and one associated with risk exposure. This integrates two lines of research explored in previous literature: economies of scope and risk management. We examine the roles played by complementarity, scale and concavity effects in economies of diversification. The approach is applied to diversification decisions made on Ethiopian farms, with a focus on production uncertainty. The econometric analysis finds large complementarity benefits, providing incentives to diversify. But this is tempered by (non)-concavity effects that provide incentives to specialise. The analysis also documents how risk affects diversification, including both variance and skewness effects. It provides new insights on economic tradeoffs between farm diversification and specialisation.

Keywords: *Complementarity; concavity; diversification; risk; scale; scope.*

JEL classifications: *D21, D8, G11.*

1. Introduction

Much research has been conducted on the economics of farm diversification. Diversification has often been studied in the context of risk management. Under uncertainty, risk-averse decision makers have incentives to diversify (e.g., Heady, 1952; Markowitz, 1959; Tobin, 1958; Samuelson, 1967; Johnson, 1967). This is illustrated

[1] Jean-Paul Chavas is Professor, Department of Agricultural and Applied Economics, Taylor Hall, University of Wisconsin, Madison, USA. Salvatore Di Falco is Professor, London School of Economics, Houghton Street, London, UK. E-mail: s.difalco@lse.ac.uk for correspondence. We would like to thank Alemu Mekonnen, Gunnar Köhlin, Menale Kassie, Mintewab Bezabih, Liyou Borga and the Ethiopian Development Research Institute for their support. We also would like to thank Brad Barham and two anonymous reviewers for useful comments on earlier drafts of the article.

by the rule of thumb: 'Don't put all your eggs in one basket'. Risk management has provided useful insights into financial and investment decisions under uncertainty. In particular, the presence of significant uncertainty in agriculture helps explain why most farms are multi-output diversified enterprises (Lin *et al*., 1974).

Besides risk management, there is another possible motivation for diversification: the presence of economies of scope. Scope economies arise when diversification implies a cost reduction associated with multi-output production processes (Baumol, 1977; Baumol *et al*., 1982; Willig, 1979).[2] This has stimulated research examining the cost properties of multi-output enterprises, with applications to the organisation and performance of many industries. There is empirical evidence that economies of scope are prevalent in farming (e.g., Chavas and Aliber, 1993; Fernandez-Cornejo *et al*., 1992; Paul and Nehring, 2005).[3] But this raises the question: What is the relative role of risk vs. economies of scope in diversification decisions? To our knowledge, this issue has not been addressed in previous literature. It indicates a need to integrate risk and scope rationales in the analysis of diversification choices. This provides the main motivation for this article.

This article is also motivated by some difficulties economists have in explaining observed diversification choices. Indeed, discrepancies between theory and observed behaviour have raised questions about our current understanding of diversification issues. One is the 'under-diversification puzzle' coming from observations that investors often hold poorly diversified portfolios (e.g., Blume and Friend, 1975; Calvet *et al*., 2007; Campbell, 2006; Goetzmann and Kumar, 2008). Explaining why so many agents are willing to hold under-diversified portfolios poses a challenge to portfolio theory that stresses the benefits of diversification. Another puzzle is the following: while economies of scope can help motivate mergers, empirical research has not found evidence that scope expansion increases the value of firms (Berger and Ofek, 1995). This seems to occur also in agriculture where asset values tend to be lower on diversified farms (Katchova, 2005). This raises the question: are there benefits of diversification? While empirical research has documented the factors affecting farm diversification (e.g., Pope and Prescott, 1980; Misra *et al*., 2004), our current understanding of the economics of diversification remains incomplete.

A final motivation for this study relates to the view that farmers are in the business of managing ecosystems to produce food. Ecologists have stressed the importance of complementarity in the functioning of ecosystems (e.g., Tillman and Kareiva, 1997). Defining complementarity as situations where an activity has a positive effect of the marginal productivity of others, complementarities contribute to economies of scope (Baumol *et al*., 1982, p. 75). This indicates that crop diversity can have two effects: a risk reducing effect, and a scope/productivity effect. There is empirical evidence that each effect can generate crop diversification benefits (e.g.,

[2] Note that Baumol *et al*. (1982) characterised economies of scope involving complete specialisation schemes. Below, we interpret economies of scope in a broader context that allows for partial specialisation (as discussed by Evans and Heckman, 1984; Berger *et al*. (1987) and Ferrier *et al*., 1993). Additional studies of the role of complementarities between production processes include Milgrom and Roberts (1990), and Topkis (1998).

[3] Studies of economies of scope have also included higher education (Cohn *et al*., 1989; De Groot *et al*., 1991), telecommunication (Evans and Heckman, 1984), banking (Berger *et al*., 1987; Ferrier *et al*., 1993), R&D (Klette, 1996), and health care (Prior, 1996).

Smale *et al.*, 1998; Di Falco and Chavas, 2009). This article goes beyond previous literature by developing a unified framework where both rationales for crop diversification are integrated in an applied microeconomics setting.

The integration of risk and scope creates several challenges. First, it requires the development of a conceptual framework where scope and risk issues can be integrated in a unified fashion. This is done relying on a 'certainty equivalent' (defined as expected income minus a risk premium), which provides the analytical framework for our analysis. Expected income captures scope effects; and the risk premium captures risk effects. Our analysis further decomposes each of these components into complementarity, scale and concavity effects. We show that, when focusing on scope economies, our approach reduces to the analysis presented by Baumol (1977), Baumol *et al.* (1982) and Willig (1979). However, the identification of scale effects and concavity effects related to risk are apparently new results in the diversification literature. This generates useful insights about the factors influencing diversification decisions.

A second challenge is that the analysis needs to be empirically tractable. Analysing scope effects requires the investigation of multi-output production processes. This is done by estimating mean productivity in a multi-output context. And analysing risk effects requires an empirical assessment of the distribution of risk. While originally developed in a mean-variance context (Markowitz, 1959; Tobin, 1958; Johnson, 1967), the risk-based analysis of diversification has been extended to capture the role of skewness and downside risk exposure (e.g., Mitton and Vorkink, 2007). This motivates our reliance on a moment-based approach first proposed by Antle (1983). In the context of production uncertainty, our analysis includes mean, variance and skewness. The inclusion of skewness extends the standard mean-variance approach (e.g., Johnson, 1967; Just and Pope, 1979).

The usefulness of the approach is illustrated in an analysis of the benefits of crop diversity on Ethiopian farms. Ethiopian agriculture is a valuable case study for two reasons. First, the historical record of Ethiopian famines underscores the importance of exposure to production uncertainty. Second, Ethiopian farm households are typically producing for their own consumption. In the absence of (both formal and informal) insurance mechanisms, production uncertainty has large effects on household welfare. It means that production uncertainty is managed privately, and on-farm diversification is an important part of household risk management. Using panel data, the econometric estimation of a multi-output stochastic production function gives the basis for evaluating both scope and risk effects on Ethiopian farms. The estimates show strong evidence of complementarities among crops, which contribute to scope economies and give incentives to diversify. But this is tempered by (non)-concavity effects that provide an incentive to specialise. Risk management issues are evaluated using both variance and skewness (capturing the role of downside risk exposure). It finds that skewness effects (reflecting downside risk) are more important than variance effects in diversification choices. The analysis provides new insights on economic tradeoffs between farm diversification and specialisation.

The article is organised as follows. The conceptual model is presented in section 2. Using a 'certainty equivalent' approach, diversification economies are defined in section 3. In section 4, we present a decomposition of diversification economies. It distinguishes between the role of scope economies (*via* expected income) and a risk premium (capturing the role of risk management). Section 5 explores the steps required to make the analysis empirically tractable, including both productivity assessment and risk assessment. After discussing the Ethiopian data in section 6, the econometric analysis is presented in section 7. Implications for the benefits of crop diversification (and their sources) are explored in section 8. Finally, concluding remarks are presented in section 9.

2. The Model

Consider an owner-managed firm (e.g., a farm) facing a multi-output production process involving m outputs $y = (y_1, \ldots, y_m) \in R_+^m$ and n inputs $x = (x_1, \ldots, x_n) \in R_+^n$. We use the netput notation, where netputs are $z \equiv (-x, y)$, outputs being positive and inputs negative. The underlying technology is represented by the set F(e), where e is a vector of random variables (e.g., weather effects in agricultural production) representing production uncertainty, and $z \in F(e)$ means that netputs z are feasible given e.[4]

The firm faces output prices $p \in R_{++}^m$, generating firm revenue $p \cdot y$. Firm income is $[I(x) + p \cdot y]$, where I(x) denotes other net income which includes other sources of income as well as input cost (treated as negative income). The firm manager has a subjective probability distribution of all random variables characterising his/her uncertain environment. Under the expected utility hypothesis, the decision maker has risk preferences represented by a von Neumann–Morgenstern utility function $U(I(x) + p \cdot y)$, where $U(\cdot)$ is strictly increasing. Then, the firm manager makes decisions in a way consistent with the maximisation

$$\mathrm{Max}\{EU(I(x) + p \cdot y : (-x, y) \in F(e)\},$$

where E is the expectation operator based on the subjective probability distribution of all uncertain variables.

In our analysis, it will be useful to rely on a functional representation of the technology. Let $g \in R_+^m$ be a reference output bundle satisfying $g \neq 0$. Following Luenberger (1995), define the shortage function as:

$$\begin{aligned} S(-x, y, e) &= \mathrm{Inf}_\beta\{\beta : (-x, y - \beta g) \in F(e)\} \text{ if there is a } \beta \text{ satisfying} (-x, y - \beta g) \in F(e), \\ &= +\infty \text{ otherwise.} \end{aligned} \tag{1}$$

The shortage function measures the distance (measured in number of units of the reference bundle g) between point (−x, y) and the upper bound of the feasible set F(e). In general, given e, $S(-x, y, e) = 0$ means that point (−x, y) is on the frontier technology. Alternatively, given e, $S(-x, y, e) < 0$ implies that (−x, y) is technically inefficient (as it is below the frontier),[5] while $S(-x, y, e) > 0$ identifies (−x, y) as

[4] As the feasible set F(e) depends on e under production uncertainty, note that technical efficiency implies at least some netputs must be chosen *ex post*. To illustrate, consider the case where inputs x are chosen *ex ante*. The output possibility set under state e becomes $Y(x, e) = \{y: (-x, y) \in F(e)\}$. This means that being on the upper bound of Y(x, e) implies that y must vary with e, i.e., that at least some outputs must be chosen *ex post*. We present our analysis assuming implicitly that some netputs choices are 'state-dependent'.

[5] Note that S(−x, y, e) includes as special cases many measures of technical inefficiency that have appeared in the literature. For example, the directional distance function proposed by Chambers *et al*. (1996) is just the negative of S(−x, y, e). Relationships with Shephard's output distance function or Farrell's (1957) measure of technical efficiency are discussed in Chambers *et al*. (1996) and Färe and Grosskopf (2000).

being infeasible (as it is located above the frontier). Luenberger (1995, pp. 20–22) has provided a detailed analysis of the properties of $S(-x, y, e)$. First, from the definition in equation (1), $(-x, y) \in F(e)$ implies that $S(-x, y, e) \leq 0$ (since $\beta = 0$ is then feasible in equation (1)), meaning that $F(e) \quad \{(-x, y): S(-x, y, e) \leq 0\}$. Second, consider the case of a technology exhibiting free disposal in outputs y, where starting from any $(-x, y) \in F(e)$, then $(-x, y') \in F(e)$ holds for all $y' \leq y$. Note that, from equation (1), $S(-x, y, e) \leq 0$ implies that $(-x, y - S(-x, y, e)\, g) \in F(e)$. It follows that, under free disposal in y, $S(-x, y, e) \leq 0$ implies that $(-x, y) \in F(e)$, meaning that $F(e) \quad \{(-x, y): S(-x, y, e) \leq 0\}$. Combining these two properties, we obtain the following result: under free disposal in outputs y, $F(e) = \{(-x, y): S(-x, y, e) \leq 0\}$, implying that $S(-x, y, e)$ provides a complete representation of the underlying technology. Importantly, besides being convenient, this result is general: it allows for an arbitrary multi-output technology and it holds under production uncertainty. We will make extensive use of it below. The linkages between the shortage function and the more traditional production function are further discussed in section 5.

Note that $p \in R^m_{++}$, $g \in R^m_+$ and $g \neq 0$ imply that $(p \cdot g) > 0$. Using the shortage function $S(-x, y, e)$, the following result will prove useful (see the proof in the Appendix).

