Inequality Traps in Convex and Convexified Economies

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Motivation

- What explains the persistence of poverty over the life cycle and across generations?
- Does persistent poverty inhibit countries from reaching their full potential in terms of economic prosperity?

Inequality Traps and Poverty Traps

- How do "inequality traps" differ from "poverty traps" in terms of mechanisms, properties, and policy implications?
- Poverty traps involves history dependence at the micro-level
- Individuals are unable to escape poverty due to wealth constraints that limit opportunities to raise their incomes, thereby reinforcing the tendency for individuals and countries to stay poor
- Inequality traps involves history dependence at the macro-level
- For example, due to some market frictions and endogenous returns to occupations, some configurations of the initial wealth distribution hinder upward mobility for the poor and perpetuate the economic privileges of being wealthy

Why Study Inequality Traps?

- There are compelling arguments that poverty traps cannot persist indefinitely as:
  - Improvement in institutions would help individuals overcome the lumpiness of investments such as lotteries or informal credit institutions like rotating saving and credit associations (ROSCAs)
  - Also, with growth, economic reforms, technological progress etc., all individuals may experience increases in the absolute level of real incomes
  - Still, macro-level persistence may continue and so inequality traps are unlikely to go away
- Policy implications of an inequality trap world are sharply different from a poverty trap world
  - Just "lifting" one person at a time from poverty with a micro-intervention (e.g., microfinance, asset transfer) will help in the poverty trap world but not in the inequality trap world
  - More systemic interventions are needed, but they would inevitably be politically harder to implement – e.g., redistribution of income or wealth, minimum wages

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- We propose a framework where there are no poverty traps, yet inequality traps exist.
- We consider two variants of the model
  - A decreasing-returns, convex technology in which capital and labor can be arbitrarily finely divided across production units
  - Another with "small" production non-convexities that introduce non-concavities in individual value (and policy) functions that rational agents will concavify via lotteries

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- Poverty trap models may best be viewed as expressions or approximations at the individual level of forces operating at the macro level, but as such, they provide little reliable policy guidance.

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- One contribution of this paper is to provide an example of a market inequality trap with a convex technology.
- We then turn to a more conventional inequality trap model with technological non-convexities, but allow for lotteries (thus the value-function is concave). This version is especially tractable for policy analysis
  - Provide conditions under which a closed-economy, one-off redistribution of wealth can set the economy onto a path out of the inequality trap, toward a more “desirable” steady state.
  - We show that a minimum wage enforced only in the formal sector may suffice to drive the economy to the desired steady state, even if no one-off redistribution can.

Literature

- Poverty Traps: Ray-Streufert 1993; Galor-Zeira 1993
- Inequality traps: Banerjee-Newman 1993; Ghatak-Jiang 2002; Piketty 1997; Mookherjee-Ray 2003, Genicot-Ray 2017
- Evidence on the presence of persistence of poverty at the individual or household level in some settings (e.g., Balboni et al, 2022; Banerjee, Duflo, and Sharma, 2021; Banerjee et al, 2019; Lybbert et al, 2004; Antman and McKenzie, 2007; Kaboski et al, 2024)

The Basic Model: Technology

- There is a single consumption good (y) produced using labour (l) and capital (k).
- The "modern" technology is convex, with decreasing returns to scale. Capital and labour are perfect complements, up to a capacity constraint:

$$y = \min\{\bar{y}, A \min\{k, l\}\}$$

  with A as a common productivity parameter.
- "Traditional" technology requires labour only and yields $\underline{w} > 0$ per unit of labour.
- "Storage" technology requires only wealth as an input; it is perfectly divisible as well and returns $r < A$ per unit invested.
- To avoid trivial cases, we assume

Assumption 1: (a) $\underline{w} < A - r$. (b) $\bar{k} > 1$.

