Correlation Misperception in Choice

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Motivation

"The debt collectors at Deutschebank sensed the bond traders at Morgan Stanley misunderstood their own trade. They weren't lying; they genuinely failed to understand the nature of the subprime CDO. The correlation among triple-B-rated subprime bonds was not 30 percent; it was 100 percent. When one collapsed, they all collapsed, because they were all driven by the same broader economic forces."

–Michael Lewis, *The Big Short*

Motivation

- Trader chooses between:
  1. The 500 stocks of the S&P 500 (in right proportion)
  2. One share of S&P 500 index fund
- Usually, no difference (other than transaction costs) between owning the stocks and owning the index fund
- But reasonable to see strict preference between the two
- Misperceiving the correlation between assets implies not equivalent

Motivation

- Thought experiment: choice between
  1. \$100 for sure, and
  2. the combination of b_C and b_F, where

$$b_C = \begin{cases} \$100 & \text{if high temp. here tomorrow } \geq 20^\circ C \\ \$0 & \textit{otherwise} \end{cases}$$

  and

$$b_F = \begin{cases} \$100 & \text{if high temp. here tomorrow } < 68^\circ F \\ \$0 & \textit{otherwise} \end{cases}$$

Illustration

- Suppose 100 preferred to having **both** b_C and b_F ($\langle b_C, b_F \rangle$)
- Let Ω be the set of possible high temperatures
- If each portfolio were reduced to a standard act on Ω, then $\langle 100 \rangle \succ \langle b_C, b_F \rangle$ is impossible
  - $20^\circ C = 68^\circ F$, so $100 = b_C(\omega) + b_F(\omega)$ for all $\omega \in \Omega$
  - The portfolios $\langle b_C, b_F \rangle$ and $\langle 100 \rangle$ reduce to the same act, implying indifference
- One explanation: misperception of correlation
  - thinks b_C and b_F are independent instead of negatively correlated
- When can we attribute $\langle 100 \rangle \succ \langle b_C, b_F \rangle$ to misperception?
  - many alternative explanations

Illustration: Formal Setting

- Key Ingredients (primitives):
  1. State space Ω
     - ⋆ Describes objective reality in relationships between assets
  2. Set $\mathcal{A}$ of assets
     - ⋆ Each asset a gives a return of $a(\omega) \in \mathbb{R}$ in state ω
  3. Portfolios of assets, e.g. $\langle a_1, a_2, ..., a_n \rangle$
     - ⋆ cares about the overall payoff
     - ⋆ overall payoff equals the sum of returns of underlying assets
  4. Trader who maximizes preference $\succsim$ over portfolios
     - ⋆ Ranks every portfolio of assets

Illustration: Key Behavior

- $\langle 100 \rangle \succ \langle b_C, b_F \rangle$ violates "Monotonicity"
  - for **every** $\omega \in \Omega$, $b_C(\omega) + b_F(\omega) \geq 100(\omega)$
- With misperception, Monotonicity is too demanding
  - In fact, it implies reduction as above
- If misperception drives this violation, then she still satisfies "Weak Monotonicity"; for any assets a, b, c:
  - if $\langle b, c \rangle$ always yields a better outcome than $\langle a \rangle$ for **every possible joint distribution** over a, b, c, then $\langle b, c \rangle \succsim \langle a \rangle$
    - ⋆ $\min_\omega b(\omega) + \min_\omega c(\omega) \geq \max_\omega a(\omega) \implies \langle b, c \rangle \succsim \langle a \rangle$
    - ⋆ $\min_\omega a(\omega) \geq \max_\omega b(\omega) + \max_\omega c(\omega) \implies \langle a \rangle \succsim \langle b, c \rangle$
  - **any** individual violation of Monotonicity can be attributed to misperception of correlation

Illustration: Main Results

We consider a DM who satisfies Weak Monotonicity as well as order, independence, and continuity. She **acts as if** she:

1. has beliefs about joint distribution of actions described by a probability measure π defined on product state space $\Omega^{\{a,b,c\}}$
   - she thinks $\langle a, b\rangle$ returns $a(\omega_1) + b(\omega_2)$ with probability

$$\pi(\omega_a = \omega_1 \ \& \ \omega_b = \omega_2)$$

2. has tastes described by utility index u
   - risk attitude plays role in identifying π
3. maximizes expected utility, given π and u

