

# Empowerment or Financialization? The Gains from Financial Inclusion\*

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## Abstract

Expanding access to credit markets can be seen as a source of empowerment when it increases economic opportunities and changes who is able to start a new business. It can also have equilibrium effects on wages so that the gains from financial development are widely shared. But others see credit market expansion as an unwelcome process of “financialization” with many of the gains being appropriated by financial institutions, pointing to the concentration in ownership of financial intermediaries, especially banks, around the world. This paper explores these issues, investigating the consequences of financial sector expansion for profits, wages and entrepreneurial activity using a calibrated general equilibrium model with financial frictions, endogenous default, and wealth inequality. A key element of the model is to examine how the surplus created in the real economy by expanding financial markets is shared between borrowers, lenders, and workers employed by firms. We show that competition in banking can be an important determinant of both equity and efficiency, and hence the gains from financial inclusion. The framework also highlights the role that different types of contractual imperfections can play in determining the distribution of gains from expanding market access.

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# 1 Introduction

Increasing the scope and scale of markets, especially those that allocate capital, has long been seen as the *sine qua non* of economic development (Gerschenkron (1962) and Goldsmith (1969)). In pursuit of such gains, financial inclusion has become a central plank of development strategies (for example, Demirguc-Kunt et al. (2017)). In the meantime, financial access has increased sharply all over the world. The World Bank documents this through the Global Findex database and reports that the share of adults who own a bank account rose globally from 51 percent in 2011 to 69 percent in 2017—an additional 515 million people (World Bank, 2021).

A key idea is that access to the formal banking sector allows individuals to start new businesses and this in turn will raise wages and incomes throughout the economy. By expanding the scope of financial intermediation, it can also reduce the misallocation of capital across productive uses, something that is now routinely emphasised in the development literature following the seminal work of Hsieh and Klenow (2009) and Restuccia and Rogerson (2008). This positive view of the financial sector is however tempered by a more negative view that is often referred to as “financialization”, emphasising that financial intermediaries extract rents on a large scale, increasingly so as the financial sector expands. Such ideas were brought into sharp relief following the financial crisis of 2008 when market concentration in the financial sector became associated with bailouts by institutions that were deemed “too big to fail”. But the wider question is that even outside of crises, the concentration in the financial sector can affect the gains from financial inclusion; if rents accrue predominantly to financial intermediaries, this can reduce the returns to entrepreneurship and hence the economic impact from greater inclusion.

The theme that market power can attenuate the positive role credit plays in resource allocation in the presence of imperfect information and contracting frictions has echoes in an earlier literature in development (see, for example, Hoff and Stiglitz (1990), Banerjee (2002), Banerjee and Duflo (2010) for overviews). Of course, the discussion of factors that determine the distribution of the surplus created from the process of development has a long and distinguished history. A good example is Ricardo (1891) who framed the idea as follows:

“The produce of the earth - all that is derived from its surface by the united application of labour, machinery, and capital, is divided among three classes of the community; namely, the proprietor of the land, the owner of the stock or capital necessary for its cultivation, and the labourers by whose industry it is cultivated. But in different stages of the society, the proportions of the whole produce of the earth which will be allotted to each of these classes, under the name of rent, profit, and wages, will be essentially different; ... To determine the laws which regulate this distribution, is the principal problem in Political Economy.”

Applying these ideas to the expansion of financial markets, suggests focusing on how far financial intermediaries, who control the allocation of financial capital, can extract returns from the “real economy” in the process of development and growth. It also suggests a focus on how the labor share changes as wages rise as well as the level of profits outside the financial sector, i.e. the returns to entrepreneurship.

Another theme in the literature on emerging market economies is how the prevalence of contracting frictions limits the scope of financial markets. The lack of ability to collateralize assets is a major example, principally due to weak legal systems and poorly enforced property

rights. But a less explored element of financial market development in emerging economies begins with the observation that concentrated ownership in banking and finance is particularly high in poor countries. This may reflect low levels of competition and concentration of financial profits in the hands of intermediaries as financial market access expands. This may also help explain some of the more equivocal findings on the economic impact of inclusion (Barajas et al. (2020)) where the lack competitiveness of the financial sector is often discussed as a factor (Love and Peria (2015)).

Although it may seem obvious that improving credit market competition would increase the borrowers' share of the total surplus from credit contracts, how they quantitatively impact occupational choice, wages and interest rates in different contracting environments and at different levels of financial inclusion is far from obvious. Providing quantitative estimates of this and studying the mechanisms behind them in the context of increasing financial inclusion is the key aim of this paper. This requires an integrated analysis of financial inclusion and its impact on the economy recognizing that it is going to be context specific.

With these goals in mind, we develop an analytical framework for studying the role of credit market competition and inclusion in a calibrated general equilibrium model of financial frictions. In the model, entrepreneurs choose whether to become borrowers or work as labourers, and lenders design contracts that anticipate potential for moral hazard. We allow for varying degrees of bargaining power of borrowers relative to lenders, which we interpret as increasing competition. Entrepreneurs hire workers in a labour market and the level of wages is determined endogenously. We explicitly introduce parameters that capture the extent to which financial frictions exist, the degree of competition in financial markets, and the extent of financial access. We calibrate the model to data and use this calibrated version to simulate the impact of increasing financial inclusion. In particular, we study the interplay of parameters that capture these three different aspects of financial markets. The model is ideally suited to studying how financial inclusion affects the economy in different settings.

The paper explores the quantitative gains from financial inclusion both in aggregate and in terms of how they are distributed. As financial access expands, as we would expect, we see gains in wages, capital deepening alongside a structural change that sees a smaller number of large employers and decline in self-employment. However, we also look at the size of these effects with an expansion in access to finance as two dimensions of the economy are varied: financial frictions relating to collateral value of assets and the degree of competition. We find the degree of competition matters the most for the impact of financial inclusion when the institutional quality (determining financial frictions) is the poorest, suggesting that in developing countries the impact of financial access would be higher the more competitive are credit markets. In other words, we find that the biggest aggregate gains to financial inclusion come when the extent of financial frictions and the degree of competition are highest.

By having a micro-founded model of financial contracts with moral hazard, we are able to see how tangible variables such as interest rates and default probabilities vary with competition and the institutional environment. We also find that the size distribution of firms, in particular generating large firms and weeding out small low-productivity firms depend largely on expanding financial access and not so much on competition in financial markets or financial frictions. The consequent gains in expanding employment (and weeding out self-employment) and the positive effect on wages also depend largely on the expansion of financial access and not so much on the other two institutional parameters. However, the distri-

bution of gains from financial inclusion between entrepreneurs and lenders do depend on the extent of competition. Our micro-founded model of credit contracts provides a clear indication of the mechanisms at work. We find that the benefits from enhanced competition are the highest when contractual frictions are substantial and that drives both the aggregate effects, the distribution of firms and occupational structure, as well the distributional effects.

The remainder of the paper is organized as follows. In the next section, we discuss the extensive related literature on finance and development as well as some background facts on financial development. Section 3 then lays out the theoretical framework. Section 4 moves from the model to the data and shows how it can be calibrated. Section 5 develops the results looking at aggregate gains, their distribution, the structure of credit contracts and the size distribution of firms. Section 6 concludes.

## 2 Background

### 2.1 Related Literature

The idea that market imperfections due to asymmetric information and imperfect competition lie at the heart of development problems has a long history. A comprehensive development of this idea in the older literature is in [Stiglitz \(1988\)](#). Since the division of the surplus cannot be separated from the generation of surplus in the presence of incentive constraints (in effort, risk-taking, as well as technology adoption) and departures from the transferable utility assumption (due to the presence of limited wealth and limited liability or risk-sharing considerations) that prevent solutions like making agents full-residual claimants. The accompanying empirical literature in development has shown that these considerations can have significant implications for productivity. For example, [Banerjee et al. \(2002\)](#) showed in the context of sharecropping tenancy that a legal stipulation of a higher crop-share (from 50% to 75%) in a tenancy reform program for tenants increased productivity significantly. A recent field-experiment by [Burchardi et al. \(2019\)](#) in Uganda that offered tenants the same shares, found that the treatment group produced 60% more output relative to the control group. We do not have comparable studies for credit markets, but a study by [Karlan and Zinman \(2009\)](#) that reports the results of a field-experiment in South Africa where the interest rate was varied in a randomized way and using a novel method to separate out selection effects from moral hazard, the authors conclude 13-21% of default is due to moral hazard. However, in this study one cannot separate out the effect of market power from incentive problems.

Macro-models have been built to illustrate the role of market frictions due to transaction costs and informational constraints could lead to development traps (for example, [Banerjee and Newman \(1993\)](#), [Galor and Zeira \(1993\)](#), [Banerjee and Duflo \(2010\)](#) and [Townsend and Ueda \(2006\)](#)). This has also spawned a range of quantitative studies of the implications of financial frictions ([Itskhoki and Moll \(2019\)](#), [Kaboski and Townsend \(2011\)](#), [Midrigan and Xu \(2014\)](#) and [Buera et al. \(2011\)](#)).<sup>1</sup>

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<sup>1</sup>[Moll \(2014\)](#) highlights the importance of diminishing the extent of small-scale self-employment as development progresses with such individuals mainly switching to wage labour. [Paulson et al. \(2006\)](#) and [Karaivanov and Townsend \(2014\)](#) estimate models with moral hazard on Thai data with equilibrium default. [Dabla-Norris et al. \(2021\)](#) consider three financial frictions: an entry cost for credit, a collateral constraint, and an intermediation inefficiency.

The importance of capital allocation is closely linked to the idea that improving the contracting environment can reduce the misallocation of resources. This has been highlighted in a recent conceptual and empirical literature with particular focus on the consequences of capital misallocation in the spirit of the voluminous misallocation literature that began with [Hsieh and Klenow \(2009\)](#) and [Restuccia and Rogerson \(2008\)](#).<sup>2</sup> The associated development accounting literature highlights misallocation of inputs across firms and industries as a key determinant of income differences (see, for example, [Hsieh and Klenow, 2010](#)). Here we look at the role of structural factors in credit markets such as contracting frictions, increased market access and competition in affecting both aggregate productivity and distribution.

The need to move away from personalized transactions in credit market by increasing access to markets has motivated the large growth literature on financial inclusion and economic performance (for example, [Levine \(2005\)](#), [Cihak et al. \(2013\)](#), [Demirguc-Kunt et al. \(2017\)](#)). The strong correlation between measures of inclusion and development outcomes are documented in, for example, [Sethi and Acharya \(2018\)](#) and [Sarma and Pais \(2011\)](#). [Burgess and Pande \(2005\)](#) exploit a natural experiment due to bank-branching rules in India as a source of exogenous variation and find a significant impact on agricultural wages.<sup>3</sup> Micro-finance has long been seen as a way of widening financial access and has led to many studies of its impact [Augsberg et al. \(2015\)](#).<sup>4</sup>

The role of market power in financial markets has received less attention than transactions costs due to information and other enforcement constraints. However, a few recent papers have emphasized the role of market power exercised by financial intermediaries and their aggregate implications, which are more relevant to our paper here.<sup>5</sup> An example is [Cavalcanti et al. \(2023\)](#) who begin by documenting high levels of and dispersion in credit spreads among Brazilian firms and use this to motivate a model where financial frictions are modeled as limited enforcement constraints (as in [Buera et al., 2011](#)). They find that market power among financial intermediaries plays a quantitatively less important role than intermediation costs in generating high interest rate spreads and aggregate losses.<sup>6</sup> In contrast to our paper, they do not allow for equilibrium default on loans nor do they explore the link to debates about financialization.

Reforms in financial markets can have important distributional consequences. This is underlined by [Dabla-Norris et al. \(2021\)](#) who show that relaxing collateral constraint or lowering intermediation costs need not create a Pareto improvement.<sup>7</sup> Changes in occupational choice and wages play a key role in how the distribution of income changes. Market power in credit

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<sup>2</sup>See [Hopenhayn \(2014\)](#) and [Restuccia and Rogerson \(2017\)](#) for overviews of the literature.

<sup>3</sup>[Dupas et al. \(2017\)](#) looks at experimental variation in access to banking services in three developing countries. They suggest that there is a puzzlingly low take-up rate of banking services, further underlining the challenge of expanding the outreach of financial services.

<sup>4</sup>See the recent review by [Buera et al. \(2023\)](#) proposing a synthesis of micro-empirical evidence and structural modeling as an emerging toolbox for macro-development.

<sup>5</sup>An earlier paper by [Claessens and Laeven \(2005\)](#) studies the cross-country relationship between banking concentration and industry growth, and finds that greater competition in countries' banking systems allows financially dependent industries to grow faster.

<sup>6</sup>Their approach is similar to ours in having a static model of model credit contracts and occupational choice. However, they also have an endogenous savings decision that generates dynamics with the interested rate determined endogenously in a closed economy.

<sup>7</sup>They also find positive interaction effects of different forms of financial constraints, suggesting that policy reforms should start by fixing the most binding ones.



markets can also affect who gets what as market access expands. This relates to debates about so-called “financialization” of economies which [Davis and Kim \(2015\)](#) define as the “increasing importance of finance, financial markets, and financial institutions to the workings of the economy.” The negative distributional and efficiency consequences of financialization gained prominence in the run up to the global financial crisis ([Epstein \(2005\)](#)).

According to [Palley \(2007\)](#), financialization has three main elements: (i) elevating the significance of the financial sector relative to the real sector; (ii) transferring income from the real economy to the financial sector; and (iii) increasing income inequality and contributing to wage stagnation. [Kohler et al. \(2019\)](#) studies the relationship between financialization and the wage share arguing that there is a negative relationship between the wage share and financialization measures. Our approach provides a theoretical framework for looking at these issues and responds to the challenge of [Sawyer \(2014\)](#) who notes that there is no standard analytical framework for articulating ideas around financialization. Moreover, such ideas seem to have been mainly studied using heterodox rather than mainstream approaches. Our framework and quantitative approach explores some important dimensions of these arguments, while acknowledging that the critique offered those who look at the consequences of market power in financial markets, is much wider than the issues that we address here (see, for example, [Palley \(2021\)](#)). Others, for example, have expressed concern about the influence of concentrated finance over government policy making (see [Lindsey and Teles \(2017\)](#)).

## 2.2 Background Facts

To motivate the analysis that follows, we now examine three dimensions of economies that are relevant to the model and quantitative analysis developed below.

**Fact 1:** *Financial inclusion is closely related to the level of economic development.*

To see this, we measure financial inclusion as the percentage of respondents with a formal financial account in 2017, from the World Bank Global Financial Development database. Figure 1a, which graphs this against the (log of) the level of GDP at a country level shows that there is a strong positive increasing relationship between the level of development (captured by income per capita) and the level of financial inclusion. Among the poorest countries in the data, fewer than 40% of the population have a bank account. A figure of 60% is typical in middle income countries and above 90% is typical in rich countries.

**Fact 2:** *Bank concentration is (weakly) related to the level of economic development.*

This can be seen in Figure 1b which measures bank concentration using data from the World Bank Global Financial Development database. It captures the assets of the three largest commercial bank as share of total commercial banking assets. There are large differences in concentration across countries but the figure makes clear that these are only weakly related to the level of income per capita. But it does tend to show increasing concentration at high levels of income in line with the financialization hypothesis. However, the fit is poor; among the high income countries with the US having low concentration compared, for example, to Finland and Sweden. And, among low income countries, there is also a large amount of variation between Nepal and Liberia. Thus this is an important independent dimension of difference to consider when looking across countries at similar development levels, something that we will do using the model developed below.

**Fact 3:** *Contractual frictions, as reflected in insolvency regimes, is positively related to the level of economic development.*

This is shown in Figure 1c which uses a measure of contractual frictions in the form of the “resolving insolvency” indicator from the World Bank Doing Business database as measured in 2018. This gives a score for each country where higher values indicate insolvency legislation that is better designed for rehabilitating viable firms and liquidating nonviable ones. Here, we find a reasonably strong relationship with the level of income per capita compared to bank concentration. Countries like Finland, the US and Switzerland are towards the top. But even among lower income countries such as Rwanda and Cambodia, there is a fairly high score. So this also suggests that it is reasonable to allow this dimension to vary across countries that have similar levels of economic development to explore its consequences.

## 3 Theoretical Framework

### 3.1 Model Overview

Our starting point is the standard model of lending under *ex ante* moral hazard and limited liability as in Besley et al. (2012).<sup>8</sup> A group of agents who are heterogeneous in two dimensions: entrepreneurial productivity and wealth can choose one of two possible occupations: becoming an entrepreneur or being an employee. If they choose to be entrepreneurs, then they have to decide how much capital and labour to employ.

There are two phases of hiring: managerial labour input is chosen up front and determines the likelihood of creating a successful firm and workers are hired from the labour market only after it is known whether the initial venture has resulted in a successful firm. Capital can come from the entrepreneur’s own resources, i.e. their wealth, but can be augmented by borrowing if they have access to financial markets. Lending is risky because some firms are not successful.

Credit markets are subject to two key frictions, as is standard in models of moral hazard with risk neutrality and limited liability.<sup>9</sup> First, the level of managerial input which determines the likelihood of creating a successful firm is not observed by lenders. Second, some entrepreneurs lack sufficient wealth to post as collateral and there is limited liability. This rules out the possibility of making entrepreneurs full residual claimants, creating an *ex ante* moral hazard problem. Wealth that can be used as collateral can be limited either because borrowers are poor or due to imperfections in the legal system that limits the collateral value of a given amount of wealth. To capture the latter, we suppose that if a borrower pledges wealth

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<sup>8</sup>Here, we extend this to a general equilibrium framework that also allows us to examine the aggregate implications of removing different types of market “frictions”, as well as their interaction effects.

<sup>9</sup>Our approach differs from much of the existing literature which has focused on credit market frictions as *ex post* repayment constraints. For example, Buera et al. (2017) allow the possibility that borrowers may renege on their debt and keep a fraction of the capital, and the only punishment they face is their financial assets deposited with intermediaries forfeited as a result. Such models also tend to impose the counterfactual assumption that all borrowers face the same interest rate. Many such models also abstract from default, with Paulson et al. (2006) and Karaivanov and Townsend (2014) being key exceptions. In our model, *ex ante* moral hazard leads to defaults in equilibrium, the likelihood of which depends on the extent of collateral a borrower is able to put up. This leads to heterogeneity in default probabilities, and consequently in the interest rates among borrowers who differ in terms of wealth and productivity.

$a$  as collateral to become an entrepreneur, and the firm is not successful, then the bank only recovers a fraction of that collateral.

Lenders design optimal credit contracts subject to information and wealth constraints. Contracts must also respect the entrepreneur's reservation payoff, which is determined endogenously by his outside opportunities whether by borrowing from another lender or working as an employee. We begin by studying optimal credit contracts which reflect the characteristics of entrepreneurs with a fixed outside option. This illustrates how frictions in the credit market lead to misallocation of capital due to the risk premium that is charged to compensate for the probability of default. We then consider the option of borrowing from a different lender. We capture competition by varying the fraction of the surplus which goes to the borrower as opposed to the lender. A fully competitive credit market is where the entrepreneur gets all of the surplus whereas the opposite is true with competition.

Next we introduce a financial inclusion parameter which, following Jeong and Townsend (2007), denotes the fraction of individuals with access to financial markets.<sup>10</sup> We then study occupational choice for each type of entrepreneur depending on their wealth, productivity and access to financial markets. Entrepreneurs tend to be drawn from among the most wealthy and productive individuals.

Finally, we determine wages endogenously using a standard decreasing-returns Lucas "span of control" model. In general equilibrium, the equilibrium wage and the fraction of agents who become entrepreneurs are jointly determined along with the outside option of agents who borrow to become entrepreneurs. This, in turn, affects the structure of credit contracts.

## 3.2 Entrepreneurs, Managers, and Workers

The economy is populated by a continuum of agents who are endowed with a unit of time which they supply as labour inelastically regardless of their occupation. All agents are risk neutral and each individual makes a discrete occupational choice, whether to become an entrepreneur and set up a firm, or to become an employee, i.e. work for a firm. Entrepreneurs earn profits from the firm that they own while employees are paid a wage. The wealth that agents have a certain endowment of is denoted by  $a$ , which varies across the population.<sup>11</sup>

**Entrepreneurs** Entrepreneurs commit all of their labour time to their own firm and are residual claimants on the firm's profit stream. Their ability as entrepreneurs is indexed by  $\theta$ . Heterogeneous productivity can be interpreted either as entrepreneurial "ability" or having access to a particular production technology.<sup>12</sup> Entrepreneurs hire two kinds of employees: managers who contribute towards the success in the initial start-up phase, and workers who increase output in already successful firms.

We denote the level of managerial input by  $e$  and the probability that an entrepreneur creates a successful firm is given by  $g(e; \theta) \in [0, 1]$  which is an increasing function of managerial

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<sup>10</sup> Following Townsend (1978), we think of this as reflecting a fixed transaction cost that some agents face (e.g., due to their geographical location or level of knowledge). Entrepreneurs without financial access can only set up firms use their own wealth.

<sup>11</sup>The level of wealth is specified in units of labour endowment.

<sup>12</sup>If there were a frictionless market for ideas then entrepreneurial ability would no longer matter and ideas would be sold to agents with the highest wealth. Hence, we are assuming that contracting frictions prevent this from happening.



input.<sup>13</sup> We are agnostic about how  $g(e; \theta)$  depends on  $\theta$ . More able entrepreneurs could potentially enhance the productivity of managerial input. However, if high  $\theta$  entrepreneurs use more complex technologies or spread themselves more thinly over larger firms, then all else equal, this could lower the probability of creating a successful firm. If the start-up is successful and a firm is set up, output is given by  $f(k, l; \theta)$  where  $k$  is the capital employed and  $l$  is wage labour employed.

We make the following regularity assumptions:

**Assumption 1** *The following conditions hold for  $g(e; \theta)$  and  $f(k, l; \theta)$ :*

- (i)  $g(e; \theta)$  is strictly increasing, twice-continuously differentiable, strictly concave for all  $e$  with  $g(0; \theta) = 0$ .
- (ii)  $f(k, l; \theta)$  is twice-continuously differentiable and strictly increasing in  $k \in \mathbb{R}^+$  and  $l \in \mathbb{R}^+$ , strictly concave in  $l$ , and is increasing in  $\theta$  with  $f_{\theta k} > 0$  and  $f_{\theta l} > 0$ . Further  $f(k, l; \theta) \geq 0$  for all  $(k, l) \in \mathbb{R}^+ \times \mathbb{R}^+$  and  $f(0, l; \theta) = 0$ .
- (iii)  $\epsilon(e; \theta) := -\frac{g_{ee}(e; \theta)}{(g_e(e; \theta))^2}$  is continuous and increasing for all  $e$  such that  $g(e; \theta) \in [0, 1]$ .

These are more or less standard assumptions which hold in commonly-used models such as Cobb-Douglas and with constant elasticity formulations of the technology which we use in the calibration below. The last part of this assumption guarantees that the level of managerial input increases when the entrepreneur's outside option improves.

**The Production Process** There are two stages to the production process.

At stage one entrepreneurs negotiate credit contracts with lenders and hire managerial labour. The stochastic nature of firm success which generates the possibility of default. We assume that entrepreneurs submit "business plans" to potential lenders which specify  $e$  and which also reveal  $\theta$  and  $a$  to the lender. Lenders understand that, since hiring managerial input is costly and cannot be monitored *ex post*, there is the potential for moral hazard. They will anticipate this when they design contracts.

If the firm is successful, then workers,  $l$ , are hired. If the firm fails, capital as well as the managerial labour is wasted and the lender and the managers are not paid. Hence, managers and lenders need to be compensated for this risk. However, no risk is born by workers who are only hired in successful firms.

