Can Discriminatory Behaviour Persist in Competitive Markets?*

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1 Introduction

This paper analyzes how 'a taste for discrimination' on the part of some employers in a competitive model of the labor market may affect the wage and hiring strategies of other firms which are not intrinsically prejudiced. In our model monitoring is costly, and firms may base their hiring decision on a worker's past employment record with other firms. We show that the presence of prejudiced firms may *increase* the cost of hiring workers belonging to minority groups to firms who are not prejudiced due to this strategic interaction and as a result may lead to a market equilibrium involving discrimination.[1]

We depart from the existing literature in two different directions. First, we allow workers to vary according to some observable characteristic unrelated to productivity (such as race, gender or caste) and allow firms to have heterogenous tastes regarding it following the literature on 'taste-discrimination' pioneered by Becker (1957). His model of the labor market is frictionless except for the presence of some prejudiced firms who are willing to hire minority workers at a lower wage rate to compensate for their disutility of having to associate with them. We study how the wage and hiring decision of firms which differ in their degree of prejudice towards minorities interact in a market setting in the presence of frictions in the form of moral hazard, and whether the presence of enough non-prejudiced

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[1] We define discrimination as an outcome in the labor market where equally productive workers end up with different levels of welfare depending on whether they possess some characteristic unrelated with productivity.

firms is sufficient to eliminate discrimination, as suggested by Becker's original analysis. Theories of "statistical discrimination" (e.g., Phelps, 1972, Lundberg and Startz, 1983 and Coate and Loury, 1993) also show how observed discriminatory practices of a firm may not have anything to with its 'tastes' as suggested by Becker.[2] But the main force driving the results have to do with multiple expectational equilibria, which is not necessarily a market phenomenon and could arise even if there was only one firm and one worker. In contrast, we show how discriminatory outcomes can result explicitly from the market interaction of firms with different degrees of prejudice, and how the equilibrium of the labour market could involve discrimination even when there are many non-prejudiced firms.

2 Detailed Literature Review

Becker (1971) proposed a model of perfect labour markets where discrimination is driven by differential preferences for white and black workers. Specifically, employers incur disutility from employing black workers. This taste for discrimination can generate a wage gap between black and white workers if they are being employed by the same firms. However, if there are a sufficient number of unprejudiced employers to hire all the black workers, then the labour market cannot sustain a race-based wage differential. Since unprejudiced employers will also make more profits and, consequently grow more quickly, the model cannot account for a race-based wage differential in the long-run (Becker 1971, Arrow 1972).

Search models of the labour market are of interest in this literature as they can potential generate not only wage differentials but also differences in unemployment spells and turnover across social groups. The search models can be classed into 'random search' models – i.e. employers and workers are randomly matched – and 'directed search' models – where workers can choose between prospective employers.

In 'random search' models, if some employers are prejudiced against black workers –

[2] If measures of an individual worker's productivity that is available to a firm when screening her are noisy, then it optimally puts some weight on moments of the distribution of the respective group-populations. Accordingly, equally productive members of different groups may treated differently if the underlying population distributions are different or firms differ in their ability to screen an individual worker across groups. More interestingly, when worker productivity is endogenous these perceived group-differences by firms can be self confirming in equilibrium.

willing to hire them only at a reduced wage or not at all – this can generate differential behaviour between black and white workers regarding current offers. Specifically, since black workers have a lower chance of a future job offer (or a lower expected wage from a future job offer), they are more inclined to accept a current offer than white workers at any given wage. Thus, all employers – not just the prejudiced ones – have the incentive to offer lower wages to black workers (Black 1995). Note that the result depends on employers having some monopsonistic power vis-à-vis workers. If multiple firms could bid for the same worker, then the wage would be driven up to the marginal product of labour (adjusting for any employer disutility due to prejudice) and so black and white workers would receive the same offers from unprejudiced firms.

Rosén (1997) develops a random search model with no taste for discrimination among employers; yet, only discriminatory equilibria are stable. When matched with a potential employer, a worker receives private information about the quality of the match. Then, for each worker, there is a threshold, such that they apply for the job only if the signal exceeds the threshold. Therefore if black workers, say, face worse prospects in the future labour market, they choose a lower threshold. Thus prospective employers have lower expectations about the quality of the match when the applicant is black; which, in turn, translates into worse labour market prospects for black workers. Of course, the same story can be told for white workers but Rosén shows that the non-discriminatory equilibrium – where whites and blacks face the same job market prospects – is unstable, because a slight preference by a single firm will cause all the others to discriminate in the same direction.

Holden and Rosén (2014) tell a similar story in the context of employment protection legislation. Unlike Rosén (1997), the model assumes symmetric information about match quality. Additional information about the match quality is revealed after the worker is hired; and if the signal is sufficiently negative, it may be in the interest of both parties to break up the relationship. But the worker will not quit until he or she has found another job and the employment protection legislation prevents the employer from firing the worker. Workers from discriminated groups will take longer to find another job and, therefore, remain with the current employer for a longer period, even after the match is revealed to be of bad quality.

Therefore, employers will be reluctant to hire workers from discriminated groups (even when there is no prejudice); thus, workers from these social groups will indeed take longer to quit a job after a bad match quality has been revealed.