Lemma 1. Given $(p \cdot g) > 0$,

$$\mathrm{Max}\{EU[I(x) + p \cdot y] : (-x, y) \in F(e)\} = \mathrm{Max}\{EU[I(x) + p \cdot y - S(-x, y, e)(p \cdot g)]\}. \tag{2}$$

Lemma 1 provides two equivalent formulations for expected utility maximisation. It applies under general conditions, including a multi-output firm facing both price and production uncertainty. Note that feasibility constraint $\{(-x, y) \in F(e)\}$ is imposed on the left-hand side of equation (2), but not on its right-hand side. Thus, equation (2) shows that subtracting the term $[S(-x, y, e)\,(p \cdot g)]$ from income is equivalent to imposing the feasibility constraint. Given $(p \cdot g) > 0$, the shortage function $S(-x, y, e)$ in equation (2) provides a formal linkage between the productivity/efficiency of point $(-x, y, e)$ and its welfare evaluation under uncertainty. We will make extensive use of Lemma 1 in our analysis.

Using the right-hand side of equation (2) and following Arrow (1965) and Pratt (1964), define the 'certainty equivalent' CE as follows

$$EU[I(x) + p \cdot y - S(-x, y, e)(p \cdot g)] = U(CE), \tag{3}$$

which implies that $CE = U^{-1}\, EU(\pi)$, with $\pi = I(x) + p \cdot y - S(-x, y, e)\,(p \cdot g)$. The certainty equivalent CE is the smallest sure amount of money the decision maker is willing to receive to give up the uncertain income $\pi = [I(x) + p \cdot y - S(-x, y, e)\,(p \cdot g)]$. Being evaluated *ex ante*, CE in equation (3) depends on $z \equiv (-x, y)$: CE(z). Through the shortage function in equation (3), CE(z) captures the effects of efficiency/productivity. Indeed, given x and e, note from definition equation (1) that $[y - S(-x, y, e)\, g]$ is located on the upper bound of the production technology. Thus, finding that $S(-x, y, e) < 0$ means that point $(-x, y, e)$ is technically inefficient, in which case subtracting $S(-x, y, e)\,(p \cdot g)$ in equation (3) corresponds to an efficiency-improving move to the frontier technology and an increase in the certainty equivalent CE(z). Alternatively, finding that $S(-x, y, e) > 0$ means that point $(-x, y, e)$ is infeasible, in which case subtracting

$S(-x, y, e)$ $(p \cdot g)$ in equation (3) corresponds to a move to feasibility and a decrease in the certainty equivalent CE(z). The certainty equivalent CE(z) also depends on the probability distribution of e and on risk preferences. The utility function $U(\cdot)$ being strictly increasing, it is clear from equations (2) and (3) that maximising expected utility is equivalent to maximising CE(z). As such, CE(z) provides a basis for evaluating the economic performance of the owner-managed firm under risk.

3. Diversification

We want to investigate the economics of farm diversification. Under what conditions would a farm benefit from being diversified? To analyse this issue, consider a scenario where the farm/firm reorganises its activities to become more specialised. Start with an original firm producing netputs $z \equiv (-x, y) \in F(e)$.[6] Then, split this firm into K specialised firms, where the *k*th firm produces netputs $z^k \equiv (-x^k, y^k)$, $k = 1, \ldots, K$. We make two assumptions. First, we assume that $z = \sum_{k=1}^{K} z^k$, so that aggregate netputs are being held constant. Second, we assume that $z^k \neq z/K$, so that each of the K firms exhibits some form of relative specialisation.[7]

Definition 1. Economies of diversification (diseconomies of diversification) exist if

$$D \equiv CE(z) - \sum_{k=1}^{K} CE(z^k) > 0 (< 0), \tag{4}$$

where $\sum_{k=1}^{K} z^k = z$.

Equation (4) measures the change in certainty equivalent due to a move toward greater specialisation, holding aggregate netputs constant ($\sum_{k=1}^{K} z^k = z$). It shows that economies of diversification exist ($D > 0$) when the certainty equivalent of producing netputs z is higher from an integrated firm compared to K more specialised firms. This identifies the presence of synergies or positive externalities across activities in the production process. Alternatively, diseconomies of diversification exist ($D < 0$) when the certainty equivalent of producing netputs z is lower from an integrated firm compared to K more specialised firms. This indicates the presence of negative externalities across activities in the production process.

Equation (4) provides a monetary measure of diversification benefits. Assuming that $CE(z) > 0$, a relative measure can be defined as

---

[6] As noted in footnote 3, under production uncertainty, technical efficiency means that at least some of the netputs z are chosen *ex post*.

[7] Note that, except for these two assumptions, our analysis applies to general specialisation schemes for the K farms: z^k, $k = 1, \ldots, K$. In particular, we allow some z^k to be either infeasible or technically inefficient. The case of infeasibility could occur when the associated specialisation scheme generates productivity losses. It would force the *k*th specialised farm to purchase additional resources to restore feasibility, thus lowering CE. Alternatively, the case of technical inefficiency would arise if specialisation yields productivity gains, thus increasing CE. A practical way of capturing these effects is discussed in section 5 below.

$$D' \equiv \left[\mathrm{CE(z)} - \sum_{k=1}^{K} \mathrm{CE}(z^k)\right] / \mathrm{CE(z)}, \tag{5}$$

where $\sum_{k=1}^{K} z^k = z$. Then, D′ in equation (5) provides a unit-free measure: it is the proportional increase in the certainty equivalent obtained by producing z in a single integrated firm vs. K more specialised firms. Again, $D' > 0(<0)$ identifies economies (diseconomies) of diversification.

Given $\sum_{k=1}^{K} = z$ and that $z^k \neq \sum_{k=1}^{K} z/K$, equations (4) and (5) allow for various forms of specialisation among the K firms. For example, the *k*th firm could be completely specialised in the *j*th output, with $y_j^k = y_j$ and $y_j^{k'} = 0$ for $k' \neq k$. In this case, the *k*th firm is the only specialised firm producing the *k*th output. Alternatively, our definition of economies of diversification allows for partial specialisation. Assuming that $y_j > 0$, having $y_j^k > 0$ for all j implies that each firm continues to produce each of the m outputs. With $z^k \neq \sum_{k=1}^{K} z/K$, this implies only partial specialisation among the K firms. In general, economies of specialisation in equations (4) or (5) depend on the patterns of specialisation among the K firms.

4. The Components of Diversification Economies

This section investigates the sources of the benefit/cost of diversification. We start with a decomposition of the certainty equivalent CE(z). Following Arrow (1965) and Pratt (1964), we define the risk premium as the value R that satisfies

$$\mathrm{EU}(\pi) = \mathrm{U}(\mathrm{E}(\pi) - \mathrm{R}), \tag{6}$$

where $\pi = I(x) + p \cdot y - S(-x, y, e)\ (p \cdot g)$. Equation (6) implies that $R = E[\pi] - U^{-1}\ EU(\pi)$. It defines the risk premium R as the smallest sure amount of money the decision maker is willing to pay to replace the risky prospect $\pi = I(x) + p \cdot y - S(-x, y, e)\ (p \cdot g)$ by its expected value $E(\pi)$. As discussed by Arrow (1965) and Pratt (1964), R provides a monetary measure of the private cost of risk bearing. Being evaluated *ex ante*, R in equation (6) depends on $z \equiv (-x, y)$: R(z). It also depends on the probability distribution of all uncertain variables and on risk preferences. The sign of the risk premium R has been used to characterise the nature of risk behaviour. Following Arrow (1965) and Pratt (1964), the decision maker is said to be risk averse, risk neutral or risk loving when $R > 0$, $R = 0$ or $R < 0$, respectively.

Combining equations (3) and (6) gives the standard decomposition of the certainty equivalent:

$$\mathrm{CE(z)} = \mathrm{E}\pi(\mathrm{z}) - \mathrm{R(z)}, \tag{7}$$

where $\pi(z) \equiv I(x) + p \cdot y - S(-x, y, e)\ (p \cdot g)$ denotes income, $E\pi(z)$ is expected income, and $R(z) = E[\pi(z)] - U^{-1}\ EU[\pi(z)]$ is the risk premium. Equation (7) shows that the certainty equivalent can always be written as the sum of expected income $E\pi(z)$, minus the risk premium R(z). This shows that both expected income $E\pi(z)$ and the cost of private risk bearing R(z) affect the welfare of the decision maker. Under risk aversion (where $R(z) > 0$), the latter provides an incentive to reduce risk exposure.

Consider the case where risk preferences $U(\pi)$ remain constant for all farm types. Let $R(z^k) = E[\pi(z^k)] - U^{-1}\ EU[\pi(z^k)]$ denote the risk premium for the *k*th firm/farm, $k = 1, \ldots, K$. Combining equations (4) and (7) gives the following result:

Proposition 1. Economies of diversification (diseconomies of diversification) exist if

$$D \equiv D_\pi + D_R > (<0), \tag{8}$$

where

$$D_\pi = E\pi(z) - \sum_{k=1}^{K} E\pi(z^k), \tag{9a}$$

$$D_R = -[R(z) - \sum_{k=1}^{K} R(z^k)], \tag{9b}$$

with $\sum_{k=1}^{K} z^k = z$.

Equation (8) identifies two additive components of the benefits of diversification: the expected income component D_π, and the risk premium component D_R. Noting that $R = 0$ in the absence of risk, it follows that $D \equiv D_\pi$ in a riskless world, where the expected income component D_π captures all the economic effects of diversification. Such effects have been analysed in the literature on economies of scope (see below). In a risky world, equation (8) shows that the risk component D_R also plays a role. As discussed below, such effects have been analysed in the literature on the role of risk in diversification strategies. Thus, equation (8) provides a step toward integrating these two literatures.

Note that the decomposition given in equation (8) applies in general for any pattern of specialisation among the K firms. Next, we explore further the benefit/cost generated by diversification strategies. We focus our attention on more specific patterns of diversification/specialisation among outputs. For that purpose, let $I = \{1, …., m\}$ be the set of outputs. Consider a partition of the set of outputs $I = \{I_1, I_2, …., I_K\}$ where I_k is the subset of outputs that *k*th firm specialises in, with $2 \leq K \leq m$. Let $\beta_k \in (1/K, 1]$ be the proportion of the original outputs $\{y_i: i \in I_k\}$ produced by the *k*th firm, $k = 1, …, K$. As discussed below, the β_k's reflect degrees of specialisation (as different choices of the β_k's capture different specialisation schemes). Then, given $z^k \equiv (-x^k, y^k)$ for the *k*th firm, consider choosing

$$x^k = x/K, \tag{10a}$$

$$y_i^k = y_i^+ \equiv \beta_k y_i \text{ if } i \in I_k, \tag{10b}$$

$$= y_i^- \equiv y_i(1-\beta_{k\prime})/(K-1) \text{ if } i \in I_{k\prime} \neq I_k, \tag{10c}$$

for some $\beta_k \in (1/K, 1]$, $k = 1, …, K$. First, note that equations (10a)–(10c) always satisfy $z \equiv (-x, y) = \sum_{k=1}^{K} z^k$. This keeps the aggregate netputs constant. Second, equation (10a) divides the inputs x equally among the K firms. This means 'no specialisation' in inputs across the K firms, as our analysis focuses on output specialisation. Third, equations (10b)–(10c) establish the patterns of specialisation for outputs y. To illustrate, consider the case where $m = K = 3$. Then, equations (10b)–(10c) yield $y^1 = (\beta_1\, y_1,\ ½\,(1-\beta_2)\, y_2,\ ½\,(1-\beta_3)\, y_3)$, $y^2 = (1-\beta_1)\, y_1,\ \beta_2\, y_2,\ ½\,(1-\beta_3)\, y_3)$, and $y^3 = (½\,(1-\beta_1)\, y_1,\ ½\,(1-\beta_2\, y_2),\ \beta_3\, y_3)$, which satisfies $y = \sum_{k=1}^{3} y^k$. In general, having $\beta_k = 1$ means that the *k*th firm produces the same outputs in I_k as the original firm. If $\beta_k = 1$ for all k, then each of the K firms becomes completely specialised in a subset of outputs. Alternatively, $\beta_k \in (1/K, 1)$

represents partial specialisation in outputs. It allows for varying amounts of specialisation, as a rise in β_k from 1/K toward 1 means that the *k*th firm becomes more specialised in $\{y_i: i \in I_k\}$, $k \in K$.

4.1. Expected income and diversification

The expected income component D_π in equations (8) and (9a) identifies the role of mean income in the benefit of diversification. Assume that the outputs are ordered such that $(y_1, y_2, \ldots, y_m) = (\{y_i: i \in I_1\}, \{y_i: i \in I_2\}, \ldots, \{y_i: i \in I_K\})$. Given $y_i^+ \equiv \beta_k y_i$ with $i \in I_k$, let $y^+ = (y_1^+, \ldots, y_m^+)$, $y_k^+ = \{y_i^+ : i \in I_k\}$, and for $k < k'$, $y_{k:k'}^+ = (y_k^+, y_{k+1}^+, \ldots, y_{k'}^+)$. Similarly, given $y_i^- \equiv y_i\ (1-\beta_{k'})/(K-1)$ with $i \in I_{k'}$, let $y^- = (y_1^-, \ldots, y_m^-)$, $y_k^- = \{y_i^- : i \in I_k\}$, and for $k \leq k'$, $y_{k:k'}^- = (y_k^-, y_{k+1}^-, \ldots, y_{k'}^-)$. Then, from equations (10b) to (10c), the outputs of the *k*th firm are $y^k = (y_{1:k-1}^-, y_k^+, y_{k+1:K}^-)$, $k = 1, \ldots, K$. The following decomposition of D_π will prove useful. See the proof in the Appendix.