**Employees** Agents who choose not to become entrepreneurs are employees. We assume that they are equally productive in managerial task (supplying  $e$ ) or as wage labour (supply  $l$ ). A wage labourer earns  $p_l$  while a manager earns  $p_m$ . As long as they are compensated for the risk of being a manager due not being paid if the firm fails, a risk neutral agent will be indifferent between workers and managers and will also not care which firm they work for. Since the risk is firm specific this means that observable wages for managers will vary by firm productivity,  $\theta$ .

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<sup>13</sup>This formulation generalizes the standard agency formulation where success depends only on an entrepreneur's own unobserved effort. Hiring managers to increase  $e$  allows the entrepreneur to spread her talent to a wider span of control.

**The Price Vector** The output price is  $p_y$ . Henceforth, let  $\mathbf{p} = (p_y, p_l, p_m)$  denote the price vector. Without loss of generality, and for notational compactness, we allow *all* of the functions that depend on *any* price to be functions of the *entire* price vector even if only some prices are relevant for some specific decisions. In general equilibrium, these prices will be determined endogenously.

### 3.3 Lenders and Credit Contracts

**Wealth and Collateral** Consider a slight variation of the model of Besley et al (2013). Let  $x$  be the loan size. The total capital invested in the project is  $k := x + \psi a$ . If the project succeeds (with probability  $g(e; \theta)$ ), the lender receives a gross *payment* of  $r$  (which means that the implied net interest rate is  $\frac{r}{x} - 1$ ). If the project fails (with probability  $1 - g(e; \theta)$ ) the lender captures  $c$ . The opportunity cost per unit of capital for the lender is  $\gamma \geq 1$  (with  $\gamma - 1$  being net rate of interest). Lenders can all access funds at a constant marginal opportunity cost  $\gamma$ .<sup>14</sup>

A fraction  $\psi$  of the borrower's assets  $a$  can be liquidated at no cost and invested in the project. Any assets that have not been liquidated yield a return  $\gamma$  (the same as the rate of return on liquid assets).<sup>15</sup> Any assets that have not been liquidated,  $(1 - \psi)a$ , can be pledged as collateral. Given the rate of return  $\gamma$ , the potential collateral value to the lender is  $\gamma(1 - \psi)a$ . Let  $\tau_1$  be the probability the lender is able to seize this form of collateral. Without any frictions,  $\tau_1 = 1$ . The fact that  $\tau_1 < 1$  reflects possible frictions associated with liquidating the entrepreneur's personal assets, giving an expected value of collateral of  $\tau_1 \gamma(1 - \psi)a$  from this source.

Suppose also that a fraction  $\delta$  of the firm's capital can be liquidated in case of failure. Again, these assets can be pledged as collateral to the lender. The potential collateral value from liquidating the capital invested in the firm is  $\delta(x + \psi a)$ . Let  $\tau_2$  be the fraction of the liquidated value of the capital invested in the firm that the bank can seize as collateral, giving an expected value of collateral from this source of  $\tau_2 \delta(x + \psi a)$ . The borrower's expected payoff from liquidating the firm's assets is  $(1 - \tau_2) \delta(x + \psi a)$ .

Let  $\pi(k; \theta, \mathbf{p})$  denote the conditional profit function given an allocation of capital  $k$  which will be defined below.

**First-best** In the first-best, the allocation decision consists of choosing effort  $e$  and capital  $k$  to maximize expected total surplus:

$$\max_{e, k} S(e, k; \theta, \mathbf{p}) := g(e; \theta) \pi(k; \theta, \mathbf{p}) + [1 - g(e; \theta)] \delta k - \gamma k - p_m e.$$

How  $k = x + \psi a$  is split between self-financing by the borrower ( $\psi a$ ) and borrowing from the lender ( $x$ ) does not matter, i.e., the first-best total surplus does not depend on  $\psi$ .

The first-order conditions with respect to  $e$  and  $k$  are:

$$g_e(e; \theta) [\pi(k; \theta, \mathbf{p}) - \delta k] = p_m \quad (1)$$

$$g(e; \theta) \pi_k(k; \theta, \mathbf{p}) + [1 - g(e; \theta)] \delta = \gamma. \quad (2)$$

<sup>14</sup>This could be justified by supposing that this is a small open economy that faces a given international interest rate. Otherwise, we would have to close the model with an endogenous  $\gamma$  which equated the demand and supply of loanable funds.

<sup>15</sup>We assume that illiquid assets earn the same market return  $\gamma$  as liquid capital which is consistent with the fact that they can be liquidated costlessly at any point. In a world where houses can be bought and sold with no transactions costs or risks or value appreciation, the returns on them and the interest rate should be the same.

The second order conditions for the existence of a unique maximum require that the functions  $g(e; \theta)$  and  $\pi(k; \theta, \mathbf{p}) - \delta k$  are not just concave but are “sufficiently” concave.<sup>16</sup> For example, for  $g(e) = \theta e^\alpha$ , the condition  $\frac{-g''(e)g(e)}{(g'(e))^2} > 1$  is equivalent to  $\alpha < \frac{1}{2}$ .<sup>17</sup>

Let us denote by  $e_{FB}$  and  $k_{FB}$  the solution to the pair of equations given by the first-order conditions. Then expected total surplus is  $S(e_{FB}, k_{FB}; \theta, \mathbf{p})$ . For the first condition to yield an interior solution, we require  $\pi(k; \theta, \mathbf{p}) - \delta k > 0$ . Notice that this, together with our assumption that  $\pi(k; \theta, \mathbf{p})$  is concave, the second first-order condition and  $g(e; \theta) < 1$  imply<sup>18</sup>

$$\pi_k(k; \theta, \mathbf{p}) > \gamma > \delta.$$

Any interior solution hence requires that  $\gamma > \delta$ . As  $\delta$  is the fraction of the capital that can be salvaged from a failed project,  $\delta \leq 1$  and so this condition holds given our assumption  $\gamma \geq 1$ , which is reasonable as it implies that the net return on capital is non-negative.

**Second-best** Credit contracts are described by a vector  $(x, r, c, \psi a)$  comprising (i) an amount borrowed,  $x$ , (ii) an amount to be repaid if the firm is successful,  $r$ , (iii) an amount of financial collateral  $c$ , (iv) the borrower’s equity  $\psi a$ . For notational simplicity we will use  $\mathbf{t} = (x, r, c, \psi a)$  to denote a credit contract.

A lender’s expected profit when agreeing to lend to an entrepreneur with collateral  $c$  is therefore:

$$\Pi(e, \mathbf{t}; \theta) = g(e; \theta) r + [1 - g(e; \theta)]c - \gamma x. \quad (3)$$

There is a finite set of lenders with whom entrepreneurs can contract. To model competition between lenders, we suppose that there is a Bertrand-style price setting game. Imagine that there are two lenders with identical access to the capital market,  $\gamma$  and the same enforcement technologies. In principle this should lead to borrowers capturing all of the surplus as lenders compete for borrowers until *ex ante* payoffs are zero. However, there are good reasons to doubt that this is a reasonable model and there are likely to be costs of switching between lenders. Rather than being specific about the friction, we capture imperfectly competitive

<sup>16</sup>The second derivatives with respect to  $e$  and  $k$  are  $S_{ee} = g_{ee}(e; \theta) \{\pi(k; \theta, \mathbf{p}) - \delta k\}$  and  $S_{kk} = g(e; \theta) \pi_{kk}(k; \theta, \mathbf{p})$  and the cross partial derivative with respect to  $e$  and  $k$  is  $S_{ek} = g_e(e; \theta) \{\pi_k(k; \theta, \mathbf{p}) - \delta\}$ . The second-order conditions are:  $S_{ee} < 0$ ,  $S_{kk} < 0$  and  $S_{ee}S_{kk} > (S_{ek})^2$ . The first two conditions are satisfied for any strictly concave function. The third one implies

$$\frac{\{-g_{ee}(e; \theta)\}g(e; \theta)}{\{g_e(e; \theta)\}^2} \frac{\{-\pi_{kk}(k; \theta, \mathbf{p})\} \{\pi(k; \theta, \mathbf{p}) - \delta k\}}{\{\pi_k(k; \theta, \mathbf{p}) - \delta\}^2} > 1.$$

If this holds for all  $e$  and  $k$  then the objective function is globally concave and therefore the second-order conditions are also sufficient to guarantee the existence of a global optimum.

<sup>17</sup>A similar condition is required in the standard textbook two input profit-maximization problem of the following nature  $\max_{k,l} = Ak^\alpha l^\beta - rk - wl$  with  $\alpha + \beta < 1$ . Namely, a sufficient condition for the second-order condition to hold globally is  $\frac{1-\alpha}{\alpha} \frac{1-\beta}{\beta} > 1$ .

<sup>18</sup>We can rewrite  $\pi(k; \theta, \mathbf{p}) - \delta k > 0$  as  $\frac{\pi(k; \theta, \mathbf{p})}{k} > \delta$ . As  $\pi(k; \theta, \mathbf{p})$  is concave (due to diminishing returns with respect to capital) and  $\pi(0; \theta, \mathbf{p}) = 0$ :

$$\pi_k(k; \theta, \mathbf{p}) > \frac{\pi(k; \theta, \mathbf{p})}{k}.$$

Therefore at the first-best it must be the case that  $\pi_k(k; \theta, \mathbf{p}) > \delta$ , which together with the second first-order condition and  $g(e; \theta) < 1$  implies:

$$\pi_k(k; \theta, \mathbf{p}) > \gamma > \delta.$$

credit markets by supposing that an alternative lender provides an outside option worth a share  $\phi$  of the total surplus created by their lending contract. If  $\phi = 1$ , then all of the surplus over and above the entrepreneur's outside option accrues to the entrepreneur rather than the lender. This is the competitive benchmark.<sup>19</sup> On the other hand, if  $\phi$  is small, then the lender has a lot of market power.

**Timing** The timing of production for a type  $(a, \theta)$  is as follows.

1. Workers choose whether to become an entrepreneur or worker.
- 2a. If she chooses to become a worker, she inelastically supplies one unit of labour to the labour market.
- 2b. If she is an entrepreneur, then each lender offers her a contract  $(x, r, c, \psi a)$ . After deciding whether to accept this contract, she chooses the level of managerial input,  $e$ .
- 3a. With probability  $g(e; \theta)$ , the firm is successful and then she chooses how much labour to hire,  $l$ . Output is realized, wages are paid to managers and workers, and the loan repayment,  $r$ , is made.
- 3b. With probability  $1 - g(e; \theta)$ , an entrepreneur produces nothing and forfeits collateral,  $c$ .

We now work backwards through these decisions to determine the optimal contract. Here, we suppose that the prices,  $\mathbf{p}$ , are fixed. We then explore the general equilibrium where these are determined.

**Labour Hiring** With probability  $g(e; \theta)$ , the firm is successful in which case it decides how many workers to hire to maximize profits, i.e.

$$l^*(k; \theta, \mathbf{p}) = \arg \max_l \{p_y f(k, l; \theta) - p_l l\} \quad (4)$$

and define  $\pi(k; \theta, \mathbf{p}) := f(k, l^*(k; \theta, \mathbf{p}); \theta) - p_l l^*(k; \theta, \mathbf{p})$  as the conditional profit function given an allocation of capital  $k$ . Throughout we make the following assumption, that ensures well-defined interior solutions.

**Assumption 2** *The following conditions hold for  $g(e; \theta)$  and  $\pi(k; \theta, \mathbf{p})$ :*

- (i)  $\pi(k; \theta, \mathbf{p})$  is strictly concave for all  $k \in \mathbb{R}^+$ .
- (ii)  $g(e; \theta)(\pi(k; \theta, \mathbf{p}) - \delta k)$  is strictly concave for all  $(e, k) \in [0, 1] \times \mathbb{R}^+$ .
- (iii)  $\lim_{e \rightarrow 0} g_e(e; \theta)(\pi(k; \theta, \mathbf{p}) - \delta k) - (1 + g(e)\epsilon(e))p_m > 0$  for all  $k > 0$ ;  
 $\lim_{k \rightarrow 0} g(e; \theta)(\pi_k(k; \theta, \mathbf{p}) - \delta) > \gamma - \tau_2 \delta$  for all  $e > 0$ .

These regularity assumptions guarantee that there is a unique global maximum level of managerial input and capital with an interior solution. The last part of the assumption are Inada-like conditions. They are satisfied by the constant elasticity model used in the calibration below.

<sup>19</sup>It could also represent the case where lenders are not-for-profit NGOs or government banks.

**Choice of Managerial Input** We allow lenders to offer credit to entrepreneurs which are tailored to an entrepreneur's characteristics,  $(a, \theta)$ . Since managerial input is costly and unobserved, the level of such input chosen by the entrepreneur has to be incentive compatible.

The expected payoff of an entrepreneur who borrows under contract  $\mathbf{t}$  is given by:

$$V(e, \mathbf{t}; a, \theta, \mathbf{p}) = g(e; \theta) (\pi(x + \psi a; \theta, \mathbf{p}) - r) + [1 - g(e; \theta)] (\delta(x + \psi a) - c) - p_m e + \gamma(1 - \psi)a. \quad (5)$$

This reflects the fact that, with probability  $g(e; \theta)$ , the lender is repaid and with probability  $(1 - g(e; \theta))$  there is default in which case the lender seizes the entrepreneur's collateral. This is decreasing in the amount of collateral, all else equal. The borrower receives returns on part of her assets that are not invested in the project and not collected as collateral by the lender in the event of the project failing, namely  $\gamma(1 - \psi)a$ .

The first-order condition for managerial input is:

$$g_e(e; \theta) [\pi(x + \psi a; \theta, \mathbf{p}) - \delta(x + \psi a) - r + c] = p_m. \quad (6)$$

The level of such input is increasing in collateral  $c$ , equity  $\psi a$ , and the amount borrowed,  $x$ . However, it is decreasing in  $r$  all else equal, i.e. asking for a higher loan repayment blunts incentives and increases the default rate. Equation (6) is an incentive-compatibility constraint on credit contracts.

Workers who are employed as managers face a risk since the firm may turn out to be unsuccessful. The managerial wage rate must therefore be set such that:  $p_m = p_l / g(e, \theta)$  which will vary with  $e$  reflecting the fact that riskier firms will have to pay managers a higher premium when hiring managers.

**Acceptable Credit Contracts** The limited liability constraint (LLC) with respect to  $r$  says that what the lender can take from the borrower is restricted by the net profits of the firm in the event of success plus the expected liquidation value of the borrowers' assets that are not invested in the project:

$$r \leq \pi(x + \psi a) + \tau_1 \gamma(1 - \psi)a. \quad (7)$$

We do not expect this constraint to necessarily bind as the lender has to respect the participation constraint of the borrower. Also, there is the incentive-compatibility constraint, to be formally introduced below, that takes into account the effect of the contractual terms on the borrower's choice of effort and a high value of  $r$  will tend to reduce  $e$ .

The LLC with respect to  $c$  is

$$c \leq \tau_1 \gamma(1 - \psi)a + \tau_2 \delta(x + \psi a). \quad (8)$$

As much as choosing a high value  $r$  reduces  $e$ , choosing a high value of  $c$  tends to increase  $e$ . Therefore, this will be the relevant constraint for most of our analysis. We show below that if this constraint does not bind, we will have the first-best. This is intuitive, since with all parties being risk neutral, full residual-claimancy (which in this context means  $r = c$ ) gives the efficient level of  $e$  and so would be chosen by the lender if consistent with profit-maximization and feasible given the various constraints.

As well as the level of managerial input being incentive compatible, entrepreneurs must choose to enter lending contracts voluntarily at stage 2, i.e. the contract offered to an entrepreneur of type  $(a, \theta)$  must generate a payoff which exceeds what is available elsewhere



which we denote by  $u$ . This yields a participation constraint:

$$V(e, \mathbf{t}; a, \theta, \mathbf{p}) \geq u(a, \theta, \mathbf{p}). \quad (9)$$

In equilibrium,  $u$  is determined endogenously and depends on  $\theta$ ,  $a$  and  $\mathbf{p}$ . It can be thought of as a price which endogenously clears the credit market given outside opportunities available to an entrepreneur. In other words, it determines the expected returns from entrepreneurship striking a balance between the demand and supply for different occupations in the economy, which in turn depends on economic fundamentals, such as the distribution of talent and wealth and prices. Below we will determine  $p_m$  and  $p_l$  endogenously but all individuals take prices as given when making their decisions.

### 3.4 Credit Contracts in Partial Equilibrium

In this section, we explore access to credit holding fixed who decides to become an entrepreneur and the price vector  $\mathbf{p}$ . We characterize the form of optimal lending contract  $\mathbf{t} = (x, r, c, \psi a)$  by considering two scenarios, i.e. the participation constraint is binding or not.

We begin with a key observation on the properties of such contracts.

**Lemma 1** *Under the optimal contract  $\psi = 1$ .*

The lender will always choose  $\psi = 1$ , i.e. the borrower's equity participation is at the highest possible level. This is driven by the assumption that in our model outside collateral can be transformed into equity at no cost. The entrepreneur's resources are more valuable as equity – which allows to reduce the loan amount – than as collateral, given the inefficiencies associated with lending contracts. As a result there is only inside collateral and the contracting friction  $\tau_1$  does not matter for the allocation. The limited liability constraint given  $\psi = 1$  is

$$c \leq \tau_2 \delta (x + a).$$

Note that the limited liability constraint will always be binding in the second best case, i.e. as long as  $c < r$ . The reason is that increasing the collateral value dominates increasing the repayment burden in case of success: both transfer resources to the lender, but the former has a positive incentive effect, while the latter has a negative incentive effect.

**When the Participation Constraint is not Binding** Suppose the participation constraint is not binding, then using the incentive compatibility constraint for the borrower and the fact that  $k = x + a$ , we can rewrite from (3) the optimal contracting problem as<sup>20</sup>:

$$\max_{e, k} g(e; \theta) \left( \pi(k; \theta, \mathbf{p}) - \frac{p_m}{g_e(e; \theta)} - \delta k \right) + \tau_2 \delta k - \gamma(k - a). \quad (10)$$

The first-order necessary conditions characterizing an interior optimum  $(e_0(\theta, \mathbf{p}), k_0(\theta, \mathbf{p}))$  are

$$g_e(e_0) [\pi(k_0; \theta, \mathbf{p}) - \delta k_0] = [1 + g(e_0; \theta) \epsilon(e_0; \theta)] p_m \quad (11)$$

$$g(e_0) [\pi_k(k_0; \theta, \mathbf{p}) - \delta] = \gamma - \tau_2 \delta \quad (12)$$

<sup>20</sup>We have shown in the appendix that in the second best case, the limited liability constraint is binding, i.e.  $c = \tau_2 \delta k$ , in a way similar to Besley et al. (2012).

By Assumption 2(iii), the unique global maximum  $(e_0, k_0)$  is an interior solution. Note that in this case, the optimal managerial input,  $e_0(\theta, \mathbf{p})$ , and capital level,  $k_0(\theta, \mathbf{p})$ , are independent of  $u$ . The intuition behind equation (11) is that, it is “as if” the cost of managerial input is increased by the term  $g(e_0; \theta) \epsilon(e_0; \theta)$  which represents the marginal “agency cost” due to moral hazard.

Equation (12) determines capital allocation. Two forces lead to an allocation of capital that is different from first best. First, the firm’s capital is depressed as default risk increases. However, conditional on default probabilities, this re-allocation of capital is efficient in the sense that expected marginal returns are equalized. Second, firm capital will be depressed when  $\tau_2 < 1$ . This distortion is caused by the lenders’ anticipation that they will not be able to recover the full collateral in case of default, effectively raising the costs of funds. Equation (12) emphasizes the role that equilibrium default has on the capital available to a firm. Unlike most of the existing literature, (e.g., Buera et al. (2011) and Buera et al. (2015)), the credit market friction affecting capital allocation is determined in equilibrium as a function of the equilibrium price vector and outside option in addition to borrower characteristics  $a$  and  $\theta$ . This will also be a feature of the calibration of the model below and we will explore heterogeneity in default rates in this setting.

Any credit contract with  $(e_0(\theta, \mathbf{p}), k_0(\theta, \mathbf{p}))$  yields the same surplus, and we will show that it is the optimal contract when the borrower has an outside option below some threshold  $\underline{u}(\theta, \mathbf{p})$ .

**When the Participation Constraint is Binding** Using the binding participation constraint and the incentive compatibility constraint, we can characterize the implicit equation characterizing the borrower’s optimal effort level as

$$p_m \left[ \frac{g(e; \theta)}{g_e(e; \theta)} - e \right] + (1 - \tau_2) \delta k = u. \quad (13)$$

This gives the optimal effort level as a function of  $k$ ,  $e^* = v(u, k; \theta, \mathbf{p})$ , and by partial differentiation, we know  $v(u, k; \theta, \mathbf{p})$  is increasing in  $u$  and decreasing in  $k$ .

Now using the binding participation constraint, the optimal contracting problem becomes

$$\max_k g(v(u, k; \theta, \mathbf{p})) (\pi(k; \theta, \mathbf{p}) - \delta k) + \delta k - p_m v(u, k; \theta, \mathbf{p}) - \gamma(k - a) - u$$

The first-order necessary condition characterizing the optimal capital level  $k^*$  is given by

$$g(v(u, k^*; \theta, \mathbf{p})) (\pi_k(k^*; \theta, \mathbf{p}) - \delta) + v_k(u, k^*; \theta, \mathbf{p}) [g_e(v(u, k^*; \theta, \mathbf{p})) (\pi(k^*; \theta, \mathbf{p}) - \delta k^*) - p_m] = \gamma - \delta \quad (14)$$

This equation captures the direct effect of equilibrium default on capital allocation, i.e. the same term  $g(e; \theta)(\pi_k(k; \theta, \mathbf{p}) - \delta)$  also appeared in equation (12). In addition, higher capital will increase default and hence decrease surplus, which is the term in square brackets.<sup>21</sup> This is because the increase in capital corresponds to lump-sum transfer to the borrower: in the case of default  $\delta k$  can be liquidated, but this can only partly be recovered by the lender as long as  $\tau_2 < 1$ . The lender will want to increase the repayment burden over and above what he would do to extract additional surplus generated by additional capital, and this depresses managerial input. Note that this distortion is amplified the lower is  $\tau_2$ , as can be seen from equation (13):

<sup>21</sup>It can be shown that the term in square brackets is positive. Further note that  $v_k(u, k^*; \theta, \mathbf{p}) < 0$ .

for  $\tau_2 < 1$  any increase in capital will only partially increase the collateral value of the project, which requires an even higher increase in the repayment burden to be beneficial to the lender; that however is detrimental to the provision of managerial input.

Further note that the optimal capital level can be written as an increasing function of outside option,  $\varsigma(u; \theta, \mathbf{p})$ . The optimal managerial input level is defined as  $\xi(u; \theta, \mathbf{p}) := \nu(u, \varsigma(u; \theta, \mathbf{p}); \theta, \mathbf{p})$ . We show in the appendix that both the capital level and the optimal managerial input are interior solutions, and both are increasing function of  $u$ . Any improvement in an entrepreneur's outside option will require the lender to impose a lower repayment burden, which increases managerial inputs, reduces default, and consequently increases the level of capital in the firm.