Lang, Manove and Dickens (2005) develop a directed search model where firms announce a single wage (not conditional on race) and workers decide whether to apply for a particular job. All workers are equally productive but firms may experience a small disutility from hiring black workers. Then, because of the cost associated with a job application (in the form of foregone opportunities) black workers will avoid openings where white workers are likely to apply. In equilibrium, there are firms that offer a high wage and attract only white workers and firms that offer a lower wage and attract only black workers (who wish to avoid competition with whites). Thus the Lang, Manove and Dickens model can generate wage differentials driven by mild discriminatory preferences. But a potential shortcoming is that there is no wage differential within firms since whites and blacks always work in different firms.

While the models above all rely on (at least some element of) discriminatory preferences (among potential employers) to generate differences in labour market outcomes across social groups, in models of statistical discrimination these differences arise primarily because of employers' use of race (or some other group identity) for statistical inference about the quality of the worker.

A simple example would be the case where employers have greater difficulty in inferring quality within certain social groups, which means that they will favour workers from social groups where information quality is better, even if the quality distribution is identical across groups (Cornell and Welch 1996). However, such a setting immediately raises the possibility that social groups where the information quality is poorer will make observable effort to signal their true quality as in Spence's signalling model; therefore, at least at some levels of quality, the discriminated group should exert more effort in signalling.

The literature on statistical discrimination originates with Phelps (1972) and Arrow (1973). In Arrow's model, workers can make a human capital investment to become 'qualified' but prospective employers can detect qualification only by making a costly investment.

Workers can be assigned to skilled and unskilled jobs but only qualified workers can do the skilled job. 'Hiring' a worker in this model means undertaking the costly investment to detect the worker's skill level and then assigning him or her to a job accordingly. If employers expect black workers to have a lower probability of being qualified, then they will have a lower expected return of 'hiring' the black worker. Arrow argues that this will mean that qualified black workers are paid less than qualified white workers (while all unqualified workers, assigned to unskilled jobs, are paid the same). Then black workers have less incentive to make the necessary investment in human capital in the first place.

An important concern with Arrow's model is that employers pay qualified black workers less even though their qualification becomes known – and they are assigned to skilled or unskilled jobs accordingly – during the hiring process. As Coate and Loury (1993) notes "Discriminatory wages for the same work is a flagrant violation of equal-employment laws, and relatively easy to detect." In particular, a law which requires wages to be a function of qualifications and not race would remove the wage discrimination among qualified workers, and thus equalise the incentives for human capital investments for black and white workers.

Coate and Loury (1993) builds on Arrow's discriminatory model in the following manner: workers can invest in their human capital but employers receive only a noisy signal of this investment (the noisiness of the employer signal is a distinct departure from Arrow's model). In the event of an ambiguous signal, employers make inferences based on the worker's social group. Thus workers from social groups with worse human capital are at a disadvantage, which weakens their incentive to invest in the first place. This behaviour "justifies" the employers' stereotyping of social groups. The model potentially has multiple equilibria including one in which there is no negative stereotyping of any social group. However, a negative stereotyping equilibrium may be justified if a particular social group initially had low levels of human capital for (exogenous) historical reasons. One of the key results in this model is that, for certain parameter values, affirmative action policies can adversely affect investment incentives within the disadvantaged groups, such that differential levels of human capital across social groups is maintained in equilibrium.

Cavounidis and Lang (2015) is a recent example of a model of statistical discrimination that aims to reproduce a number of stylised facts regarding labour market discrimination, including a wage differential, as well as longer unemployment spells and longer employment spells for black workers. The basic idea is that if worker quality is unobserved, and employers can assess quality with noise at a cost, then they will engage in costly assessment only if their prior beliefs about the worker quality is sufficiently low. Workers who are detected to be of low quality are fired and returned to the pool of unemployed workers. If employers start off with very negative beliefs about the quality of workers from a certain social group, they will undertake costly investments to observe their quality which, in turn, will worsen the pool of unemployed workers from that social group. This can result in an equilibrium where black and white workers with the same quality distribution are treated differently, e.g. white workers do not face costly assessment, and the pool of unemployed workers is of high quality, and black workers face costly assessment and the pool of unemployed workers is of low quality.

Our proposed model is about taste discrimination and how it affects the incentives of workers to undertake some actions. Unlike Becker's taste discrimination model, we introduce endogenous behavioural choice by workers through a principal-agent relationship with hidden action. The core model has no asymmetric information about worker type or their choice of investment (as in the models by Arrow 1973 and Coute and Loury 1993; also Cornell and Welch 1996), or noisy signal on investment/action of workers where the noisiness differs in degree by social group affiliation (as in Phelps 1972), or the quality of the match between the worker and the employer (Rosén 1997). We assume imperfect labour markets but no market power for employers that would enable them to extract surplus from disadvantaged social groups (as in Black 1995). A snapshot of our contribution is that we take Becker's model, add a labour market friction in the form of moral hazard and show that taste discrimination is not eliminated by market forces.