Proposition 2. Given equation (9), the expected income component D_π in equations (9a) can be written as

$$D_\pi = D_{\pi C} + D_{\pi S} + D_{\pi V}, \tag{11}$$

where

$$D_{\pi C} = \sum\nolimits_{k=1}^{K-1} E_\pi(-x/K, y_{1:k-1}^-, y_k^+, y_{k+1:K}^+) - \sum\nolimits_{k=1}^{K-1} E_\pi(-x/K, y_{1:k-1}^-, y_k^-, y_{k+1:K}^+)$$

$$-[\sum\nolimits_{k=1}^{K-1} E\pi(-x/K, y_{1:k-1}^-, y_k^+, y_{k+1:K}^-) - \sum\nolimits_{k=1}^{K-1} E\pi(-x/K, y_{1:k-1}^-, y_k^-, y_{k+1:K}^-)] \tag{12a}$$

$$D_{\pi S} = E\pi(z) - KE\pi(-x/K, y/K), \tag{12b}$$

$$D_{\pi V} = K\ E\pi(-x/K, y/K) - E\pi(-x/K, y^+) - (K-1) - E\pi(-x/K, y^-). \tag{12c}$$

Proposition 2 decomposes the expected income component D_π into three additive parts: $D_{\pi C}$ given in equation (12a), $D_{\pi S}$ given in equation (12b), and $D_{\pi V}$ given in equation (12c). As discussed below, $D_{\pi C}$ is a complementarity component, $D_{\pi S}$ is a scale component, and $D_{\pi V}$ is a concavity component related to expected income.

First, consider $D_{\pi C}$. The term $D_{\pi C}$ in equation (12a) depends on how y_k affects the marginal expected income of other outputs. It reflects the presence of complementarity among outputs. To see that, consider the case where the expected income is twice continuously differentiable in y. Then, equation (12a) can be written as

$$D_{\pi C} \equiv \sum\nolimits_{k=1}^{K-1} \int_{y_{k+1:K}^-}^{y_{k+1:K}^+} \int_{y_{k^-}}^{y^{k^+}} \frac{\partial^2 E\pi}{\partial\gamma_1 \partial\gamma_2}(-x/K, y_{1:k-1}^-, \gamma_1\gamma_2) d\gamma_1 d\gamma_2, \tag{12a$'$}$$

where γ_1 and γ_2 are dummies of integration for y_k *and* $y_{k+1:K}$, respectively.

Equation (12a′) shows that the sign of $D_{\pi C}$ is the same as the sign of $\partial^2 E\pi/\partial y_k \partial y_{k'}$, $k' \neq k$. In this context, define complementarity between y_k and $y_{k'}$ as any situation where the expected income function satisfies $\partial^2 E\pi/\partial y_k \partial y_{k'} > 0$, $k' \neq k$. This means that, under complementarity, y_k has positive effects on the marginal expected income of $y_{k'}$, implying positive synergies between y_k and $y_{k'}$, $k' \neq k$. Then, it is clear from equation (12a) that $D_{\pi C} > 0$ if the expected income function exhibits complementarity between y_k and $y_{k'}$, for all $k' \neq k$. Thus, Proposition 2 establishes that complementarity among outputs (as reflected by the term $D_{\pi C}$) is one of the components of D. It shows that complementarity is one the factors contributing to economies of diversification.

Second, consider $D_{\pi S}$. Note from equation (12b) that $D_{\pi S} = 0$ when $E\pi(z)$ is linearly homogenous in z. Then, $D_{\pi S} = 0$ corresponds to situations where $[E\pi(\lambda\, z)/\lambda]$ is a constant for all $\lambda > 0$. Define the ray-average expected income as $RAE(\lambda, z) \equiv [E\pi(\lambda\, z)/\lambda]$, where λ is a positive scalar reflecting the scale of operation. Define $\left\{\begin{matrix}\text{increasing returns to scale (IRTS)}\\ \text{constant returns to scale (CRTS)}\\ \text{decreasing returns to scale(DRTS)}\end{matrix}\right\}$ as situations where the ray-average expected income $RAE(\lambda, z)$ is $\left\{\begin{matrix}\text{increasing}\\ \text{constant}\\ \text{decreasing}\end{matrix}\right\}$ in $\lambda > 0$. It follows that $D_{\pi S} = 0$ under CRTS. Alternatively, $D_{\pi S}$ in equation (12a) is non-zero only when there is a departure from CRTS, with $D_{\pi S} > 0$ (<0) under IRTS (DRTS). This makes it clear that $D_{\pi S}$ captures scale effects. It implies from equation (11) that IRTS contributes to economies of diversification (with $D_{\pi S} > 0$).[8]

The term $D_{\pi V}$ in equation (12c) reflects a concavity effect. To see that, note that $E\pi(\sum_{k=1}^{K} \theta_k z^k) \left\{\begin{matrix}\geq\\ =\\ \leq\end{matrix}\right\} \sum_{k=1}^{K} \theta_k E\pi(z^k)$ if the function $E\pi(z)$ is $\left\{\begin{matrix}\text{concave}\\ \text{linear}\\ \text{convex}\end{matrix}\right\}$ in z, for any $\theta_k \in [0,1]$ satisfying $\sum_{k=1}^{K} \theta_k = 1$. Choosing $\theta_j = 1/K$, $z^1 = (-x/K,\ y^+)$ and $z^k = (-x/K,\ y^-)$ for $k = 2, \ldots, K$, it follows from equation (12c) that $D_{\pi V} \left\{\begin{matrix}\geq\\ =\\ \leq\end{matrix}\right\} 0$ if the function $E\pi(z)$ is $\left\{\begin{matrix}\text{concave}\\ \text{linear}\\ \text{convex}\end{matrix}\right\}$ in z. In other words, from equation (11), the concavity of $E\pi(z)$ in z contributes to economies of diversification. The concavity of $E\pi(z)$ in z reflects diminishing marginal productivity in netputs. Thus, diminishing marginal productivity plays a role in economies of diversification. In addition, note that the concavity of $E\pi(z)$ in equation (12c) is evaluated along a hyperplane (since $z = \sum_{k=1}^{K} z^k$). Following Baumol *et al.* (1982, p. 81), a function is said to be trans-ray concave (trans-ray convex) if it is concave (convex) along a hyperplane. Thus, the concavity (convexity) properties just discussed are in fact trans-ray concavity (trans-ray convexity) of the expected income function $E\pi(z)$. It follows from equation (12c) that trans-ray concavity of $E\pi(z)$ contributes to economies of diversification.

---

[8] It is well known that the existence of fixed costs can contribute to IRTS for small farms. Thus, the presence of fixed costs in production/marketing/investment decisions can imply $D_{\pi S} > 0$, meaning that the scale effect would provide an incentive for farms to diversify.

How does Proposition 2 relate to previous research? Evans and Heckman (1984), Baumol (1977), Baumol *et al*. (1982), Willig (1979), and others have investigated the role of economies of scope. Our analysis of the role of scale and of trans-ray concavity effects reduces to the analysis of diversification presented by Baumol (1977) and Baumol *et al*. (1982). Indeed, Baumol (1977) and Baumol *et al*. (1982) showed that complementarity among outputs contributes to the presence of economies of scope. This is captured by the complementarity effect $D_{\pi C}$ in equations (11) and (12a). And they showed that IRTS contributes to the presence of economies of scope. This is captured by the scale effect $D_{\pi S}$ in equations (11) and (12b). This shows how both returns to scale and output complementarity can affect economies of diversification. It illustrates how our approach extends previous literature on the economics of the multiproduct firm.

4.2. Risk and diversification

The risk premium component D_R in equations (8) and (9b) identifies the role of risk and risk aversion in the benefit of diversification. Such a role has been examined in classical contributions by Heady (1952), Markowitz (1959), Tobin (1958), Samuelson (1967), Johnson (1967) and others. As we show below, our approach refines these analyses. We start with the following decomposition of D_R. (The proof is similar to Proposition 2 and is omitted).

Proposition 3. Given equation (9), the risk component D_R in equations (9b) can be written as

$$D_R \equiv D_{RC} + D_{RS} + D_{RV}, \tag{13}$$

where

$$\begin{aligned} D_{RC} = -\Big\{ & \Big[\sum_{k=1}^{K-1} R(-x/K, y^-_{1:k-1}, y^+_k, y^+_{k+1:K}) - \sum_{k=1}^{K-1} R(-x/K, y^-_{1:k-1}, y^-_k, y^+_{k+1:K})\Big] \\ & - \Big[\sum_{k=1}^{K-1} R(-x/K, y^-_{1:k-1}, y^+_k, y^-_{k+1:K}) - \sum_{k=1}^{K-1} R(-x/K, y^-_{1:k-1}, y^-_k, y^-_{k+1:K})\Big]\Big\}, \end{aligned} \tag{14a}$$

$$D_{RS} = -[R(z) - K\ R(-x/K, y/K)], \tag{14b}$$

$$D_{RV} = -[K\ R(-x/K, y/K) - R(-x/K, y^+) - (K-1)R(-x/K, y^-)], \tag{14c}$$

Proposition 3 decomposes the risk effect D_R into three additive components: D_{RC}, D_{RS} and D_{RV}, where D_{RC} is given in equation (14a), D_{RS} is given in equation (14b), and D_{RV} is given in equation (14c). As discussed below, the term D_{RC} reflects risk complementarity effects, D_{RS} reflects risk scale effects, and D_{RV} reflects risk concavity/convexity effects.

First, consider D_{RC} in equation (14a). The term D_{RC} depends on how y_k affects the marginal risk premium of other outputs. In a way similar to equation (12a′), D_{RC} is of the opposite sign of $\partial^2 R/\partial y_k \partial y_{k'}$, $k' \neq k$. Under risk aversion (where $R(z) > 0$), define risk complementarity between y_k and $y_{k'}$ as any situation where the risk premium satisfies $\partial^2 R/\partial y_k \partial y_{k'} < 0$, $k' \neq k$. This means that, under risk complementarity, y_k has negative effects on the marginal risk premium of $y_{k'}$, $k' \neq k$. Then, it is clear from equation (14a) that $D_{\pi R} > 0$ if the risk premium exhibits risk complementarity between y_k and $y_{k'}$, for all $k' \neq k$. And Proposition 2 establishes that risk complementarity among outputs (as reflected by the term D_{RC}) is one of the components of D. This shows how risk complementarity can contribute to the economies of diversification.

Second, consider D_{RS}. Note from equation (14b) that $D_{RS} = 0$ when R(z) is linearly homogeneous in z. Then, $D_{RS} = 0$ corresponds to situations where $[R(\lambda z)/\lambda]$ is a constant for all $\lambda > 0$. Define the ray-average risk premium as $RAR(\lambda, z) \equiv R(\lambda z)/\lambda$, where λ is a positive scalar reflecting the scale of operation. It follows that $D_{RS} = 0$ when the ray-average risk premium $RAR(\lambda, \cdot)$ is constant. And under a U-shape $RAR(\lambda, \cdot)$, being in the region where the ray-average risk premium is declining (increasing) implies $D_{RS} > 0 (< 0)$. Thus, an increasing ray-average risk premium implies that $D_{RS} < 0$, i.e., that the scale of operation provides a disincentive for risk diversification. This shows that D_{RS} captures how scale affects risk incentives for diversification.

The term D_{RV} in equation (14c) reflects a trans-ray concavity/convexity effect. To see that, note from equation (14c) that $D_{RV} \begin{Bmatrix} \leq \\ = \\ \geq \end{Bmatrix} 0$ if the function R(z) is $\begin{Bmatrix} \text{concave} \\ \text{linear} \\ \text{convex} \end{Bmatrix}$ in z.[9] Since the concavity/convexity is evaluated along a hyperplane (where $\sum_{k=1}^{K} z^k = z$), it follows from equation (14c) that the trans-ray convexity (trans-ray concavity) of R(z) in z implies that $D_{RV} \geq 0$ (≤ 0).