The Basic Model: Demographics

- Time is discrete, indexed by $t = 0, 1, 2....$ Each period there is a unit measure continuum of risk-neutral agents, each endowed with a unit of labour.
- They may differ in initial endowments a with the (endogenous) c.d.f. as $F_t(a)$.
- Within a period, an agent inherits wealth $a \geq 0$ from its parent, then chooses how to invest its labour and wealth across the three technologies.
- Once the returns from these activities have accrued, the agent gives birth to one offspring and divides its income between consumption and a bequest so as to maximize the "warm glow bequest" utility (Andreoni, 1989)

$$u(c, b) \propto c^{1-\beta} b^{\beta}. \tag{1}$$

The Basic Model: Demographics

- The degree of warmth (β) varies across individuals in the population, independently of wealth, and independently over time within lineages.
- Specifically, $\beta = \overline{\beta} \in (0, 1)$ with probability q, and $\underline{\beta}$ otherwise with $\overline{\beta} > \underline{\beta} \in [0, 1)$.
- For multiple steady state distributions, we impose

Assumption 2: $\overline{\beta}(A - \underline{w}) > 1 > \overline{\beta}r$.

The Basic Model: Markets

- Labour markets are competitive and subject to no distortions. Since anyone, regardless of initial wealth, has access to the traditional technology, its return $\underline{w}$ is the lower bound for the market wage rate.
- Capital markets are imperfect and it is costly to enforce credit contracts. In its extreme form, an agent is constrained by the amount of wealth he or she has, subject to the maximum capital stock per firm that the technology permits, $\overline{k}$:

$$k \leq \min\{a, \overline{k}\}.$$

- With imperfect but not absent capital markets, with standard enforceability constraints, the relevant wealth thresholds change but nothing substantive is affected
- Finally, we assume there is no market in which agents can insure against having stingy parents (i.e., $\beta = \underline{\beta}$).

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- With imperfect but not absent capital markets, with standard enforceability constraints, the relevant wealth thresholds change but nothing substantive is affected
- Finally, we assume there is no market in which agents can insure against having stingy parents (i.e., $\beta = \underline{\beta}$). In our model this is a driver of downward mobility to counter the upward mobility from production and accumulation.

Wealth and Payoffs

- For $a < \overline{k}$, an agent's payoff is:

$$V(a) = (A - r - w)\, a + w + ar = (A - w)\, a + w.$$

- For $a \geq \overline{k}$, it is:

$$V(a) = ar + (A - r - w)\, \overline{k} + w$$

- The marginal return to a for agents with $a < \overline{k}$ is $A - w$, and r for a richer agent.
- Unlike in many of the existing models of occupational choice, the payoff function here is continuous (indeed, concave) in a, as there is no lumpiness in the production technology.
- For any investment in the modern technology to take place at all, it is necessary that $A - w \geq r$. there is an upper bound on the wage rate: $w \leq \bar{w} \equiv A - r$.

Labour Market Equilibrium

- Everyone inelastically supplies their unit of labour, so total labour supply $L^S \equiv 1$.
- For the modern technology, labour demand L^D is equal to the amount of overall capital investment in the modern sector:

$$L^D = \left(1 - F\left(\overline{k}\right)\right)\overline{k} + \int_0^{\overline{k}} af\left(a\right) da = \overline{k} - \int_0^{\overline{k}} F\left(a\right) da$$

- Since $\overline{k} > 1$, there can be potentially "excess demand" as well as "excess supply" of workers, depending on the wealth distribution. If $L^D > L^S$, then the equilibrium wage rate is $\bar{w}$, and if $L^D < L^S$, then it is $\underline{w}$.

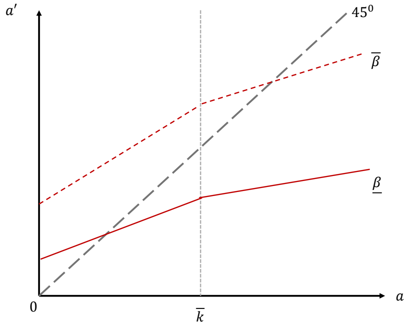
Dynamics of Wealth Distribution

- The wealth of a lineage follows the stochastic recursion $a \mapsto a'$:

$$a' = h_w(a;\beta) := \begin{cases} \beta\left[(A-w)\,a+w\right], & a < \overline{k} \\ \beta\left[(A-w)\,\overline{k}+w+\left(a-\overline{k}\right)r\right], & a \geq \overline{k} \end{cases}$$