Formally, $\succsim$ is represented by V where

$$V(\langle a, b\rangle) = \int_{\Omega^{\{a,b,c\}}} u(a(\omega_a) + b(\omega_b))\pi(\vec{\omega})$$

Illustration: Main Results

Equivalent procedure easier to apply and allows tighter identification

- Trader **endogenously** splits assets into "understanding classes"
  - In basic representation, the trader "has" $|\mathcal{A}|$ copies of the original Ω; now, she "has" many fewer copies
- She has beliefs about the correlation between classes of assets
  - Correlation **within** a class **correctly perceived**
  - Correlation **across** classes (potentially) **misperceived**
  - π defined on product space indexed by classes rather than assets
  - If two assets belong to the same understanding class, then they depend on the same "copy" of Ω
- If each class contains **diverse enough** assets, then uniquely identified "coarsest" understanding classes and beliefs

Related literature

- Failure of logical omniscience: Lipman (1999)
- Complexity via preference for flexibility: Al-Najjar et al. (2003)
- Unforeseen contingencies: Kochov (2015)
- Framing effects: Tversky-Kahneman (1981), Ahn-Ergin (2010), Salant-Rubinstein (2008)
- Failures of inference: Piccione-Rubinstein (2003), Eyster-Rabin (2005), Jehiel (2005), Esponda (2008), Eyster-Piccione (2012), Spiegler (2014)
- Correlation misperception: DeMarzo et al. (2003), Eyster-Weizsacker (2010), Levy-Razin (2015a,b), Rubinstein-Salant (2015), Ortoleva-Snowberg (2015)
- Models related but not covered: Barberis et al (2006), Rabin-Weizsacker (2009), Esponda (2008), Spiegler (2014), Levy-Razin (2015c)

Preview

1. Formal framework
2. Behavior of interest within this framework
3. Foundations
4. Main results
5. Identification and Understanding

Framework

- An exogenous state space Ω that determines objective relationship between actions
  - e.g. payoffs in a financial market
  - e.g. structure of an incomplete info. game
- An exogenous set $X = \mathbb{R}$ of consequences
- A set $\mathcal{A}$ of actions, mappings from Ω to X (caveats)
  - e.g. security or behavioral strategy
- The set of all **action profiles** $\mathcal{F}$ over $\mathcal{A}$
  - "multi-sets" of actions (order does not matter and same action may enter many times)
  - Take actions a and b: $\langle a, b \rangle$ or $\langle b, a \rangle$
  - Take actions $a_1, a_2, ..., a_n$ is $\langle a_1, a_2, ..., a_n \rangle = \langle a_i \rangle_{i=1}^n$
- Preference $\succsim$ on $\Delta\mathcal{F}$, the set of all (finite support) **lotteries over action profiles**

Behavior of Interest

- DM fails to reduce profiles to acts
- If $\sum_{i=1}^{n} a_i(\omega) = \sum_{i=1}^{m} b_i(\omega)$, then the Savage act corresponding to $\langle a_i \rangle_{i=1}^{n}$ equals the Savage act corresponding to $\langle b_i \rangle_{i=1}^{m}$
  - $100 = b_C(\omega) + b_F(\omega)$ for all ω but $100 \succ \langle b_C, b_F \rangle$
- Observed violation of following axiom

Axiom: Reduction to Acts

If $\sum_{i=1}^{n} a_i(\omega) = \sum_{i=1}^{m} b_i(\omega)$ for all ω, then $\langle a_i \rangle_{i=1}^{n} \sim \langle b_i \rangle_{i=1}^{m}$

- Reduction to Acts implied by usual Monotonicity assumption:

Axiom: Monotonicity

If $\sum_{i=1}^{n} a_i(\omega) \geq \sum_{i=1}^{m} b_i(\omega)$ for all ω, then $\langle a_i \rangle_{i=1}^{n} \succsim \langle b_i \rangle_{i=1}^{m}$

Weak Monotonicity

- Set of **plausible realizations** of $\{c_1, ..., c_n\}$ equals

$$range(c_1) \times range(c_2) \times ... \times range(c_n).$$

- Vector of outcomes $\vec{x} = (x^a)$ s.t. a **could, in isolation,** yield x^a
  - There exists a correlation structure in which every $a \in \{c_1, ..., c_n\}$ simultaneously gives x^a with positive probability
- $\vec{x}$ is a **plausible realization of lotteries** p **and** q if it is a plausible realization of the set of all the actions included in profiles that are assigned positive probability by either p or q
  - Formally, of $\{a_j \in \{a_1, ..., a_n\} : p(\langle a_i \rangle_{i=1}^n) + q(\langle a_i \rangle_{i=1}^n) > 0\}$
- Assigns outcome to each action that arises in some profile $\langle a_i \rangle$ with $p(\langle a_i \rangle) > 0$ or $q(\langle a_i \rangle) > 0$