We can work out the case of contractible effort (as in Becker) for our setup. Here, our conjecture is that Becker's insight should hold: enough neutral firms would ensure workers have the same expected payoff (even though there may be segregation). We can also add elements of statistical discrimination by making the cost of effort unobservable and heterogeneous across workers. In this case, our conjecture is that we should obtain a 'Colin Powell' effect (aka 'Obama effect'), i.e. those minority workers who make the cut despite the noise must be better quality than majority workers (this is akin to the effect highlighted by Fryer 2007, see below).

One could criticise our core model by saying taste discrimination is like a tax - if it exists, it affects incentives negatively. Our point is not that, but rather to say: in a world where people could move to other places where there are no taxes (analogy to neutral firms) and yet, because of the presence of some places where there is this tax targeted at specific people, other places also effectively tax them. The core insight is not how taste for discrimination affects behaviour of minority workers but rather, how neutral firms treat minority workers in the presence of firms with a taste for discrimination and so how a small amount of "prejudice" can be subject to a multiplier effect. If these neutral firms could commit to keep these workers forever, our conjecture is that the effect would go away because of Becker's point that you have workers who actually value you as an employer because of the presence of discriminators out there in the labour market.

As in our model, Black (1995) and Rosén (1997) obtain, within random search models, the outcome where a small fraction of prejudiced employers leads to large wage differentials between social groups. In Holden and Rosén (2014), a small fraction of prejudiced employers can generate differences in unemployment rates across social groups (but wage discrimination is ruled out by assumption) But the policy implications for how to address discrimination in the labour market would be different in each case. In the case of Black's model, a more competitive market structure, or at least one where workers have greater bargaining power, would reduce the wage differential. In the case of Rosén (1997) and Holden and Rosén (2014), labour market discrimination would decline if firms have better information about the match quality (prior to hiring). In the case of a wage differential generated by our proposed mechanism, the above policies would not be effective, but lowering the cost of monitoring effort would induce unprejudiced employers to hire workers who face discrimination and, consequently, improve worker incentives and reduce the wage differential.

In the case of affirmative action policies that introduce hiring quotas for disadvantaged social groups, this would invariably improve incentives, in our model, for workers who were previously discriminated against. This is in contrast to the result obtained (under some conditions) in Coate and Loury (1993) but introducing affirmative action policies in Black (1995) or Rosén (1997) is likely to generate similar results.

It is worth mentioning that our proposed model generates an equilibrium with negative stereotypes about a certain social group – being lazy, dishonest, etc. – as documented by Lang and Lehmann (2012) for inner-city black men in the United States. These negative stereotypes do not arise in the model by Black (where black workers simply have less bargaining power) or Rosén (where black workers are more keen in pursuing job opportunities).

Coate and Loury's model generates negative stereotypes as well but note that these stereotypes relate to the human capital of workers (from a certain social group) rather than their expected effort on the job. Indeed, in their model, the workers from the discriminated group who are hired have a lower cost of human capital investments than non-discriminated workers (because they make the human capital investments in spite of the discrimination). Therefore, if the cost of human capital investment is correlated with the cost of effort, and effort on the job is unobservable, then the black workers would exert more effort than the white workers in the discriminatory equilibrium.

Thus, while our basic argument that the presence of discriminatory employers would reduce the incentives of black workers, etc. may also work in a model of human capital investments à la Coate and Loury; this story may no longer hold true if monitoring effort on the job is costly: because black workers who are hired have more 'grit' than the white workers.

Fryer (2007) makes a similar point in a dynamic model of statistical discrimination. The first stage of the game is akin to the Coate and Loury model. But hired workers again have the opportunity to make human capital investments. In this second stage, it is profitable for the employer to 'promote' workers who have made an additional human capital investment but the employer observes this human capital investment noisily as in the Coate and Loury model. Fryer (2007) derives conditions for a 'belief-flipping' equilibrium where black workers are subject to initial discrimination but, once hired, are more likely to be promoted.

3 Setup

Consider a labour market where workers take up one-period employment contracts every period. They can be employed to undertake two kinds of work, which we call 'managerial' and 'menial'. Once employed, a worker chooses a level of effort $e \in [0,1]$. In the case of managerial work, an effort level $e \in [0,1]$ produces high output $(y = 2)$ with probability e^{α} and low output $(y = 1)$ with probability $1 - e^{\alpha}$. In the case of menial work, output is low $(y = 1)$ regardless of effort. Here, $\alpha > 0$ is a fixed parameter.

Achieving high output in one period makes the worker 'skilled' $(k = 1)$ during the next period. Otherwise, the worker will become 'unskilled' $(k = 0)$. Only skilled workers can undertake managerial work.

The worker's effort level is not observed by the employer. The worker's output realisation becomes public information after the end of the contract. Thus, the wage cannot be contingent on effort or output generated over the course of the contract. But future hiring and wages can be made contingent on previous output. The worker's per-period utility is given by

$$u(e, w) = w - \frac{1}{\gamma} e^{\gamma}$$

where $\gamma > 0$. Each worker has an outside option to the labour market that can generate a utility of 1 in each period. Future utility is discounted at a rate $\beta \in (0,1)$ per period. The question we address – in the next section – is how much effort the worker will exert in equilibrium.