To illustrate, consider situations where the risk premium takes the form

$$R(z) = \alpha [y_1 \quad y_2 \quad \cdots] \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots \\ \sigma_{12} & \sigma_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \end{bmatrix}, \tag{15}$$

where $\alpha > 0$ reflects risk aversion and $\begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots \\ \sigma_{12} & \sigma_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$ is the positive semi-definite variance-covariance matrix of net returns per unit of $y = (y_1, y_2, \ldots)$. Under the expected utility model, the specification equation (15) corresponds to situations of normal distributions and constant absolute risk aversion (see Freund, 1956; Pratt, 1964). More generally, equation (15) applies as a 'local measure' of the risk premium in the neighbourhood of the riskless case (Pratt, 1964). Under the specification equation (15), we have $D_{RS} = R(z)(1 - K)/K \leq 0$. Thus, under risk aversion, a local measure of the risk premium implies $D_{RS} \leq 0$. In this case, with $D_{RS} \leq 0$,

---

[9] Note that $R(\sum_{k=1}^{K} \theta_k z^k) \begin{Bmatrix} \geq \\ = \\ \leq \end{Bmatrix} \sum_{k=1}^{K} \theta_k R(z^k)$ if the function R(z) is $\begin{Bmatrix} \text{concave} \\ \text{linear} \\ \text{convex} \end{Bmatrix}$\ in z, for any $\theta_k \in [0, 1]$ satisfying $\sum_{k=1}^{K} \theta_k = 1$. Choosing $\theta_j = 1/K$, $z^1 = (-x/K, y^+)$ and $z^k = (-x/K, y^-)$ for k = 2, …, K, gives the desired result.

scale effects tend to have a negative effect on diversification incentives. To the extent that such local results may apply globally, this would mean that scale tends to decrease the motivation for risk diversification.

Note that R(z) in equation (15) is convex in z. It follows that $D_{RV} \geq 0$ under the local approximation given in equation (15), implying that risk exposure provides an incentive to diversify. To the extent that this local result may apply globally, this would mean that the risk premium R(z) is trans-ray convex (along the hyperplane satisfying $\sum_{k=1}^{K} z^k = z$). Then, $D_{RV} \geq 0$ and risk aversion generates incentives for diversification. In this case, the scale effect D_{RS} and the trans-ray concavity effect D_{RV} work against each other: the latter in favour of diversification, the former against it. Which one dominates depends on the nature of risk exposure. Under the specification equation (15) where the z^k's involve complete specialisation (with $y^1 = (y_1, 0, 0, \ldots)$, $y_1{}^2 = (0, y_2, 0, \ldots)$, etc.), note that $D_R = D_{RC} + D_{RS} + D_{RV} = -\alpha \sum_j \sum_{j' \neq j} \sigma_{jj'}\, y_j\, y_{j'}$. Given $\alpha > 0$ and $y > 0$, it follows that D_R is positive (negative) when all covariances $\sigma_{jj'}$ are negative (positive). This gives the well-known result that, among risk-averse decision makers, negative (positive) covariances tend to stimulate (dampen) the incentive to diversify (e.g., Markowitz, 1959; Tobin, 1958; Samuelson, 1967). Thus, the net effect of D_{RC}, D_{RS} and D_{RV} on diversification incentives is largely an empirical matter. This shows how our approach extends previous analyses of the role of risk in diversification strategies.

4.3. A synthesis

From equation (8) and using Propositions 2 and 3, we obtain our main result.

Proposition 4. Given equation (9), economies of diversification (diseconomies of diversification) exist if

$$D \equiv D_C + D_S + D_V > 0 \ (<0), \tag{16}$$

with

$$D_C = D_{\pi C} + D_{RC}, \tag{17a}$$

$$D_S = D_{\pi S} + D_{RS}, \tag{17b}$$

$$D_V = D_{\pi V} + D_{RV}, \tag{17c}$$

where

$$\begin{aligned} D_C \equiv & \sum_{k=1}^{K-1} CE(-x/K, y^-_{1:k-1}, y^+_k, y^+_{k+1:K}) - \sum_{k=1}^{K-1} CE(-x/K, y^-_{1:k-1}, y^-_k, y^+_{k+1:K}) \\ & - \left[\sum_{k=1}^{K-1} CE(-x/K, y^-_{1:k-1}, y^+_k, y^-_{k+1:K}) - \sum_{k=1}^{K-1} CE(-x/K, y^-_{1:k-1}, y^-_k, y^-_{k+1:K})\right] \end{aligned}$$

represents complementarity effects, $D_S \equiv CE(z) - K\ CE(z/K)$ represents scale effects, and $D_V \equiv K\ CE(-x/K, y/K) - CE(-x/K, y^+) - (K-1)\ CE(-x/K, y^-)$ represents trans-ray concavity effects.

Proposition 4 decomposes the economies of diversification D into all its components. Along the lines discussed in Propositions 2 and 3, equation (16) shows that D can be decomposed into three components: the complementarity effect D_C, the scale effect D_S, and the trans-ray concavity effect D_V. Equation (17a) shows that the complementarity effect D_C is the sum of the complementarity effects associated with expected income $D_{\pi C}$ and risk D_{RC}. This indicates that complementaries can affect the motivation for diversification in two ways: through their effects on expected income, and their effects on the risk premium. Equations (17b) and (17c) show similar results for scale effects and complementarity effects. In equation (17b), D_S is the sum of the scale effects associated with expected income $D_{\pi S}$ and risk D_{RS}. This indicates that the scale of operation can affect the motivation for diversification in two ways: through their effects on expected income, and their effects on the risk premium. And equation (17c) implies that the trans-ray concavity effect D_V is the sum of the corresponding effects associated with expected income $D_{\pi V}$ and risk D_{RV}. Again, this means that the trans-ray concavity effects matter in two ways: through their effects on expected income, and their effects on the risk premium. While the role of trans-ray convexity of the cost function has been identified in the literature on scope economies (e.g., Baumol, 1977; Baumol *et al.*, 1982), our analysis shows that such effects are also relevant in assessing the role of risk in diversification strategies.

5. Toward Empirical Analysis

Equations (2) and (3) rely on the shortage function $S(-x, y)$ defined in equation (1). As discussed in section 2, the shortage function measures the distance (measured in number of units of the reference bundle g) between point $(-x, y)$ and the upper bound of the feasible set $F(e)$. It provides a convenient representation of the underlying production technology. Below, we focus our attention on the case where $g = (1, 0,\ldots, 0)$. Assuming that y_1 is an output satisfying free disposal, the production technology $F(e)$ can then be written as $F(e) = \{(-x, y)\text{: } S(-x, y_1, \ldots, y_m, e) \leq 0, (-x, y) \in R^{m+n}\}$, where $S(-x, y_1, \ldots y_m, e) \equiv y_1 - h(y_2, \ldots, y_m, x, e)$ is the shortage function defined in equation (1), and $h(y_2, \ldots, y_m, x, e)$ is the stochastic multi-output production frontier satisfying $h(y_2, \ldots, y_m, x, e) = \max_{y1} \{y_1\text{: } (-x, y) \in F(e)\}$. This shows how the traditional production function approach arises as a special case of the shortage function. Then, equations (2) and (3) establish linkages between the certainty equivalent, the shortage function and the more traditional production function. As noted above, these linkages hold under general conditions, including a multi-output technology and a firm facing both price and production uncertainty.[10]

To assess production uncertainty, we need to evaluate the probability distribution of $h(y_2, \ldots, y_m, x, e)$. This can be done by relying on a moment-based approach (Antle, 1983; Antle and Goodger, 1987). For that purpose, consider the specification[11]

[10] Note that these arguments hold even if the firm does not produce output y_1. Indeed, even if $y_1 = 0$, the shortage function $S(-x, y, e)$ still measures the distance (measured in units of y_1) between point $(-x, y)$ and the upper bound of the feasible set $F(e)$. And given $g = (1, 0, \ldots, 0)$ and $S(-x, y_1, \ldots, y_m, e) \equiv y_1 - h(y_2, \ldots, y_m, x, e)$, the same arguments apply to the production function $h(y_2, \ldots, y_m,, x, e)$.

[11] Note that, while $v_2(e)$ and $v_3(e)$ enter equation (18) in an additive form, the random variables e can still enter the production function $h(y_2, \ldots, y_m, x, e)$ in a flexible way. As discussed in section 2, $h(y_2, \ldots, y_m, x, e)$ gives a valid representation on an arbitrary multi-output technology under production uncertainty. In this context, $v_2(e)$ and $v_3(e)$ can be non-linear functions of e, allowing equation (18) to provide a flexible representation of the effects of production uncertainty. And equation (18) allows the effects of $v_2(e)$ and $v_3(e)$ to depend on inputs and outputs (through the functions $h_2(y_2, \ldots, y_m, x)$ and $h_2(y_2, \ldots, y_m, x)$).

$$h(y_2,\ldots,y_m,x,e) = h_1(y_2,\ldots,y_m,x) + [h_2(y_2,\ldots,y_m,x) - (h_3(y_2,\ldots,y_m,x)/\kappa)^{2/3}]^{1/2}v_2(e) + [h_3(y_2,\ldots,y_m,x)/\kappa]^{1/3}v_3(e), \quad (18)$$

where $v_2(e)$ and $v_3(e)$ are independently distributed error terms satisfying $E[v_i(e)] = 0$, $E[v_i(e)^2] = 1$, $E[v_2(e)^3] = 0$, and $E[v_3(e)^3] \equiv \kappa > 0$.[12] This means that the error terms $v_2(e)$ and $v_3(e)$ are normalised (i.e., they are each distributed with mean zero and variance 1). In addition, $v_2(e)$ has zero skewness, $E[v_2(e)^3] = 0$, while $v_3(e)$ has positive skewness, $E[v_3(e)^3] \equiv \kappa > 0$. Note that equation (18) implies that $E(h) = h_1(y_2, \ldots, y_m, x)$, $E[(h - h_1)^2] = h_2(y_2, \ldots, y_m, x)$, and $E[(h - h_1)^3] = h_3(y_2, \ldots, y_m, x)$. Thus, h_1, h_2 and h_3 are the first three moments (i.e., mean, variance and skewness) of the distribution of h.[13] It follows from equation (18) that, for given distributions of $v_2(e)$ and $v_3(e)$ and conditional on $(y_2, \ldots, y_m, x)$, the first three moments h_1, h_2 and h_3 are sufficient statistics for the distribution of the stochastic production function g. And the certainty equivalent can be written as $CE(z) = CE[y_1, h_1(y_2, \ldots, y_m, x), h_2(y_2, \ldots, y_m, x), h_3(y_2, \ldots, y_m, x)]$. This shows that, under the specification equation (18), knowing the first three moments h_1, h_2 and h_3 provides the information necessary to evaluate the implications of production uncertainty for diversification strategies.

The next step is to estimate the stochastic production function equation (18). Consider the parametric specifications $f_i(y_2, \ldots, y_m, x, \beta_i) \equiv h_i(y_2, \ldots, y_m, x)$, $i = 1, 2, 3$, where the β_i's are parameter vectors to be estimated. This can provide a flexible representation of the production technology, including the effects of $(y_2, \ldots, y_m, x)$ on mean production $f_1(y_2, \ldots, y_m, x, \beta_1)$, on the variance of production $f_2(y_2, \ldots, y_m, x, \beta_2) > 0$, and on the third central moment (measuring the skewness of production) $f_3(y_2, \ldots, y_m, x, \beta_3)$. For example, it allows the *i*th output to be variance increasing, variance neutral, or variance decreasing depending upon whether $\partial f_2/\partial y_i > 0$, $= 0$, or < 0, respectively. Similarly, the *i*th output can affect downside risk exposure through its effect on skewness: it would contribute to decreasing (increasing) downside risk exposure if $\partial f_3/\partial y_i > 0$ (< 0). The specification equation (18) goes beyond the standard mean-variance analysis analysed by Just and Pope (1979), and Meyer (1987): it also considers the effects of skewness and downside risk exposure. Indeed, the two-moment models are obtained as special case of equation (18) when f_3 is constant. This shows that the specification equation (18) provides a generalisation of the mean-variance approach (e.g., as analysed by Just and Pope (1979), and Meyer (1987)) to three-moment models. As such, equation (18) will allow us to investigate the

---

[12] We assume that $\kappa > 0$ is chosen to be large enough so that $[h_2 - (h_3/\kappa)^{2/3}] > 0$ in equation (18).

[13] This implies that the (conditional) mean, variance and skewness of $h(\cdot)$ in equation (18) are identified. Yet, in the presence of multiple sources of production uncertainty (where e is a vector of random variables), the moments of each element of e would typically not be identified. This is a significant difference between the economic model (involving e) given on the left-hand side of equation (18) and the econometric model (involving v_2 and v_3) given on the right-hand side of equation (18). Our empirical analysis below focuses on the econometric model (i.e., the right-hand side of equation (18)) which does not suffer from identification issues.

effects of both variance and skewness of production risk on diversification incentives.