Weak Monotonicity

- for a plausible realization $\vec{x}$ of p and q, p **induces the lottery**

$$\left(p\left(\langle a_i\rangle_{i=1}^n\right), \langle\sum_{i=1}^n x^{a_i}\rangle\right)_{p(\langle a_i\rangle)>0} \equiv p_{\vec{x}}$$

- outcome yielded by the profile $\langle a_i\rangle_{i=1}^n$, $\sum_{i=1}^n x^{a_i}$ according to $\vec{x}$, occurs with the probability of that profile, $p\left(\langle a_i\rangle_{i=1}^n\right)$
- similarly q induces the lottery $q_{\vec{x}}$

Axiom: Weak Monotonicity

For any $p, q \in \Delta\mathcal{F}$, if for **every** plausible realization $\vec{x}$ of p and q $p_{\vec{x}} \succsim q_{\vec{x}}$, then $p \succsim q$.

- Very weak when comparing $\langle a, b\rangle$ with $\langle c\rangle$
  - Becomes: $\min a + \min b \geq \max c \implies \langle a, b\rangle \succsim \langle c\rangle$
  - Independence, and lotteries, make it a stronger assumption

Weak Monotonicity

How does this apply to:

- $\langle 100\rangle$ vs $\langle b_C, b_F\rangle$?
  - for $\vec{x} = (100, 100, 100)$: $\langle 100\rangle$ induces 100, $\langle b_C, b_F\rangle$ induces 200
  - for $\vec{x} = (100, 0, 0)$: $\langle 100\rangle$ induces 100, $\langle b_C, b_F\rangle$ induces 0
  - Weak Monotonicity **does not** impose a ranking
- $\langle 100\rangle$ vs $\langle b_C\rangle$?
  - $\langle 100\rangle$ induces 100, $\langle b_C\rangle$ induces 100 or 0
  - Weak Monotonicity implies $\langle 100\rangle \succ \langle b_C\rangle$
- $p = \frac{1}{2}\langle b_F, b_C\rangle + \frac{1}{2}0$ vs $q = \frac{1}{2}\langle b_C\rangle + \frac{1}{2}\langle b_F\rangle$?
  - p and q induce same lottery for $\vec{x} \in \{(100, 0), (0, 100), (0, 0)\}$
  - for $\vec{x} = (100, 100)$: $p_{\vec{x}} = (\frac{1}{2}, 200; \frac{1}{2}, 0)$ and $q_{\vec{x}} = (1, 100)$
  - Risk-averse DM expresses $q \succ p$ and risk-loving expresses $p \succ q$

Standard Axioms

Axioms: Mixture Space

$\succsim$ satisfies the vN-M/Herstein-Milnor Mixture space axioms:

1. $\succsim$ is complete and transitive
2. $p \succsim q \iff \alpha p + (1-\alpha) r \succsim \alpha q + (1-\alpha) r$ for $1 \geq \alpha > 0$
3. The sets $\{\alpha \in [0,1] : \alpha p + (1-\alpha) q \succsim r\}$ and $\{\alpha \in [0,1] : r \succsim \alpha p + (1-\alpha) q\}$ are closed

Representation

- Uncertainty beyond that captured by Ω relevant
- Represent by expanding the "dimension" of uncertainty

Theorem

$\succsim$ satisfies the Mixture Space Axioms and Weak Monotonicity if and only if there exists:

- *a utility index $u : X \to \mathbb{R}$ and*
- *a probability measure π on $\Omega^{\mathcal{A}}$ (with an appopriate σ-algebra)*

such that for any $p, q \in \Delta\mathcal{F}$, $p \succsim q$ if and only if

$$\sum_{p(\langle a_i \rangle)>0} V(\langle a_i \rangle) p(\langle a_i \rangle) \geq \sum_{q(\langle b_j \rangle)>0} V(\langle b_j \rangle) q(\langle b_j \rangle)$$

where

$$V(\langle a_i \rangle_{i=1}^{n}) = \int_{\Omega^{\mathcal{A}}} u\left(\sum_{i=1}^{n} a_i(\omega^{a_i})\right) \pi(d\vec{\omega})$$