4 A Labour Market Without Discrimination

Let V_1 be the continuation utility of a worker engaged in managerial work and let V_0 be the continuation utility of a worker engaged in menial work. Let w_1 be the wage offered for managerial work and w_0 the wage offered for menial work in equilibrium. Then a skilled worker in managerial work solves the following optimisation problem:

$$\max_{e} u(e, w) + \beta \{ e^{\alpha} V_1 + (1 - e^{\alpha}) V_0 \}$$

From the first-order condition, we obtain

$$-e^{\gamma-1} + \alpha\beta e^{\alpha-1}(V_1 - V_0) = 0$$

$$\Longrightarrow e^{\gamma-\alpha} = \alpha\beta(V_1 - V_0)$$

$$\Longrightarrow e_1^* = \{\alpha\beta(V_1 - V_0)\}^{\frac{1}{\gamma-\alpha}} \tag{1}$$

$$\Longrightarrow (V_1 - V_0) = \frac{1}{\alpha\beta}(e_1^*)^{\gamma-\alpha} \tag{2}$$

A worker engaged in menial work gains nothing from exerting effort as he produces an output of 1 in any case. Therefore, his effort level is given by $e_0^* = 0$.

We assume that employers offer contracts where workers receive their marginal product. We can justify this assumption on the basis of perfectly competitive labour markets (this will require reconsideration when we introduce discrimination). Because potential employers can observe the skill level of workers (given that their past output is public information), skilled workers will be offered the following wage for managerial work:

$$\begin{aligned} w_1 &= 1 + e_1^* \\ &= 1 + \{\alpha\beta(V_1 - V_0)\}^{\frac{1}{\gamma-\alpha}} \end{aligned}$$

All workers will be offered the following wage for menial work (recall that unskilled workers cannot undertake managerial work):

$$w_0 = 1$$

$$\Longrightarrow w_1 - w_0 = e_1^*$$

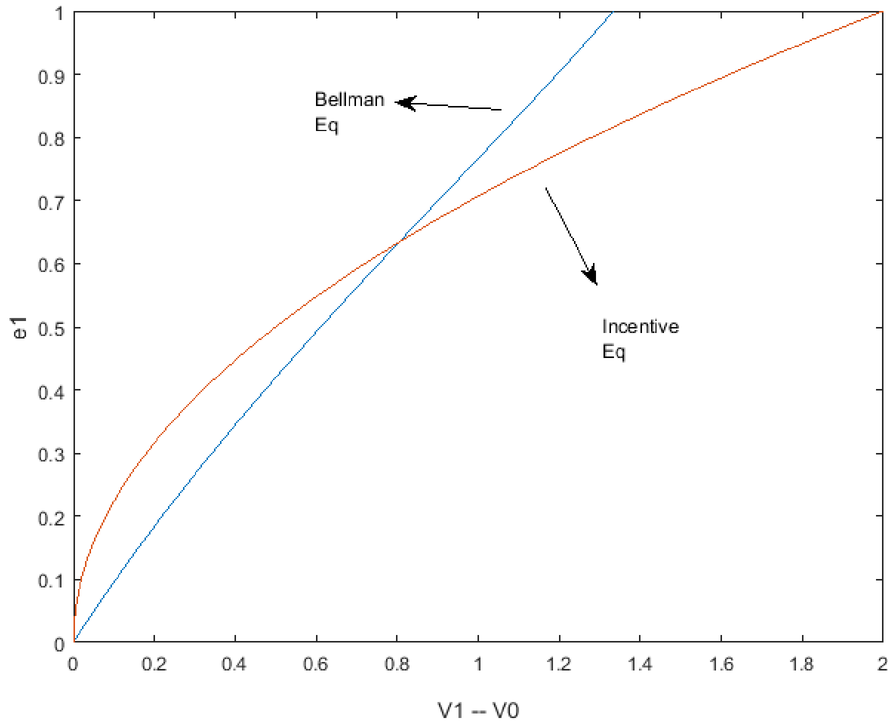
Using the definition of V_0 and V_1, we can write

$$\begin{aligned} V_1 &= u(e_1^*, w_1) + \beta\{(e_1^*)^\alpha V_1 + (1 - (e_1^*)^\alpha) V_0\} \\ V_0 &= u(e_0^*, w_0) + \beta V_0 \end{aligned}$$

$$\Longrightarrow \quad V_1 - V_0 = e_1^* - \frac{1}{\gamma}(e_1^*)^\gamma + \beta(e_1^*)^\alpha (V_1 - V_0)$$

$$\Longrightarrow \quad (V_1 - V_0)\{1 - \beta(e_1^*)^\alpha\} = e_1^* - \frac{1}{\gamma}(e_1^*)^\gamma$$

$$\Longrightarrow \quad (V_1 - V_0) = \frac{e_1^* - \frac{1}{\gamma}(e_1^*)^\gamma}{1 - \beta(e_1^*)^\alpha} \tag{3}$$



Equations (2) and (3) each provide a relationship between the level of effort and the difference in the continuation payoffs of workers presently in managerial versus menial work. Each intersection of the two resulting curves generates a potential equilibrium. The figure above shows two such possible equilibria when $\alpha = 1, \beta = 0.5$ and $\gamma = 3$.