After substituting $f_i(y_2, \ldots, y_m, x, \beta_i)$ for $h_i(y_2, \ldots, y_m, x)$, equation (18) gives

$$y_1 = f_1(y_2, \ldots, y_m, x, \beta_1) + u, \tag{19}$$

where $u \equiv [f_2(y_2, \ldots, y_m, x, \beta_2) - (f_3(y_2, \ldots, y_m, x, \beta_3)/\kappa)^{2/3}]^{1/2}\, v_2(e) + [f_3(y_2, \ldots, y_m, x, \beta_3)/\kappa]^{1/3}\, v_3(e)$ is an error term with mean zero, variance $f_2(y_2, \ldots, y_m, x, \beta_2)$, and skewness $f_3(y_2, \ldots, y_m, x, \beta_3)$. Thus, the error term u in equation (19) exhibits heteroscedasticity (through its conditional variance $f_2(y_2, \ldots, y_m, x, \beta_2)$) as well as conditional skewness. Following Antle (1983), and Antle and Goodger (1987), the parameters β_i's in equation (19) can be consistently estimated. First, we obtain a consistent estimate β_1^e of β_1 in equation (19), yielding an estimate of the conditional mean $f_1(y_2, \ldots, y_m, x, \beta_1^e)$, Second, we use the estimated error term in equation (19) $u^e \equiv y_1 - f_1(y_2, \ldots, y_m, x, \beta_1^e)$ to obtain consistent estimates of the variance and skewness. Following Antle (1983), we estimate β_2^e and β_3^e using $E(u)^2 = f_2(y_2, \ldots, y_m, x, \beta_2)$ and $E(u)^3 = f_3(y_2, \ldots, y_m, x, \beta_3)$, respectively. This gives the conditional variance $f_2(y_2, \ldots, y_m, x, \beta_2^e)$ and the conditional skewness $f_3(y_2, \ldots, y_m, x, \beta_3^e)$.[14] In turn, these estimates can be used to evaluate firm welfare and diversification economies.

The estimation of equation (19) poses at least three econometric challenges. First, we would like $\{f_i(y_2, \ldots, y_m, x, \beta_i), i = 1, 2, 3\}$ to provide a flexible representation of the effects of outputs $(y_2, \ldots, y_m)$ on productivity and welfare. This is feasible when the number of outputs is small. However, this becomes problematic if the number of outputs becomes large. Indeed, a flexible representation of output effects with a large number of outputs requires a large number of parameters, implying the prospects of facing severe collinearity problems. Our empirical analysis focuses on a situation with five outputs (see below). Five is 'large enough' to allow the investigation of the benefit of diversification across processes, yet 'small enough' to avoid collinearity problems. In this context, with five outputs, we specify the f_i's in equation (19) to be a quadratic function of outputs y. This provides a parsimonious specification allowing for a flexible representation of the effects of each output on the marginal product of other outputs and on risk exposure. Finally, we assume that inputs x enter f_1 in equation (19) in log form.[15] This keeps the number of parameters reasonably small while allowing for curvature in input effects.

Second a potential problem arises from the effects of unobserved heterogeneity across firms. This can arise when managerial abilities and/or environmental conditions vary across firms in a way that is not observed by the econometrician. This can generate biassed estimation of the model (due to omitted variable bias). We therefore need to control for time invariant firm characteristics. This is especially important to make sure that our empirical analysis does not confuse unobserved heterogeneity with risk exposure. When using panel data, let the error term in equation (19) for the *h*th firm at time t take the form $u_{ht} = \mu_h + w_{ht}$, where μ_h is the time invariant unobserved characteristics of the *h*th firm. Then, equation (19) can

[14] Note that Antle (1983) shows that, in general, the moment functions are heteroscedastic.

[15] When some of the inputs are observed to be 0, we followed Battese (1997) and used the following specification: $[\alpha_0\, D + \alpha_1\, \ln(x + D)]$, where D is a dummy variable satisfying $D = 1$ if $x = 0$ and $D = 0$ if $x > 0$, and α_0 and α_1 are parameters.

be estimated using a fixed-effect model, which generates consistent parameter estimates. Standard fixed-effect models rely on data transformation (i.e., first time-differencing) that removes the individual effect (μ_h) as well as time invariant variables (such as quality of inputs). Alternatively, one could use a random effect model that would capture efficiency gains. Below, we follow the approach proposed by Mundlak (1978). It allows the firm-specific unobservable effects to be correlated with the explanatory variables. In this context, denote by $\bar{X}_h$ the mean of the time-varying explanatory variables of the *h*th farm in equation (19). Following Mundlak (1978), let $\mu_h = \bar{X}_h \alpha + \eta_h$, where α is a vector of parameters capturing possible correlation between the unobservable farm-specific effects μ_h and the explanatory variables, and η_h is an error term that is uncorrelated with $\bar{X}_h$. Then, for the *h*th farm at time t, substituting $u_{ht} = \mu_h + w_{ht}$ and $\mu_h = \bar{X}_h \alpha + \eta_h$ into equation (19) yields the Mundlak specification

$$y_{1ht} = f_1(y_{2ht}, \ldots, y_{mht}, x_{ht}, \beta_1) + \bar{X}_h\alpha + \varepsilon_{ht}, \tag{19'}$$

where $\varepsilon_{ht} = \eta_h + w_{ht}$ (where $\eta_h \sim \text{iid}(0, \sigma_\eta^2)$) and $\bar{X}_h\alpha$ controls for the relevant farm-specific unobserved heterogeneity. Equation (19′) is the one used in the estimation of the mean effects, yielding consistent estimates of β_1. The estimated error term $\varepsilon_{ht,}$ is in turn used to estimate the second and third moment of the error term in equation (19′). Again, with panel data, these moments would have a farm-specific component. This implies a need to control for farm-specific components in the estimation of production risk. Again, we rely on the Mundlak approach to control for the effects of farm-specific components in the estimation of the variance and skewness equations, while generating consistent estimates of the effects of time-varying components on variance and skewness. The results are shown below.[16]

The Mundlak approach is also very useful to tackle a third challenge we face in the estimation of equation (19): endogeneity. This is expected to arise if outputs (used as right-hand side variables) are correlated with the error term u in equation (19), with $\text{cov}(y_n, u) \neq 0$. Given $u_{ht} = \mu_h + w_{ht}$ and $\mu_h = \bar{X}_h\alpha + \eta_h$, the Mundlak approach in equation (19′) effectively deals with the correlation between μ_h and the time-varying right-hand side variables (as captured by α). When this correlation is the reason why $\text{cov}(y_n, u) \neq 0$, the Mundlak approach solves this endogeneity problem and generates consistent parameter estimates. In our setting this is particularly relevant as many right-hand side variables are potentially correlated with the individual component of the error term. Finally, we dealt with heteroscedasticity by using White's robust standard errors.[17]

6. Data

Agriculture is the mainstay of the Ethiopian economy. The agricultural sector accounts for about 40% of national GDP and 85% of employment. The performance

[16] For an alternative error component specification of the stochastic production function, see Griffiths and Anderson (1982).

[17] As noted by a reviewer, while our parameter estimates and their standard errors are consistent, we do not claim that they are asymptotically efficient (as more refined weighting schemes could possibly generate gains in asymptotic efficiency).

of the Ethiopian agricultural sector, however, has been dismal. Real agricultural GDP and per capita cereal production have been falling over the last 40 years with cereal yield stagnant at about 1.2 tons per hectare (World Bank, 2005). This is further compounded by extreme land shortages in the highlands: per capita land cultivated has fallen from 0.5 ha in the 1960s to only 0.2 ha by 2005. Neither fertiliser nor improved seeds are used much (Demeke *et al.*, 1998). Chemical fertilisers are currently employed on only 13% of the total cultivated area. Poor infrastructure, limited extension services and poorly functioning market institutions result in high transaction costs and low productivity (World Bank, 2005).

As in many developing countries, Ethiopian agriculture is very diversified. It exhibits considerable diversity in food crops (Gebre Egziabher, 1991; Unruh, 2001). Ethiopia is a recognised centre of genetic diversity for several crops (Vavilov, 1951; Harlan, 1992). It is widely believed that farmers diversify to exploit positive synergies across crops and to hedge against risk. Our analysis will provide new insights into these economic rationales.

Our empirical research examines the economics of diversification using farm level panel data from Ethiopia. The agricultural production data were collected at the household level as well as the plot level. The data were gathered biennially from household surveys in two districts of the Amhara National Regional State, between 2000 and 2005. The region encompasses part of the Northern and Central Highlands of Ethiopia. One of the Zones (Districts), East Gojjam is a fertile plateau receiving good average rainfall while the South Wollo zone is characterised by degraded hill side plots receiving lower and highly erratic rainfall.

A total of 855 farms were observed consistently in three rounds of surveys. About 24% of the original sample was unfortunately lost due to missing values and outliers. A total of 1,922 observations remained and were used in the empirical analysis presented below. The resulting data provided detailed information on socioeconomic and physical farm characteristics of the households. In addition, data on crop types, production and inputs were also collected. Descriptive statistics on the variables used in our analysis are presented in Table 1. The main crops are cereals and pulses and include: teff (grown by 70% of the sample), wheat (grown by 43% of the sample), sorghum (grown by 35% of the sample), barley (grown by 25% of the sample) and beans and pulses (grown by 30% of the sample). These are all food crops. They are typically produced by the farm household for home consumption. Conventional inputs are land, labour, oxen use, fertiliser and manure. Due to limited off-farm and migration opportunities, there is little labour mobility outside the village. And there is a little change in physical characteristics of each farm over time. This reflects very limited land redistribution taking place within the study period. Fertiliser and manure use has increased over time.[18]

7. Econometric Results

We use the Ethiopian farm panel data to estimate equation (19). As mentioned above, the analysis considers teff, barley, wheat, sorghum, beans and ‘other output’ as outputs. Teff is the dominant crop on Ethiopian farms. It is taken to be the first crop and as the reference bundle, meaning that y_1 in equation (19) is the quantity

[18] Mechanisation and irrigation are not used on any farm in our sample.

Table 1
Descriptive statistics

Variables	Sample mean	Std deviation	Min	Max
Teff in kg	417.13	712.97	0	12,951.86
Wheat in kg	204.26	529.21	0	13,288.70
Sorghum in kg	227.73	658.67	0	8,653.87
Barley in kg	103.18	347.64	0	7,321.25
Land in hectares (ha)	1.23	2.17	0.02	20.68
Labour (number of working-age family members)	5.30	2.09	1	16.00
Oxen (number)	1.55	1.413	1	5.99
Fertiliser in kg	28.43	51.03	0	411.45
Manure in kg	91.30	144.27	0	496.00
Other crops in kg	613.90	780.24	6.64	12,951.86
Plot slope (proportion of plots with flat slope)	0.27	0.32	0	1
Plot fertility (proportion of plots with fertile soil)	0.39	0.37	0	1

of teff produced. The econometric results for the mean equation are reported in Table 2. The panel data cover three years: 2000, 2002 and 2005. As discussed above, we estimated our model using the fixed-effect approach proposed by Mundlak (1978). The results are reported in column a) of Table 2. To investigate endogeneity issues, we used as instruments: the age of the household head, gender of the household head, and geographical distance of farm plots from the farm. Moreover, the multi-year data allowed us to use lagged values of the endogenous variables to generate additional instruments. Lagged values are relevant instruments in the presence of adjustment costs which create dynamics in firm decisions (e.g., Hamermesh and Pfann, 1996). We first implemented a Wu–Hausman endogeneity test on the pooled data (i.e., neglecting fixed effects). The Wu–Hausman test statistic was 32.57, with a corresponding P-value of 0.001. This provides strong evidence of endogeneity when fixed effects are neglected. Then, we implemented the same testing procedure based on the Mundlak fixed-effect estimator. The test results are reported at the bottom of Table 2. Both a Wu–Hausman test and a C test do not show evidence of endogeneity bias after controlling for fixed effects. The Hansen test for overidentifying restrictions indicates that the instruments are not correlated with the error term. The set of instruments was also tested for their relevance *via* a Anderson–Rubin Wald test and a Stock–Wright Lagrange Multiplier test. Both testing procedures supported the validity of the instruments. Finally, the F tests results from the first stage regressions are all large, indicating that the instruments are relevant.