Representation

- $\Omega^{\mathcal{A}}$ captures all possible correlations between actions
  - DM attaches a (possibly zero) probability to receiving $b_F(\tau_F) + b_C(\tau_C)$ from $\langle b_C, b_F\rangle$ for each τ_F, τ_C
- $\pi(\cdot)$ assigns probabilities to correlations
  - if $\pi(\tau_C \neq \tau_F) > 0$ for some τ, then DM does not think temp in Celsius perfectly correlated with temp in Fahrenheit
  - Allows $100 \succ \langle b_F, b_C\rangle$ or $\langle b_F, b_C\rangle \succ 100$
- Special cases: $\pi(\times_{i=1}^{m} E_{\nu_i} \times \Omega^{\mathcal{U}\setminus\{\nu_1,\ldots,\nu_m\}}) =$
  - $q_{SEU}(\cap_{i=1}^{m} E_{\nu_i})$ is standard model
  - $\prod_{i=1}^{m} q_{Prod}(E_{\nu_i})$ is correlation neglect model
  - $\chi q_{Prod} + (1-\chi) q_{SEU}$ is "χ-cused" model
  - $\sum_{E\in\mathcal{Q}} \prod_{i=1}^{m} q(E_{\nu_i} \cap E) q(E)$ is $\mathcal{Q}$-analogical model
    - ($\mathcal{Q}$ is a partition of Ω)
- Caveats: π might not be unique and $\Omega^{\mathcal{A}}$ far from parsimonius

Specific Instance

Representation: Equivalent procedure

DM acts as if she does the following:

1. Divides assets into subsets that are easy to understand
   - Such a subset of assets called an "understanding class"
   - DM reduces any portfolio of assets in same class to act
   - Let $\mathcal{U}$ be the set of such classes
     - ★ e.g. $\mathcal{U} = \{B_C, B_F\}$ where B_C are actions understood in terms of Celsius and B_F are actions understood in terms of Fahrenheit
2. Assigns probabilities to returns across classes
   - π defined on $\Omega^{\mathcal{U}}$ rather than Ω or $\Omega^{\mathcal{A}}$
     - ★ State is "(temp. in $^\circ F$, temp. in $^\circ C$)" rather than "temp."
     - ★ If $\pi(\tau_{B_F} = \tau_{B_C}) < 1$, then DM acts as if uncertain (or wrong) about conversion for Celsius to Fahrenheit
3. Maximizes expected utility, where all the assets in a given understanding class use the same coordinate of $\Omega^{\mathcal{U}}$

Equivalent Representation, formally

Definition

$\succsim$ has a **probabilistic correlation representation (PCR)** if

- $\mathcal{U}$ is a set of "understanding classes", subsets of $\mathcal{A}$
  - let Σ_C be the σ-algebra generated by the actions in $C \in \mathcal{U}$
- π is a probability measure defined on $(\Omega^{\mathcal{U}}, \otimes_{C\in\mathcal{U}}\Sigma_C)$
- u is a utility index

and $\succsim$ has an EU representation with utility index $V : \mathcal{F} \to \mathbb{R}$ where

$$V(\langle a_i \rangle_{i=1}^n) = \int_{\Omega^{\mathcal{U}}} u\left(\sum_{i=1}^n a_i(\omega^{C_i})\right) \pi(d\vec{\omega})$$

for any $C_1, ..., C_n \in \mathcal{U}$ with $a_i \in C_i$

- $\succsim$ has a PCR $\iff$ $\succsim$ satisfies the Mixture Space Axioms and Weak Monotonicity (equivalent representation)

Identification: How do parameters affect behavior?

1. What does DM believe about the joint distribution of actions?
2. Can we precisely characterize the extra dimensionality needed to represent the preferences of the DM?

Advantage of PCR: can provide tighter answer to these questions

- In basic representation, every action has its own understanding class; finest possible grouping
  - Set of profiles "sparse" in domain of π; no hope for uniqueness
- PCR allows more action per "dimension"
- If sufficient diversity, we can uniquely identify both coarsest correlation cover and beliefs (with caveats)

Identification

Definition

- A set $B \subset A$ is **rich** if for any $f : \Omega \to X$, there exists $c \in B$ s.t. $c(\omega) = f(\omega)$ for all ω.
- The PCR $(\mathcal{U}, \pi, u)$ is **rich** if every $C \in \mathcal{U}$ is rich.