- Each $h_w(\cdot;\beta)$ has a unique non-negative fixpoint
- Since bequest parameters realize independently across lineages, wealth, and time, we start with a given $w \in \{\underline{w}, \overline{w}\}$, and follow the corresponding lineage stochastic dynamics.
- These will each have a globally stable stationary distribution (they are irreducible Markov chains). Then we check whether w is the market clearing wage for the stationary distribution.
- Each distribution that is stationary given the wage and clears the market at that wage will be locally stable

“Typical” Recursion Diagram



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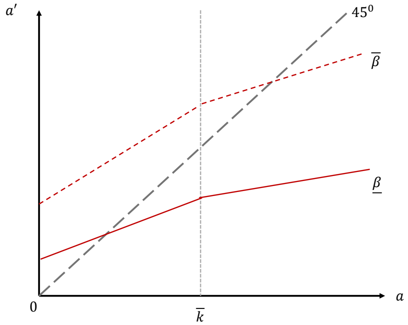

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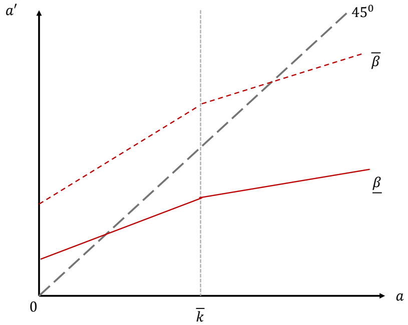

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- We set $\underline{\beta} = 0$ to simplify the explicit characterizations of stationary distributions:
  - A mass $1 - q$ at $a_0 = 0$.
  - A sequence $\{a_n(w)\}_{n=1}^{\infty}$, where $a_n = h^n_{\underline{\beta}(0;w)}$, mass at a_n is $(1-q)q^n$.

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  - A sequence $\{a_n(w)\}_{n=1}^{\infty}$, where $a_n = h^n_{\overline{\beta}(0;w)}$, mass at a_n is $(1-q)q^n$.
- **Assumption 3:** $\overline{\beta}(A - r) > 1/q > \overline{\beta}\underline{w}$.

Proposition

In the convex economy satisfying Assumptions 1, 2, and 3, there exist $\overline{k} \in (1/q, \overline{\beta}(A - r))$ *such that two distinct, locally stable wealth distributions exist, one with a high wage and one with a low wage. All points in the support of each distribution are reachable from any other in finite time.*

Dynamics of Wealth Distribution

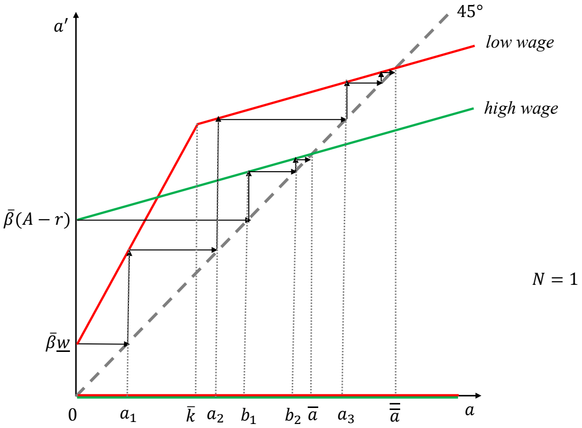

Figure 2: Recursions for construction of two steady states

Proof Sketch

- From Assumptions 3, if $w = \overline{w}$, in steady state, q of the population inherit wealth at least $\overline{\beta}(A - r) > \overline{k}$. Labor demand for any $w < \overline{w}$ is therefore $q\overline{k} > 1$, so the market clearing wage is indeed $w = \overline{w}$.
- If $w = \underline{w}$, since $1 > q\overline{\beta}\underline{w}$, there is $N \geq 1$ such that $a_N < \overline{k} < a_{N+1}$. Demand is

$$\overline{k}[1 - (1 - q)\sum_{n=0}^{N} q^n] + (1 - q)\sum_{n=0}^{N} q^n a_n = q\overline{k} + (1 - q)\sum_{n=0}^{N} q^n(a_n - \overline{k}) < q\overline{k}$$

  and < 1 for $\overline{k}$ close enough to $1/q$, so market clears at $w = \underline{w}$.
- This establishes existence of two distinct steady-state distributions, without non-convexities, and without "poverty traps" (there is mobility over all wealth levels in the respective supports)

Inequality Trap - Aggregate Performance

- Quite generally, whenever the two types of steady state coexist:

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Mean wealth and income are lower in the low-wage steady state than in the high-wage steady state.