4.1 Corner Solutions

It is evident from equations (2) and (3) that $e_1^* = 0$ is always a solution; i.e. if workers never exert any effort, then no-one will ever be offered managerial work, and if no-one is ever offered managerial work then workers have no incentive to exert effort. But this is a relatively uninteresting equilibrium. Another potential corner solution at $e_1^* = 1$ occurs if the difference $(V_1 - V_0)$ at $e_1^* = 1$ is sufficiently large to incentivise maximum effort. This occurs if and only if

$$\frac{e_1 - \frac{1}{\gamma}(e_1)^{\gamma}}{1 - \beta (e_1)^{\alpha}} \geq \frac{1}{\alpha\beta}(e_1)^{\gamma-\alpha} \text{ at } e_1 = 1$$

$$\text{i.e. } \frac{1 - \frac{1}{\gamma}}{1 - \beta} \geq \frac{1}{\alpha\beta}$$

$$\iff \left(\frac{\gamma-1}{\gamma}\right) \geq \left(\frac{1-\beta}{\alpha\beta}\right) \tag{4}$$

4.2 Interior Solutions

We obtain an interior solution if both equations (2) and (3) are satisfied; i.e.

$$\frac{e_1 - \frac{1}{\gamma}(e_1)^{\gamma}}{1-\beta(e_1)^{\alpha}} = \frac{1}{\alpha\beta}(e_1)^{\gamma-\alpha} \text{ for some } e_1 \in (0,1)$$

Rearranging the equation, we obtain

$$\alpha\beta\left(\frac{1}{\gamma}-\frac{1}{\alpha}\right)(e_1^*)^{\gamma} - (e_1^*)^{\gamma-\alpha} + \alpha\beta e_1^* = 0$$

To facilitate the analysis, we consider the particular case where $\gamma = 3$ and $\alpha = 1$. These parameter values generates a quadratic equation for interior solutions:

$$2\beta(e_1^*)^2 + 3e_1^* - 3\beta = 0$$

$$\Longrightarrow e_1^* = \frac{-3 \pm 3\sqrt{1+2\beta^2}}{4\beta}$$

Because e_1^* cannot take a negative value, we obtain a unique interior solution at

$$e_1^* = \frac{-3 + 3\sqrt{1+2\beta^2}}{4\beta} \tag{5}$$

4.3 Comparative Statics

Next, we investigate how effort and wages vary according to the discount factor β. As per the discussion above, the labour market potentially has multiple equilibria. To make the comparative statics exercise precise, we need to focus on a particular type of equilibrium. We consider the 'best' equilibrium in terms of the level of effort and wages. As noted above, if condition (4) holds, then there is a corner solution at $e_1^* = 1$ and, therefore, changes in β would have no effect on the best equilibrium unless it causes the condition to be violated. The more interesting case occurs when the condition in (4) does not hold and the best equilibrium is given by an interior solution. If $\gamma = 3$ and $\alpha = 1$, this occurs for $\beta \leq \frac{3}{5}$; and, as shown above, there is a unique interior equilibrium given by (5).

Differentiating throughout (5) w.r.t. β, we obtain

$$\frac{de_1^*}{d\beta} = \frac{3}{2\left(1+2\beta^2\right)^{\frac{1}{2}}} + \frac{3-\left(1+2\beta^2\right)^{\frac{1}{2}}}{4\beta^2}$$

It is straightforward to check that, for $\beta \in \left(0, \frac{3}{5}\right)$, we have $\frac{de_1^*}{d\beta} > 0$. Therefore, a higher rate of effort can be sustained in equilibrium when workers are more patient.

5 A Labour Market with Discrimination

We modify the setup above as follows:

1. There are two types of workers A and B, who differ in terms of some observable trait such as race, gender, etc. but are identical in terms of ability, preferences, etc.

2. There are two types of employers: unprejudiced employers who do not distinguish between workers of type A and B and offer them contracts on the basis of their skill level only as described above, and prejudiced employers who offer managerial work only to workers of type A, with type B workers being offered menial work only.

3. In each period, workers find themselves faced with a collection of potential employers who simultaneously offer them contracts. Therefore, the workers can choose among multiple offers. But, workers of type B face a probability $\lambda \in [0,1]$ of being faced with prejudiced employers who would not offer them managerial work; and a probability $1-\lambda$ of being faced with at least two unprejudiced employers.[3]

To investigate how discrimination affects wages and effort by workers, we denote the equilibrium wage and continuation utility of a worker of type $r \in \{\text{A,B}\}$ in a current position m (1=managerial, 0=menial) by w_{mr} and V_{mr} respectively (henceforth, when the worker type appears in the subscript, we use lower case 'a' and 'b' to facilitate reading).

It is straightforward to show that V_{ma} and w_{ma} are the same as V_m and w_m derived above for $m \in \{0,1\}$. And that the effort level of a worker of type A is the same as e_0^* and e_1^* derived above.

[3] We assume, for ease of exposition, that the event where the worker faces exactly one unprejudiced employer other prejudiced employers has zero probability.