As a further check, we reestimated equation (19) using alternative estimation methods. This includes the Mundlak scheme estimated using instrumental variables (IV) and reported in column b) of Table 2). It allows us to investigate whether the endogenous variables may be correlated with the stochastic component of error term (and not just through its individual effect). We also implemented a first-difference approach to remove the time invariant unobserved heterogeneity

Table 2

Estimation of the production function: Fixed effects, fixed-effects IV, and first-difference IV

	(a) Mundlak fixed effect		(b) Mundlak fixed-effects IV		(c) First-difference IV	
Variables	Coeffs	Std errors	Coeffs	Std errors	Coeffs	Std errors
Wheat	−0.565***	0.029	−0.81***	0.07	−1.017***	0.27
Wheatˆ2	0.000012***	0.000003	−0.0004***	0.00002	−0.0001**	0.00006
Sorghum	−0.7426***	0.028	−0.83***	0.086164	−0.66***	0.18
Sorghumˆ2	−0.000037**	0.000006	−0.00003	0.00002	−0.0001**	0.00004
Barley	−0.978***	0.046	−1.02***	0.085	−1.62***	0.34
Barleyˆ2	−0.00012***	0.000011	−0.0001***	0.00002	−0.0002***	0.0001
Wheat * Barley	0.000083**	0.00004	0.00009	0.00006	0.0004*	0.0002
Wheat * Sorghum	0.00058***	0.000062	0.00058***	0.00004	0.0005**	0.0003
Wheat * Beans	0.00030***	0.000039	0.0003***	0.00003	0.0003*	0.0002
Barley * Sorghum	0.00014***	0.000028	0.00002	0.00007	0.00001	0.0001
Sorghum * Beans	0.00023***	0.000078	0.00018***	0.00006	0.0006*	0.0003
Barley * Beans	0.00016***	0.00004	0.0002***	0.00004	0.0000001	0.0001
Beans	−0.46***	0.035	−0.49***	0.036	−0.46***	0.16
Beansˆ2	−0.00011***	0.000013	−0.0001***	0.00002	−0.0001	0.0001
Land	40.14***	16.8	29.62***	5.41	147.35**	70.4
Labour	−21.04	47.196	−13.4	15.34	40.61	198
Dummy oxen	18.1	29.85	−4.42	10.71	4.11	92.97
Oxen	−14.7	40.34	−9.99	13.44	−43.69	115.6
Dummy fertiliser	59.46	63.96	31.7	27.79	8.52	188.53
Fertiliser	27.1	17.95	15.47*	8.31	0.81	48.94
Dummy manure	13.498	97.618	13.24	36.48	239.28	257.62
Manure	10.642	20.191	6.51	6.92	60.15	50.05
Other crops	1.192***	0.029	1.18***	0.03	1.35***	0.18
Other cropsˆ2	−.000015***	0.000004	0.000007	0.00002	−0.000017	0.000013
Plot slope	−15.92	28.99	−12.5	11.03	−131.69	128.29
Fertility	−23.72	25.18	−32.9	8.048	55.82	89.51
Constant	−31.8	126.34	50.55	53.99	8.43	48.08

Notes: Number of observations: 1,922 for (a), (b) and (d) and 780 for (c). Endogeneity testing for the outputs and interaction terms (after the fixed-effect transformation): C statistic (exogeneity): 13.313; χ^2 (14) *P*-value = 0.5; Wu–Hausman F test: 1.24, *P*-value = 0.246; Hansen J statistic: 39.824, χ^2 (31) *P*-value = 0.2; Anderson–Rubin Wald test χ^2 (17) = 126.34, *P*-value = 0; Stock-Wright LM S statistic χ^2 (17) = 108.73, *P*-value = 0. The adjusted R^2 for a) is 0.7. Significance levels are denoted by one asterisk (*) at the 10% level, two asterisks (**) at the 5% level, three asterisks (***) at the 1% level. Robust standard errors are used.

(Wooldridge, 2002). It is reported in column c) of Table 2. It provides an alternative way of controlling for individual effects.[19]

The mean results are reported in Table 2: Mundlak fixed effects in column a), Mundlak fixed-effects IV in column b), and first-difference IV in column c). The

[19] Treating all outputs as endogenous in a simultaneous equations system, we also estimated the model using three-stage least squares (3SLS). This gave results similar to the ones reported in Table 2. The 3SLS estimates are available upon request.

estimates are similar and appear robust across different estimation methods. Most of the estimated coefficients in the mean equation are statistically significant. Note that all coefficients of the interaction terms $\{(y_j\ y_k),\ j \neq k\}$ are positive and statistically significant. This implies that each crop has positive effects on the marginal product of others. As discussed in section 4.1, this points to the presence of output complementarities reflecting productivity-improving synergies across crops. Note that the positive coefficients of $\{(y_j\ y_k),\ j \neq k\}$ tend to be much larger than the corresponding coefficients of y_j^2 and y_k^2. This indicates that the multi-output mean production function is not concave in outputs. As discussed in section 4.1, this is relevant in the analysis of the value of diversification. The implications of these output effects are further evaluated in section 8 below. Table 2 shows that the coefficient for land is positive and statistically significant. The coefficients of other inputs are often not statistically significant. This may reflect the limited use of inputs and low agricultural productivity on Ethiopian farms. To deal with heteroscedasticity, we used White's robust standard errors.

Table 3 reports the estimation results for the variance and the skewness functions estimated under the Mundlak specification.[20] Many outputs are statistically significant in the variance equation, including interaction effects between outputs. Evaluated at sample means, the elasticities of variance with respect to output are 0.15, 0.18, 0.001 and 0.16 for wheat, sorghum, barley and beans, respectively. They indicate that, in general, outputs are variance increasing. The cross-output effects on variance vary with the crops: they are positive for sorghum-beans, but negative for barley-sorghum and barley-beans. Implications of these patterns for risk diversification are evaluated in section 8 below. Finally, the estimates in the variance equation show that fertiliser is variance increasing. Note that the variance equation does not distinguish between upside and downside risk. But the skewness equation does. From Table 3, the estimates of the skewness equation show that the interaction between wheat and sorghum is positive and significant. This indicates that combining these two crops tends to reduce downside risk exposure. The skewness estimates also show that fertiliser is skewness increasing. The implications of these estimates for diversification incentives are explored next.

8. Implications for Diversification

Sections 3 and 4 provided analytical results on economies of diversification and their determinants. The analysis relied on the certainty equivalent (CE), on expected income ($E\pi$) and on the risk premium (R). In the previous section, we reported estimates of the first three moments of a stochastic production function for a sample of Ethiopian farmers. Below, we establish linkages between these two parts of the article to explore the empirical implications for economies of diversification on Ethiopian farms.

Using equation (3), the estimation of expected income $E\pi(z)$ is straightforward. However, the evaluation of the risk premium is more complex (as it involves risk

---

[20] The estimated coefficients for the Mundlak fixed effect were jointly significant. This indicates that controlling for fixed effects in the estimation of higher moments of the distribution of y is appropriate.

Table 3
Estimation results: Variance and skewness functions (Mundlak fixed-effects model)

	Variance		Skewness	
Variables	Coeffs	Std errors	Coeffs	Std errors
Wheat	125.336***	29.049	−0.191**	0.082
Wheat^2	−0.017***	0.003	0.00001	0.000009
Sorghum	108.221***	27.706	0.081	0.074
Sorghum^2	−0.023***	0.006	−0.000039***	0.000015
Barley	−66.088	47.373	−0.167	0.12
Barley^2	−0.068***	0.011	−0.000057*	0.000032
Wheat * Barley	−0.070*	0.041	0.000072	0.00011
Wheat * Sorghum	0.020	0.062	0.000376**	0.00019
Wheat * Beans	−0.065*	0.039	0.000159	0.00011
Barley * Sorghum	−0.150***	0.029	−0.000031	0.00008
Sorghum * Beans	0.227**	0.079	0.000185	0.00021
Barley * Beans	−0.242***	0.041	0.000039	0.00011
Beans	123.79***	35.600	−0.063	0.095
Beans^2	0.038***	0.014	−0.000042	0.00004
Land	−5945.75	16670.48	63.571	48.05
Labour	21551.53	13647.52	51.27	39.73
Dummy oxen	12918.52	21101.37	−15.639	87.743
Oxen	801.09***	302.23	29.26	118.69
Dummy fertiliser	−126.46*	67.22	374.198*	187.30
Fertiliser	253.25***	18.63	122.383***	52.2
Dummy manure	−8017.22	29473.73	−130.6895	262.95
Manure	2476.13	25369.84	−31.43	50.13
Other crops	−26569.93	28305.61	0.17**	0.083
Other crops^2	−539.45	392.72	−0.000013	0.00001
Plot slope	−21311.33	14355	57.82	79.33
Fertility	−15937.5	25888.27	21.68	68.28
Constant	−35554.2	30986.68	−14.674	83.4

Notes: Number of observations: 1,922. The adjusted R^2 is 0.36 and 0.38 for variance and skewness, respectively. Significance levels are denoted by one asterisk (*) at the 10% level, two asterisks (**) at the 5% level, three asterisks (***) at the 1% level. Robust standard errors are used.

preferences). In general, we can write the risk premium as $R(z) = R[x, f_1(\cdot), f_2(\cdot), f_3(\cdot)]$, where $\{f_i(\cdot): i = 1, 2, 3\}$ are the estimates of the first three moments of the stochastic production function given in equation (19). Below, we consider the following specification for the risk premium

$$R(z) = r_2 \mathrm{Var}(\pi)/E\pi + r_3 \mathrm{Skew}(\pi)/[(E\pi)^2], \tag{20}$$

where $\mathrm{Var}(\pi)$ and $\mathrm{Skew}(\pi)$ are respectively the variance and skewness of income $\pi(z)$, and r_2 and r_3 are risk aversion parameters reflecting the decision maker's risk preferences. The risk premium R in equation (20) can be decomposed into two parts:

$$R = R_{var} + R_{skew}, \tag{20a}$$

where $R_{var} = r_2 \, Var(\pi)/E\pi$ is the variance component of R, and $R_{skew} = r_3$ Skew $(\pi)/[(E\pi)^2]$ is the skewness component of the risk premium. We argue below that equations (20)–(20′) provide a reasonable and flexible specification for our analysis of diversification economies. Under equation (20), note that the relative risk premium defined as $R/E\pi$ is homogenous of degree zero in income π. It follows that the moment-based specification equation (20) corresponds to constant relative risk aversion (CRRA) (Pratt, 1964). Given $\pi > 0$ and under the expected utility model, the CRRA utility function $U(\pi)$ in equation (2) takes the form $U(\pi) = \pi^{1-\alpha}/(1-\alpha)$ where α is the Arrow–Pratt relative risk aversion coefficient (Pratt, 1964).[21] While the risk premium in equation (20) applies globally, its local characterisation under CRRA can give useful additional information on the risk aversion parameters r_2 and r_3 in equation (20). Defined in the neighbourhood of the riskless case, the local risk premium can be written as $R \approx -(1/2)\,[(\partial^2U/\partial\pi^2)/(\partial U/\partial\pi)]\,Var(\pi) - (1/6)\,[(\partial^3U/\partial\pi^3)/(\partial U/\partial\pi)]\,Skew(\pi)$ (Chavas, 2004, p. 49). This includes as a special case the local characterisation of the risk premium analysed by Pratt (1964) when $Skew(\pi) = 0$. Thus, 'in the small', it follows from equation (20) that $r_2/E\pi = (1/2)\,[(\partial^2U/\partial\pi^2)/(\partial U/\partial\pi)]$ and $r_3/[(E\pi)^2] = (1/6)\,[(\partial^3U/\partial\pi^3)/(\partial U/\partial\pi)]$. Under CRRA preferences (where $U(\pi) = \pi^{1-\alpha}/(1-\alpha)$), this implies $r_2 = \alpha/2$ and $r_3 = -\alpha\,(\alpha + 1)/6$.

The empirical evidence indicates that most decision makers exhibit risk aversion ($R > 0$), decreasing absolute risk aversion (where R declines with a rise in income), and downside risk aversion (where R increases with a higher exposure to downside risk) (e.g., Chavas, 2004). Under CRRA preferences, risk aversion implies $\alpha > 0$. Also, Pratt (1964) and Menezes *et al.* (1980) have shown that CRRA implies decreasing absolute risk aversion and downside risk aversion. This indicates that equation (20) provides a reasonably good global measure of the risk premium when $r_2 = \alpha/2$, $r_3 = -\alpha\,(\alpha + 1)/6$ and $\alpha > 0$. Indeed, $r_2 = \alpha/2 > 0$ means that, under risk aversion, any increase in variance f_2 increases the risk premium. And $r_3 = -\alpha\,(\alpha + 1)/6 < 0$ means that, under downside risk aversion, any increase in skewness (i.e., a decrease in downside risk exposure) reduces the risk premium. Finally, while risk preferences can vary significantly across individuals, the empirical evidence indicates that the relative risk aversion coefficient α typically varies between 1 and 4 (Gollier, 2001, p. 31). On that basis, we assume below that $\alpha = 2$, corresponding to a moderate level of risk aversion. Given $r_2 = \alpha/2$, $r_3 = -\alpha\,(\alpha + 1)/6$, this means that we pursue our analysis using the risk premium R given in equation (20) with $r_2 = 1$ and $r_3 = -1$.[22]

In our simulations of Ethiopian farms, we make two simplifying assumptions. First, we assume that $I(x) = 0$. This means that there is no off-farm income, and that the cost of input use is negligible. This reflects the fact that, due to poor infrastructure, high transaction costs and poorly functioning markets, most Ethiopian

[21] When $\alpha = 1$, the CRRA utility function takes the form $U(\pi) = \ln(\pi)$.

