Condorcet Meets Ellsberg

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Background

Strategic voting can aggregate information:

Theorem

Suppose that
(i) voters are subjective expected utility maximizers with common values and a common prior,
(ii) there is an informative signal with conditionally independent distribution, and
(iii) the common prior assigns positive probability to all states.
Then there is an equilibrium in which the correct candidate is elected with arbitrarily high probability as the number of voters goes to ∞.

Versions of this proved by Feddersen-Pesendorfer (1997), Myerson (1998), Wit (1998), etc.

Ambiguous Policies

Subjective expected utility (SEU) fails to allow concern for "ambiguity," "vagueness" or "robustness" (Ellsberg Paradox)

- A policy to cap carbon emissions deals with unknown base case emissions, unknown costs, and unknown tails of the probability distribution
- The recession of 2008-2009 resulted at least in part from an unprecedented event (systematic default in AAA rated bonds) in the credit market. Decision to bail out companies based on poorly understood connection between this event, these companies, and the financial system as a whole
- The 2003 invasion of Iraq based on information about the presence of WMDs of dubious quality

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Preference

- Solution: allow for ambiguity aversion. Adopt "max-min expected utility" (MEU): voters maximize

$$\min_{p \in \Pi} \mathbb{E}_p[u(f)]$$

- Properties of MEU:
  - Allows for Ellsberg-reversals
  - Subjective expected utility is a special case
  - Value to certainty across states; this can lead to "hedging" and *strict* preference for randomization

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Example

- Suppose that $n = 11$ voters must decide between policies convict (C) or acquit (A)
- Two states (Guilty (G) or Not Guilty (N)) and two signals $(1, 2)$
  - Each voter observes signal 1 with probability of .6 in state G and .4 in state N independently of the realizations of all other signals
- Common values: everyone agrees convicting if guilty and acquitting if not guilty are best
- CJT says that they can do better by having an election than having a benevolent but privately informed dictator

Example (SEU)

- Suppose prior q s.t. $q(G) = q(N) = .5$
  - Posterior after observing 1:

$$q_1(G) = \frac{r(1|G)q(G)}{r(1|G)q(G) + r(1|N)q(N)} = \frac{.6 * .5}{.6 * .5 + .4 * .5} = .6$$

  - Posterior after observing 2: $q_2(G) = .4$
- Consider strategies so that $\sigma_1^*(C) = 1$ and $\sigma_2^*(C) = 0$ ("Sincere voting")
- In each state, probability of selecting right policy is

$$\sum_{x=6}^{11} \binom{11}{x} .6^x .4^{11-x} \approx 0.753$$

  - Ex ante, expected utility is $\frac{1}{2}0.753 + \frac{1}{2}0.753 = 0.753$
- This maximizes ex-ante "social" welfare. By McLennan (1998) Theorem 1, this is an equilibrium

Example (MEU)

- Suppose set of priors, $\Pi = \{q : q(G) \in [.39, .61]\} \equiv [.39, .61]$
- Bayesian update all measures in Π to form a set of posteriors after observing signal (voters are dynamically consistent)
- Sets of posteriors: $\Pi_1 = [.49, .7]$ and $\Pi_2 = [.3, .51]$
- σ^* still maximizes ex-ante welfare, but σ^* is not an equilibrium
    - Note that if all signals were observed, everyone would be almost certain which state it is

Example (MEU, con't)

Fix any voter; suppose all other voters play σ^*

- Voter only changes outcome if she's pivotal: there are exactly 5 votes for C and 5 for A
- Pivotal in either state with probability

$$p = \binom{10}{5} .6^5 .4^5 \approx .201$$

- Correct candidate elected independent of her vote with probability

$$\theta = \sum_{x=6}^{10} \binom{10}{x} .6^x .4^{10-x} \approx .633$$

Example (MEU, con't)

Suppose the voter observes signal 1:

- If she votes for C, she gets

$$\min_{q\in[.49,.7]} q(p+\theta)+(1-q)(\theta) = .49p+\theta$$

- If she votes for A, she gets

$$\min_{q\in[.49,.7]} q(\theta)+(1-q)(\theta+p) = .3p+\theta$$

- If she votes for each with equal probability, she gets

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Result

Theorem

There are games of common interest with MEU players where a strategy profile maximizes the ex-ante utility function but is not an equilibrium.