Next, we turn to the worker of type B. The worker, if in a managerial position, solves:

$$\max_e u(e,w) + \beta \{(e^\alpha - e^\alpha \lambda) V_{1b} + (1 - e^\alpha + e^\alpha \lambda) V_{0b}\}$$

because the worker, even if he/she achieves high output, will be offered only menial work in the next period with probability λ. From the first-order condition, we obtain

$$-e^{\gamma-1} + \alpha\beta e^{\alpha-1} (1-\lambda)(V_{1b} - V_{0b}) = 0$$

$$\Longrightarrow \quad (e_{1b}^*)^{\gamma-\alpha} = \alpha\beta(1-\lambda)(V_{1b} - V_{0b})$$
$$\Longrightarrow \quad e_{1b}^* = \{\alpha\beta(1-\lambda)(V_{1b} - V_{0b})\}^{\frac{1}{\gamma-\alpha}} \tag{6}$$

And a worker of type B in a menial job will choose effort $e_{0b}^* = 0$.

When a skilled worker of type B is faced with multiple unprejudiced employers, they will offer him or her managerial work, and the competition will ensure that the equilibrium wage is equal to the worker's expected marginal product:

$$w_{1b} = 1 + e_{1b}^*$$

When a skilled worker of type B is faced with prejudiced employers, the wage will equal the marginal product of menial work.[4] An unskilled worker of type B will only be offered menial work. In all these situations, the equilibrium wage will be given by

$$w_{0b} = 1$$

Following the reasoning in Section 4, we can write

$$V_{1b} = u(e_{1b}^*, w_{1b}) + \beta [\{(e_{1b}^*)^\alpha - (e_{1b}^*)^\alpha \lambda\} V_{1b} + \{1 - (e_{1b}^*)^\alpha + (e_{1b}^*)^\alpha \lambda\} V_{0b}]$$
$$V_{0b} = u(e_{0b}^*, w_{0b}) + \beta V_{0b}$$

$$\Longrightarrow V_{1b} - V_{0b} = e_{1b}^* - \frac{1}{\gamma}(e_{1b}^*)^\gamma + \beta[(e_{1b}^*)^\alpha (1-\lambda)(V_{1b} - V_{0b})]$$

Rearranging terms, we obtain

$$(V_{1b} - V_{0b}) = \frac{e_{1b}^* - \frac{1}{\gamma}(e_{1b}^*)^\gamma}{1 - \beta (e_{1b}^*)^\alpha (1-\lambda)} \tag{7}$$

[4] Even if there is one unprejudiced employer who offers the worker managerial work, the wage can be pushed down to the worker's best alternative wage which is that for menial work.

5.1 Equilibrium

Note that equations (6) and (7) are identitical to (1) and (3) except that β is replaced by $\beta(1-\lambda)$. Therefore, the equilibrium analysis for the setup without discrimination still applies with a suitable change of parameters. In particular, if $\gamma = 3$, $\alpha = 1$, and $\beta(1-\lambda) \leq \frac{3}{5}$, then the best equilibrium is given by an interior solution given by

$$e_1^* = \frac{-3 + 3\sqrt{1 + 2\beta^2 (1-\lambda)^2}}{4\beta (1-\lambda)^2}$$

Therefore, the wage received by a high-skilled worker of type B, when faced with (multiple) unprejudiced employers is given by

$$w_{1b} = 1 + \frac{-3 + 3\sqrt{1 + 2\beta^2 (1-\lambda)^2}}{4\beta (1-\lambda)^2}$$

Thus we see that, even when a higher-skilled worker of type B works for a unprejudiced employer, the wage offered is a function of the probability of discrimination. This is because the worker's effort in a managerial position depends on the probability of experiencing future discrimination in the labour market. Furthermore, using the comparative statics result in (4.3), we can conclude that the type B high-skilled worker's effort and wage in a managerial position (i.e. when working for an unprejudiced employer) is decreasing in the probability of future discrimination. Therefore, if $\lambda > 0$, the wage is lower than that offered to high-skilled workers of type A.

We can argue that discrimination has a multiplier effect in the following sense. The expected wage of a high-skilled worker of type B is equal to $\lambda(w_{0b}) + (1-\lambda)(w_{1b})$ while the expected wage of a low-skilled worker of type B is w_{0b}. Therefore, the difference in the expected wage between a high-skilled and low-skilled worker of type B is

$$\begin{aligned} & (1-\lambda)(w_{1b} - w_{0b}) \\ & (1-\lambda)\left\{1 + \frac{-3 + 3\sqrt{1 + 2\beta^2 (1-\lambda)^2}}{4\beta (1-\lambda)^2} - 1\right\} \\ = \ & (1-\lambda)\left\{\frac{-3 + 3\sqrt{1 + 2\beta^2 (1-\lambda)^2}}{4\beta (1-\lambda)^2}\right\} \end{aligned}$$

If the expression $(w_{1b} - w_{0b})$ were independent of λ, then an x% increase in discrimination would lead to a proportional decline in the wage gap between high-skilled and low-skilled workers. Because $(w_{1b} - w_{0b})$ is decreasing in λ, an x% increase in discrimination leads to a more than proportional decline in the wage gap.