[22] We conducted some sensitivity analysis on the relative risk aversion parameter α. As expected increasing (decreasing) risk aversion α made the risk premium R larger (smaller), thus making the risk diversification component D_R larger (smaller). Yet, other effects remained qualitatively similar to the ones reported below.

farms are subsistence farms relying very little on purchased inputs and having few opportunities for non-farm activities.[23] Second, we neglect output price uncertainty. This reflects that, under difficult agro-climatic conditions, production uncertainty is the most important concern in Ethiopian agriculture (as exemplified by the many famines that have occurred in Ethiopia over the last few decades). The analysis is conducted assuming output prices p of 2.5, 2, 1, 1.1 and 0.92 bir/kg for teff, wheat, sorghum, barley and bean, respectively. In this context, our econometric estimates are used to evaluate the expected income $E\pi$ and risk exposure (as given by our variance and skewness estimates).

Given $r_2 = 1$ and $r_3 = -1$, the components of the risk premium in equation (20′) take the form: $R_{var} = Var(\pi)/E\pi$ as the variance component of R, and $R_{skew} = -Skew(\pi)/[(E\pi)^2]$ as the skewness component of the risk premium. This means that the certainty equivalent in equation (7) can now be decomposed into three additive terms: $CE = E\pi - R_{var} - R_{skew}$. These three terms can be evaluated empirically from the econometric estimates of the first three moments reported above.

First, we evaluated these terms at sample means. The sample mean estimates are: $E\pi = 1{,}936$ birr, $R_{var} = 498$ birr, $R_{skew} = -109$ birr, $R = 388$ birr, and $CE = 1{,}550$ birr. This shows that the cost of risk is important: at sample means, the relative risk premium is $R/E\pi = 0.20$, implying that the implicit cost for risk amounts to 20% of income.[24] Most of this cost is due to the variance component. However, the skewness component also plays a role. At sample means, the skewness is positive, implying that R_{skew} is negative and contributes to reducing the overall cost of risk.

Next, we simulated $E\pi$, R_{var} and R_{skew} for six farms exhibiting different levels of specialisation. We started with an integrated farm defined to be three times as large as the average farm in our sample. And with a focus on five outputs (teff, wheat, sorghum, barley and bean), we examined five specialised farms, each one specialising in one output under the specialisation schemes given in equation (9). The simulations reported below focus on the case where $\beta_i = \beta = 0.8$ in equation (9).[25] This corresponds to a case of partial output specialisation where each of the five specialised farms produces 80% of one of the outputs of the integrated farm, and 5% of the remaining outputs. In this context, we evaluated $E\pi$, R_{var} and R_{skew} for each farm. To support hypothesis testing, we bootstrapped these simulated variables. In turn, using Propositions 1–4, this allows the investigation of economies of diversification D and its various components, including the complementarity, scale and concavity components associated with $E\pi$, R_{var} and R_{skew}. The results are presented in Table 4.

[23] Our conceptual analysis considers the general case where diversification can involve both on-farm and off-farm activities. While our simulation focuses on on-farm diversification, the importance of off-farm income has also been noted (e.g., Ellis, 2000; Barrett *et al.*, 2000; Misra *et al.*, 2004).

[24] This is fairly similar to estimates of the cost of risk found in other studies in low-income situations. It suggests that our risk aversion assumptions are reasonable.

[25] We chose the parameter $\beta = 0.8$ based on the following arguments: it is 'high enough' to give significant diversification; and it is 'not too high' to avoid getting the simulations outside the sample data. Sensitivity analysis on β showed that the value of diversification D increased with β.

Table 4

Economies of diversification, $\beta = 0.8$

Monetary value of diversification benefits (in birr)[a,b]	Value of diversification	Complementarity effect	Scale effect	Concavity effect
Expected income, $E\pi$	D_{π}	$D_{\pi C}$	$D_{\pi S}$	$D_{\pi V}$
	339.39	1,363.54***	222.95	−1,247.10***
Standard error	(901.17)	(219.45)	(972.87)	(267.06)
Variance component	$D_{R,var}$	$D_{RC,var}$	$D_{RS,var}$	$D_{RV,var}$
of R, R_{var}	199.11	−134.22	−194.38	527.71
Standard error	(825.21)	(1,051.39)	(625.49)	(1,157.31)
Skewness component	$D_{R,skew}$	$D_{RC,skew}$	$D_{RS,skew}$	$R_{RV,skew}$
of R, R_{skew}	605.40***	72.81***	584.87***	−52.27**
Standard error	(44.23)	(27.04)	(39.42)	(22.17)
Risk Premium,	D_R	D_{RC}	D_{RS}	D_{RV}
$R = R_{var} + R_{skew}$	804.51	−61.41	390.49	475.44
Certainty equivalent,	D	D_C	D_S	D_V
$CE = E\pi + R_{var} + R_{skew}$	1,143.90	1,302.13	613.44	−771.67

Notes: [a]Bootstrapped standard errors are presented in parentheses below the corresponding estimates. Significance levels are denoted by one asterisk (*) at the 10% level, two asterisks (**) at the 5% level, three asterisks (***) at the 1% level.
[b]The exchange rate was around 0.12 US dollar for 1 Ethiopian birr during the study period.

First, start with the effects of mean income on diversification incentives, as measured by D_{π}. Using equations (11) and (12), Table 4 shows the three components of D_{π}: $D_{\pi} = D_{\pi C} + D_{\pi S} + D_{\pi V}$. The mean complementarity component $D_{\pi C}$ is estimated to be positive and large: $D_{\pi C} = 1{,}363.54$. It is statistically significant at the 1% level. This value amounts to 70% of the expected revenue for an average farm in our sample. It indicates that complementarity generates very large mean productivity gains, as each crop significantly increases the marginal products of other crops. This provides a strong incentive for farms to diversify. The mean scale component $D_{\pi S}$ is positive but not statistically significant. As discussed in section 4.1, this lack of significance can be interpreted as a failure to reject the null hypothesis of CRTS. It means that, as far as expected income is concerned, farm scale does not have a major impact on diversification incentives. Finally, the mean concavity component $D_{\pi V}$ is estimated to be negative and large: $D_{\pi V} = -1{,}247.10$. It is statistically significant at the 1% significance level. As discussed in section 4.1, $D_{\pi V}$ being negative and significant means that the underlying technology does not exhibit diminishing marginal productivity (as mentioned in section 7, the multi-output production function is not concave in outputs). It also indicates a strong incentive to specialise. As such, the concavity component $D_{\pi V}$ works against the complementarity component $D_{\pi C}$. Yet, the complementarity component dominates, thus generating a small but positive total value D_{π}: $D_{\pi} = 339.39$. From an expected income viewpoint, this implies a weak incentive to diversify. However, this effect is not statistically significant, reflecting the fact that the complementarity component $D_{\pi C}$ and the concavity component almost cancel each other.

Second, Table 4 reports the effects of variance on diversification incentives, as measured by D_{Rvar} obtained from equation (9b) using the variance component of the risk premium R_{var}. Following the decomposition in equations (13) and (14), Table 4 shows the three variance components of D_{Rvar}: $D_{Rvar} = D_{RCvar} + D_{RSvar} + D_{RVvar}$. Note that none of these estimated effects are found to be statistically significant. It indicates that the effects of production decisions on variance outcomes are not an essential part of farm diversification strategies. This is an important result. It applies to a risk-averse decision maker exhibiting $R_{var} > 0$. It means that, in the context of managing production risk, the effects of diversification on R_{var} are not particularly strong. This raises questions about the focus on variance effects in much of previous literature on risk diversification.

Third, consider the effects of skewness on diversification incentives, as measured by $D_{R,skew}$ obtained from equation (9b) using the skewness component of the risk premium R_{skew}. Following equations (13) and (14), Table 4 reports the three skewness components of D_{Rskew}: $D_{Rskew} = D_{RCskew} + D_{RSskew} + D_{RVvar}$. Each of these effects is found to be statistically significant at the 1% level. This provides strong evidence that skewness effects are important in the analysis of farm diversification strategies. Both the complementarity component and the scale component are found to be positive: $D_{RCskew} = 72.81$ and $D_{RSskew} = 584.87$. This latter effect indicates that scale effects matter in diversification decisions. As larger farms face greater downside risk exposure, smaller and more diversified farms manage to reduce their downside risk exposure, thus contributing to diversification incentives. However, the estimated concavity component $D_{RVskew} = -52.27$ is negative, contributing to specialisation incentives. The net effect is positive and statistically significant, $D_{RCskew} = 605$. This is due mostly to the dominating scale effect. This establishes positive linkages between downside risk management and diversification incentives.

Table 4 also reports the combined risk effects $D_R = D_{Rvar} + D_{Rskew}$, along with their complementarity, scale and concavity components. The estimated $D_R = 804.51$ is positive, due in large part to the skewness scale effect, $D_{RSskew} = 584.87$. It shows that, overall, risk considerations provide positive diversification benefits. Such benefits get added to the mean diversification benefits, $D_\pi = 339.39$, to generate positive economies of diversification, $D = 1{,}143.90$ (see Table 4). This estimated diversification benefit D amounts to 17% of the expected revenue for an average farm in our sample. This indicates an overall incentive to diversify. This is consistent with the fact that most Ethiopian farms are highly diversified. Our estimates show useful information on the sources of diversification benefits. These benefits come in large part from two sources: large mean complementarity effects, $D_{\pi C} = 1{,}363.54$, and skewness scale effects, $D_{RSskew} = 584.87$. However, these positive effects are somewhat muted by negative mean (non)-concavity effects, $D_{\pi V} = -1{,}247.10$, that work against them and reduce the incentive to diversify.

9. Conclusion

This article has investigated the economics of farm diversification. Its main contribution is the integration of two economic rationales for diversification: economies of scope, and risk management. Our integration defines economies of diversification using a certainty equivalent. The analysis distinguishes between the role of expected profit (or scope economies), and a risk premium (capturing the role of risk management). We further decompose each of these components into complementarity, scale and concavity effects. We show that, when focusing on scope economies, our approach reduces to the analysis presented by Baumol (1977), Baumol *et al.* (1982) and Willig (1979). Our approach identifies scale effects and concavity/convexity effects related to risk. These effects are apparently new in the diversification literature.

This article illustrates the usefulness of the approach in an empirical application to diversification decisions on Ethiopian farms. In this context, we present an empirically tractable method to assess diversification benefits, including productivity/scope as well as risk effects. The historical record of Ethiopian famines motivates our focus on production uncertainty. It makes diversification on Ethiopian farms a valuable case study. Our econometric analysis evaluates both the productivity and risk aspects of diversification decisions. And our risk assessment involves variance as well as skewness (capturing the role of downside risk exposure). On the productivity/scope side, we find large complementarity benefits, providing incentives to diversify. This is tempered by (non)-concavity effects that provide incentives to specialise. The complementarity component is found to dominate, thus generating a (weak) incentive to diversify. The analysis also documents how risk affects diversification. It finds that skewness effects (related to downside risk management) are more important than variance effects in diversification decisions. The estimated diversification benefit amounts to 17% of the expected revenue for an average farm. This indicates an overall incentive to diversify. The analysis provides new and useful insights on existing economic tradeoffs between farm diversification and specialisation.

References

Antle, J. M. 'Testing the stochastic structure of production: A flexible moment-based approach', *Journal of Business and Economic Statistics*, Vol. 1, (1983) pp. 192–201.

Antle, J. M. and Goodger, W. J. 'Measuring stochastic technology: The case of Tulare milk production', *American Journal of Agricultural Economics*, Vol. 69, (1987) pp. 509–522.

Arrow, K. J. *Aspects of the Theory of Risk-Bearing*. (Helsinki: Yrjo Jahnsson Saatio, 1965).

Barrett, C. B., Reardon, T. and Webb, P. 'Nonfarm income diversification and household livelihood strategies in rural Africa: Concepts, dynamics and policy implications', *Food Policy*, Vol. 26, (2000) pp. 315–331.

Battese, G. E. 'A note on the estimation of Cobb–Douglas production functions when some explanatory variables have zero values', *Journal of Agricultural Economics*, Vol. 48, (1997) pp. 250–252.

Baumol, W. J. 'On the proper cost tests for natural monopoly in a multiproduct industry', *American Economic Review*, Vol. 67, (1977) pp. 809–822.

Baumol, W. J., Panzar, J. C. and Willig, R. D. *Contestable Markets and the Theory of Industry Structure* (New York: Harcourt Brace Jovanovich, Inc., 1982).

Berger, P. G. and Ofek, E. 'Diversification's effects on farm value', *Journal of Financial Economics*, Vol. 37, (1995) pp. 39–65.

Berger, A. N., Hanweck, G. A. and Humphrey, D. B. 'Competitive viability in banking: Scale, scope and product mix economies', *Journal of Monetary Economics*, Vol. 20, (1987) pp. 501–520.