- Contrasts with McLennan (1998), Theorem 1, which says that in *any* game of common interest with *SEU* players, *any* strategy profile that maximizes the ex-ante utility function is an equilibrium
  - Preferences are dynamically consistent and consequential
  - McLennan's result uses sure thing principal or law of iterated expectations, neither of which necessarily hold for MEU

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An Equilibrium

- One equilibrium is given by

$$\sigma_t(C) = \sigma_t(A) = \frac{1}{2}$$

for both $t = 1$ and $t = 2$

- In this equilibrium, defendant convicted and acquitted with equal probability regardless of which state obtains
- No information aggregation
- Main result will show that no equilibrium does better in both states, even as $n \to \infty$

Game

Static game, but can be thought of as an abstraction of:

1. Nature chooses number of players via a Poisson distribution with mean n
2. Nature chooses $\omega \in \{a, b\}$ and picks a signal $t \in T$ independently for each player according to $r(\cdot|\omega)$
3. A player of type t has a set Π_t of posteriors over $\{a, b\}$
4. Each player votes for better candidate $c \in \{A, B\}$ according to her (correct) beliefs about how the others will vote and her set of posteriors Π_t
5. Candidate with the most votes wins (tie: coinflip) and implements policy
6. State realized and good or bad policy obtains

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Preferences

- "Instrumental voters": only outcome matters
- Two consequences: good policy ($u(Good) = 1$) or bad policy ($u(Bad) = 0$)
  - In state a, everyone prefers candidate A wins
  - In state b, everyone prefers candidate B wins
- Utility of playing σ when others play σ^* is

$$V_t(\sigma;\sigma^*) = \min_{q \in [p_t, q_t]} [q \sum_{c \in \{A,B\}} \sigma(c) Pr(A \text{ wins} | a, vote_c, \sigma^*) + \\ + (1-q) \sum_{c \in \{A,B\}} \sigma(c) Pr(B \text{ wins} | b, vote_c, \sigma^*)]$$

Equilibrium

A strategy profile σ^* is an equilibrium for Γ if

$$\sigma^*(t) \in \arg\max_{\sigma \in \Delta\Omega} V_t(\sigma; \sigma^*)$$

for every $t \in T$.

Theorem

An equilibrium exists for any Γ.

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Information aggregation

For rest of talk, consider only sequences $(\Gamma_n)_{n=1}^{\infty}$ so that Γ_n and $\Gamma_{n'}$ differ only in the expected number of players

Definition

A sequence of ambiguous voting games $(\Gamma_n)_{n=1}^{\infty}$ satisfies *Full Information Equivalence (FIE)* if there exists a sequence of strategy profiles $(\sigma_n)_{n=1}^{\infty}$ so that σ_n is an equilibrium for Γ_n and for any $\varepsilon > 0$ there exists an N so $n > N$ implies the correct candidate is elected in each state with probability higher than $1 - \varepsilon$ when σ_n is played.

Baseline

Theorem

Consider a sequence of ambiguous voting games $(\Gamma_n)_{n=1}^{\infty}$*. If*
(i) all voters are SEU,
(ii) there is a common prior Q *so that* $Q(A), Q(B) > 0$*, and*
(iii) $r(t|A) \neq r(t|B)$ *for some* $t \in T$ *and* $r(t|\omega) > 0$ *for every* $t \in T$
then $(\Gamma_n)_{n=1}^{\infty}$ *satisfies FIE.*

(See Myerson (1998), Theorem 2)

- Common priors is not essential. As long as each signal leads to an interior posterior distribution over the states (that assigns positive probability to both states), the theorem holds

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Main Result

Theorem

If $(\Gamma_n)_{n=1}^{\infty}$ is a sequence of ambiguous voting games so that $p_t < \frac{1}{2} < q_t$ for all $t \in T$, then $(\Gamma_n)_{n=1}^{\infty}$ does not satisfy FIE. In particular, for n large enough, there is no equilibrium so that A is the expected winner in state a and B is the expected winner in state b.

- For the second part, n need not be "too large"
- In "symmetric" case, any n will do

- Regardless of the population size, the outcome of the election does not contain as much information about true state of the world:

$$Pr(a|A_{wins}) \nrightarrow 1$$

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Failure of FIE

- No condition on distribution of signals or number of signals
- Assumption that $p_t < \frac{1}{2} < q_t$ for all $t \in T$ drives result
  - Each voter expresses likelihood judgments via betting preferences: $a \succ_t b$, $a \sim_t b$ or $b \succ_t a$
  - Voters "lack confidence" in this judgment:
    - $\frac{1}{2}a + \frac{1}{2}b \succ_t a$ and $\frac{1}{2}a + \frac{1}{2}b \succ_t b$
    - Suppose that some voter were made a dictator. This voter would strictly prefer to pick the policy implemented by flipping a fair coin regardless of the signal she observes
    - If we offer the voter a bet that the state is a or b with odds sufficiently close to fair, she would refuse to take either side of the bet (regardless of the stakes)

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FIE without SEU

Definition

An ambiguous Poisson game has *disjoint* posteriors* if $[p_t, q_t] \cap [p_{t'}, q_{t'}]$ (is either empty or) is contained in the boundary of both sets for all t and t' in T.