Similarly, the gap in expected wage between a high-skilled worker of type A and a high-skilled worker of type B can be written as

$$\begin{aligned} & w_{1a} - \lambda\left(w_{0b}\right) - (1-\lambda)\left(w_{1b}\right) \\ = & w_{1a} - (1-\lambda)\left(w_{1b} - w_{0b}\right) \\ = & \frac{-3+3\sqrt{1+2\beta^2}}{4\beta} - (1-\lambda)\left\{\frac{-3+3\sqrt{1+2\beta^2(1-\lambda)^2}}{4\beta(1-\lambda)^2}\right\} \end{aligned}$$

Again, if the expression $(w_{1b} - w_{0b})$ were independent of λ, then an x% increase in discrimination would lead to a proportional increase in the wage gap between high-skilled workers of type A and B. Because $(w_{1b} - w_{0b})$ is decreasing in λ, an x% increase in discrimination leads to a more than proportional increase in the wage gap.

6 A Labour Market with Contractible Effort

We analyse here whether and how the wage discrimination results obtained above would change in a setting where the worker's effort is observable and contractible.

Socially Optimal Effort: For a benchmark, we obtain, first, the socially optimal level of effort, taking into account both expected output and the agent's disutility from effort. We denote by Y_0 the discounted value of output generated by a worker in menial work every period, and by $Y_1(e)$ the corresponding expected value generated by a skilled worker in managerial work (who will remain in managerial work in future periods as long as he remains skilled), given an level of effort e in each period. We have

$$Y_0 = \frac{1}{1-\beta} \tag{8}$$

$$Y_1(e) = 1 + e^{\alpha} - \frac{1}{\gamma}e^{\gamma} + \beta\left\{e^{\alpha}\left(Y_1(e) - Y_0\right) + Y_0\right\} \tag{9}$$

The first equation simply captures the discounted present value of an output of 1 in each period. The second equation shows the expected value of the output from managerial work in the current period given an effort level of e, the disutility of effort e, and the gain in continuation value from the second period onwards if high output is achieved in the current period (an event which occurs with probability e^{α}). Rearranging the second equation, we obtain

$$Y_1(e) = \frac{1 + e^{\alpha} + \beta Y_0 (1 - e^{\alpha}) - \frac{1}{\gamma} e^{\gamma}}{1 - \beta e^{\alpha}}$$

Therefore,

$$Y_1(e) - Y_0 = \frac{1 + e^{\alpha} + \beta Y_0 (1 - e^{\alpha}) - \frac{1}{\gamma} e^{\gamma}}{1 - \beta e^{\alpha}} - Y_0$$

Rearranging and substituting for Y_0 using (8), we obtain

$$Y_1(e) - Y_0 = \frac{e^{\alpha} - \frac{1}{\gamma} e^{\gamma}}{1 - \beta e^{\alpha}}$$

Using this last expression to substitute in (9), we obtain

$$\begin{aligned} Y_1(e) &= 1 + e^{\alpha} - \frac{1}{\gamma} e^{\gamma} + \beta \left\{ e^{\alpha} \left(\frac{e^{\alpha} - \frac{1}{\gamma} e^{\gamma}}{1 - \beta e^{\alpha}} \right) + Y_0 \right\} \\ &= Y_0 + \frac{e^{\alpha} - \frac{1}{\gamma} e^{\gamma}}{1 - \beta e^{\alpha}} \end{aligned}$$

Therefore, the socially optimal effort level is given by

$$e_{so} = \arg \max_{e \in [0,1]} Y_1(e) = \arg \max_{e \in [0,1]} \frac{e^{\alpha} - \frac{1}{\gamma} e^{\gamma}}{1 - \beta e^{\alpha}}$$

Next, we show that, when effort is contractible, the socially optimal level of effort will be specified in equilibrium contracts in a competitive labour market without discrimination. This will be the case both when permanent labour contracts are possible and when contracts are restricted to one-period contracts.

Permanent Labour Contract with Contractible Effort: If effort is contractible, then the contract will specify both the wage and the effort level. In a permanent labour contract, the marginal product of a skilled worker is $Y_0 + \frac{e^{\alpha}}{1 - \beta e^{\alpha}}$. Such a contract also entails an expected sum of effort cost equal to $\frac{\frac{1}{\gamma} e^{\gamma}}{1 - \beta e^{\alpha}}$ for the worker. Suppose that that contract offered in equilibrium specifies an effort level e^* and a wage $w_1(e^*)$. Then the employer's expected profit equals

$$Y_0 + \frac{(e^*)^\alpha}{1-\beta(e^*)^\alpha} - w_1(e^*)$$

If multiple firms simultaneously bid for the worker, then the logic of Bertrand competition implies that, at the equilibrium wage, the firm will make zero expected profits from hiring a skilled worker (for managerial work); i.e. $w_1(e^*) = Y_0 + \frac{(e^*)^\alpha}{1-\beta(e^*)^\alpha} = Y_1(e^*)$. If not, a different firm can raise the wage slightly, draw away the worker and still make positive profits. Additionally, the effort level specified in the equilibrium contract must be the socially optimal effort level, i.e. $e^* = e_{so}$. If not, a different firm can offer a contract with the socially optimal effort level, pay wages that leaves the worker with slightly higher utility and still make positive profits. Therefore, we have

$$\begin{aligned} e^* &= e_{so} \\ w_1(e^*) &= Y_1(e_{so}) \end{aligned}$$

Similarly, we can argue that the equilibrium contract for menial work will specify a wage $w_0 = Y_0$ and zero effort (as output is independent of effort).