Blume, M. and Friend, I. 'The asset structure of individual portfolios and some implications for utility functions', *Journal of Finance*, Vol. 30, (1975) pp. 585–603.

Calvet, L. E., Campbell, J. Y. and Sodini, P. 'Down or out: Assessing the welfare cost of household investment mistakes', *Journal of Political Economy*, Vol. 115, (2007) pp. 707–747.

Campbell, J. Y. 'Household finance', *Journal of Finance*, Vol. 61, (2006) pp. 1553–1604.

Chambers, R. G., Chung, Y. and Färe, R. 'Benefit and distance functions', *Journal of Economic Theory*, Vol. 70, (1996) pp. 407–419.

Chavas, J. P. *Risk Analysis in Theory and Practice* (London: Elsevier Academic Press, 2004).

Chavas, J. P. and Aliber, M. 'An analysis of economic efficiency in agriculture: A nonparametric approach', *Journal of Agricultural and Resource Economics*, Vol. 18, (1993) pp. 1–16.

Cohn, E., Rhine, S. L. and Santos, M. C. 'Institutions of higher education as multi-product farms: Economies of scale and scope', *Review of Economics and Statistics*, Vol. 71, (1989) pp. 284–290.

De Groot, H., McMahon, W. W. and Volkwein, J. F. 'The cost structure of American research universities', *Review of Economics and Statistics*, Vol. 73, (1991) pp. 424–431.

Demeke, M., Kelly, V., Jayne, T. S., Said, A., LeValee, J. C. and Chen, H. 'Agricultural Market Performance and Determinants of Fertilizer Use in Ethiopia', Grain Market Research Project, Working paper No. 10 (Ethiopia: Addis Ababa, 1998).

Di Falco, S. and Chavas, J.-P. 'On crop biodiversity, risk exposure and food security in the highlands of Ethiopia', *American Journal of Agricultural Economics*, Vol. 91, (2009) pp. 599–611.

Ellis, F. 'The determinants of rural livelihood diversification in developing countries', *Journal of Agricultural Economics*, Vol. 51, (2000) pp. 289–302.

Evans, D. S. and Heckman, J. J. 'A test for subadditivity of the cost function with an application to the bell system', *American Economic Review*, Vol. 74, (1984) pp. 615–623.

Färe, R. and Grosskopf, S. 'Theory and application of directional distance functions', *Journal of Productivity Analysis*, Vol. 13, (2000) pp. 93–103.

Farrell, M. 'The measurement of productive efficiency', *Journal of the Royal Statistical Society*. Series A, General, Part 3, Vol. 120, (1957) pp. 253–281.

Fernandez-Cornejo, J., Gempesaw, C. M. II, Elterich, J. G. and Stefanou, S. E. 'Dynamic measures of scope and scale economies: An application to German agriculture', *American Journal of Agricultural Economics* Vol. 74, (1992) pp. 329–342.

Ferrier, G. D., Grosskopf, S., Hayes, K. J. and Yaisawarng, S. 'Economies of diversification in the banking industry', *Journal of Monetary Economics*, Vol. 31, (1993) pp. 229–249.

Freund, R. J. 'The introduction of risk into a programming model', *Econometrica*, Vol. 24, (1956) pp. 253–263.

Gebre Egziabher, T. B. 'Ethiopian vegetation: Past, present and future trends', in *Proceedings of the Second International Symposium on the Flora of Ethiopia and Plants of Eastern and North Eastern Africa and Southern Arabia* (Addis Ababa University, 1991).

Goetzmann, W. N. and Kumar, A. 'Equity portfolio diversification', *Review of Finance*, Vol. 12, (2008) pp. 433–463.

Gollier, C. *The Economics of Risk and Time* (Boston: The MIT Press, 2001).

Griffiths, W. E. and Anderson, J. R. 'Using time-series and cross-section data to estimate a production function with positive and negative marginal risks', *Journal of the American Statistical Association*, Vol. 77, (1982) pp. 529–536.

Hamermesh, D. S. and Pfann, G. A. 'Adjustment costs in factor demand', *Journal of Economic Literature*, Vol. 34, (1996) pp. 1264–1292.

Harlan, J. R. *Crops and Man* (Madison, Wisconsin: American Society of Agronomy, 1992).

Heady, E. O. 'Diversification in resource allocation and minimization of income variability', *Journal of Farm Economics*, Vol. 34, (1952) pp. 482–496.

Johnson, S. R. 'Reexamination of the farm diversification problem', *Journal of Farm Economics*, Vol. 49, (1967) pp. 610–621.

Just, R. E. and Pope, R. D. 'Production functions estimation and related risk considerations', *American Journal of Agricultural Economics*, Vol. 61, (1979) pp. 276–284.

Katchova, A. L. 'The farm diversification discount', *American Journal of Agricultural Economics*, Vol. 87, (2005) pp. 984–994.

Klette, T. J. 'R&D, scope economies and plant performance', *Rand Journal of Economics*, Vol. 27, (1996) pp. 502–522.

Lin, W., Dean, G. W. and Moore, C. V. 'An empirical test of utility vs. profit maximization in agricultural production', *American Journal of Agricultural Economics*, Vol. 56, (1974) pp. 497–508.

Luenberger, D. *Microeconomic Theory* (New York: McGraw-Hill, Inc., 1995).

Markowitz, H. *Portfolio Selection: Efficient Diversification of Investment* (New York: John Wiley & Sons, 1959).

Menezes, C., Geiss, G. and Tressler, J. 'Increasing downside risk', *American Economic Review*, Vol. 70, (1980) pp. 921–932.

Meyer, J. 'Two-moment decision models and expected utility', *American Economic Review*, Vol. 77, (1987) pp. 421–430.

Milgrom, P. and Roberts, J. 'The economics of modern manufacturing, technology, strategy and organization', *American Economic Review*, Vol. 80, (1990) pp. 511–528.

Misra, A. K., El-Osta, H. S. and Sandretto, C. L. 'Factors affecting farm enterprise diversification', *Agricultural Finance Review*, Vol. 64, (2004) pp. 151–166.

Mitton, T. and Vorkink, K. 'Equilibrium underdiversification and the preference for skewness', *Review of Financial Studies*, Vol. 20, (2007) pp. 1255–1288.

Mundlak, Y. 'On the pooling of time series and cross section data', *Econometrica*, Vol. 46, (1978) pp. 69–85.

Paul, C. J. M. and Nehring, R. 'Product diversification, production systems, and economic performance in U.S. agricultural production', *Journal of Econometrics*, Vol. 126, (2005) pp. 525–548.

Pope, R. D. and Prescott, R. 'Diversification in relation to farm size and other socio-economic characteristics', *American Journal of Agricultural Economics*, Vol. 62, (1980) pp. 554–559.

Pratt, J. W. 'Risk aversion in the small and in the large', *Econometrica*, Vol. 32, (1964) pp. 122–136.

Prior, D. 'Technical efficiency and scope economies in hospitals', *Applied Economics*, Vol. 28, (1996) pp. 1295–1301.

Samuelson, P. A. 'General proof that diversification pays', *Journal of Financial and Quantitative Analysis*, Vol. 2, (1967) pp. 1–13.

Smale, M., Hartell, J., Heisey, P. W. and Senauer, B. 'The contribution of genetic resources and diversity to wheat production in the Punjab of Pakistan', *American Journal of Agricultural Economics*, Vol. 80, (1998) pp. 482–493.

Tillman, D. and Kareiva, P. M. *Spatial Ecology: The Role of Space in Population Dynamics and Interspecific Interactions* (Princeton, NJ: Princeton University Press, 1997).

Tobin, J. 'Liquidity preference as behavior toward risk', *Review of Economic Studies*, Vol. 67, (1958) pp. 65–86.

Topkis, D. M. *Supermodularity and Complementarity* (Princeton, NJ: Princeton University Press, 1998).

Unruh, J. D. 'The dilemma of African agrobiodiversity: Ethiopia and the role of food insecurity in conservation', in *Proceedings of the International Symposium Managing Biodiversity in Agricultural Ecosystems* (Canada: Montreal, November 2001).

Vavilov, N. I. 'The origin, variation, immunity breeding of cultivated plants. Translated from the Russian by K. S. Chester', *Chronica Botanica*, Vol. 13, (1951) pp. 1–364.

Willig, R. D. 'Multiproduct technology and market structure', *American Economic Review*, Vol. 69, (1979) pp. 346–351.

Wooldridge, J. M. *Econometric Analysis of Cross Section and Panel Data* (Cambridge, MA: MIT Press, 2002).

World Bank. *Ethiopia: Risk and Vulnerability Assessment*. Report No. 26275-ET (Washington, DC: World Bank, 2005).

Appendix

Proof of Lemma 1
Note that $(-x, y) \in F(e)$ implies that $S(-x, y, e) \leq 0$. It follows that

$$\mathrm{Max}\{EU(I + p \cdot y) : (-x, y) \in F(e)\} \leq \mathrm{Max}\{EU(I + p \cdot y - S(-x, y, e)(p \cdot g))\}.$$

We now need to show that the reverse inequality holds:

$$\mathrm{Max}\{EU(I + p \cdot y) : (-x, y) \in F(e)\} \geq \mathrm{Max}\{EU(I + p \cdot y - S(-x, y, e)(p \cdot g))\}.$$

Note that this inequality holds when $S(-x, y\ e) = \infty$. Next, consider the case where $S(-x, y, e) < \infty$. Then, we have $[-x, y - S(-x, y, e)\ g] \in F(e)$. It follows that

$$\mathrm{Max}\{EU(I + p \cdot y : (-x, y) \in F(e)\} \geq EU(I + p \cdot y - S(-x, y, e)(p \cdot g)),$$

which concludes the proof.

Proof of Proposition 2
The method of proof is to use some identities to decompose expression equation (9a) into components representing the complementarity, scale and concavity effects of diversification. Starting from equation (9a) and using equations (10a)–(10c), we have

$$\begin{aligned} D_\pi &= E\pi(z) - \sum_{k=1}^{K} E\pi(z^k), \\ &= E\pi(z) - \sum_{k=1}^{K-1} E\pi(-x/K, y^-_{1:k-1}, y^+_k, y^-_{k+1:K}) - E\pi(-x/K, y^-_{1:k-1}, y^+_k). \end{aligned} \tag{A1}$$

Note that

$$\begin{aligned} \sum_{k=1}^{K-1} E\pi(-x/K, y^-_{1:k-1}, y^+_k, y^+_{k+1:K}) = &\sum_{k=1}^{K-1} E\pi(-x/K, y^-_{1:k-1}, y^-_k, y^+_{k+1:K}) \\ &+ E\pi(-x/K, y^+) - E\pi(-x/K, y^-_{1:k-1}, y^+_k), \end{aligned} \tag{A2}$$

$$\sum_{k=1}^{K-1} E\pi(-x/K, y^-_{1:k-1}, y^-_k, y^-_{k+1:K}) = (K-1)E\pi(-x/K, y^-). \tag{A3}$$

Adding equations (A2) and (A3) to (A1) gives

$$\begin{aligned} D_\pi = E\pi(z) &- \sum_{k=1}^{K-1} E\pi(-x/K, y^-_{1:k-1}, y^+_k, y^-_{k+1:K}) - E\pi(-x/K, y^-_{1:K-1}, y^+_K) \\ &- \sum_{k=1}^{K-1} E\pi(-x/K, y^-_{1:k-1}, y^-_k, y^+_{k+1:K}) - E\pi(-x/K, y^+) + E\pi(-x/K, y^-_{1:K-1}, y^+_K) \\ &+ \sum_{k=1}^{K-1} E\pi(-x/K, y^-_{1:k-1}, y^+_k, y^+_{k+1:K}) - (K-1)E\pi(-x/K, y^-) \\ &+ \sum_{k=1}^{K-1} E\pi(-x/K, y^-_{1:k-1}, y^-_k, y^-_{k+1:K}). \end{aligned}$$

This can be alternatively written as

$$\begin{aligned} D_\pi = & \sum_{k=1}^{K-1} E\pi(-x/K, y^-_{1:k-1}, y^+_k, y^+_{k+1:K}) - \sum_{k=1}^{K-1} E\pi(-x/K, y^-_{1:k-1}, y^-_k, y^+_{k+1:K}) \\ & - \left[\sum_{k=1}^{K-1} E\pi(-x/K, y^-_{1:k-1}, y^+_k, y^-_{k+1:K}) - \sum_{k=1}^{K-1} E\pi(-x/K, y^-_{1:k-1}, y^-_k, y^-_{k+1:K})\right] \\ & + E\pi(z) - KE\pi(-x/K, y/K) + KE\pi(-x/K, y/K) \\ & - E\pi(-x/K, y^+) - (K\text{-}1)E\pi(-x/K, y^-), \end{aligned} \tag{A4}$$

which gives equations (11) and (12).