Theorem

Consider a sequence of ambiguous voting games $(\Gamma_n)_{n=1}^{\infty}$. If
(i) each Γ_n has disjoint posteriors,*
(ii) each posterior has full support, and
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Comparison

Consider some ambiguous voting game Γ:

1. If Γ has singleton posteriors, then all voters act as SEU maximizers and none strictly prefer to randomize for any strategy profile
2. If Γ has disjoint* posteriors, then at most one type of voter strictly prefers to randomize
3. If Γ has voters who lack confidence, then there is a strategy profile so that all voters strictly prefer randomizing to playing a pure strategy

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Assume prior-by-prior Bayesian updating

- Posterior "beliefs" determined by both prior beliefs and precision of information
- Suppose $T = \{1,2\}$ and $r(1|a) = r(2|b)$
  - If $\Pi = [.49, .51]$, voters lack confidence whenever $r(1|a) \in (.49, .51)$ and have disjoint* posteriors otherwise
  - If $\Pi = [.01, .99]$, voters lack confidence whenever $r(1|a) \in (.01, .99)$ and have disjoint* posteriors otherwise
- Very precise signals ($\frac{r(t|b)}{r(t|a)}$ very high or very low for some t) or little ambiguity (Π "close" to singleton) make it likely that information can aggregate

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Intuition

- Need election to be "close enough" that worst case scenario for the voter changes with her vote
  - Otherwise, voters act as if SEU and Myerson's Theorem 2 applies
  - Similar to Epstein and Wang (1994)'s condition for indeterminacy of asset prices
- Recall example: minimizing measure different when voting for A than when voting for B
  - $p_t < \frac{1}{2} < q_t$ for all $t \in T$ implies that this happens in any equilibrium

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- If worst case scenario changes with vote, magnitude of "swing voter's curse" (Feddersen and Pesendorfer (1996)) increases
  - In SEU elections, swing voter's curse says that less informed voters strictly prefer to abstain rather than vote
  - Abstention increases efficiency of election (higher percentage of "informed" votes)
- With ambiguity, all voters (not just the more informed) want to abstain
  - Instead, each randomizes to insure herself against altering the outcome of the election for the worse
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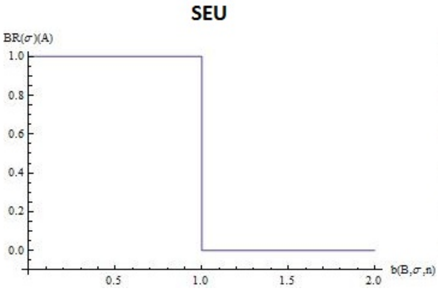
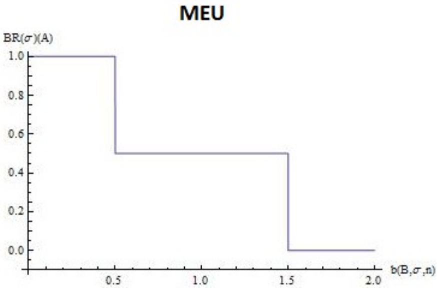
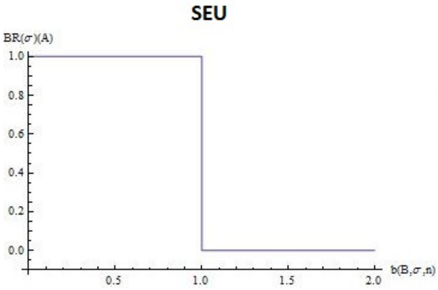


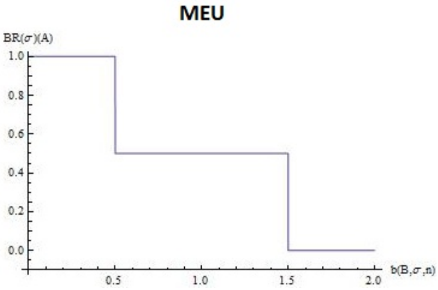


$$b(B,\sigma_n,n) = \frac{Pr(Piv_B|b,\sigma,n) + Pr(Piv_A|b,\sigma,n)}{Pr(Piv_B|a,\sigma,n) + Pr(Piv_A|a,\sigma,n)}$$