One-Period Contracts with Contractible Effort: Suppose labour contracts can last for one period only (as assumed in Sections 3-5). Then, a contract that specifies effort level e yields an expected output $1 + e^\alpha$ for the firm, a disutility cost $-\frac{1}{\gamma}e^\gamma$ for the worker and expected future benefit of $\beta e^\alpha \{W_1(e) - W_0\}$ for the worker (where $W_1(e)$ and W_0 denote the expected future income streams of skilled and unskilled workers respectively minus the expected cost of effort, and we assume that future spot contracts for skilled work also specify an effort level e). If multiple firms simultaneously bid for the worker, then the logic of Bertrand competition, once again, implies that at the equilibrium wage the firm will make zero expected profits from hiring a worker, i.e. $w_1(e^*) = 1 + (e^*)^\alpha$ and $w_0 = 1$. In the case of managerial work, the effort level specified in the equilibrium contract e^* must equal

$$e' = \arg\max_{e\in[0,1]} 1 + e^\alpha - \frac{1}{\gamma}e^\gamma + \beta e^\alpha \{W_1(e^*) - W_0\}$$

If not, a different firm can offer a contract that specifies effort level e' and pay wages that leaves the worker with slightly higher utility and still make positive profits. Because the firms make zero profits, all the surplus generated by production must accrue to the workers; thus $W_1(e^*) = Y_1(e^*)$ and $W_0 = Y_0$. Therefore, we have

$$\begin{aligned} e^* &= \arg\max_{e\in[0,1]} 1 + e^\alpha - \frac{1}{\gamma}e^\gamma + \beta e^\alpha \{Y_1(e^*) - Y_0\} \\ &= \arg\max_{e\in[0,1]} 1 + e^\alpha - \frac{1}{\gamma}e^\gamma + \beta e^\alpha \left\{\frac{e^\alpha - \frac{1}{\gamma}e^\gamma}{1-\beta e^\alpha}\right\} \\ &= \arg\max_{e\in[0,1]} 1 + \frac{e^\alpha - \frac{1}{\gamma}e^\gamma}{1-\beta e^\alpha} \\ &= e_{so} \text{ by definition} \end{aligned}$$

Therefore, we have

$$\begin{aligned} e^* &= e_{so} \\ w_1(e^*) &= 1 + (e_{so})^\alpha \end{aligned}$$

Labour Market with Discrimination: Next, we consider the case where there are two types of workers, A and B, and prejudiced and unprejudiced employers as in Section 5. Suppose that permanent labour contracts are feasible. Unskilled workers of either type A or B will be offered a wage $w_0 = Y_0$ for zero effort as per the reasoning above. The reasoning above also applies to skilled workers of type A. If a skilled worker of type B is faced with at least two unprejudiced employers, they will receive the same offer as workers of type A because the expected income stream that they generate are identical. But prejudiced employers will only offer menial work to skilled workers of type B and so if the worker is faced with prejudiced employers only, this will lead to a wage $w_0 = Y_0$.

If only one-period contracts are feasible, unskilled workers all receive a contract that pays a wage $w_0 = 1$ for zero effort according to the reasoning provided above. For skilled workers of type A,.the reasoning above also applies. Therefore, they will be offered a managerial contract that specifies an effort level e_{so} for a wage of $w_{1a} = 1 + (e_{so})^\alpha$. For skilled workers of type B, with probability λ they are faced with prejudiced workers only, in which case they are offered the standard contract for menial work: $w_0 = Y_0$ for zero effort. If there are two or more unprejudiced firms available,[5] then, as in the case of skilled workers of type A, they will be offered a managerial contract that specifies the socially optimal effort level. But the socially optimal effort level is *lower* in their case than for skilled workers of type A. This is because they face a *future* probability λ of being matched with prejudiced employers only, in which case high skill has no value. Formally,

$$\begin{aligned} e_b(\lambda) &= \arg\max_{e\in[0,1]} \frac{e^{\alpha} - \frac{1}{\gamma}e^{\gamma}}{1-(1-\lambda)\beta e^{\alpha}} \\ &< e_{so} \text{ for } \lambda > 0 \end{aligned}$$

The contract will pay them their marginal product in skilled work, i.e. $w_{1b} = 1 + \{e_b(\lambda)\}^{\alpha}$ which is less than w_{1a} for $\lambda > 1$. Thus, they are paid a lower wage than skilled workers of type A.

7 Conclusion

We have established that when effort is contractible, permanent labour contracts are feasible, and there are a sufficient number of unprejudiced firms (more precisely, workers of type B always face at least two unprejudiced prospective employers), then the employment contracts offered to workers of type A and B are identical. This is akin to the result obtained by Becker (1971).

However, if labour contracts last for one period only, then skilled workers of type B are offered a lower wage than skilled workers of A, even when they are faced with unprejudiced employers. This is because they have a positive future probability of being faced with prejudiced employers who will not value their current investment in skill (generated by their current effort). Thus, it is more 'costly' for them to exert current effort than it is for skilled workers of type A. Therefore, unprejudiced workers assign them to managerial work but ask for lower effort and pay a lower wage.

[5] Recall that we are abstracting away from the case where they are faced with exactly one unprejduced employer and other, prejudiced employers, for ease of exposition.