- Rich if there are "diverse enough" actions in each class
  - e.g. trader understands connection between a stock and any of its derivatives but not necessarily between two distinct stocks
- Similar spirit to Savage assumption that all acts are conceivable
- Rich PCR allows for unique identification and exists under weak additional conditions (in paper)

Identification

Theorem

If the preference $\succsim$ has a rich PCR $(\mathcal{U}, \pi, u)$, then:

- *there exists a unique coarsest correlation cover, and*
- *π is unique if u is not a polynomial.*

- $\mathcal{U}$ is **coarsest** if there is a rich PCR with correlation cover $\mathcal{U}$ and if $(\mathcal{U}', \pi', u')$ is also a rich PCR of $\succsim$, then for any $B' \in \mathcal{U}'$, there exists $B \in \mathcal{U}$ with $B' \subseteq B$.
- Coarsest $\mathcal{U}$ is not a partition
  - every $x \in X$ belongs to every $C \in \mathcal{U}$
  - if DM knows that $0^\circ C = 32^\circ F$, then any action measurable w.r.t. freezing or not is in both Celsius and Fahrenheit classes
- When u is polynomial, uniqueness of π typically fails
  - e.g. for risk-neutral DM, only marginals matter

Implications

- Fixed DM undervalues certain profiles while overvaluing others
  - rich PCR, strictly risk-averse with same marginals
  - Fix assets a, b, c and event E so that

	$a(\cdot)$	$b(\cdot)$	$c(\cdot)$	$d(\cdot)$
$\omega \in E$	1	1	-1	2
$\omega \notin E$	-1	-1	1	-2

  - Correct evaluations: $\langle a, c\rangle \sim \langle 0\rangle$ and $\langle d\rangle \sim \langle a, b\rangle$
  - If $\langle b, c\rangle \sim \langle 0\rangle$, then $\langle a, c\rangle \succ \langle 0\rangle \iff \langle d\rangle \succ \langle a, b\rangle$
  - If underestimates safety of $\langle a, c\rangle$, underestimates risk of $\langle a, b\rangle$
- Independence requires that the DM is unsophisticated
  - If $\langle b, c\rangle \sim 0$ and $\langle a, b\rangle \sim \langle a, c\rangle$, then DM misperceives relationship between the assets
  - Sophisticated DM, recognizing misperception, may express $\frac{1}{2}\langle a, b\rangle + \frac{1}{2}\langle a, c\rangle \succ \langle a, b\rangle \sim \langle a, c\rangle$

Implications: Structured Finance

- Misperception allows tranching to alter the evaluation of CDO
- Untranched CDO: return equals the sum of underlying assets
  - Any two traders that agree on the expected value of each component asset also agree on value of the untranched CDO
  - even if they disagree about the correlation between the assets.
- Tranching changes the calculations
- Consider two tranches: senior has a claim on the first y dollars of return, junior the return in excess of y
- The expected returns calculated using indexes $u^J(x) = \max\{x - y, 0\}$ and $u^S(x) = \min\{x, y\}$
- neither is a polynomial, so all correlations relevant
- distinct assessments, even when each of the underlying assets is evaluated correctly

Implications: Structured Finance

- Consider a trader with a PCR $(\{C_i\}_{i=1}^N, u, \pi^\chi)$ where π^χ satisfies

$$\pi^\chi(\omega^{C_1}, ..., \omega^{C_N}) = \chi q(\bigcap_{i=1}^N \{\omega^{C_i}\}) + (1-\chi) \prod_{i=1}^N q(\{\omega^{C_i}\})$$

  for some probability measure q over Ω
  - q interpreted as objective distribution on Ω
  - $\chi = 1$ implies no misperception, $\chi = 0$ implies independence
- CDO is a profile $\langle a_1^n, ..., a_n^n \rangle$, where:
  - each a_i^n is a $\frac{1}{n}$ share of an asset a_i,
  - $a_i^n \in C_i$ for each i and n,
  - a_i is identical to a_j: each $a_i^n(\omega) = \frac{1}{n} a(\omega) \geq 0$ for all ω