Note that when  $u$  is large enough, i.e.  $u \geq \bar{u} = p_m \left( \frac{g(e_{FB}; \theta)}{g_e(e_{FB}; \theta)} - e_{FB} \right) + (1 - \tau_2) \delta k_{FB}$ , the first best outcome is achieved, where managerial input level is chosen to set the marginal benefit equal to the marginal cost when the entrepreneur is a full residual claimant. At the other extreme, for low  $u$ , the participation constraint will not binding. The value of  $u$  in determining the optimal contract is summarized in our next result:

**Proposition 1** *There exists  $[\underline{u}(\theta, \mathbf{p}), \bar{u}(\theta, \mathbf{p})]$  such that optimal lending contracts yield managerial input  $\hat{e}$  and total lending  $\hat{x}$ , as follows:*

$$\hat{e}(u; \theta, \mathbf{p}) = \begin{cases} e_0(\theta, \mathbf{p}) & \text{for } u < \underline{u}(\theta, \mathbf{p}) \\ \xi(u; \theta, \mathbf{p}) & \text{for } \underline{u}(\theta, \mathbf{p}) \leq u < \bar{u}(\theta, \mathbf{p}) \\ e_{FB}(\theta, \mathbf{p}) & \text{for } u \geq \bar{u}(\theta, \mathbf{p}) \end{cases}$$

where  $e_0(\theta, \mathbf{p})$  is a constant, defined in the case where the participation constraint is not binding;  $e_{FB}(\theta, \mathbf{p})$  is a constant equal to first best managerial input;  $\lim_{u \rightarrow \underline{u}(\theta, \mathbf{p})} \xi(u; \theta, \mathbf{p}) = e_0(\theta, \mathbf{p})$  and  $\lim_{u \rightarrow \bar{u}(\theta, \mathbf{p})} \xi(u; \theta, \mathbf{p}) = e_{FB}(\theta, \mathbf{p})$ .

$$\hat{x}(u; a, \theta, \mathbf{p}) = \begin{cases} k_0(\theta, \mathbf{p}) - a & \text{for } u < \underline{u}(\theta, \mathbf{p}) \\ \varsigma(u; \theta, \mathbf{p}) - a & \text{for } \underline{u}(\theta, \mathbf{p}) \leq u < \bar{u}(\theta, \mathbf{p}) \\ k_{FB}(\theta, \mathbf{p}) - a & \text{for } u \geq \bar{u}(\theta, \mathbf{p}) \end{cases}$$

where  $k_0(\theta, \mathbf{p})$  is a constant, defined in the case where the participation constraint is not binding;  $k_{FB}(\theta, \mathbf{p})$  is a constant equal to first best managerial input;  $\lim_{u \rightarrow \underline{u}(\theta, \mathbf{p})} \varsigma(u; \theta, \mathbf{p}) = k_0(\theta, \mathbf{p})$  and  $\lim_{u \rightarrow \bar{u}(\theta, \mathbf{p})} \varsigma(u; \theta, \mathbf{p}) = k_{FB}(\theta, \mathbf{p})$ .

The gross repayment  $r$  is pinned down by the binding incentive compatibility constraint and limited liability constraint as

$$r(u; a, \theta, \mathbf{p}) = \pi(\hat{x}(u; a, \theta, \mathbf{p}) + a; \theta, \mathbf{p}) - (1 - \tau_2) \delta(\hat{x}(u; a, \theta, \mathbf{p}) + a) - \frac{p_m}{g_e(\hat{e}(u; \theta, \mathbf{p}); \theta)}. \quad (15)$$

Notice that while  $\hat{x}(u; a, \theta, \mathbf{p})$  does depend on wealth  $a$ , the optimal capital level  $\hat{k} := \hat{x}(u; a, \theta, \mathbf{p}) + a$  does not. We can then define the indirect total surplus in the lending relationship as

$$\hat{S}(u; \theta, \mathbf{p}) := S(\hat{e}(u; \theta, \mathbf{p}), \hat{x}(u; a, \theta, \mathbf{p}) + a; \theta, \mathbf{p}) \quad (16)$$

Observe that surplus does not depend on wealth  $a$  directly, other than through any effect it might have on  $u$ . Further, for the first best case and the case where participation constraint is not binding, the optimal managerial input and capital level are independent from  $u$ , so  $u$

will not affect total surplus, i.e.  $S_u = 0$ , where  $S_u$  denotes the partial derivative of the surplus function with respect to  $u$ . The following result gives a characterization of the ranges in which  $u$  can fall in terms of the surplus function:

**Corollary 1** *The indirect surplus function,  $\hat{S}(u; \theta, \mathbf{p})$ , is increasing in  $u$  for all  $u \in (\underline{u}(\theta, \mathbf{p}), \bar{u}(\theta, \mathbf{p}))$ . For  $u \geq \bar{u}(\theta, \mathbf{p})$  or  $u \leq \underline{u}(\theta, \mathbf{p})$ ,  $\hat{S}(u; \theta, \mathbf{p})$  is constant.*

Credit contracts will implement first best level of managerial input as long as entrepreneurs can provide sufficient collateral, i.e. has high  $a$ , or a high outside option. The first of these is standard feature of existing models of *ex post* enforcement constraints. What the general equilibrium contracting model emphasizes is that whether the first-best is attainable also depends on an endogenously determined outside option which affects the equilibrium default rate. In the intermediate  $u$  range, greater collateral allows for more efficient lending since it relaxes (6). The lender then offers a higher  $x$ , which amplifies the effect of collateral on the incentive compatibility constraint. Similarly, a higher outside option increases lending efficiency. The lender has to transfer a greater share of surplus to the entrepreneur, and this is optimally implemented by reducing  $r$  and increasing  $x$ , which in turn increase managerial input. However, for  $u \leq \underline{u}(\theta, \mathbf{p})$  the lender will always implement  $e_0(\theta, \mathbf{p})$ . In this range – due to the concavity of  $g(e; \theta)$  – a reduction in  $r$  increases surplus by more than it transfers surplus to the entrepreneur. Therefore it is in the interest of the lender to offer a contract which leaves the entrepreneur with an expected income greater than the outside option. It is optimal to transfer surplus by decreasing  $r$ . In this region, the lender reacts to an increased  $c$  by increasing  $r$  by the same amount, and leaving both  $e$  and  $x$  unchanged. Surplus stays unchanged, but is transferred from the borrower to the lender.

**The Lender's Participation Constraint** Whether a lender wishes to lend to an entrepreneur of type  $(a, \theta)$  depends upon whether they can make a profit by doing so. Hence for an entrepreneur of type  $(a, \theta)$  to be offered any credit requires that

$$\hat{\Pi}(u; a, \theta, \mathbf{p}) := \Pi(\hat{e}(u; \theta, \mathbf{p}), \mathbf{t}(u; a, \theta, \mathbf{p}); \theta) \geq 0.$$

**Determining the Entrepreneur's Outside Option** The final part of the partial equilibrium analysis is to determine the entrepreneur's outside option endogenously. This will be the maximum of three things: (i) what she can obtain by borrowing from another lender, (ii) self-financing the project with the (limited) wealth owned and (iii) working for a wage. We now explore this in detail.

Let  $\hat{u}(\phi; \theta, \mathbf{p})$  be defined by:

$$\phi \cdot \hat{S}(\hat{u}(\phi; \theta, \mathbf{p}); \theta, \mathbf{p}) = \hat{u}(\phi; \theta, \mathbf{p}). \quad (17)$$

This implicitly defines the equilibrium payoff of an entrepreneur if the only outside option is to receives a share  $\phi$  of the surplus in a lending relationship. Note that this is not the payoff from borrowing since the efficiency utility in Proposition 1 bounds the borrower's payoff from below when the outside option is low, and in particular when  $\phi$  is low.<sup>22</sup>

<sup>22</sup>Note that even with  $\phi = 0$ , the lender does not necessarily receive  $u$  since, as we observed Proposition 1, the entrepreneur's participation constraint might not be binding.

Now consider the payoff where the agent chooses to self-finance, i.e. use only his own wealth. This is given by

$$V^{self}(a, \theta, \mathbf{p}) = \max_{(e, k)} [g(e; \theta) (\pi(k; \theta, \mathbf{p}) - \delta) - p_m e + \gamma(a - k) : k \leq a]. \quad (18)$$

Let  $\{e^{self}(a, \theta, \mathbf{p}), k^{self}(a, \theta, \mathbf{p})\}$  denote the solutions to the maximization problem (18). Lastly the entrepreneur could choose to become a wage labourer. The entrepreneurs outside option will therefore be given by

$$u(a, \theta, \mathbf{p}) = \max\{V^{self}(a, \theta, \mathbf{p}), \hat{u}(\phi; \theta, \mathbf{p}), p_l + \gamma a\}.$$

**Comparative Statics** We now have the following result for the payoff of entrepreneurs:

**Proposition 2** *The entrepreneur's expected profit increases with more competition ( $\phi$ ) and greater wealth ( $a$ ). Without further assumptions, the effect of productivity ( $\theta$ ) on entrepreneur's expected profit is indeterminate.*

Thus entrepreneurs benefit from increased competition since they get a larger share of the surplus in the credit market. They also do better when they have more collateral to post. Increasing productivity has competing effects which explains the ambiguous effect on total surplus. On the one hand, profits are higher as firms are more productive. However, the effect on the repayment probability is ambiguous since the cost of managerial input is larger in larger firms.

### 3.5 General Equilibrium

So far, we have taken the price vector  $\mathbf{p}$  and the occupational structure as given. Our general equilibrium analysis determines these endogenously.

**Financial Market Access** We assume that a fraction  $z(a, \theta) \in [0, 1]$  of agents of type  $(a, \theta)$  has access to financial markets.<sup>23</sup> Denote with  $\chi \in \{0, 1\}$  whether any given individual has access to credit markets. Let  $h(a, \theta)$  denote the joint density associated with the distribution of  $(a, \theta)$ . Total financial inclusion in the economy is defined by

$$\bar{\chi} := \int \int z(a, \theta) h(a, \theta) da d\theta,$$

i.e. as the proportion of agents who have market access. If they have access then they can access credit markets as described in the previous section.

**Occupational Choice** Let  $\sigma \in \{0, 1\}$  denote whether an agent becomes an entrepreneur, with  $\sigma = 1$  indicating entrepreneurship and  $\sigma = 0$  indicating becoming a worker. An agent will choose entrepreneurship when the expected payoff from being an entrepreneur exceeds that from being a wage labourer. Formally,

$$\sigma(\chi, a, \theta, \mathbf{p}) = \begin{cases} 1 & \text{if } \chi = 1 \text{ and } \hat{\Pi}(u(a, \theta, \mathbf{p}); a, \theta, \mathbf{p}) \geq 0, \text{ or, if } V^{self}(a, \theta, \mathbf{p}) \geq p_l + \gamma a. \\ 0 & \text{otherwise.} \end{cases}$$

<sup>23</sup>We assume that autarky is the only alternative to credit market access. An interesting extension in future work would be to allow an informal sector which could be characterized by a higher cost of funds,  $\gamma$  and would be another potential outside option for the borrower.



The borrower will always choose to become an entrepreneur if the autarchy payoff is bigger than the wage. If she has access to credit markets, she will also become an entrepreneur if the lender can offer a profitable credit contract (satisfying the borrower's outside option and incentive constraint). Clearly this depends on the individuals type  $(a, \theta)$ . Moreover, since the payoff from entrepreneurship is increasing in  $a$  and  $\theta$ , if a type  $(a, \theta)$  becomes an entrepreneur then so do all individuals with higher wealth and productivity. Hence, there will be critical values of wealth and productivity that define the entrepreneurial class. How dense this is depends on the joint distribution of wealth and productivity.

**Equilibrium Wages** To determine equilibrium wages, we need to solve for aggregate labour supply and demand in the economy. This means aggregating over the distribution of wealth and productivity. Aggregate labour supply is determined by the fraction of individuals who choose not to become entrepreneurs, i.e.

$$L^S(\mathbf{p}) = \int \int [z(a, \theta) \{1 - \sigma(1, \theta, a, \mathbf{p})\} + (1 - z(a, \theta)) \{1 - \sigma(0, \theta, a, \mathbf{p})\}] h(a, \theta) da d\theta. \quad (19)$$

Denote managerial labour demand, conditional on becoming entrepreneur, as

$$\hat{e}(\chi, a, \theta, \mathbf{p}) := \chi(\hat{e}(u(a, \theta, \mathbf{p}); a, \theta, \mathbf{p}) + (1 - \chi)e^{self}(a, \theta, \mathbf{p}),$$

and firm capital, conditional on becoming entrepreneur, by

$$\hat{k}(\chi, a, \theta, \mathbf{p}) := \chi(\hat{k}(u(a, \theta, \mathbf{p}); a, \theta, \mathbf{p}) + (1 - \chi)k^{self}(a, \theta, \mathbf{p}).$$

To solve for aggregate labour demand we need to take into account the fraction of firms that are operational given the equilibrium default probability which we denote by

$$\hat{g}(\chi, a, \theta, \mathbf{p}) = g(\hat{e}(\chi, a, \theta, \mathbf{p}); \theta).$$

Note that this also depends on  $\mathbf{p}$  through its effect on profits and the cost of managerial input. Labour demand also depends on the amount of labour hired by each firm, conditional on producing. We will denote this by

$$\hat{l}(\chi, a, \theta, \mathbf{p}) = l^*(\hat{k}(\chi, a, \theta, \mathbf{p}); \theta, \mathbf{p})$$

using (4).

Aggregate labour demand is then given by

$$\begin{aligned} L^D(\mathbf{p}) &= \int \int z(a, \theta) \left[ \sigma(1, a, \theta, \mathbf{p}) \cdot \left( \hat{l}(1, a, \theta, \mathbf{p}) \cdot \hat{g}(1, a, \theta, \mathbf{p}) + \hat{e}(1, a, \theta, \mathbf{p}) \right) \right] h(a, \theta) da d\theta \\ &+ \int \int (1 - z(a, \theta)) \left[ \sigma(0, a, \theta, \mathbf{p}) \cdot \left( \hat{l}(0, a, \theta, \mathbf{p}) \cdot \hat{g}(0, a, \theta, \mathbf{p}) + \hat{e}(0, a, \theta, \mathbf{p}) \right) \right] h(a, \theta) da d\theta. \end{aligned} \quad (20)$$

This is the sum over the labour demand functions of individuals who choose to become entrepreneurs at prevailing prices  $\mathbf{p}$ , characterized by  $(a, \theta, \chi)$ .

The equilibrium prices  $\hat{\mathbf{p}}$  now equates supply and demand, i.e. solves

$$L^S(\hat{\mathbf{p}}) = L^D(\hat{\mathbf{p}}).$$

This depends implicitly on all dimensions of choice: occupational choice, credit contracts which determines use of capital and labour demand. It also depends on the extent of financial access since this will affect who becomes an entrepreneur and the amount of labour demand among those who do, depending on whether they can access financial markets.

## 4 From Theory to Data

We use the model to produce a range of calibrated counterfactuals to explore the model's predictions quantitatively. The framework allows us to think about two main things. First, we can think about the effect of credit market frictions on optimal credit contracts. We can explore the effect of two specific frictions as represented by  $\phi$  and  $\tau_2$ . For the remaining of this paper, we simplify the notations by denoting  $\tau = \tau_2$ . Second, we can look at impact of changing market access as represented by  $z(a, \theta)$ .

Changing market frictions affects labour demand for a given wage in (20) through three channels. First, it increases access to capital and this increases labour demand since capital and labour are complements. Second, it reduces the default probability by increasing managerial input. Third, it lowers the threshold productivity and wealth levels at which agents choose to become entrepreneurs. Increasing  $z(a, \theta)$  has a direct effect on labour demand since some entrepreneurs now get access to more capital.

General equilibrium effects are largely driven by shifts in labour demand and occupation choice which affect the wage which, in turn, feeds back on to the participation constraint of entrepreneurs and hence to the terms of credit contracts. Wages also affect the amount of managerial labour applied by changing profitability and the amount of capital used.

The model is able to give a clear sense of the different “moving parts” that affect credit market frictions in a general equilibrium model with endogenous occupational choice. Our next step is to put the model to work by exploring different aspects of its quantitative predictions. For this, we will need to give a specific parametrization and simulate the model's predictions which will give insights in three main areas.

We next describe the specific functional forms that use and then discuss how various key parameters are calibrated.

### 4.1 Parametrization

The production function,  $f(k, l; \theta)$  is Cobb-Douglas with diminishing returns:

$$f(k, l; \theta) = \theta^{1-\eta-\alpha} \left( l^{1-\beta} k^\beta \right)^\eta, \quad (21)$$

where  $\theta$  is the firm specific productivity parameter and  $\alpha, \beta$ , and  $\eta$ , all of them belonging to the interval  $(0, 1)$ , are parameters governing the shape of the production function. Thus the model is essentially a classic Lucas-style “span of control” model  $\eta$  representing the extent of diminishing returns and pure profits can be thought of as payment to an untraded factor such as technology or ability.

Using this, a firm's labour demand, conditional on  $k$ , is given by:

$$l^*(k; \theta, \mathbf{p}) = \left[ \eta (1 - \beta) \frac{p_y}{p_l} \theta^{1-\eta-\alpha} k^{\eta\beta} \right]^{\frac{1}{1-\eta(1-\beta)}} \quad (22)$$

and the conditional profit function is

$$\pi(k; \theta, \mathbf{p}) = (1 - \eta (1 - \beta)) \left[ p_y \left( \frac{\eta (1 - \beta)}{p_l} \right)^{\eta(1-\beta)} \theta^{1-\eta-\alpha} k^{\eta\beta} \right]^{\frac{1}{1-\eta(1-\beta)}} \quad (23)$$

The marginal product of capital is therefore given by:

$$\pi_k(k; \theta, \mathbf{p}) = \eta\beta \left[ p_y \left( \frac{\eta(1-\beta)}{p_l} \right)^{\eta(1-\beta)} \theta^{1-\eta-\alpha} k^{\eta-1} \right]^{\frac{1}{1-\eta(1-\beta)}} \quad (24)$$

For the success technology, we use a constant-elasticity functional form where:<sup>24</sup>

$$g(e; \theta) = \lambda [e/\theta^\mu]^\alpha \text{ with } \mu \geq 0.$$

The parameter  $\mu$  governs the dependence of the cost of managerial input on  $\theta$ , i.e. the link between this and firm size. If  $\mu = 0$ , then the cost of securing a given level of default does not depend on firm size whereas  $\mu > 0$  means that achieving the same default in a large firm requires more managerial input. The parameter  $\alpha$  in the technology above governs the elasticity of the success probability with respect to managerial input.<sup>25</sup> Together with the assumption in (21) this functional form implies that output has constant returns to scale in managerial input ( $e$ ), capital ( $k$ ), labour ( $l$ ), and entrepreneurial talent ( $\theta$ ). Finally, the parameter  $\lambda$  captures the general productivity of managerial input in achieving project success. In the next section, we will show how to use data on the firm size distribution and heterogeneous default probabilities by firm size to calibrate  $(\mu, \alpha, \lambda)$ . Each agent who works as an employee is indifferent between being a worker (providing input  $l$ ) and managerial labour; they are paid at rate  $p_l$ , or - alternatively - at a risk-adjusted wage rate  $p_m (= p_l/g(e; \theta))$  in case of success.

## 4.2 Calibration

Without loss of generality, we will take the output price to be the same across countries and choose the unit of measurement such that  $p_y = 1$ . Since we will think of the price of capital goods (but not necessarily the rental rate) to be equal across countries, we measure capital,  $k$  in value terms. Further we will assume  $p_m/g(e; \theta) = p_l$ .

**Model Parameters** We calibrate a subset of the model parameters using evidence from existing studies. First, we assume that  $\beta$ , which in first best measures the share of output paid to capital relative to labour<sup>26</sup>, is 1/3 in line with standard calibrations used in the macro-economic literature. Second, we set  $\eta$  to 3/4, following the assumption of Bloom (2009) in a related context. Third, we assume  $\lambda = 1.0$  for both calibration and simulation exercises. Fourth, we take the marginal cost of capital  $\gamma$  to be 1.02, which roughly corresponds to long run real interest rates in the US since the 1980s (Yi and Zhang (2016)) with an allowance for capital depreciation.<sup>27</sup> Fifth,  $\delta$  is the recovery rate in case of default. We back this out from data on the

<sup>24</sup>An alternative isomorphic specification in terms of effort choice would assume that the same level of effort is more costly in large firms:  $g(e; \theta) = \lambda e^\alpha$  and cost of effort  $p_e \theta^\mu e$ .

<sup>25</sup>The parameter  $\alpha$  will be chosen such that first best default probabilities  $g(e^*(\theta, \mathbf{p}); \theta)$  match their empirical counter-part, which is particularly high for large firms, i.e. at the highest level of  $\theta$ . For lower levels of  $\theta$  and in second-best default probabilities will be higher, i.e. success probabilities will be lower. Therefore no additional assumption is required to guarantee that  $g(e; \theta) \in [0, 1]$ .

<sup>26</sup>Note that this only holds when defining the labor income share as payments to  $l$ , not  $e$ .

<sup>27</sup>The rates paid to depositors in developing countries are of little guidance to calibrate  $\gamma$  if depositors are not the marginal source of funding, or there are transaction costs in financial intermediation.

charge-off rate (0.0084) and delinquency rate (0.0279) for corporate loans since 1985 in the US. Those imply  $\delta = 0.711$ , or a loss of 28.9% of the loan value in case of corporate defaults.<sup>28</sup>

We normalize, without loss of generality, the US wage to be one, and therefore any wage or income level in the distorted model is measured relative to the US wage.

**Entrepreneurship and the Distribution of Productivity** The remaining parameters are  $\alpha$  and  $\mu$ , the distribution of  $\theta$  as well as the competitiveness of credit market  $\phi$ . We pin down those parameters jointly by calibrating the model to US data, assuming that this is an example of well-functioning credit markets. While this assumption is somewhat extreme, it may still serve as a reasonable approximation of the difference between US credit markets and credit markets in developing countries which is our main focus of attention. What makes this assumption convenient is that all of the model's predictions are independent of the wealth distribution, allowing to calibrate the unknown parameters without knowledge of the wealth distribution. We can then specify any hypothetical marginal wealth distribution and its correlation with the productivity distribution when we simulate second best outcomes.

The parameters  $\alpha$  and  $\mu$  and the distribution of  $\theta$  jointly determine the pattern of corporate default rates across firm sizes and the firm size distribution and therefore need to be chosen *jointly* to match those moments in the US data. However, to clarify which variation in the data pins down which parameters, it is instructive to consider how the pattern of corporate default rates across firm sizes depends on  $\alpha$  and  $\mu$ , conditional on the  $\theta$  distribution; and how the firm size distribution is linked to the  $\theta$  distribution conditional on  $\alpha$  and  $\mu$ .

First, we can solve for the first best level of managerial labour  $e_{FB}$  in closed form as:

$$e_{FB}(\theta, \mathbf{p}) = \left[ p_y \theta^{1-\eta-\alpha} \left( \frac{\eta(1-\beta)}{p_l} \right)^{\eta(1-\beta)} \left( \frac{\eta\beta}{\gamma} \right)^{\eta\beta} \left( \frac{\lambda\alpha(1-\eta(1-\beta))}{p_l\theta^\mu} \right)^{1-\eta} \right]^{\frac{1}{(1-\alpha)(1-\eta)-\alpha\eta\beta}} \quad (25)$$

Generally a tuple  $(\mu, \alpha)$  implies that first best default probabilities increase or decrease with first best firm size: the parameter  $\alpha$  govern the general level of default, and the parameters  $\mu$  governs the rate at which default probabilities increase/decrease with firm size. We choose  $\alpha$  and  $\mu$  such that the (loan value weighted) average default probability amongst firms with less than 250 employees and firms with more than 250 employees matches the average default probabilities of 0.105 and 0.062 for those respective firm size categories reported in an early version of Besley et al. (2020).