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- In the former case, the imperfect credit market is not channeling all wealth into the most productive activities, so that aggregate production, income, and wealth are lower.
- The argument does not rely on the specific stochastic specification of the bequest propensity β, only on labour and capital being "underemployed" (misallocated away from the most productive technology) in the inequality trap, so aggregate production and income are commensurately lower.

Inequality Trap - the name fits!

- The high-wage steady state is also “more equal” than the low-wage steady state (same probability masses are pushed toward middle):

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- To say more about policy, we want to go beyond existence and characterization– need global analysis.

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- Moreover, it causes the poverty trap to disappear, along with most of its macro implications. By contrast, inequality traps may remain intact.
- The “lotterized” version of the inequality trap affords a huge dimensional reduction, and we shall use it to carry out global policy analysis.

Non-convexities

- We make the following modifications to the modern productive technology.
- First, we assume that the agent operating the productive technology does not hire out its own endowment of labor on the market but rather uses it to operate its own enterprise in some other set of “entrepreneurial” tasks than what workers perform – this is not essential for our conclusions, but greatly simplifies the algebra of our policy thought experiments.
- Second, the modern technology now yields $\hat{A}$ only if $\overline{k} \equiv 1$ units of capital and $n \geq 1$ units of labor are invested, with
$$\hat{A} - n\underline{w} - r > \underline{w};$$
- As before, no further returns are generated by investing more of either factor, but now returns are zero if investment is less than these thresholds.

Lotterized Value Function

- Indifference between entrepreneurship and working again implies an upper bound for equilibrium wage $\overline{w} = \frac{\hat{A}-r}{n+1} > \underline{w}$. We further impose

Assumption 4: $\overline{\beta}\overline{w} > 1$.

- The (pre-lottery) value function at equilibrium wage w is: $\phi w + (1-\phi)\underline{w} + ar$ when $a < 1$, and $\hat{A} - r - nw + ar$ when $a \geq 1$. ϕ is the probability of getting a "good" (formal sector) job ($\phi < 1$ only if $w = \underline{w}$, else $\phi = 1$).

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- When wage is low, an agent with $a \in [0, 1)$ would accept, prior to entering the labor market, a (fair) lottery that pays 1 with probability a, and 0 with probability $1 - a$, in exchange for its inheritance.
- The new lotterized value function is concave:

$$\hat{V}(a) = \begin{cases} (1-a)\underline{w} + a(\hat{A} - n\underline{w}), & a < 1 \\ \hat{A} - n\underline{w} + (a-1)r, & a \geq 1 \end{cases},$$

Dynamics: High Wage

- For the high wage, we can study lineage dynamics by reducing the number of states to just two: treat the interval $[0, 1)$ as one state, and $[1, \overline{a}]$ as the other, where $\overline{a}$ is a finite upper bound on wealth.

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- An agent born with a in the in the first state leaves its offspring there (in fact at 0) with probability $1 - q$, and in the other state (at $\overline{\beta}(a + \overline{w}) > 1$) with probability q.
- Those born in the second state leave their offspring in the first and second states respectively with the same probabilities.
- Thus a (unique) stationary distribution $\overline{\mathbf{p}} = (1 - q, q)$ is reached immediately from any initial condition. Since demand in the labor market is nq and supply is $1 - q$, this distribution is an equilibrium steady state provided $nq > 1 - q$.