Implications: Structured Finance

- risk neutral trader correctly evaluates an untranched $\langle a_1^n, \ldots, a_n^n \rangle$ as exactly $E_q[a]$
- Suppose CDO is tranched as above, i.e. senior tranche claims first y dollars, junior everything else u^J and u^S
- Junior tranche undervalued and Senior tranche overvalued
  - Misvaluation monotonic in n and in χ
- Profit opportunity: short the senior and go long on the junior
- Lewis (2010) reports the story of a Morgan Stanley trader adopting the opposite trade strategy and losing over $9 billion

Thank you

Representation

- Uncertainty beyond that captured by Ω relevant
- Represent by expanded the "dimension" of uncertainty
  - Cartesian product of Ω
  - Assigns each action to a copy
- Event "action a yields x and action b yields y" is

$$a^{-1}(x) \times b^{-1}(y) \times \Omega \times \Omega \times \ldots$$

- DM assigns a probability to each event as above

The Model

- For the thought experiment, the state space can equal

$$\Omega^* = \Omega^{\{C,F\}}$$

with $b_C \in C$ and $b_F \in F$

- The probability measure π defined on Ω^*, includes

$$E = \{\vec{\tau} : \tau_C \geq 0^\circ C \text{ and } \tau_F < 0^\circ C\}$$

- Chooses profile that maximizes EU with π and Ω^*
  - $V(\langle b_C, b_F \rangle) = \int_{\Omega^*} u(b_C(\tau_C) + b_F(\tau_F))\pi(d\omega^*)$
  - $V(100) = \int_{\Omega^*} u(100)\pi(d\omega^*) = u(100)$
  - If $\pi(E) \neq 0$ and DM risk averse, then $100 \succ \langle b_C, b_F \rangle$

Return I Return II

Framework: Consequences

There is a set X of outcomes with an operation $+$

- $+ : X \times X \to X$, with $+(x, y) \equiv x + y$
- There exists $0 \in X$ and for all $x \in X$, $0 + x = x$
- $+$ commutative and associative
  - $(x + y) + z = x + (y + z)$
  - $x + y = y + x$
- Subset of algebraic group, closed under $+$
  - might not include inverses

◂ Framework

Framework: Actions

There is a set $\mathcal{A}$ of **actions** (securities)

- Function ρ maps action a and state ω to consequence $\rho(a, \omega)$
- **Assume** $\rho(a, \cdot)$ is finite ranged
  - **Assume** for every $x \in X$ there is $x \in A$ s.t. $\rho(x, \omega) = x$ (constant action that gives x for sure) and write as x
- Notation: $a(\omega)$ for $\rho(a, \omega)$ and $\sigma(a)$ for $\sigma(\rho(a, \cdot))$coarsest σ-algebra by which a is measurable
- Do not need to have every possible action in $\mathcal{A}$ but require that all constant outcomes are in $\mathcal{A}$

◂ Framework

Framework

- $\mathcal{F}$ not mixture space
- Typical trick: consider horse race - roulette wheel acts
  - Does not help: hard to define mixtures and must add lotteries
- In usual framework, can replace ex-post lotteries (Fishburn, 1970) by ex-ante lotteries (Kreps, 1988; Battigali et al, 2013)
  - Monotonicity assumption less elegant to state
  - Explicit "reduction of compound uncertainty"
- $\Delta\mathcal{F}$ is a mixture space, and:
  - No addition take place over lotteries
  - Mixing between profiles does not create/destroy connections
  - Do not have to specify mixtures of actions
  - Easy to interpret; allows simple axiomatization
- Other papers use ex-ante mixtures as well, e.g. Anscombe-Aumann (1963); Seo (2009); Saito (2013/15)

◂ Framework

Understanding

- Two assumptions imply existence of rich PCR
  1. [Non-Singularity] For every a, there exists $B_a \subseteq \mathcal{A}$ where
     1. B_a is understood:
        $p \succsim q$ if $p_{\vec{x}} \succsim q_{\vec{x}}$ for every plausible realization $\vec{x}$ of p and q with the property that for $x^c = c(\omega)$ for each $c \in B_a$ for some ω
     2. B_a is rich
  2. [Strict Concavity] X is a convex set, and for any $x \neq y \in X$ and $\lambda \in (0,1)$, $(1, \lambda x + (1-\lambda) y) \succ (\lambda, x; (1-\lambda), y)$.

◂ Rich PCR