Second, the marginal distribution of  $\theta$  can be backed out from data on the distribution of firms sizes, conditional on  $\alpha$  and  $\mu$ . Plugging  $k_{FB}(\theta, \mathbf{p})$  into (22) we can write equilibrium labour demand,  $l_{FB}(\theta, \mathbf{p}) := l^*(k_{FB}; \theta, \mathbf{p})$  as a monotonically increasing function of  $\theta$ . Empirically the distribution of firm sizes measured in terms of the size of the labour force. This corresponds to  $e_{FB}(\theta, \mathbf{p}) + l_{FB}(\theta, \mathbf{p})$  in the model, assuming that only successful firms are recorded in the data. Both  $e_{FB}(\theta, \mathbf{p})$ , given the above calibration, and  $l_{FB}(\theta, \mathbf{p})$  are monotonically increasing in  $\theta$ . Empirically the firm size distribution is known to be well approximated by a Pareto distribution, with shape parameter  $\sigma_l = 1.059$  (Axtell (2001)). The monotonic relationship described above allows, for any size of the labour force observed in the data, to back

<sup>28</sup>Data from Board of Governors of the Federal Reserve (2016, <https://www.federalreserve.gov/releases/chargeoff/>).

out  $\theta$ .<sup>29</sup> Further we can calculate the associated probability of success and hence back out the density of that  $\theta$  in the population given the empirical frequency of that firm size.

However, this procedure only works for any  $\theta$  such that  $\sigma(a, \theta, \mathbf{p}) = 1$ , i.e. values of  $\theta$  for which individuals choose to operate the project. To pin down the full  $\theta$  distribution we assume that also the distribution of successful firms that would be observed if all individuals decided to operate their project follows a Pareto distribution with shape parameter  $\sigma_l = 1.059$ . Note that this assumption is consistent with the data, since a lower-truncated Pareto distribution is again a Pareto distribution with the same shape parameter. We then find the scale parameter of that firm size distribution, and more importantly the associated full  $\theta$  distribution, such that labour markets clear at the observed US wage, i.e. solve  $L^S(\mathbf{p}) = L^D(\mathbf{p})$  at  $p_l = 1$ .

**Financial Frictions** The rating agency Moody's has calculated the recovery rate for of US 1st Lien Loan which is on average 63.39% over 2017-2022. The counterpart moment in our framework is the composite of  $\delta$  and  $\tau$ . We have chosen  $\delta = 0.711$  based on the charge-off rate and delinquency rate for corporate loans since 1985 in the US. And we will assume  $\tau = 0.9$  to capture the US benchmark. Besides, we choose  $\chi = 0.95$  to match the US level of financial access.

We calibrate the degree of competition in the credit market to the share of financial profits in total profits for the US as a point of departure. Philippon (2015) examines the evolution of the share of the finance industry in the US economy over the past century and we use the value added and employee compensation by industry data from Bureau of Economic Analysis (BEA) to get a measure of the share of financial industry's profits in total US GDP, which is on average 4% over the past 7 years.

**The Distribution of Wealth** We can specify the marginal asset distribution to follow any observed or hypothetical wealth distribution. For our calibration exercise we choose the marginal distribution of assets to approximate the wealth distribution in the US, whereas our simulation exercises will be based on the Indian wealth distribution. We obtained data on the Indian wealth distribution from the Global Wealth Report 2015 (Credit Suisse, 2015).

This report provides information on the Gini coefficient of the Indian wealth distribution, mean wealth, median wealth and the fraction of the population in four wealth classes: 0-10k, 10k-100k, 100k-1m and over 10m USD. The median wealth in India is 1.75% of median wealth in the US, and the mean wealth is 1.24% of mean wealth in the US.

We assume the US and Indian wealth distribution to be of the Pareto family, which has been shown to be a reasonable approximation in a number of countries. This reduces the calibration to choosing a shape and scale parameter of that distribution. Moreover, given the Pareto assumption, the shape parameter has a known monotonic relation to the Gini coefficient. We use this relation together with the aforementioned data on the empirical Gini coefficient to back out the shape parameter. Specifically, the scale parameter is chosen to minimize the sum of squared differences between the empirical probability mass and the probability mass of the calibrated Pareto distribution in each of the four wealth categories, where the summation is across wealth categories.

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<sup>29</sup>This implies that in the first best scenario, the largest firm in our simulations is as large as the largest US firm in our data. In second best simulations larger firms might emerge to the extent that high productivity entrepreneurs have access to capital and wages are depressed.



Lastly, we need to specify the joint distribution  $h(a, \theta)$  of assets and productivities.<sup>30</sup> This is difficult to back out non-parametrically from data. In a world with first best credit contracts, knowledge of individual wealth levels, occupational status, and the size of the labour force of firms held by entrepreneurs, would be sufficient to back out the joint distribution of  $a$  and  $\theta$  for the subset of individuals with a  $\theta$  high enough to become entrepreneurs. However, for all individuals with a value of  $\theta$  that does not lead to them becoming entrepreneurs,  $\theta$  is fundamentally unobserved. In our simulations we therefore work with several hypothetical joint distributions.

To this end, we can specify a pattern of dependency between  $a$  and  $\theta$  using the statistical concept of copulas.<sup>31</sup> According to Sklar’s theorem (Sklar (1959)), the multivariate density function  $h(a, \theta)$  can be rewritten as  $h(a, \theta) = h_a(a) \cdot c(H_a(a), H_\theta(\theta)) \cdot h_\theta(\theta)$ , where  $H_a(\cdot)$  and  $H_\theta(\cdot)$  are the cumulative density functions of the marginal distribution of  $a$  and  $\theta$ , respectively,  $h_a(\cdot)$  and  $h_\theta(\cdot)$  are the corresponding probability density functions, and  $c : [0, 1]^2 \rightarrow R^+$  is the density function of the copula. We assume that the dependency between  $a$  and  $\theta$  is characterized by a Normal copula. This implies that the only free parameters that have to be specified is the covariance which we choose such that the induced correlation between  $a$  and  $\theta$  matches one of a range of “target values” of the correlation. As we increase  $\rho$  we are postulating a stronger link between productivity and wealth. Using this approach, we can simulate our model given each value of  $\rho$  to trace out the implications of different degrees of correlation between  $a$  and  $\theta$  for credit market outcomes. For our main simulation results below, we will take  $\rho = 0$ .

### 4.3 Computation

In order to compute the model, we approximate the continuous distribution of  $a$  and  $\theta$  by a distribution with 1000 and 5000 discrete values, respectively, both in the calibration and the subsequent simulations. These discrete values represent equally spaced centiles of the continuous distribution.

When calibrating the model we solve jointly for the distribution of productivities,  $\alpha$ ,  $\mu$  and  $\phi$  using an iterative process as follows. We start from an initial trial value of the parameters affecting default risk, the cost of managerial input and financial profit share of GDP,  $(\mu, \alpha, \phi)$ , and then find the distribution of productivity,  $\theta$ , to match the empirical firm size distribution and ensure that the labour markets clear at a wage ( $p_l$ ) of one as described above. Conditional on this distribution of  $\theta$  we then update the value of  $(\mu, \alpha, \phi)$  to generate the empirical average default probabilities for small and large firms which are active in the equilibrium, financial profit shares, etc. We then iterate this process until the values of  $\alpha$ ,  $\mu$  and  $\phi$  converge in the sense that their values change each by less than 0.1 percentage points relative to the previous iterations.

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<sup>30</sup>Here we take the joint distribution of productivity and wealth as a primitive. This contrasts with the approach taken in Buera et al. (2011) and Buera et al. (2017) where the distribution of productivity is the only primitive,  $h(a, \theta)$  then being determined endogenously through the agents’ saving behavior. In a model with default, we would not expect the distribution of wealth to be pinned down only by  $\theta$  since it would depend on the history of default which would wipe out an entrepreneur’s wealth in our framework. Introducing savings into our framework is an important future extension. More generally, the framework that we are proposing could handle shocks to the value of wealth due, for example, to asset price fluctuations which hit agents heterogeneously.

<sup>31</sup>See Nelsen (1999) and Trivedi and Zimmer (2007) for accessible introductions.

The core problem of the simulations is to find the equilibrium wage at each level of  $\tau_2$ ,  $\chi$  and  $\phi$ . We implement this computationally using the bisection method. A wage is accepted as a solution once labour demand relative to labour supply deviates by less than 0.001 from 1. Given any  $\tau_2$ ,  $\chi$ ,  $\phi$  and wage, the simulations involve computing the credit contracts for each of the  $1000 \times 5000$  tuples for  $(a, \theta)$ . In order to speed up the computation, we make use of the result that if a potential entrepreneur decides to become a worker at  $(a, \theta)$ , all individuals with the same productivity and lower wealth will also choose to become workers.

## 5 Results

We now put the model to work to explore the impact of financial deepening by expanding access to financial markets, and examining whether and how those effects vary with the extent of competition ( $\phi$ ) in financial markets and differences in the contracting frictions ( $\tau$ ). In all cases, we will look at aggregate implications as well as distributional effects and the impacts on the firm size distribution. The model will allow us to examine the mechanism by looking at how the credit contracts depend on different factors.

### 5.1 Alternative Scenarios

Recall that our calibration yields  $\alpha = 0.021$ ,  $\mu = 0.422$  and  $\phi = 0.873$  for the US benchmark economy. We capture different kinds of economies based on their levels of competition, quality of the institutional environment, and the level of financial inclusion. In each case, we pick a range of parameters to examine the impact of financial deepening through expanding inclusion in different environments. We begin by describing the rationale for the parameter ranges that we consider.

**Competition** We will consider three levels of credit market competition:  $\phi \in \{0.5, 0.7, 0.9\}$ . All else equal, those correspond to a share of the financial sector of total profits of 15%, 8%, and 4%, respectively.<sup>32</sup> The lowest competition case corresponds to the highest financial share of profits amongst OECD countries, observed in Luxembourg.<sup>33</sup> The middle case corresponds to the finance share in total value added for South Africa.<sup>34</sup> The highest competition case corresponds roughly to the United States.

**Contractual Frictions** In parallel with the levels of competition, we also consider three values of contractual frictions:  $\tau \in \{0.3, 0.6, 0.9\}$ . These capture the different legal institutional environments that affect frictions and characterize differences in economies across the world. Low  $\tau$  represents a case where borrowers struggle to offer up collateral to obtain loans while

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<sup>32</sup>It is important to note that the finance industry share of total value added is often used to capture the level of financial depth/development, which is different from our banking concentration measure. However, in our theoretical framework, the share of lender in total GDP is a sufficient statistics of credit market competition  $\phi$ , i.e. higher  $\phi$  implies lower share of lender profits in total GDP in our model.

<sup>33</sup>Luxembourg's finance and insurance industry's share of total value added is around 30%, the highest among OECD countries. We therefore take 15% to capture only the finance industry share of value added. Data source: <https://data.oecd.org/natincome/value-added-by-activity.html>.

<sup>34</sup>Again, we obtain the finance and insurance share of total value added in South Africa to be around 16% from the original dataset. We choose the target finance only share to be around 8%.

lenders have a limited capacity to recover assets if a loan fails. By looking at how this varies with  $\tau$ , we can explore the impact of contractual frictions in the gains from financial inclusion.

**Financial Inclusion** To look at increases in financial inclusions Figure 1a suggests reasonable parameter values. It depicts variation in the percentage of respondents with a formal financial account in 2017 and suggests that considering values of  $\chi$  between 0.3 and 0.9 captures most of the variation in financial access observed across countries.

## 5.2 Aggregate Effects of Financial Inclusion

We now use the model to explore aggregate implications of improving financial access and competition in a world where wages and occupational choice are endogenous. We will examine three main outcomes: the level of the wage (relative to the US wage), the fraction of the population that works as an entrepreneur, and the level of capital-output ratio in the economy.

For each variable of interest, we allow the level of financial market access to vary in terms of the fraction of the population that has access to financial markets. At the same time, we vary  $\phi$  and  $\tau$  along the lines described above. This will enable us to understand what it means to have access to the financial markets in environments that vary in terms of how surplus is shared between borrowers and lenders, as well as the presence of contractual frictions.

The results are presented in the nine panels of Figure 2 where the top row shows wages, the middle row is the fraction of entrepreneurs and the bottom row is the capital-output ratio. In all cases, the horizontal axis varies the extent of financial inclusion. The different curves in each sub-figure are for different values of  $\phi$  and the columns represent three values of  $\tau$ .

A common feature across all the scenarios is that financial inclusion increases wages, reduces the fraction of self-employment, and increases capital intensity. Our model suggests that these outcomes are interrelated. Increasing financial access allows a wider group of individuals to leverage their wealth through credit market access and either expand the scale of their businesses or start a business. Increased labour demand causes higher wages. Higher wages in turn change the outside options of borrowers, and are a key driver of credit contracts. They also push out marginal entrepreneurs.

Despite these common features, the precise quantitative picture varies substantially across environments. In the left column (i.e. Figures 10a, 10d and 10g) we present results for the most weakly institutionalized environment typical of many low income countries ( $\tau = 0.3$ ). Here, a change in financial sector competition has large effects. For example, moving from low to high competition when there is high inclusion leads to wages rising by about 3% relative to the US wage. Since the vast majority of the economy is dependent on wage labour, this also has a distinct distributional effects as we shall see below. Moreover, when financial frictions are high, the effect of financial inclusion is also amplified by financial sector competition.

As we move across the columns, it is clear that as the contracting environment improves, there is a smaller and smaller effect from increasing competition on aggregate outcomes. In the far right column of Figure 2 (i.e. Figure 10c, 10f and 10i) where contracting frictions are low, the aggregate gains from competition are small. This is a core result from our model.

At the heart of the result is the fact that in our model of financial contracting there is a rent-extraction vs efficiency trade-off, which implies that the reduction of financial frictions and increases in competition are substitutes in encouraging the efficient provision of effort.

To see this, recall that lenders are extracting surplus from borrowers through the collateral ( $c$ ) and the repayment ( $r$ ). As long as  $c < r$  lenders prefer to extract surplus by increasing the collateral requirement and decreasing the repayment, as this encourages the provision of effort by the borrower. Contracting frictions (and limited asset values) constraint their ability to do so. The greater are contracting frictions, the lower is the collateral value of wealth, i.e. the tighter is the limited liability constraint. Similarly, when lenders are forced to transfer surplus to the borrowers to satisfy their participation constraint, lenders will do so by lowering  $r$ , not  $c$ . So increasing competition and decreasing contracting frictions have a similar impact on encouraging the provision of effort. Notice however that effort is having a decreasing marginal impact on the success probability. Therefore the degree of competition matters more the greater are financial frictions and financial frictions matter more the lower is competition.

But that should not make us lose sight of the large aggregate effects from financial inclusion with wages going from around 75% of US wages to 95% in all cases purely from extending the range of people who have access to capital, i.e. with nothing else in the economy varying. This partly allays the concern that we mentioned earlier that gains from financial inclusion could get dissipated in the presence of market power and financial frictions.

The story for wages plays out similarly for entrepreneurship. The second row (i.e. Figure 10d, 10e and 10f) illustrates the classic structural transformation story of development but told through the lens of declining reliance on self-employment and moving towards wage labor. As more individuals have access to capital markets wages are pushed up and drive out marginal firms. As we show below, this also seems more large scale employers emerging and few very small firms. In quantitative terms financial inclusion drives down the fraction of self-employment from around 10% of the economy to around 5% as financial inclusion goes from low to high.

These findings are consistent with empirical findings in the literature on economic gains from financial development, which has been growing substantially in recent years. In an early contribution, [Jayaratne and Strahan \(1996\)](#) find that bank branch deregulations in the US – which can be thought of as reducing contracting frictions and competition – led to an increase of the per capita income growth rate of about 0.6 percentage points. Reanalysing their data with a modern econometric approach suggests that the impact on the per capita income growth rate might have been as high as 2.7 percentage points.<sup>35</sup> [Burgess and Pande \(2005\)](#) examine the gains from bank branch expansion in India – which can be interpreted as a combination of increased competition and financial access in a high contracting frictions environment. They show that in rural areas one additional bank branch per 100,000 increases wages by about 8 percentage points. Also in India, [Breza and Kinnan \(2021\)](#) find that the reduction of micro-credit supply by 26 percentage points is associated with a 5% decrease in labour earnings and 2% decrease in casual daily wages. At the same time, individuals in the affected areas are 1.3 percentage points more likely to operate a business (over a mean of 58.4%) and 0.3 percentage points less likely to be employer (over mean of 3.5%). The decrease in wages and increase in small-scale entrepreneurship are very much consistent with the predictions of our model. In ongoing work [Barboni et al. \(2024\)](#) report results from the experimental placement of private sector bank branches. The arrival of such branches in a rural area led to substantially higher business investments amongst better off households, a 33% increase in the probability that

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<sup>35</sup>Their analysis exploited the staggered adoption of bank branch deregulation policies using a two-way fixed effects estimator. We re-analysed their data using the estimator proposed by [Borusyak et al. \(2024\)](#).

households employ non-household members in business activities, and 6% higher wages after two years. Again, those findings are consistent with what our model would predict. And [Fonseca and Matray \(2023\)](#) present evidence from Brazilian cities. They document that an expansion of bank branches by publicly owned banks increased the number of firms, employment and wages (by 4.1%). Those average effects mask substantial heterogeneity: in places where a private bank existed at baseline, wages increase by only 1.2%; but in places without a private bank at baseline, wages increase by 7.0%. These results are consistent with the idea that financial access can generate quantitatively important gains that are heterogeneous by institutional environment and competition.

Our findings are also consistent with the capital deepening story of development that goes back, at least, to [Lewis \(1954\)](#) and is also apparent in the third row of Figure 2 (i.e. Figure 10g, 10h and 10i). It is striking that the capital output ratio goes from a little above 30% to around 45% just off the back of increasing financial inclusion with reducing either contracting frictions or increasing competition.

In all cases, there is a strong economic empowerment effect coming from financial inclusion which has aggregate benefits for the economy in the form of higher wages and more capital installed. It also allows successful entrepreneurs to expand their scale of production which shifts the labour demand curve out. Our analysis clearly highlights the importance of using a general equilibrium framework to capture these effects.<sup>36</sup>

But although expanded access to financial markets create changes in aggregate outcomes, Figure 2 gives away little about distributional outcomes when it comes to sharing the surplus from improved financial market access. We now turn to the distributional effects of financial inclusion.

### 5.3 Distributional Effects of Financial Inclusion

The distribution of the aggregate gains that we found in Figure 2 vary according to the level of credit market competition. This chimes with some older debates about the determinants of the functional distribution of income such as [Kaldor \(1955\)](#) and [Kalecki \(1938\)](#). These authors suggested that monopoly power is associated with a higher share of profit in national income. Given our focus on financialization, we disaggregate profits into those that accrue to entrepreneurs (the so-called “real economy”) and those that accrue to lenders (the “financiers”) To home in on this, we will fix  $\tau = 0.6$  since there is very little sensitivity of these distributional effects to the level of contracting frictions.<sup>37</sup>

The main findings are in Figure 3, where the left panel shows how the share of total surplus is distributed between lenders, entrepreneurs and workers as the level of financial inclusion varies and as competition varies. Figure 3a, 3c, and 3e present results for increasing levels of competition,  $\phi$ . As a point of comparison we also present the first-best case where all surplus is earned by the entrepreneur, which corresponds to the case where entrepreneurs are self-financed and do not need to borrow. Perhaps not surprisingly, the distribution of profits between lenders and borrowers (entrepreneurs) varies in line with  $\phi$ , with high competition leading to lower gains to lenders compared to borrowers and with low competition, the gains accruing more to lenders.

<sup>36</sup>Appendix C demonstrates the robustness of those conclusions to a 25% increase or decrease of the calibrated values of  $\alpha$  and  $\mu$ .

<sup>37</sup>See Figure 6 and Figure 7 in the Appendix.



A different way of looking at this is in the right panel of Figure 3 that considers a series of ten percentage point increases in financial inclusion and how the gains are distributed. The most striking finding is that most of the gains actually accrue to wage laborers who receive wage increases. This is consistent with the evidence in Kohler et al. (2019). In line with the financialization hypothesis, it is lenders that appropriate more gains when financial market access is increased in low competition environments. Indeed, in low competition environments, nearly all of the gain accrues in the form of financial sector profits. Such low competition, is consistent with a narrative that emphasises that the “takers” are receiving more benefits from expanding market access than the “makers”. Moreover, as Figure 2 showed, this is also associated with lower wages.

## 5.4 The Level of Interest Rates

It is common to think of lack of competition as a markup of the interest rate that borrowers pay over the funding rate that lenders pay to secure the funds. However, our contracting model cautions against such a simplistic view, as highlighted by equation (15): credit contracts are indexed according to the productivity of borrowers, their wealth and their outside option. The resulting distribution of interest rates charged to borrowers depends on the amount that they borrow and the (equilibrium) probability of default. This feature of the model parallels the heterogeneity in lending rates found in real world credit markets, especially in less developed economies: for example, Bhattacharjee and Rajeev (2010) finds that the distribution of interest rates charged by professional moneylenders to poor households are more skewed to the left in developed areas than in less developed ones. The recent paper by Cavalcanti et al. (2023) also documents the variations of interest rate spreads among Brazilian firms with different sizes and ages.

Being able to capture this richness differentiates our approach with commonly used models of financial frictions and approaches to credit market competition that abstract from default and the competitive environment in financial markets. We now look at how the distribution of interest rates changes as competition and contracting frictions vary. The interest rate does reflect how the surplus is divided between lenders and borrowers. However, there is no immediate relationship between the interest rate and the allocation of capital which depends on the default rate and lender’s funding rate.

Figure 4 illustrates the distribution of interest rates for different values of  $\phi$ ,  $\tau$  and  $\chi$ . Figure 4a is a case of low competition and high frictions ( $\tau = 0.3$  and  $\phi = 0.5$ ) which leads to a distribution of high interest rates between 10 percent and 100 percent. These are the kinds of rates that are associated with institutions like money lending in low income countries where competition is extremely low in village economies Aleem (1990). The level of interest rates does though vary, as we would expect, with the borrower’s characteristics, such as their wealth. Figure 4b shows the same level of  $\tau$  and  $\phi$ , but  $\chi$  is now higher. Interestingly, the level of financial inclusion does not affect the equilibrium interest rate distribution in any substantive way.

Moving to the second row in Figure 4 where  $\tau = 0.3$  and  $\phi = 0.9$  so that competition is higher than in the top row has two effects. First, it reduces the average interest rate, which is consistent with an earlier finding by Iqbal (1988) that formal lending expansion reduces market power of moneylenders in local credit market and their interest rates. However, it also shrinks the distribution of interest rates, closer to the law of one price. But it is important to note

that, even though competition has increased, this would be poorly represented as a uniform reduction in the markup of interest rates over bank funding rates. It does reduce very high interest rates suggesting that the effects of competition that we saw above can be thought of as part of a process of lowering interest rates at which borrowers can access credit even though this is the product of a change in credit contracts when outside options improve. Moving to Figure 4d does not show much of change from increased financial inclusion.

Figure 4e ( $\tau = 0.9$  and  $\phi = 0.5$ ) illustrates a pure effect of reducing contracting frictions compared to Figure 4a. Here too, there is a notable leftward shift in the distribution of interest rates, now leading to interest rates below 10 percent and fewer very high interest rates. Once again the extent of financial inclusion does not have any significant effect on this distribution (compared to Figure 4f). Figure 4g illustrates the distribution of interest rate for high competition and low contracting frictions ( $\tau = 0.9$  and  $\phi = 0.9$ ). This has both lower interest rates and a tighter distribution. These findings, which reflect the richness of the contracting environment with heterogeneous default rates by type of borrower, are of particular interest given how often interest rates charged to borrowers are casually used as a barometer of the competitiveness of credit markets. The shift to the left in the distribution shows that there is some validity in this. But the gains will vary and to understand this, one needs to delve into the sources of heterogeneity in rates. Even though financial intermediation has an effect on the level of wages in the model, comparing the left and right hand columns in Figure 4 shows that this not a particularly significant factor in the determination of the equilibrium distribution of interest rates.