Recursion Diagrams

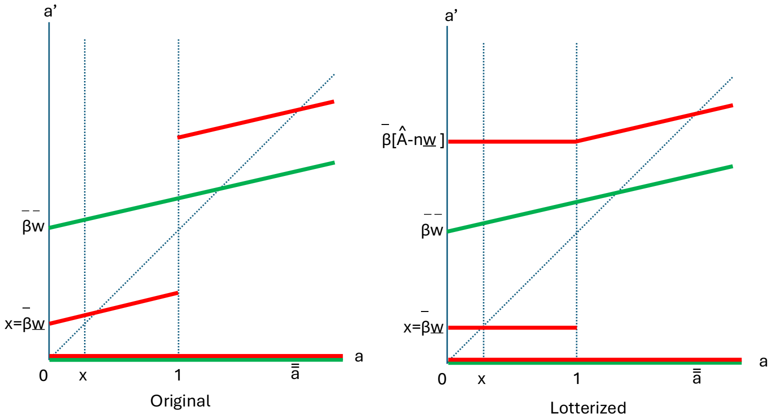

Figure 3: Recursion diagrams for non-convex technology: green/light $\overline{w}$, red/dark $\underline{w}$.

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- Agents born with $a \in [1, \overline{\overline{a}}]$ leave their offspring at 0 or in $[1, \overline{\overline{a}}]$
- Thus we can reduce the state space to $\{0, x, [1, \overline{\overline{a}}]\}$. The corresponding transition probabilities are depicted in the matrix:

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- The unique stationary distribution is $\underline{\mathbf{p}} = (1-q, \frac{q(1-q)}{1-q+qx}, \frac{q^2 x}{1-q+qx})$.

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- Thus, depending on whether the economy starts out near enough to $\underline{\mathbf{p}}$ or $\overline{\mathbf{p}}$, it will converge to one or the other steady state, with corresponding differences in the income and wealth distributions, including in the first moments.

Dynamics

- As long as $x < \frac{1-q}{nq}$, formal sector demand is inadequate to bid the wage up beyond $\underline{w}$, and this steady state sustains the low-wage labor market equilibrium.
- Thus, depending on whether the economy starts out near enough to $\underline{\mathbf{p}}$ or $\overline{\mathbf{p}}$, it will converge to one or the other steady state, with corresponding differences in the income and wealth distributions, including in the first moments.
- Notice that both distributions display mobility among the states. But compared to the high-wage steady state, income and wealth in the low-wage steady state are more unequally distributed, a true "inequality trap."
- It is the low wages resulting from competition among the large number of poor that make it difficult (albeit not impossible) for any of them to move up the wealth distribution. They are replaced by others moving down the ladder after experiencing bad luck.

Contrast with Poverty Traps

- A poverty trap model would exogenously fix the wage at $\underline{w}$ and rule out lotteries.
- It would also have to rule out the random bequest we have specified in order to stave off total collapse of the economy: with every lineage eventually born with 0, the (unique) distribution would be supported on $\{0, x\}$.
- Only then would we have the (extreme) path dependence at the lineage and aggregate levels, where any initial distribution of wealth supported on the two states (low wealth, high wealth) would be stationary.
- Convexifying such a model would result in a unique steady state distribution, with either everyone at high wealth (without the random bequests) or at $\underline{\mathbf{p}}$ (with random bequests).
- Poverty trap models, while descriptive of individual experience in an economy with persistent poverty, seem rather unsatisfactory for analysing aggregate performance.

Policies to Escape Inequality Traps

- Can wealth redistribution or a minimum wage pull a whole economy out of an inequality trap, i.e., away from one steady state and toward another?
- Labor demand is maximized if exactly 1 unit of wealth is distributed to as many people as possible. This may not be enough to sustain $\overline{w}$ if average wealth is less than 1!
- Assuming that the economy was at (or close to) $\underline{\mathbf{p}}$ before the redistribution, we can state

Proposition

A closed one-off redistribution can succeed in pulling the economy from the low-wage steady state $\underline{\mathbf{p}}$ *to the high wage steady state* $\overline{\mathbf{p}}$ *if and only if*

$$\frac{xq(1-q+q\beta(\hat{A}-r)-nqx)}{(1-q+xq)(1-q\beta r)} \geq \frac{1-xq(n+1)}{(1-x)(n+1)}. \tag{3}$$

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- In particular if x is low enough (equivalently, the informal sector is sufficiently unproductive) so that condition (3) fails, no closed redistribution can succeed.
- On the other hand, properly designed one-off redistributions can be effective for larger values of x.
- Raising informal productivity x or formal sector productivity A increases the likelihood that (3) is satisfied. So redistribution may be complementary to productivity enhancement in eliminating inequality traps.