- Flat section implies strict preference for randomization
- Voter insures herself by mixing so that conditional expected utilities are equal

Illustration





$$b(B,\sigma_n,n) = \frac{Pr(Piv_B|b,\sigma,n) + Pr(Piv_A|b,\sigma,n)}{Pr(Piv_B|a,\sigma,n) + Pr(Piv_A|a,\sigma,n)}$$

- Flat section implies strict preference for randomization
- Voter insures herself by mixing so that conditional expected utilities are equal

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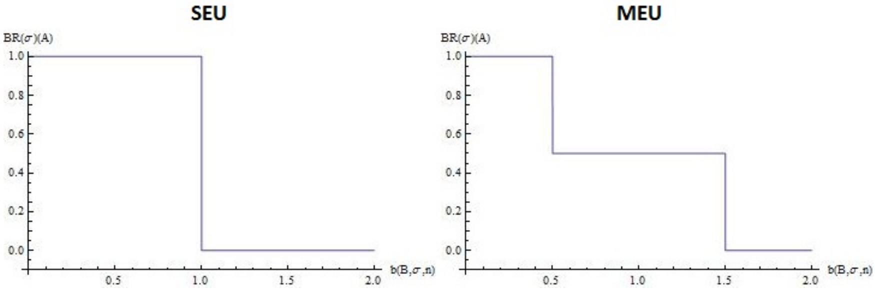

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Extension: Strategic Abstention

- Previous result assumes that all voters must vote
- Allowing voters to abstain "strategically" typically improves outcome of election
- Can I obtain an analogous result when voters may opt to abstain?

Setup

- Define an *ambiguous voting game with abstention (AVGA)* as above, except the action set is $C = \{A, B, \emptyset\}$, where:
  - $\emptyset$ is abstain
  - A is vote for A
  - B is vote for B
- Simplifying assumptions:
  - Bayesian posteriors: for every $t \in T$,

$$\Pi_t = \{\frac{r(t|a)\pi(a)}{r(t|a)\pi(a) + r(t|b)\pi(b)} : \pi \in \Pi\}$$

  - There are two types ($T = \{1, 2\}$)

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Result

Theorem

If $(\Gamma_n)_{n=1}^{\infty}$ is a sequence of AVGAs with voters who lack confidence, then $(\Gamma_n)_{n=1}^{\infty}$ does not satisfy FIE for "most" signal structures

- Non-aggregation result robust to strategic abstention

Theorem

If Γ is an AVGA that has voters who lack confidence, the strategy profile σ^* defined by $\sigma^*(t)(\emptyset) = 1$ for every $t \in \{1, 2\}$ is an equilibrium for Γ

- Extreme swing-voter's curse

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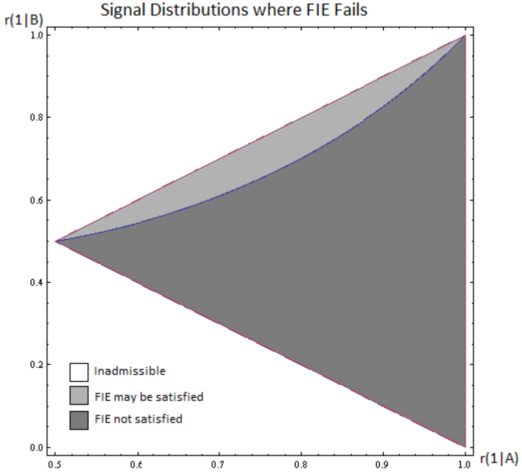
Clarifying "most"

Normalize so that $r(1|a) + r(1|b) \geq r(2|a) + r(2|b)$ and $r(1|a) \geq r(1|b)$:

Figure 1:



Related Literature

- CJT: Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1996,1997,1999), Myerson (1998), McLennan (1998), Wit (1998), many others
- Failures of CJT: Feddersen and Pesendorfer (1997), Bhattacharya (2008), Mandler (2011)
- Political economy applications of ambiguity aversion: Berliant and Konishi (2005), Ghirardato and Katz (2006), Ashworth (2007), Bade (2010)
- Failure of information transmission: Condie and Ganguli (2011)