## 5.5 Structural Change via the Firm-Size Distribution

We have already seen that as credit markets are rolled out, competition is increased and contracting frictions diminish, there is a concomitant decline in the number of firms as “marginal” unproductive firms are being driven out as a consequence of rising wages. An important dimension of structural change is there being more large firms in the economy who tend to expand. Such changes alter the allocation of capital in the economy, an issue that has been prominently explored by Restuccia and Rogerson (2008), Hsieh and Klenow (2009) and the subsequent literature. This literature looks at how access to capital can distort the firm size distribution by “misaligning” productivity and the usage of factors of production. More specifically, there have also been debates about whether market imperfections lead to a “missing middle” (Hsieh and Olken (2014)) where the biggest hit is to mid-size firms. Our model allows to unpack the structure of these kind of issues.

We explore the overall consequences for the distribution of firm sizes when credit market frictions are lower and competition is higher. We can compare the resulting distribution of firms sizes to a hypothetical “first-best” distribution of firm size, i.e. where underlying productivity mainly determines capital usage. The results are presented in Figure 5 where we compare two levels of financial inclusion: low ( $\chi = 0.3$ ) on the left and high ( $\chi = 0.9$ ) on the right. Within each of these, we consider different values of  $\phi$  and  $\tau$  reflecting the competition and contracting environment, giving eight panels in total.

The top row in Figure 5 has the lowest level of competition and the highest contractual frictions. Subfigure 5a has the lowest level of financial inclusion. Here there are substantially more small firms compared to the first-best and substantially fewer medium-sized and large firms in the economy. This reflects that a large fraction of the population are self-employed,

which is facilitated by low wages allowing marginal firms to survive. Moreover, all firms may be unable to access capital to become large even if they are of high potential productivity.

At the other extreme is Subfigure 5h which has high financial inclusion, high competition and low contractual frictions. Now the firm size distribution looks very similar to what we would expect to see in the absence of contractual frictions with very few tiny firms and more large firms reflecting the opportunities available to high productivity firms when they are empowered through financial market access. Our results suggest that promoting financial development can be conducive to bringing up more large firms in the economy. This is also related to [Cagetti and De Nardi \(2006\)](#) who argue that tighter borrowing constraints lead to reduction in average firm size, as well as [Hopenhayn \(2016\)](#) who provides a stylized framework to discuss how higher average firm size in an economy is correlated with higher stages of development (i.e. higher GDP per capita).

As we move around the rows and columns in Figure 5, we are able to see the marginal impact of changing the level of financial inclusion, competition and contractual variations alone. The figure shows that, when comparing the two columns, it is financial inclusion that plays the largest role in driving out small scale self-employment which is driven by the fact that wages are higher.

It is interesting to note that our model predicts a missing middle in the size distribution of firms when competition is low, contractual frictions are high, and financial inclusion is high. This gradually changes as we increase competition and reduce financial frictions. But consistent with the findings of [Hsieh and Olken \(2014\)](#), our model does not generally predict that market distortions result in a “missing middle” with a shrinkage in firm sizes across the piece. This is particularly noticeable in the left column of Figure 5.

Through the panels in the left column, a substantial fraction of small firms survive with a concomitant smaller fraction of medium- and large-sized firms even with greater competition and lower contractual frictions. Those very small firms are not self-financed entrepreneurs without access to the financial markets; they are predominantly borrowers. Low wages allow them to operate inefficiently small businesses. Comparing the left and right panels of Figure 5 shows that it is mainly financial inclusion that is driving structural change in the firms size distribution leading to more large firms. With financial inclusion labour demand and wages rise, driving out small scale businesses. Taken together, the figure suggests that the empowerment effect that comes from increasing market access through financial inclusion is more important than whether there are high financial profits when it comes to the structural changes in the economy that financial development brings.

More generally, the approach gives insight into the underlying structural factors due to frictions, competition and inclusion that drive the size distribution of firms. Standard analyses of the quantitative consequence of misallocation do not consider what the factors are that lead to underlying misallocation of resources. We have been able to look at these both together and selectively. We have also highlighted the central role of endogenously changing wages in affecting resource allocation, leading to heterogeneous effects across firms of different levels of wealth and productivity.

## 5.6 Summary

We have exploited the role of contracting frictions and competition for assessing the gains from financial inclusion and understanding the underlying economic mechanism. Our model also

casts light on the distribution of gains between workers and firms but also across different types of firms. The model has generated three core insights.

First, there are aggregate effects that mirror what we might see in the macro-economy as financial inclusion increases with gains in wages, changes in occupational choice and capital deepening. Competition matters most for the size of these effects when contractual frictions are large with very little impact on aggregate outcomes when contractual frictions are small. So we expect the smallest aggregate gains from financial inclusion when contractual frictions are large *and* borrowers capture a smaller share of the surplus from their investments.

Second, the functional distribution of income changes with financial inclusion. Gains in wages do not depend so much on either competition or contractual frictions. However, the distribution of gains between entrepreneurs (the real economy) and lenders (the financial sector) are affected by competition and we see some evidence of the a distributional impact of the financialization hypothesis whereby it is those with financial wealth who can leverage this in financial markets who are the primary beneficiaries of financial inclusion when competition is low. It is perhaps not surprising that there is emphasis on the role of large financial firms in restricting competition in financial markets.

Third, studying the micro underpinnings of the findings and the mechanisms at work, we find that changing both competition and contractual frictions changes the nature of the credit contracts that people face, which accounts for the aggregate and distributional effects. One of the key elements underlying these effects is the firm size distribution as this affects the size of the labor demand shift from financial inclusion and hence what happens to wages. Our results indicate that it is financial inclusion *per se* that matters for creating structural transformation in the form of more large employers and weeding low productivity micro-firms. Although competition and contractual frictions matter, their quantitative impact is less important.

Taken together these results give a vivid illustration of a heterogeneous impact from expanding credit market access. Market access is not sufficient to empower firms if those who access credit markets do not get a significant share of the surplus that they create. The results also underline the value of allowing not just access to credit to vary, but also the nature of the relationship between the lender and borrowers and the institutional environment in which they interact.

## 6 Concluding Comments

Expanding credit market access is a cornerstone of development strategies and this has led to a focus on trying to break down financial inclusion as well as improving the institutional environment on which credit markets operate by reducing contractual frictions and improving property rights protection. The role of increasing competition has had, by comparison, less attention. Yet, recent debates have stressed the perils of financialization whereby financial market development has led to greater rents available to lenders.

Examining these ideas requires a framework where surplus sharing of the gain from credit market access is studied along with aggregate implications for wages. This paper has developed a model of credit market frictions where we can look at expanding access to credit markets for different levels of competition and institutional development. We have shown that the impact depends on the extent to which there are private and aggregate gains to financial inclusion depends on how surplus is shared between lenders and borrowers. It underpins

the proposition that the gains from financial access are highly heterogeneous. Our model has demonstrated that the empowerment effect of market access is much greater when more of the surplus accrues to the borrower. This has important efficiency implications which affect the general equilibrium consequences of financial inclusion.

Expanding access to credit markets has the potential to be empowering in two ways. First, it can allow potential entrepreneurs to trade in markets limiting the extent to which becoming an entrepreneur is limited by their personal wealth. Second, markets can reduce monopoly power allowing entrepreneurs to capture a large share of the surplus. We have shown that, in a world of moral hazard, competition reduces default risk when it increases the share of the surplus that the entrepreneur keeps. This expands the amount of capital used and also increase labor demand. In general equilibrium, this leads to higher wages which has a selection effect on occupational choice with less able marginal entrepreneurs being squeezed out so that capital and labor are reallocated to more efficient firms.

Our contracting model is key to generating these effects since we can separately understand different elements of the process. In addition, our model offers insights into the pattern of default risk and the array of heterogeneous interest rates offered to different borrowers. These are affected by the empowerment effect which comes from improving outside options.

The paper has mainstreamed issues that have mainly been regarded as heterodox, including debates about the “financialization” of capitalist economics due to a larger share of profits being earned in the financial sector. It is novel in having a systematic quantitative analysis of why the distribution of the surplus between the lender and the borrower is important in understanding financial frictions. It makes clear why having less competitive credit markets has an impact both at the firm level and on equilibrium outcomes such as wages and occupational choice. In our model, this arises when lenders can earn a large share of the profits from entrepreneurship due to their market power. Not only does it allow owners of financial capital to earn profits, it also has real effects on the economy reducing the labor share and reducing efficiency by increasing default risk. Financial inclusion is still valuable in such cases but some of the gains benefit lenders who earn a larger share of profits in national income. So this has both efficiency and distributional consequences for expanding financial access.

The paper has posited a specific technology for providing credit where limits on conventional collateral create a friction, conditional on having access to credit. In ongoing work we are exploring the potential gains from expanding collateral to non-pecuniary punishments, often referred to as “social collateral” which are folded into many microcredit programs. This would allow us to link the paper to discussions about the role of microcredit in development.

The framework that we have adopted has largely looked at the allocative effects of frictions, lack of competition and limited market access in credit markets. This is consonant with the development accounting literature that stresses determinants of low levels of income rather than low growth. A natural extension in future work would be to look at how these three distinct aspects of credit market development interact in affecting the expansion of aggregate capital as well as its allocation.

The distributional effects that we have stressed also suggest the scope for understanding the political economy of change in this context something that has also been stressed in critiques around market power in financial markets (Lindsey and Teles, 2017; Palley, 2021). This is also linked to an old theme in the development literature is that those with existing market power resist reform, particular that which exposes them to greater competition. However, it

less obvious that they would wish to impose frictions and to restrict market access. So there could be subtle differences in the reform packages that the financial sector will wish to see implemented in political equilibrium. This is also an important topic for future research.



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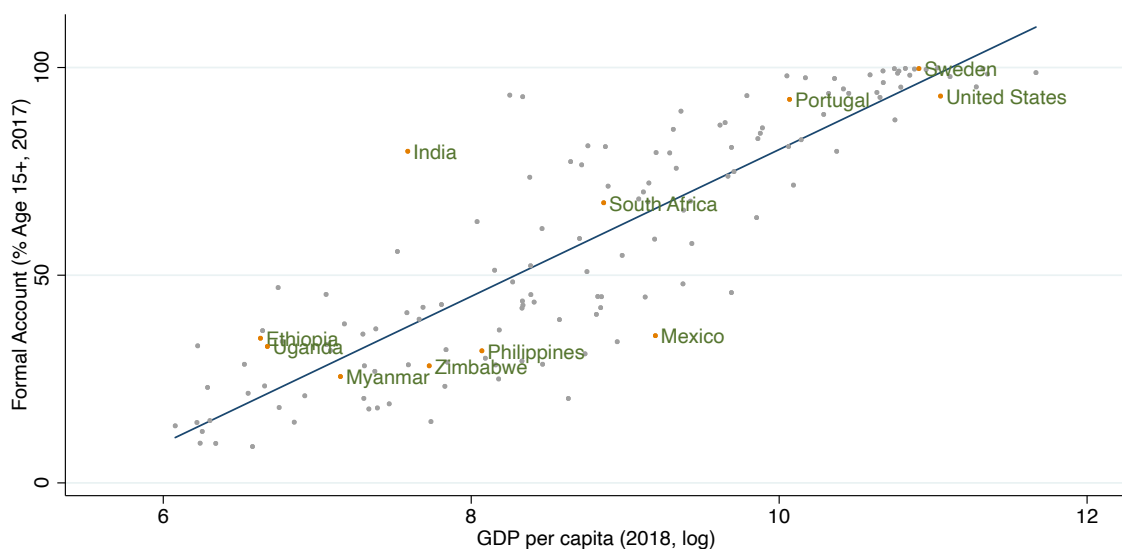
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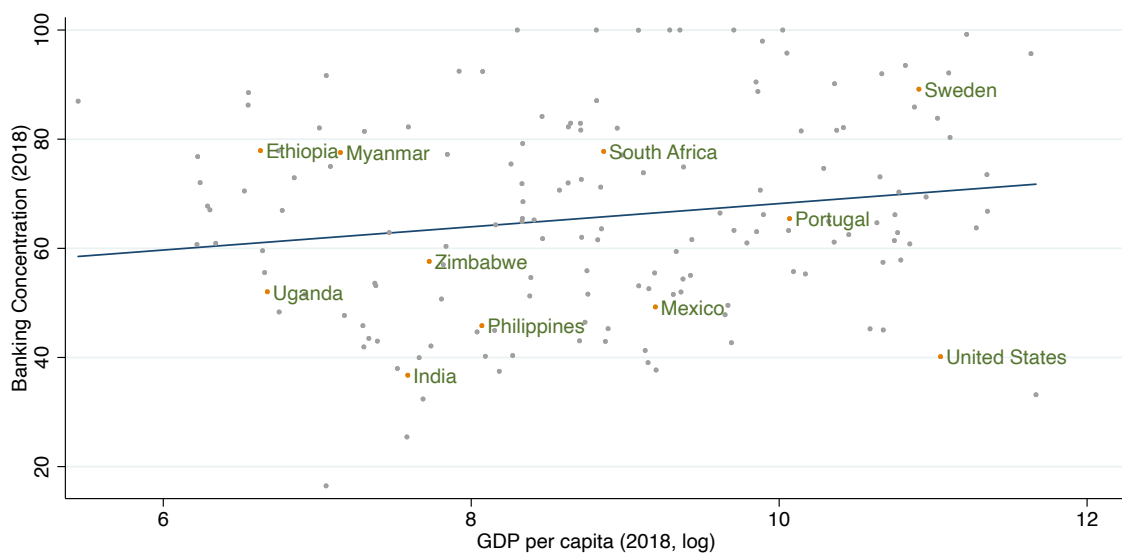
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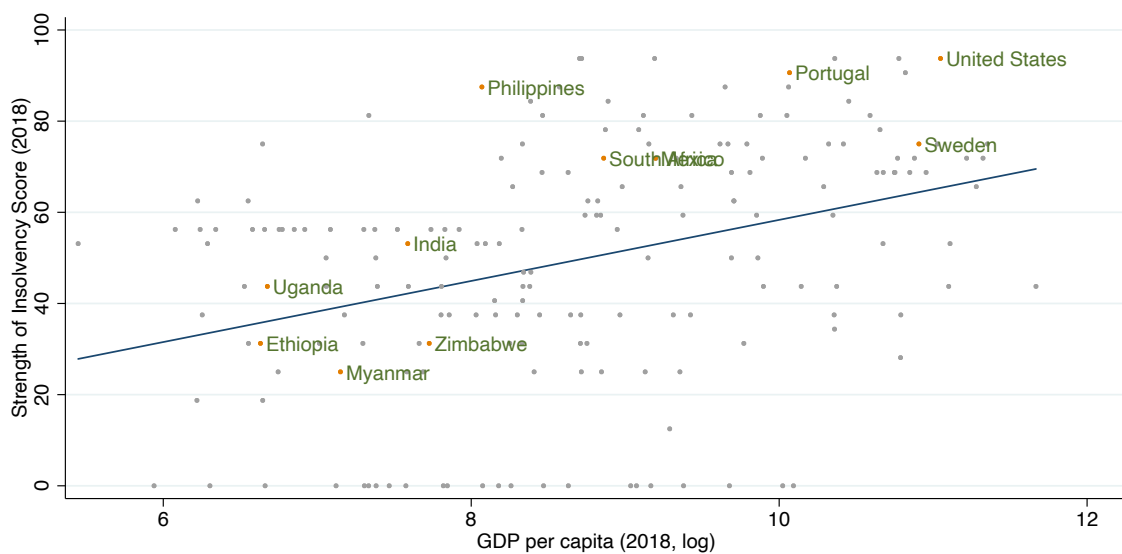
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(A) GDP & FINANCIAL INCLUSION



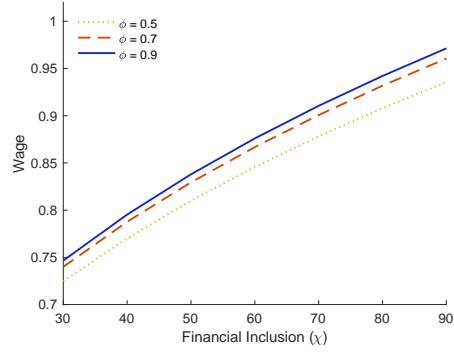
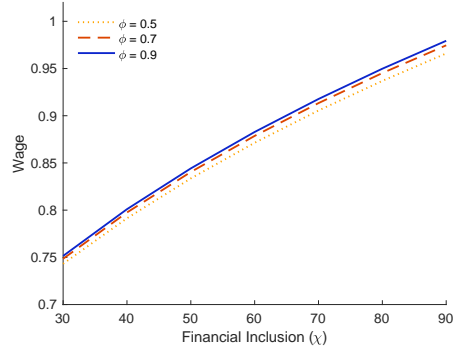
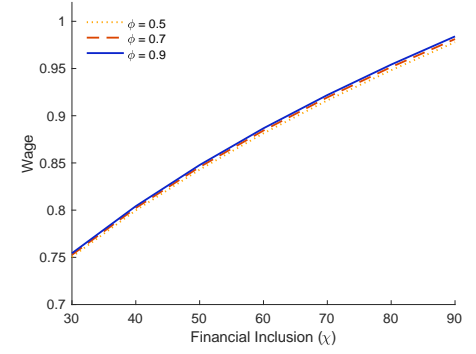
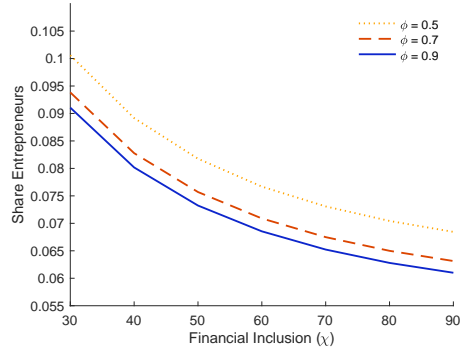
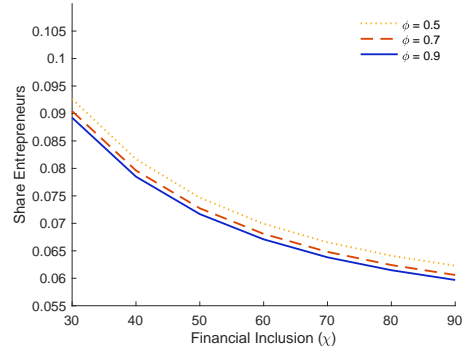
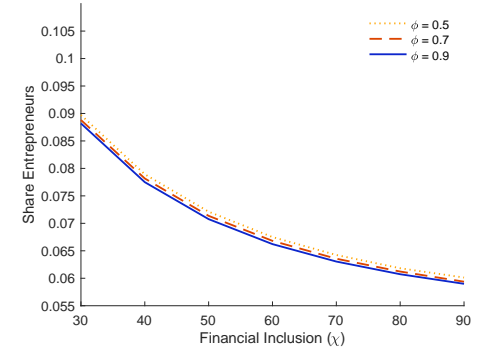
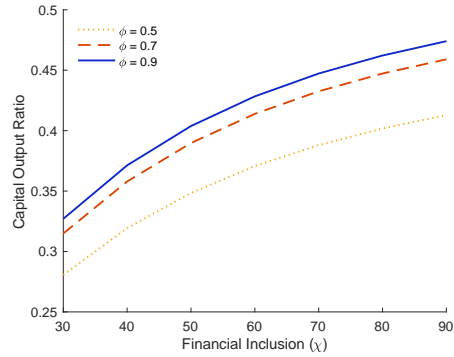
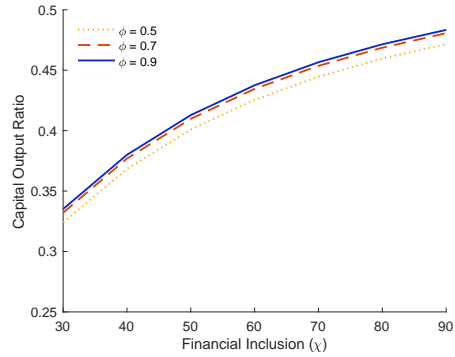
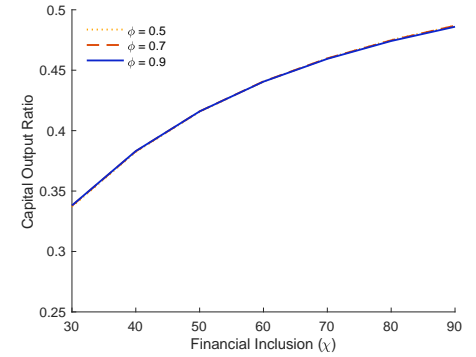
(B) GDP & COMPETITION

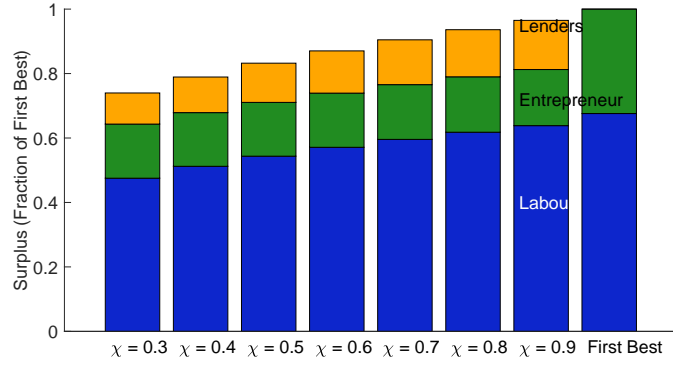
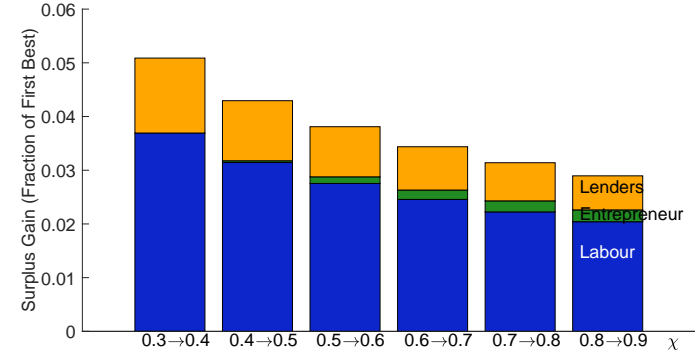
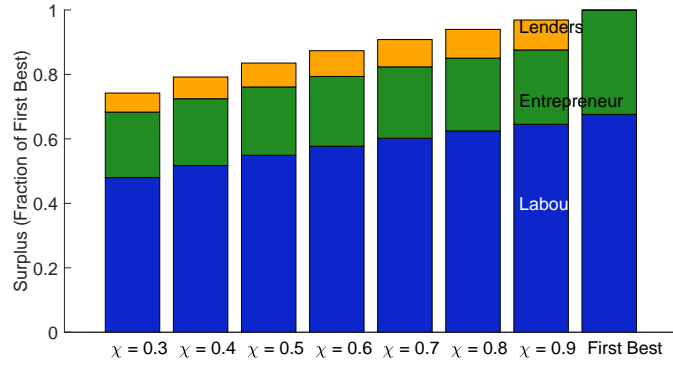
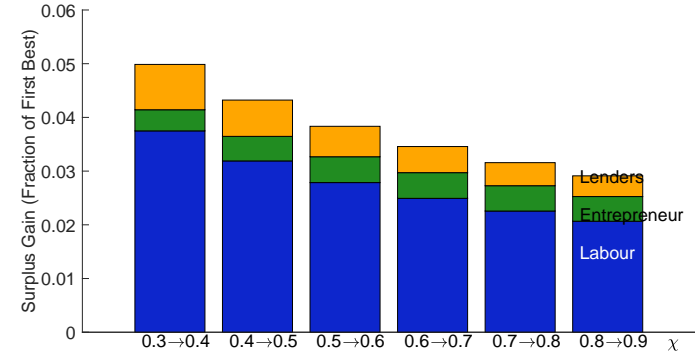
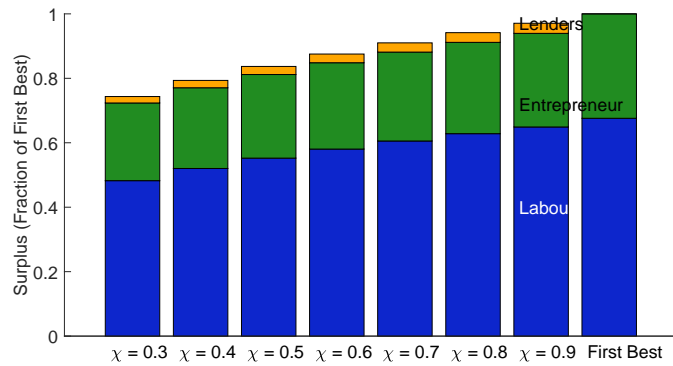
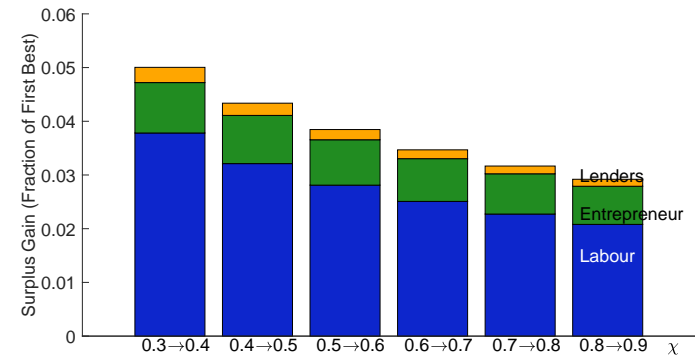