Minimum Wage

- We assume that a minimum wage $w_m < \overline{w}$ could only be enforced in the modern/formal sector. We make it low enough that children of generous formal sector workers will not automatically be born wealthy enough to be entrepreneurs:

$$y := \overline{\beta} w_m < 1. \tag{4}$$

- Can a suitably chosen minimum wage satisfying (4) pull the economy out of the trap altogether in finite time? The labor market would then clear at the high wage $\overline{w}$, and the minimum wage would not bind. There is an affirmative answer:

Proposition

If $y > \frac{1-q}{nq}$, $\overline{\mathbf{p}}$ is the unique, globally stable distribution.

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  (-2) Use D=Demand/Supply $= \frac{n(p_R + xp_x + yp_y)}{p_0 + (1-x)p_x + (1-y)p_y}$ independent of a. With this uniform-rationing rule, some canceling of terms leads to $p_y \equiv np_R$ after one period. (Helped by having the no-splitting-time-across firms assumption – else further nonlinearities to deal with!)
- Following the remaining one-dimensional dynamics is straightforward, with transition into the high wage regime occurring iff w_m exceeds the threshold in the proposition.

Minimum Wage

- In other words, with a suitably high minimum wage, even if enforced in the formal sector alone, the inequality trap vanishes.
- The minimum wage allows the economy to eventually accumulate enough agents with wealth at least 1 that there will be excess demand in the labour market and the high wage steady state will prevail.
- Just as in the convex technology case, letting rich agents instead accumulate additional wealth beyond the minimum efficient scale is wasteful of resources when there are agents below that scale, because they do not use their wealth to hire additional workers.
- So while the minimum wage diminishes the accumulation of wealth by wealthy lineages, it does so with the social benefit of allowing <u>more</u> wealthy lineages to emerge and bid up wages.

Discussion: Empirical Relevance

- A key mechanism that the policy implications of our model highlights is that policies that are aimed at the poor can have significant general equilibrium effects that influence the labour market and wages. We now discuss some recent empirical work that provides strong suggestive evidence for this channel.
- A number of papers show that policies that raise the outside option of workers have a positive effect on private sector wages.
- For example, Muralidharan et al. (2022) show that India's National Rural Employment Guarantee Scheme (NREGS) led to a 14% increase in earnings for beneficiary households and a 26% reduction in poverty, but notably, 86% of the income gains came from non-programme earnings, driven by higher private-sector wages and employment.

Discussion: Empirical Relevance

- Similarly, a number of recent papers show that directly redistributive programmes aimed at the poor also put upward pressure on wages in the labour market.
- Bandiera et al. (2017) study an asset transfer programme in Bangladesh that provided livestock assets and skills to ultra-poor women. They find that this significantly reduced their participation in casual wage labour activities. With fewer ultra-poor women participating in these labour markets, there is reduction in labour supply to casual labour activities which leads to significant wage increases.
- Another paper studying a large-scale cash transfer program in rural Kenya (Egger et al. 2022) similarly finds that wages increased as a result of the programme, particularly for non-recipient households.
- Breza and Kinnan (2021) study the microfinance crisis in Andhra Pradesh both hindered business investments reducing labor demand, and curtailed household consumption due to the resulting falling incomes, thereby reducing demand for local goods and services, putting further downward pressure on wages.

Concluding Remarks

- This paper explores the distinction between inequality traps and poverty traps, showing that inequality traps can exist even in the absence of poverty traps.
- Inequality traps arise from the interplay between the wealth distribution and endogenous returns to occupations that can lead to multiple steady states.
- We rule out poverty traps by assuming convex technology, or allowing for lotteries in case there are non-convexities.
- The paper also examines the impact of policies, and shows that inequality traps require systemic changes like wealth redistribution or minimum wage laws unlike policies that could help individuals overcome poverty traps.
- We show that a minimum wage in the formal sector can move the economy toward the more desirable steady-state, even when redistribution fails to do this for lack of adequate aggregate wealth.