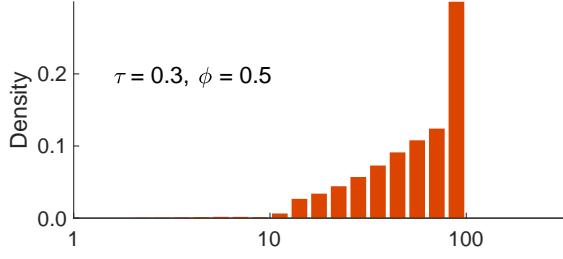
(C) GDP & PROPERTY RIGHTS

FIGURE 1: STYLIZED FACTS

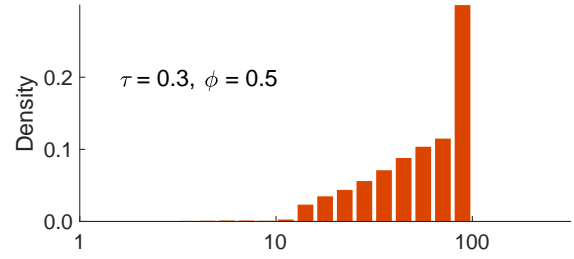


(A) WAGE ( $\tau = 0.3$ )(B) WAGE ( $\tau = 0.6$ )(C) WAGE ( $\tau = 0.9$ )(D) SHARES OF ENTREPRENEURS ( $\tau = 0.3$ )(E) SHARES OF ENTREPRENEURS ( $\tau = 0.6$ )(F) SHARES OF ENTREPRENEURS ( $\tau = 0.9$ )(G) CAPITAL OUTPUT RATIO ( $\tau = 0.3$ )(H) CAPITAL OUTPUT RATIO ( $\tau = 0.6$ )(I) CAPITAL OUTPUT RATIO ( $\tau = 0.9$ )FIGURE 2: AGGREGATE RESULTS:  $\tau = 0.3$  (LEFT),  $\tau = 0.6$  (CENTER) AND  $\tau = 0.9$  (RIGHT)

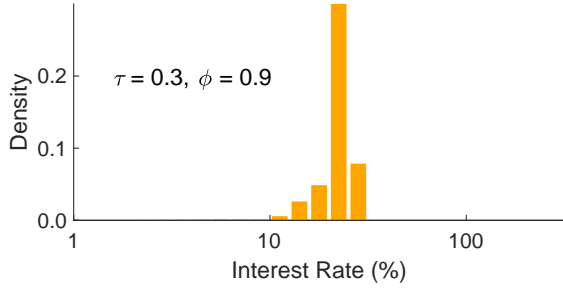
(A) SURPLUS SHARES FOR  $\phi = 0.5$ (B) SURPLUS GAINS FOR  $\phi = 0.5$ (C) SURPLUS SHARES FOR  $\phi = 0.7$ (D) SURPLUS GAINS FOR  $\phi = 0.7$ (E) SURPLUS SHARES FOR  $\phi = 0.9$ (F) SURPLUS GAINS FOR  $\phi = 0.9$ FIGURE 3: THE DISTRIBUTIONAL EFFECTS OF FINANCIAL INCLUSION ( $\tau = 0.6$ )



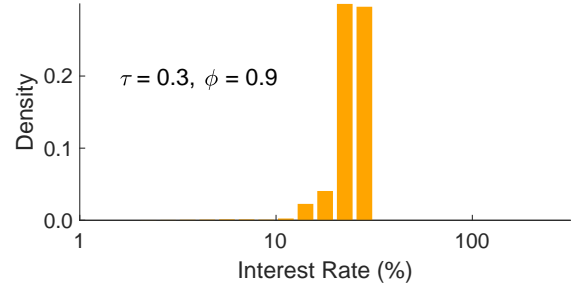
(A)  $\tau = 0.3, \phi = 0.5, \chi = 0.3$



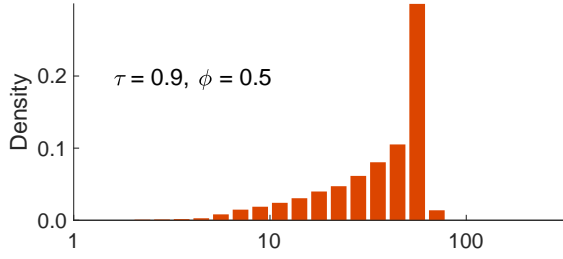
(B)  $\tau = 0.3, \phi = 0.5, \chi = 0.9$



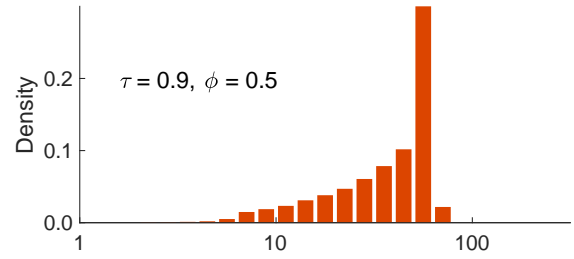
(C)  $\tau = 0.3, \phi = 0.9, \chi = 0.3$



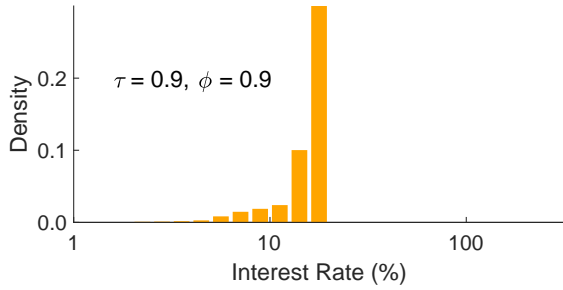
(D)  $\tau = 0.3, \phi = 0.9, \chi = 0.9$



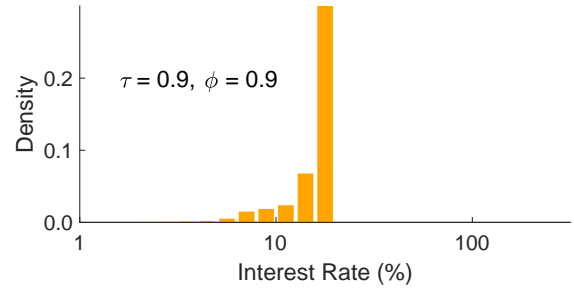
(E)  $\tau = 0.9, \phi = 0.5, \chi = 0.3$



(F)  $\tau = 0.9, \phi = 0.5, \chi = 0.9$

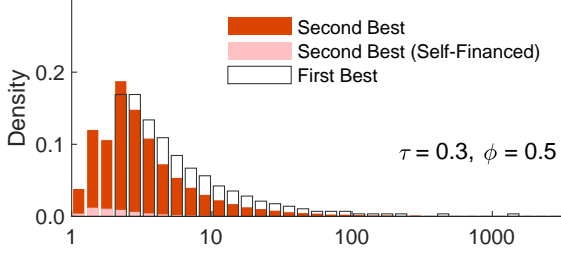


(G)  $\tau = 0.9, \phi = 0.9, \chi = 0.3$

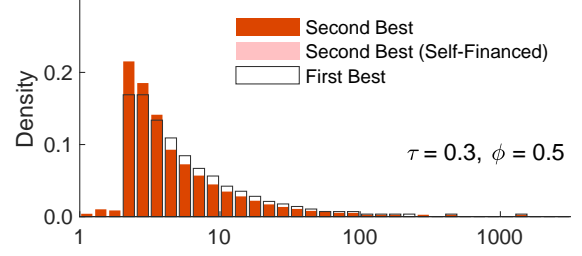


(H)  $\tau = 0.9, \phi = 0.9, \chi = 0.9$

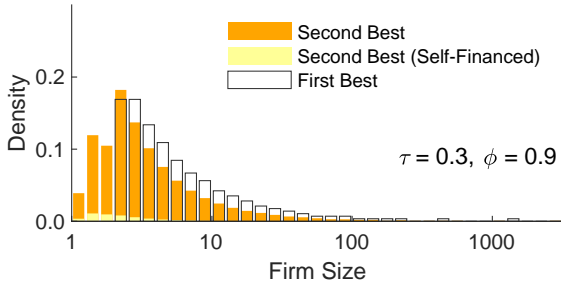
FIGURE 4: DISTRIBUTION OF INTEREST RATES



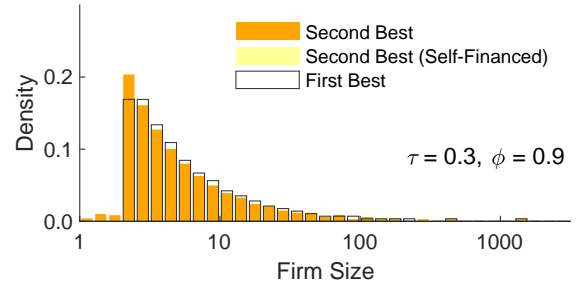
(A)  $\tau = 0.3, \phi = 0.5, \chi = 0.3$



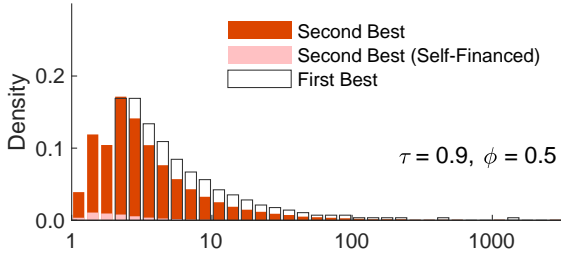
(B)  $\tau = 0.3, \phi = 0.5, \chi = 0.9$



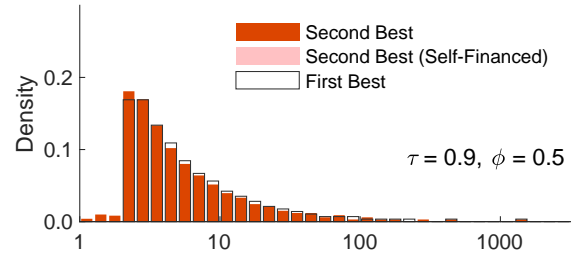
(C)  $\tau = 0.3, \phi = 0.9, \chi = 0.3$



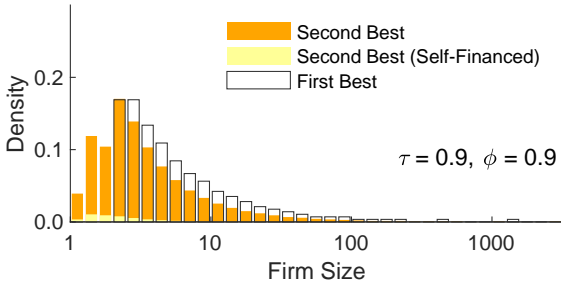
(D)  $\tau = 0.3, \phi = 0.9, \chi = 0.9$



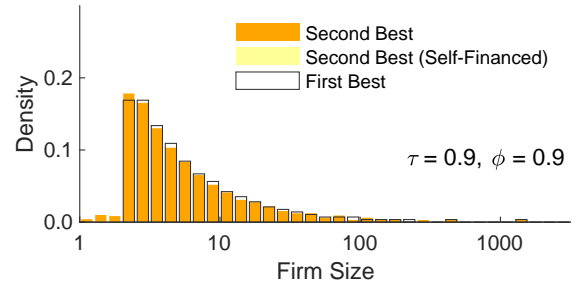
(E)  $\tau = 0.9, \phi = 0.5, \chi = 0.3$



(F)  $\tau = 0.9, \phi = 0.5, \chi = 0.9$



(G)  $\tau = 0.9, \phi = 0.9, \chi = 0.3$



(H)  $\tau = 0.9, \phi = 0.9, \chi = 0.9$

FIGURE 5: FIRM SIZE DISTRIBUTION

# Appendices

## A Proof of Propositions

This section provides the proofs of the results presented in the paper. Recall that the expected payoffs of the lender and the entrepreneur are from equations (3) and (5):

$$\Pi = g(e; \theta) r + (1 - g(e; \theta)) c - \gamma x$$

$$V = g(e; \theta) \{ \pi(x + \psi a; \theta, \mathbf{p}) - r \} + (1 - g(e; \theta)) \{ \delta(x + \psi a) - c \} - p_m e + \gamma (1 - \psi) a.$$

The relevant constraints are

(i) the participation constraint (9):

$$V(e, \mathbf{t}; a, \theta, \mathbf{p}) \geq u(a, \theta, \mathbf{p})$$

(ii) the incentive-compatibility constraint (6):

$$g_e(e; \theta) [\pi(x + \psi a; \theta, \mathbf{p}) - \delta(x + \psi a) - r + c] = p_m$$

(iii) the limited liability constraints (7) and (8):

$$\begin{aligned} r &\leq \pi(x + \psi a) + \tau_1 \gamma (1 - \psi) a \\ c &\leq \tau_1 \gamma (1 - \psi) a + \tau_2 \delta(x + \psi a). \end{aligned}$$

Notice that  $\pi(x + \psi a) > \delta(x + \psi a)$ , and so the right hand side of the limited liability constraint with respect to  $r$  is higher than that of the one with respect to  $c$ . For notational simplicity, we omit throughout the dependence of functions on  $\theta$  and  $\mathbf{p}$ .

### Proof of Lemma 1.

**Step 1.** We show that under the optimal contract  $r \geq c$ .

**Proof of Step 1.** Suppose instead  $r < c$  is optimal. Then consider a small increase in  $r$  to  $r + dr$  and a small decrease in  $c$  to  $c + dc$  that keeps the borrower's payoff constant, and hold  $k$  constant, so we have

$$g(e) dr + (1 - g(e)) dc = 0$$

The limited liability constraints will be satisfied as the one with respect to  $c$  is more stringent than the one with respect to  $r$ . Also, by construction the participation constraint is still satisfied and so this is a feasible contract. Observe that from the incentive compatibility constraint, effort level  $e$  will decrease, i.e.  $de < 0$ . By the envelope theorem we can ignore the effect of the change of  $e$  on borrower payoff. The change in lender's payoff (holding  $k$  constant) is

$$g_e(e)(r - c) de + g(e) dr + (1 - g(e)) dc = g_e(e)(r - c) de > 0$$

Since  $r < c$ ,  $de < 0$  and  $g_e(e) > 0$ , this implies that the lender is made better off, contradicting the conjecture of optimality.

**Step 2.** We show that under the optimal contract  $\psi = 1$  (and hence  $k = x + a$ ).

**Proof of Step 2.** Suppose that  $\psi < 1$ , and consider a change in contract such that  $-dx = d(\psi a) > 0$  (so the capital in the project is unchanged, but the lender's funds are 1:1 replaced by the entrepreneurs liquidated assets), collateral  $c$  is decreased by  $\tau_1 \gamma d(\psi a)$  (the amount that can no longer be seized from assets not invested in the project), and  $r$  is adjusted to keep the borrower payoff constant. Notice that the limited liability constraint on  $c$  as well as the participation constraint of the borrower are both still satisfied by construction. For the borrower's payoff to be unchanged, it needs to hold that

$$(g_e(e)[\pi(x + \psi a) - r + \tau_1 \gamma (1 - \psi)a - (1 - \tau_2)\delta k] - p_m)de - g(e)dr + [(1 - g(e))\tau_1 \gamma - \gamma]d(\psi a) = 0$$

Using the incentive compatibility constraint to simplify (envelope theorem), we find the change in  $r$  that keeps the borrower's payoff from the credit contract unchanged to be:

$$dr = \frac{(1 - g(e))\tau_1 \gamma - \gamma}{g(e)} d(\psi a) < 0.$$

Since  $dr < 0$ , also the limited liability constraint on  $r$  will be satisfied with the updated contract. The effect on effort is then found by totally differentiating the borrowers incentive compatibility constraint:

$$\left[ p_m \frac{g_{ee}(e)}{g_e(e)} \right] de - g_e(e)dr - g_e(e)\tau_1 \gamma d(\psi a) = 0.$$

The lenders change in payoff is given by

$$d\Pi = g_e(e) [r - c] de + g(e)dr + [(1 - g(e))[-\tau_1 \gamma] + \gamma] d(\psi a).$$

Using the above expressions for  $de$  and  $dr$  we can simplify to:

$$d\Pi = \frac{g_e(e)}{p_m g(e) \epsilon(e)} (1 - \tau_1)(r - c) \gamma d(\psi a).$$

As long as  $\tau_1 < 1$ , we have  $d\Pi > 0$ , a contradiction of the premise that the lender's profit was being maximized originally. We therefore conclude that the optimal contract satisfies  $\psi = 1$ .

## Proof of Proposition 1.

The proof of Proposition 1 proceeds through 5 steps, some of which are similar to the proof of Proposition 1 in Besley et al. (2012):

**Step 1.** It is never optimal for both the participation constraint and limited liability constraint to be slack.

**Proof of Step 1.** Suppose this was optimal. Then the lender could increase both  $r$  and  $c$  by the same small amount, keeping  $k$  constant. Effort  $e$  will be unchanged, the participation constraint will be satisfied, and the lender will be better off, a contradiction.

**Step 2** We show that: (i) if  $r > c$  under the optimal contract, then  $c = \tau_2 \delta (x + a)$ ; (ii) if  $c < \tau_2 \delta (x + a)$  under the optimal contract, then  $r = c$  and effort is at the first-best level.



**Proof of Step 2** (i) Suppose instead  $c < \tau_2 \delta (x + a)$  is optimal. Then by increasing  $c$  and decreasing  $r$  by small amount to keep the borrower's payoff constant, and hold  $k$  constant, we have as in Step 1 of Lemma 1,  $g(e)dr + (1 - g(e))dc = 0$ ; effort level will be higher (from the incentive compatibility constraint of the borrower), i.e.  $de > 0$ . And with  $r > c$ , the lender's payoff is  $g_e(e)(r - c)de + g(e)dr + (1 - g(e))dc = g_e(e)(r - c)de > 0$ , which implies that the lender is better off. Hence a contradiction.

(ii) Consider the contrapositive statement of (i): if  $c < \tau_2 \delta (x + a)$ , so the limited compatibility constraint is not binding, then  $r \leq c$ . From Step 1 of the proof of Lemma 1 we then have  $r = c$ . Now the lender's problem is to maximize  $c - \gamma(k - a)$  subject to the participation constraint  $g(e)(\pi(k) - \delta k) + \delta k - c - p_m e \geq u$ , as well as the incentive compatibility constraint  $g_e(e)(\pi(k) - \delta k) = p_m$ . Then the lender will want to choose  $c$  as high as possible, i.e. the participation constraint is binding. Then the first order condition with respect to  $k$  is  $g(e)(\pi_k(k) - \delta) = \gamma - \delta$ . Hence effort level and capital level are all at the first-best.

**Step 3** We show that (i) there exists  $\underline{u}$  such that for  $u \in [0, \underline{u}]$ , the optimal contract is characterized by the interior solutions  $e = e_0$ ,  $k = k_0$ ,  $r = r_0$  and the participation constraint is not binding; (ii) for  $u \geq \underline{u}$  the participation constraint is binding.

**Proof of Step 3** (i) Suppose the participation constraint is slack. Then the limited liability constraint is binding by Step 1. The lender's maximization problem is

$$\max_{e,k} g(e) \left( \pi(k) - \frac{p_m}{g_e(e)} - \delta k \right) + \tau_2 \delta k - \gamma(k - a).$$

We first show that this problem is globally concave. Denote the maximand as  $\Gamma(e, k)$ . We can derive  $\Gamma_{ee} = g_{ee}(e)(\pi(k) - \delta k) - p_m(g_e)_e(e)$ ,  $\Gamma_{kk} = g(e)\pi_{kk}$ , and  $\Gamma_{ek} = g_e(e)(\pi_k - \delta)$ . Then with  $\pi_{kk} < 0$ , the fact that  $g(e)\epsilon(e)$  is increasing in  $e$ , as well as the assumption that the second order condition is satisfied in the first best case (i.e.  $g(e)\pi_{kk}g_{ee}(e)(\pi(k) - \delta k) > (g_e(e)(\pi_k - \delta))^2$ ), we have  $\Gamma_{ee}\Gamma_{kk} = g(e)\pi_{kk}g_{ee}(e)(\pi(k) - \delta k) - g(e)\pi_{kk}p_m(g_e)_e(e) > g(e)\pi_{kk}g_{ee}(e)(\pi(k) - \delta k) > (g_e(e)(\pi_k - \delta))^2 = (\Gamma_{ek})^2$ . This implies that the problem is globally concave.

The solution is characterized by the first-order conditions

$$\begin{aligned} g_e(e_0) [\pi(k_0) - \delta k_0] &= [1 + g(e_0)\epsilon(e_0)] p_m \\ g(e_0) [\pi_k(k_0) - \delta] &= \gamma - \tau_2 \delta \end{aligned}$$

By Assumption 2 (iii) and the fact that  $g_e(e)(\pi(k) - \delta k) - p_m g(e)\epsilon(e)$  is decreasing in  $e$  and  $g(e)(\pi_k(k) - \delta)$  is decreasing in  $k$ , the unique global maximum  $(e_0, k_0)$  is an interior solution.

Using the binding limited liability constraint and the incentive compatibility constraint, we can rewrite the participation constraint in this case as

$$p_m \left( \frac{g(e_0)}{g_e(e_0)} - e_0 \right) + (1 - \tau_2) \delta k_0 \geq u$$

Note that the left hand side is strictly positive by concavity of  $g(e)$ , and upon partial differentiation, strictly increasing for  $e_0, k_0 > 0$ . Hence for any pairs of positive  $e_0$  and  $k_0$ , we can define a  $\underline{u} := p_m \left( \frac{g(e_0)}{g_e(e_0)} - e_0 \right) + (1 - \tau_2) \delta k_0$  such that for any  $u \leq \underline{u}$ , the participation constraint is indeed slack and the above contract is feasible and optimal.

(ii) Now suppose for any  $u \geq \underline{u}$ , the participation constraint is not binding. In this scenario the optimal contract is  $(e_0, k_0)$  and the limited liability constraint is binding, see Step 1. The borrower's utility from such a contract is  $p_m \left( \frac{g(e_0)}{g_e(e_0)} - e_0 \right) + (1 - \tau_2) \delta k_0 = \underline{u}$ . The participation constraint not being binding then implies  $\underline{u} > u$ , a contradiction.

**Step 4** We show (i) there exists a  $\bar{u}$  such that for all  $u \geq \bar{u}$ , the first-best  $(e_{FB}, k_{FB})$  is implemented; (ii)  $e_0 < e_{FB}$ ,  $k_0 < k_{FB}$  and  $\underline{u} < \bar{u}$  and (iii) for any  $u \leq \bar{u}$ , the optimal contract satisfies  $c = \tau_2 \delta k$ .

**Proof of Step 4** (i) The first-best effort and capital level is characterized in the main body of the paper as  $e_{FB}$  and  $k_{FB}$ . By Assumption 2 (iii) and the fact that  $g(e)\epsilon(e) \geq 0$  and  $\tau_2 \geq 0$  we have  $\lim_{e \rightarrow 0} g_e(e; \theta) (\pi(k; \theta, \mathbf{p}) - \delta k) - p_m \geq \lim_{e \rightarrow 0} g_e(e; \theta) (\pi(k; \theta, \mathbf{p}) - \delta k) - (1 + g(e)\epsilon(e)) p_m > 0$  for all  $k > 0$  and  $\lim_{k \rightarrow 0} g(e; \theta) (\pi_k(k; \theta, \mathbf{p}) - \delta) > \gamma - \tau_2 \delta \geq \gamma - \delta$  for all  $e > 0$ , which ensures that the solutions are interior.

Note that  $r = c$  is a necessary condition for the first-best to be implemented. Suppose instead  $r \neq c$  and yet the first-best is implemented. Then it follows from the incentive compatibility constraint that given  $k_{FB}$ , the borrower would not choose  $e = e_{FB}$ , a contradiction with the first-best being implemented. Now given  $r = c$ , the lender's optimization problem becomes to maximize  $c - \gamma(k_{FB} - a)$  subject to the limited liability constraint  $c \leq \tau_2 \delta k_{FB}$  and the participation constraint  $g(e_{FB})(\pi(k_{FB}) - \delta k_{FB}) + \delta k_{FB} - p_m e_{FB} - u \geq c$ . The lender will want to choose  $c$  as high as possible, subject to constraints. Define  $\bar{u} := g(e_{FB})(\pi(k_{FB}) - \delta k_{FB}) - p_m e_{FB} + (1 - \tau_2) \delta k_{FB}$ , which is the level of  $u$  such that both constraints become binding for the same  $c$ . Then  $u > \bar{u}$  together with the participation constraint imply  $\tau_2 \delta k_{FB} > g(e_{FB})(\pi(k_{FB}) - \delta k_{FB}) + \delta k_{FB} - p_m e_{FB} - u \geq c$ , i.e. the limited liability constraint is slack, and hence the participation constraint is binding. Given a binding participation constraint, the surplus maximizing  $(e_{FB}, k_{FB})$  is also maximizing lender profits. Hence for  $u \geq \bar{u}$ , the first-best outcome  $e_{FB}$  and  $k_{FB}$  is implemented, as long as the lender makes positive profits, with  $r = c = g(e_{FB})(\pi(k_{FB}) - \delta k_{FB}) + \delta k_{FB} - u - p_m e_{FB}$ .

(ii) Next we show that  $e_0 < e_{FB}$  and  $k_0 < k_{FB}$ . Consider the first-order conditions characterizing  $(e_0, k_0)$  and  $(e_{FB}, k_{FB})$ . We can write them in the more general form  $g_e(e)(\pi(k) - \delta k) = a$  and  $g(e)(\pi_k(k) - \delta) = b$ . We can then determine how the solutions to the system of two equations change with  $a$  and  $b$ . We find  $\frac{\partial e}{\partial a} = \frac{g \pi_{kk}}{g_{ee}(\pi - \delta k)g \pi_{kk} - (g_e(\pi_k - \delta))^2} < 0$ ,  $\frac{\partial k}{\partial b} = \frac{g_{ee}(\pi - \delta k)}{g_{ee}(\pi - \delta k)g \pi_{kk} - (g_e(\pi_k - \delta))^2} < 0$ , and  $\frac{\partial e}{\partial b} = \frac{\partial k}{\partial a} = \frac{g_e(\pi_k - \delta)}{(g_e(\pi_k - \delta))^2 - g_{ee}(\pi - \delta)g \pi_{kk}} < 0$ . Since  $g(e)\epsilon(e) > 0$ , it follows that  $e_0 < e_{FB}$  and  $k_0 < k_{FB}$ .

In order to show that  $\bar{u} > \underline{u}$  recall that  $\underline{u} := p_m (g(e_0)/g_e(e_0) - e_0) + (1 - \tau_2) \delta k_0$ . Further we can write  $\bar{u} = p_m (g(e_{FB})/g_e(e_{FB}) - e_{FB}) + (1 - \tau_2) \delta k_{FB}$  using the first order condition for the first-best. Since  $(g(e)/g_e(e) - e)$  is strictly increasing in  $e$  by Assumption 1 (iii),  $(1 - \tau_2) \delta k$  is trivially increasing in  $k$ , and  $e_0 < e_{FB}$  and  $k_0 < k_{FB}$  it follows that  $\bar{u} > \underline{u}$ .

(iii) Suppose the limited liability constraint was not binding for  $u \leq \bar{u}$ , i.e.  $c < \tau_2 \delta k$ . Then the participation constraint is binding by Step 1 and by Step 2 (ii) we have  $r = c$ . Further  $u \leq \bar{u}$  implies by definition  $u \leq g(e_{FB})(\pi(k_{FB}) - \delta k_{FB}) - p_m e_{FB} + (1 - \tau_2) \delta k_{FB}$  and the participation constraint is  $u = g(e_{FB})(\pi(k_{FB}) - \delta k_{FB}) + \delta k_{FB} - c - p_m e_{FB}$ . Together they imply  $\tau_2 \delta k_{FB} \leq c$ , a contradiction.

**Step 5** We show for  $u \in [\underline{u}, \bar{u})$ , (i) the optimal effort  $\xi(u)$  and capital level  $\varsigma(u)$  are increasing in  $u$ , and (ii)  $\lim_{u \rightarrow \underline{u}} \xi(u) = e_0$ ,  $\lim_{u \rightarrow \underline{u}} \varsigma(u) = k_0$ , and  $\lim_{u \rightarrow \bar{u}} \xi(u) = e_{FB}$ ,  $\lim_{u \rightarrow \bar{u}} \varsigma(u) = k_{FB}$ .

**Proof of Step 5** (i) We know that for  $u \in [\underline{u}, \bar{u})$  the limited liability constraint with respect to  $c$  is binding (from Step 4 (iii)) and the participation constraint is binding (from Step 3 (ii)). We can use the binding limited liability constraint to substitute for  $c$  everywhere, the incentive compatibility constraint to substitute for  $r$  in the lender's profit function as well as in the binding participation constraint. This reduces the problem to a maximization of lender profits over two dimensions,  $e$  and  $k$ , and subject to the participation constraint. Write the Lagrangian as:

$$\mathcal{L}(e, x) = g(e) \left[ \pi(k) - \delta k - \frac{p_m}{g_e(e)} \right] + \tau_2 \delta k - \gamma(k - a) - \lambda \left\{ g(e) \left[ \frac{p_m}{g_e(e)} \right] + (1 - \tau_2) \delta k - p_m e - u \right\} \quad (26)$$

The first-order conditions are:

$$g_e(e) [\pi(k) - \delta k] - p_m [1 + g(e) \epsilon(e)] - \lambda g(e) \epsilon(e) p_m = 0 \quad (27)$$

$$g(e) [\pi_k(k) - \delta] + \tau_2 \delta - \gamma - \lambda (1 - \tau_2) \delta = 0 \quad (28)$$

$$g(e) \left[ \frac{p_m}{g_e(e)} \right] + (1 - \tau_2) \delta k - p_m e - u = 0. \quad (29)$$

We can rewrite (27) and (28) to obtain:

$$(1 - \tau_2) \delta [g_e(e) (\pi(k) - \delta k) - p_m] - p_m g(e) \epsilon(e) [g(e) (\pi_k(k) - \delta) + \delta - \gamma] = 0. \quad (30)$$

Denoting the left hand side as  $h(e, k)$ , the total differential is  $h_e(e, k) de + h_k(e, k) dk = 0$ , with

$$\begin{aligned} h_e(e, k) &= (1 - \tau_2) \delta g_{ee}(e) (\pi(k) - \delta k) - p_m [g(e) \epsilon(e)]_e (g(e) (\pi_k(k) - \delta) + \delta - \gamma) \\ &\quad - p_m g(e) \epsilon(e) g_e(e) (\pi_k(k) - \delta) \\ h_k(e, k) &= (1 - \tau_2) \delta g_e(e) (\pi_k(k) - \delta) - p_m g(e)^2 \epsilon(e) \pi_{kk}(k). \end{aligned}$$

Notice that  $g(e)$  and  $\epsilon(e)$  are both positive and increasing in  $e$ ; that  $g_{ee} < 0$  and  $\pi_{kk} < 0$  by concavity; that  $\pi(k) - \delta k > 0$  from the incentive compatibility constraint together with  $r \geq c$  (from Lemma 1); that  $\pi_k(k) - \delta$  from the last result and the assumption  $\gamma > \delta$ . Further from the borrower's incentive compatibility constraint we have  $g_e(e) (\pi(k) - \delta k) - p_m = g_e(e) (r - c) \geq 0$  where the latter inequality follows from Step 1 of Lemma 1. By (30) this implies  $g(e) (\pi_k(k) - \delta) + \delta - \gamma \geq 0$ . Combined these imply  $h_e(e, k) < 0$  and  $h_k(e, k) > 0$ , and  $\frac{dk}{de} > 0$ .

Total differentiating (29), we get  $p_m g(e) \epsilon(e) de + (1 - \tau_2) \delta dk - du = 0$ , which together with the previous result implies

$$\frac{de}{du} = \left[ p_m g(e) \epsilon(e) - (1 - \tau_2) \delta \frac{h_e}{h_k} \right]^{-1} > 0. \quad (31)$$

Together with  $\frac{dk}{de} > 0$  this immediately also implies  $\frac{dk}{du} > 0$ . Hence we can write the optimal effort and capital both as increasing function of  $u$ , i.e.  $\xi(u)$  and  $\varsigma(u)$  respectively.

(ii) We shall prove the result by contradiction. Suppose that  $\lim_{u \rightarrow \underline{u}} \xi(u) > e_0$ . For any  $\epsilon > 0$ , this is equivalent to  $\lim_{\epsilon \rightarrow 0} \xi(\underline{u} + \epsilon) > e_0$ . Denote  $\tilde{e} := \lim_{\epsilon \rightarrow 0} \xi(\underline{u} + \epsilon)$  and  $\tilde{k} := \lim_{\epsilon \rightarrow 0} \varsigma(\underline{u} + \epsilon)$ . First observe that (30) holds for any  $(\xi(u), \varsigma(u))$  with  $u \in [\underline{u}, \bar{u})$  and hence

$\frac{d\zeta(u)}{d\tilde{\zeta}(u)} > 0$  holds too. Therefore  $\tilde{e} > e_0$  implies  $\tilde{k} > k_0$ . Further the participation constraint is binding as long as  $u \geq \underline{u}$ , and hence in particular for any small  $\epsilon$ :

$$p_m \left[ \frac{g(\tilde{\zeta}(\underline{u} + \epsilon))}{g_e(\tilde{\zeta}(\underline{u} + \epsilon))} - \tilde{\zeta}(\underline{u} + \epsilon) \right] + (1 - \tau_2) \delta \zeta(\underline{u} + \epsilon) = \underline{u} + \epsilon.$$

Taking limits on both sides as  $\epsilon$  goes to 0 we obtain:

$$\left[ \frac{g(\tilde{e})}{g_e(\tilde{e})} - \tilde{e} \right] + (1 - \tau_2) \delta \tilde{k} = \underline{u}. \quad (32)$$

Recall that by the definition we have  $\underline{u} = p_m \left( \frac{g(e_0)}{g_e(e_0)} - e_0 \right) + (1 - \tau_2) \delta k_0$ . But since  $\left[ \frac{g(e)}{g_e(e)} - e \right]$  is increasing in  $e$  and  $(1 - \tau_2) \delta k$  is trivially increasing in  $k$ ,  $\tilde{e} > e_0$  and  $\tilde{k} > k_0$  implies that the left hand side is larger than the right hand side, a contradiction. An exactly analogous argument can be constructed for  $\lim_{u \rightarrow \underline{u}} \tilde{\zeta}(u) < e_0$  and  $\lim_{u \rightarrow \underline{u}} \zeta(u) \neq k_0$ .

The argument for  $\lim_{u \rightarrow \bar{u}} \tilde{\zeta}(u) = e_{FB}$  and  $\lim_{u \rightarrow \bar{u}} \zeta(u) = k_{FB}$  follows the same steps, and noting that we can write  $\bar{u} = p_m \left( \frac{g(e_{FB})}{g_e(e_{FB})} - e_{FB} \right) + (1 - \tau_2) \delta k_{FB}$  by the definition of  $\bar{u}$ , the incentive compatibility constraint, as well as the fact that  $r = c$  for  $u = \bar{u}$ .

## Proof of Corollary 1.

First notice that  $\frac{\partial S(\hat{e}, \hat{k}; \theta, \mathbf{p})}{\partial u} = \frac{\partial S(\hat{e}, \hat{k}; \theta, \mathbf{p})}{\partial e} \frac{\partial \hat{e}}{\partial u} + \frac{\partial S(\hat{e}, \hat{k}; \theta, \mathbf{p})}{\partial \hat{k}} \frac{\partial \hat{k}}{\partial u}$ . Recall from the proof of Proposition 1 that  $(e, k) = (e_0, k_0)$  for any  $u \leq \underline{u}$  and  $(e, k) = (e_{FB}, k_{FB})$  for any  $u \geq \bar{u}$ , and hence  $\frac{\partial \hat{e}}{\partial u} = \frac{\partial \hat{k}}{\partial u} = \frac{\partial S(\hat{e}, \hat{k}; \theta, \mathbf{p})}{\partial u} = 0$  in both cases. Further, since  $S(e, k; \theta, \mathbf{p}) = g(e; \theta) \pi(k; \theta, \mathbf{p}) + [1 - g(e; \theta)] \delta k - \gamma k - p_m e$ , we have:

$$S_e(e, k; \theta, \mathbf{p}) = g_e(e; \theta) (\pi(k; \theta, \mathbf{p}) - \delta k) - p_m \quad (33)$$

$$S_k(e, k; \theta, \mathbf{p}) = g(e; \theta) (\pi_k(k; \theta, \mathbf{p}) - \delta) + \delta - \gamma. \quad (34)$$

For any  $u$  such that  $\underline{u} < u < \bar{u}$  we have  $S_e(\hat{e}, \hat{k}; \theta, \mathbf{p}) \geq 0$  and  $S_k(\hat{e}, \hat{k}; \theta, \mathbf{p}) \geq 0$ , as well as  $\frac{\partial \hat{k}}{\partial u} > 0$  and  $\frac{\partial \hat{e}}{\partial u} > 0$  by Step 5 (i) of the proof of Proposition 1, and hence  $\frac{\partial S(\hat{e}, \hat{k}; \theta, \mathbf{p})}{\partial u} \geq 0$ .

## Proof of Proposition 2.

Recall from the proof of Corollary 1 that for any  $u \in (\underline{u}, \bar{u})$  we have  $\frac{\partial S(\hat{e}, \hat{k}; \theta, \mathbf{p})}{\partial u} \geq 0$ , and  $\frac{\partial S(\hat{e}, \hat{k}; \theta, \mathbf{p})}{\partial u} = 0$  otherwise.

Now consider the comparative static on wealth  $a$ . Notice that it impacts surplus only through its effect on the outside option  $u(a, \theta, \phi) = \max\{V^{self}(a, \theta, \mathbf{p}), \hat{u}(\phi; \theta, \mathbf{p}), p_l + \gamma a\}$ . While  $\hat{u}(\phi; \theta, \mathbf{p})$  is independent of  $a$ , both  $V^{self}(a, \theta, \mathbf{p})$  and  $p_l + \gamma a$  are trivially increasing in  $a$ . Hence  $u(a, \theta, \phi)$  is weakly increasing in  $a$ , and so is  $S(e, k; \theta, \mathbf{p})$ .

For comparative statics on  $\phi$ , we differentiate (17) with respect to  $\phi$  to yield

$$\frac{\partial \hat{u}(\phi; \theta, \mathbf{p})}{\partial \phi} = \frac{\hat{S}(\hat{u}(\phi; \theta, \mathbf{p}); \theta, \mathbf{p})}{1 - \phi \hat{S}_u(\hat{u}(\phi; \theta, \mathbf{p}); \theta, \mathbf{p})} \quad (35)$$

Since  $\phi \in [0, 1]$ , we have  $\frac{\partial \hat{u}(\phi; \theta, \mathbf{p})}{\partial \phi} > 0$  if  $S_u(\hat{u}(\phi; \theta, \mathbf{p}); \theta, \mathbf{p}) \leq 1$ . To see this, note that we could also write the total surplus of a lending relationship as  $\hat{S}(\hat{u}; \theta, \mathbf{p}) = \hat{\Pi}(\hat{u}; \theta, \mathbf{p}) + \hat{u}$ , where

$\hat{\Pi}(\hat{u}; \theta, \mathbf{p})$  is the lender's profit. Then

$$\hat{S}_u(\hat{u}; \theta, \mathbf{p}) = 1 + \hat{\Pi}_u(\hat{u}; \theta, \mathbf{p}) \leq 1. \quad (36)$$

The inequality follows from the fact that the lender's profit must be non-increasing as the borrower's outside option goes up, i.e.  $\hat{\Pi}_u(\hat{u}; \theta, \mathbf{p}) \leq 0$ . If this was not true, the lender could offer another contract which makes both him and the borrower better off, contradicting that the lender is maximizing profits.

Last, we find  $\hat{S}_\theta$  as

$$\begin{aligned} \hat{S}_\theta = & g_\theta(\hat{e}; \theta)[\pi(\hat{k}; \theta, \mathbf{p}) - \delta\hat{k}] + g(\hat{e}; \theta)\pi_\theta(\hat{k}; \theta, \mathbf{p}) \\ & + [g_e(\hat{e}; \theta)[\pi(\hat{k}; \theta, \mathbf{p}) - \delta\hat{k}] - p_m]\hat{e}_\theta + [g(\hat{e}; \theta)[\pi_k(\hat{k}; \theta, \mathbf{p}) - \delta] + \delta - \gamma]\hat{k}_\theta. \end{aligned} \quad (37)$$

The sign of  $g_\theta(\hat{e}; \theta)$  and  $g_{e\theta}(\hat{e}; \theta)$  are both indeterminate, and examples can be constructed both such that  $\hat{S}_\theta > 0$  and  $\hat{S}_\theta < 0$ . In particular, we can show that for  $g(e; \theta) = \theta e^\alpha$ ,  $\hat{S}_\theta > 0$ . For  $g(e; \theta) = \lambda(\frac{e}{\theta^\mu})^\alpha$ , and  $\pi(k; \theta) = A\theta^q k^\beta$  with  $0 < \alpha, \beta, q < 1$ : if  $\mu\alpha < q$ , then  $\hat{S}_\theta > 0$ ; if instead  $\mu\alpha > q$ , then  $\hat{S}_\theta > 0$  if  $\pi_k(k_0; \theta) < \beta C$ ;  $\hat{S}_\theta < 0$  if  $\pi_k(k_{FB}; \theta) > \beta C$  and in addition  $\pi_k(k_0; \theta) > C$ , where  $C = \frac{\delta\mu\alpha}{\mu\alpha - q}$  is a constant.

## B Additional Figures

### B.1 Distributional Effects of Financial Inclusion

In Figure 3 we present the decomposition of total surplus and surplus gains (as financial access expands) among lenders, entrepreneurs and workers focusing on  $\tau = 0.6$ . Now in Figure 6 and Figure 7 we present respectively the surplus shares and surplus gains but across different levels of contracting friction  $\tau$ . From the left to the right column we show respectively  $\tau = 0.3$ ,  $\tau = 0.6$  and  $\tau = 0.9$ . The top row has our lowest level  $\phi = 0.5$  and the bottom row is for the highest  $\phi = 0.9$ . As we have argued in the main text, there is little variation across different intensities of contracting frictions.

### B.2 Equilibrium Contracts

We now look for clues about the impact of competition and inclusion on the real economy by using the model to examine how credit contracts vary across the wealth distribution as competition varies. We standardize by considering firms that are of the median entrepreneurial ability in the first-best. Since the results do not vary significantly by the level of financial inclusion (see the Appendix), we home in on a case of a medium level of financial development with  $\chi = 0.7$ . We will consider how contracts vary for the three different values of  $\tau$  representing different levels of quality of the institutional environment and home in on three dimensions of credit contracts: the interest rate that the borrower pays, the amount that a borrower can borrow relative to their assets, and the equilibrium default probability. Since the marginal product of capital is equal to the risk adjusted cost of capital of the lender, the default probability is a sufficient statistic for capital allocation

In Figure 8, the log of firm assets denominated in units of annual earnings so zero corresponds to firm assets equal to the annual earnings of a wage laborer. A firm with a lot of assets will have less need for credit. We consider three levels of competition: the lowest level of

competition ( $\phi = 0.5$ ) is where the lender can appropriate all of the surplus from the lending relation subject to possibly offering the borrower an efficiency level of utility. We also consider a middle level of surplus sharing ( $\phi = 0.7$ ) where the borrower and lender share the surplus from the lending relationship equally and finally we allow all of the surplus to go to the borrower ( $\phi = 0.9$ ), our representation of a highly competitive credit market. Recall from above that we estimated that  $\phi$  is approximately 0.87 in the US.

The columns in Figure 8 represent different values of  $\tau \in \{0.5, 0.7, 0.9\}$  and the different lines, as in Figure 2 represent different levels of competition. In all cases, the running variable is the level of (log) assets.

It is well-known that there are dramatic differences in interest rates faced by firms within the same economy. Moreover, we know that in many low and middle income countries, interest rates can be extremely high. Our model allows us to explore the determinants of this and how the interest rate faced by firms varies with wealth at different levels of competition and the institutional environment. There is no simple relationship between default probabilities and interest rates since the amount to be repaid and the amount borrowed will both vary endogenously with wealth and competition. The top row in Figure 8 gives the quantitative results and, not surprisingly, entrepreneurs with higher levels of wealth pay lower interest rates. It is striking that the level of interest rates varies with institutional environment with very high rates for low wealth borrowers when  $\tau$  is low. However, these fall dramatically as the institutional environment improves across the columns. They also vary markedly with competition. And there is a threshold at which having a decent amount of collateralizable wealth leads to a lower interest rate being charged by the lender.

The second row in Figure 8 shows that the leverage ratio declines sharply with wealth. But in quantitative terms, the pattern is not so different by institutional environment or the level of competition. Thus, the clue as to why competition or institutional environment matter is not in the amount that an agent of a given wealth level can borrow.

The third row of Figure 8 shows that the answer may lie in equilibrium default probabilities. In all cases, default falls rapidly when borrowers get a larger share of the surplus. Low competition is associated with a default probability of around 10% for firms with few assets and falls to around 7-8% under high competition. The default risk falls with increased wealth in all circumstances, but mostly for low levels of competition.

## C Robustness

We check if our main aggregate implications in Figure 2 would qualitatively change under alternative values of  $\alpha$  and  $\mu$ . We consider two alternative sets of simulation results: in Figure 9, we present the effect of financial inclusion on wage, share of entrepreneurs, as well as aggregate capital output ratio under 25% reduction of our calibrated values of  $\alpha$  and  $\mu$ . In Figure 10, we conduct the same exercise but under 25% increase of the calibrated values of  $\alpha$  and  $\mu$ . These alternative parameter values do somewhat change the level of the curves as compared to Figure 2, but do not impact the qualitative messages as stated in Section 5.2.



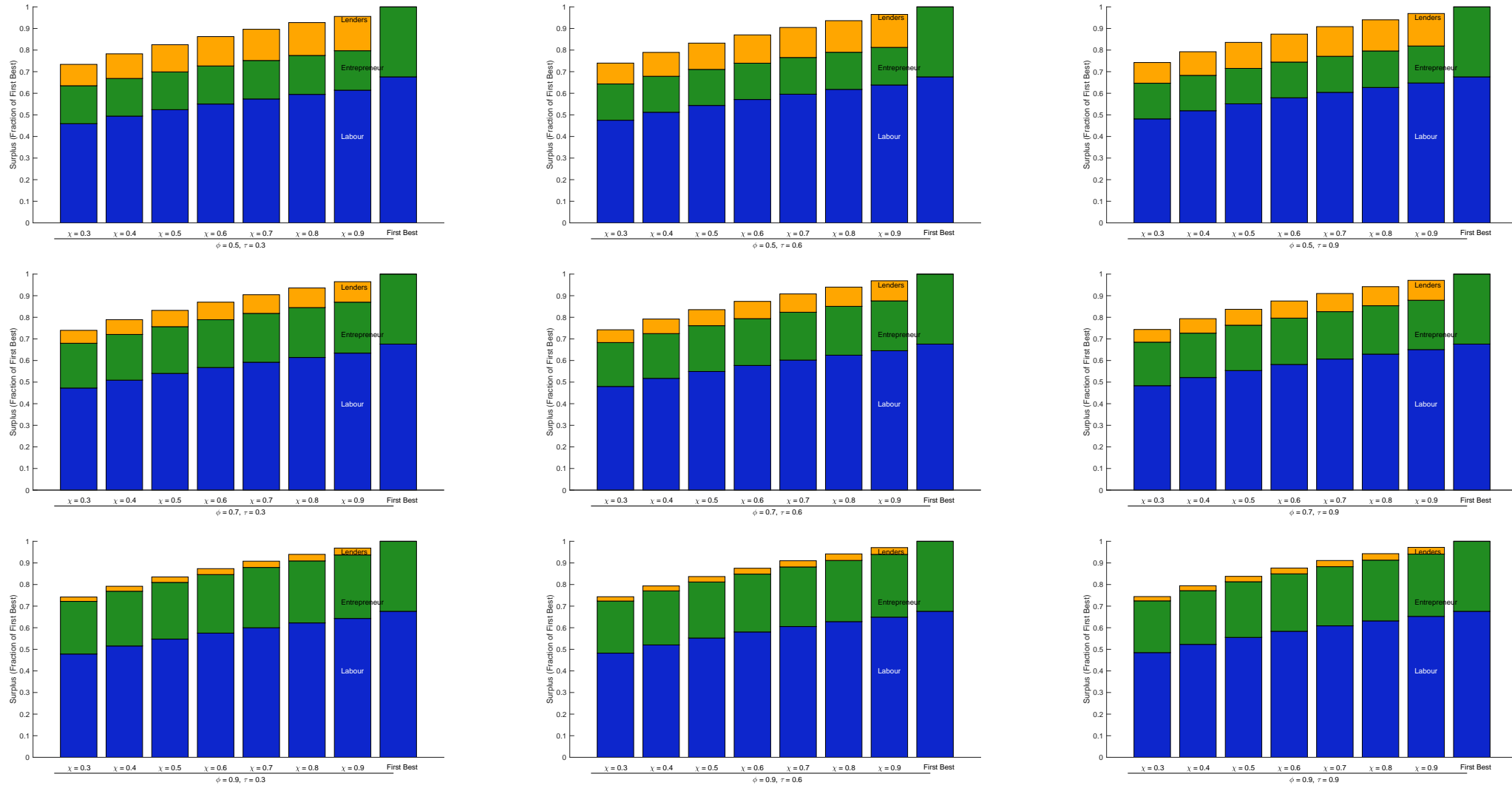
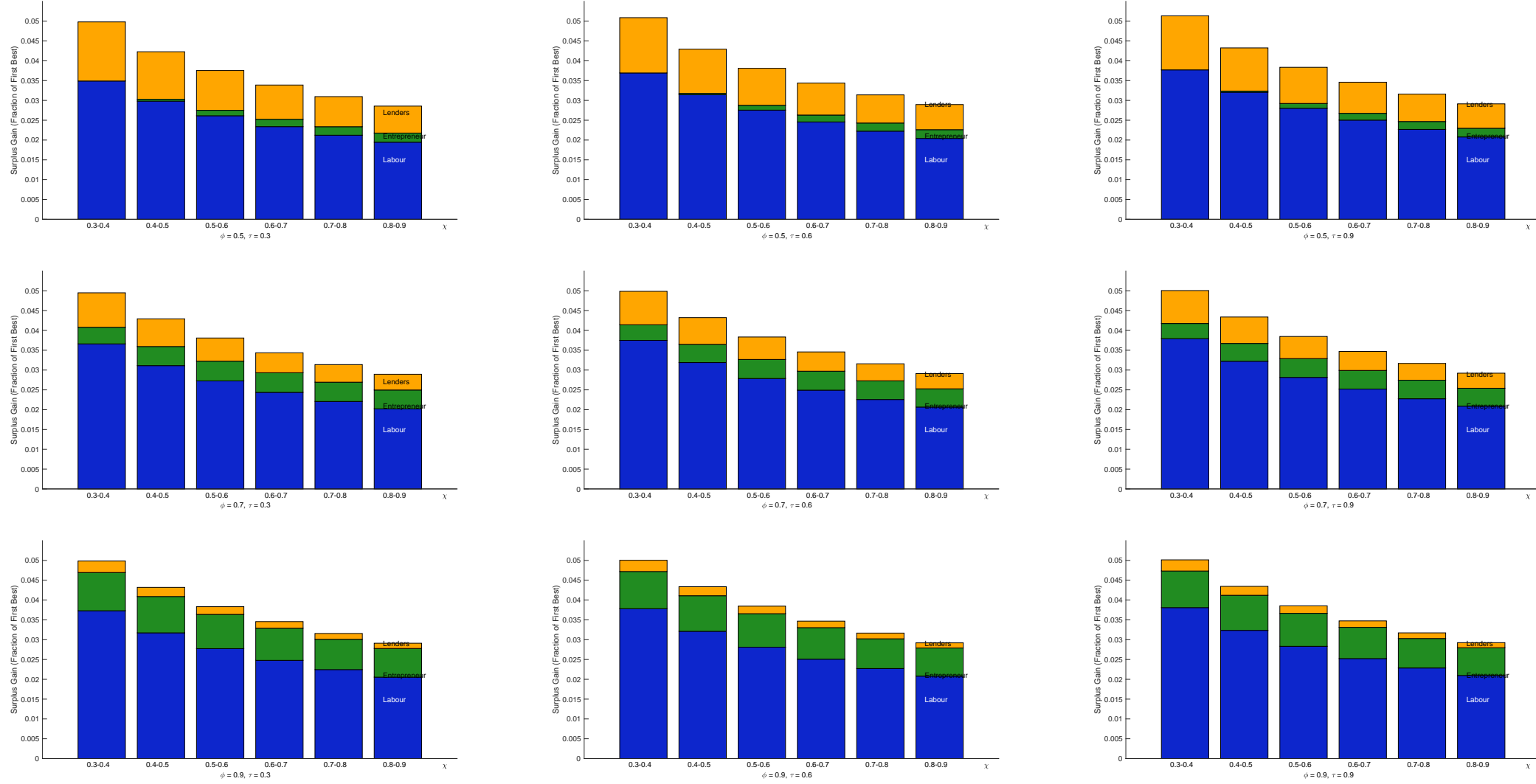


FIGURE 6: SURPLUS SHARES WITH  $\tau = 0.3$  (LEFT),  $\tau = 0.6$  (CENTER) AND  $\tau = 0.9$  (RIGHT)

FIGURE 7: SURPLUS GAINS WITH  $\tau = 0.3$  (LEFT),  $\tau = 0.6$  (CENTER) AND  $\tau = 0.9$  (RIGHT)

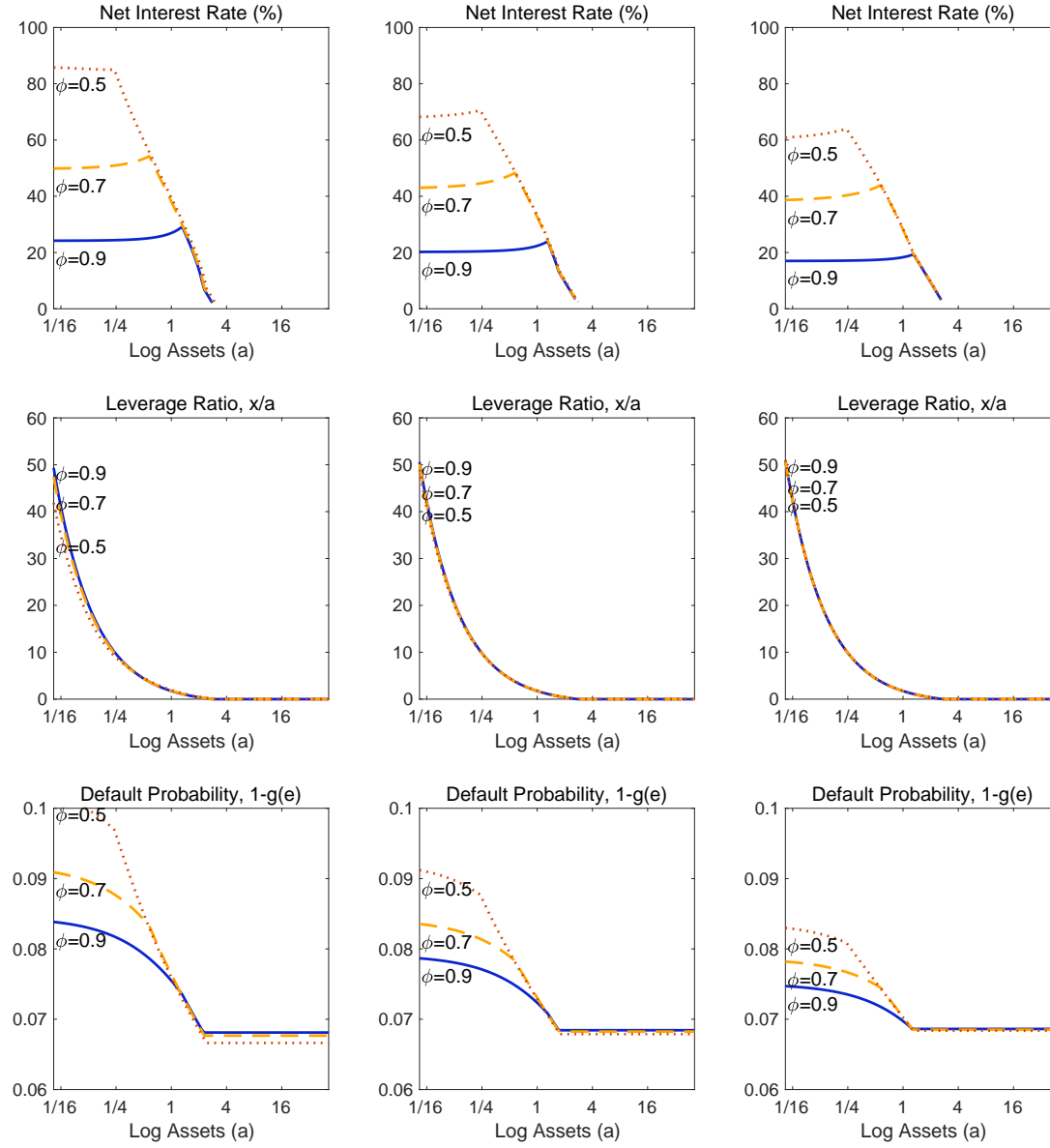
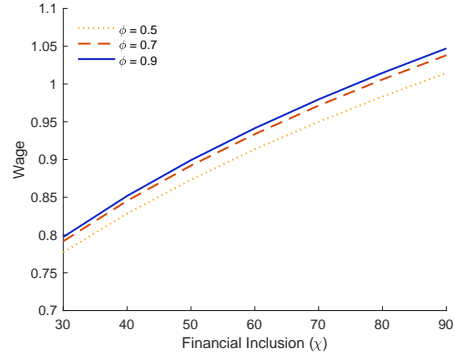
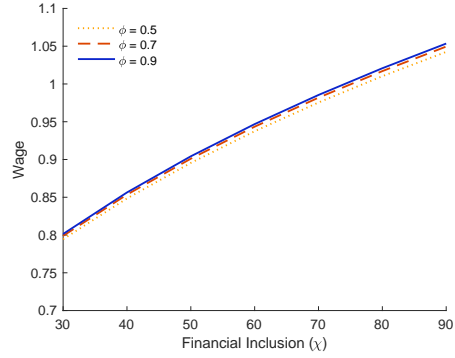
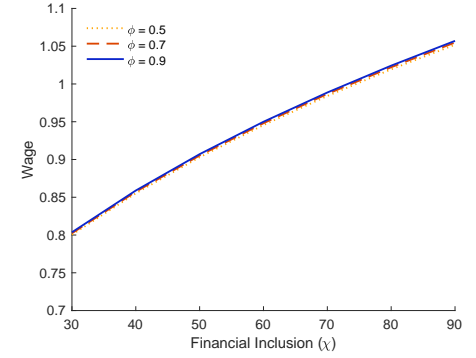
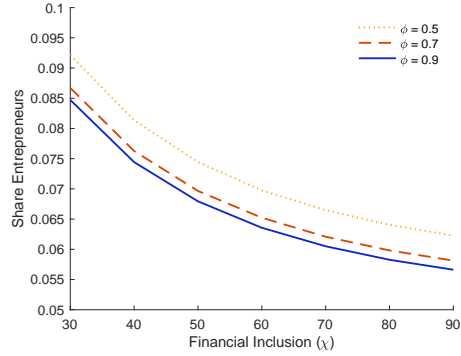
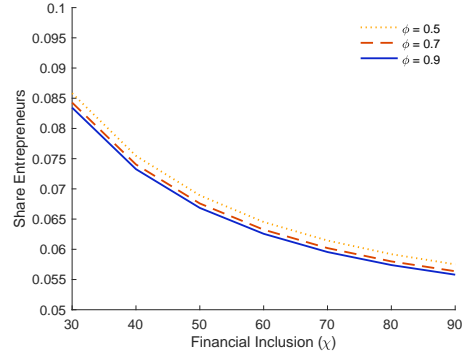
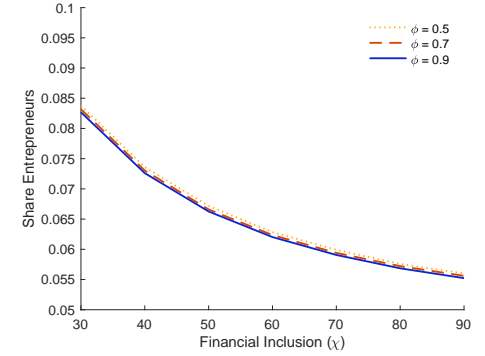
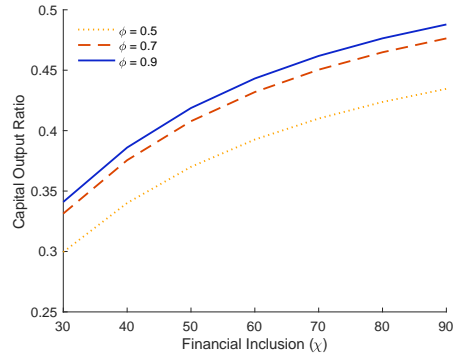
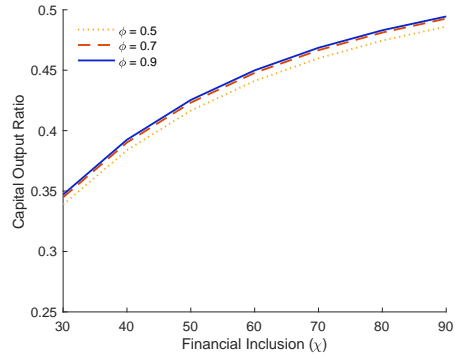
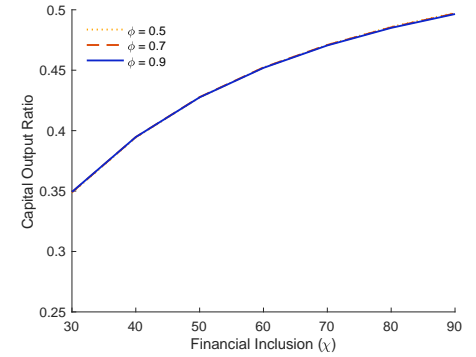
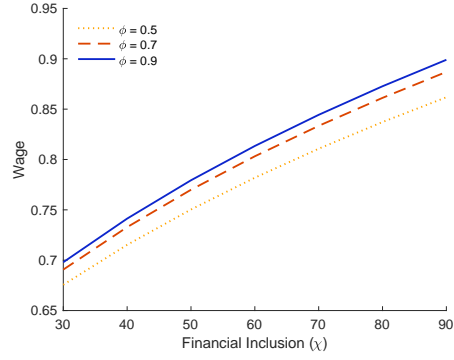
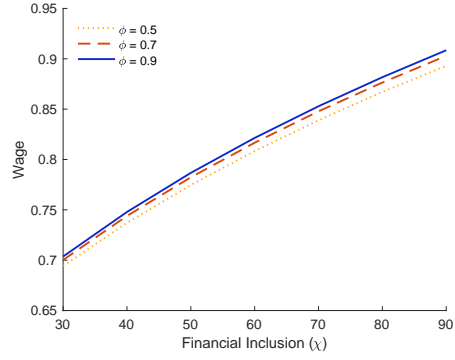
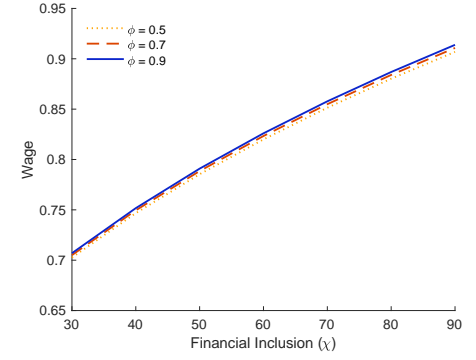
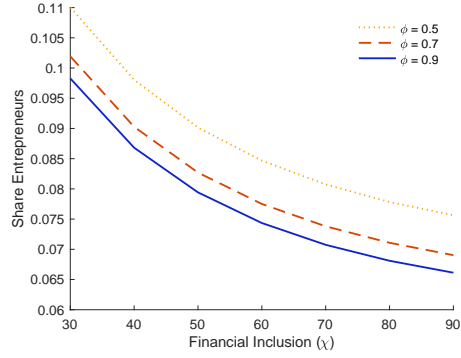
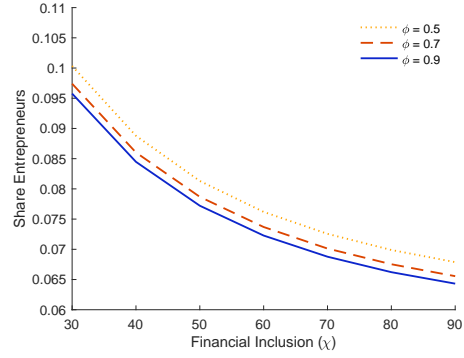
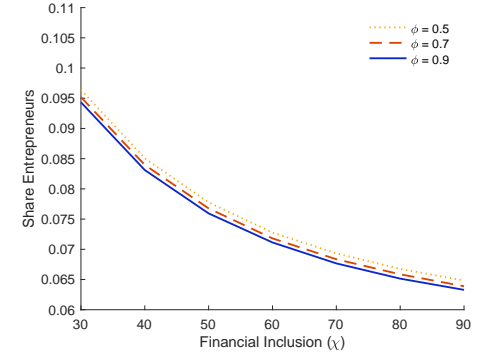
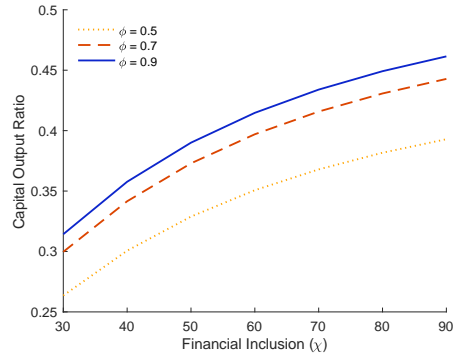
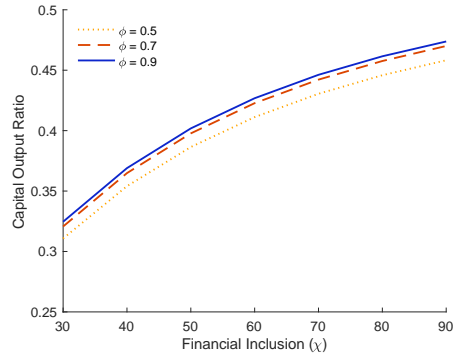
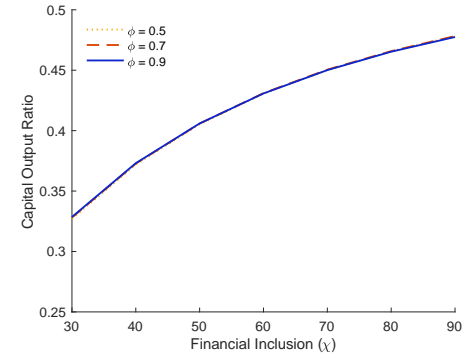


FIGURE 8: EQUILIBRIUM CONTRACTS WITH  $\chi = 0.7$ :  $\tau = 0.3$  (LEFT),  $\tau = 0.6$  (CENTER) AND  $\tau = 0.9$  (RIGHT)

(A) WAGE ( $\tau = 0.3$ )(B) WAGE ( $\tau = 0.6$ )(C) WAGE ( $\tau = 0.9$ )(D) ENTREPRENEURS SHARES ( $\tau = 0.3$ )(E) ENTREPRENEURS SHARES ( $\tau = 0.6$ )(F) ENTREPRENEURS SHARES ( $\tau = 0.9$ )(G) CAPITAL OUTPUT RATIO ( $\tau = 0.3$ )(H) CAPITAL OUTPUT RATIO ( $\tau = 0.6$ )(I) CAPITAL OUTPUT RATIO ( $\tau = 0.9$ )FIGURE 9: AGGREGATE RESULTS (USING 75% OF THE CALIBRATED VALUES OF  $\alpha$  AND  $\mu$ ):  $\tau = 0.3$  (LEFT),  $\tau = 0.6$  (CENTER) AND  $\tau = 0.9$  (RIGHT)

(A) WAGE ( $\tau = 0.3$ )(B) WAGE ( $\tau = 0.6$ )(C) WAGE ( $\tau = 0.9$ )(D) ENTREPRENEURS SHARES ( $\tau = 0.3$ )(E) ENTREPRENEURS SHARES ( $\tau = 0.6$ )(F) ENTREPRENEURS SHARES ( $\tau = 0.9$ )(G) CAPITAL OUTPUT RATIO ( $\tau = 0.3$ )(H) CAPITAL OUTPUT RATIO ( $\tau = 0.6$ )(I) CAPITAL OUTPUT RATIO ( $\tau = 0.9$ )FIGURE 10: AGGREGATE RESULTS (USING 125% OF THE CALIBRATED VALUES OF  $\alpha$  AND  $\mu$ ):  $\tau = 0.3$  (LEFT),  $\tau = 0.6$  (CENTER) AND  $\tau = 0.9$  (RIGHT)