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GUILT, ESTEEM, AND MOTIVATIONAL INVESTMENTS

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GUILT, ESTEEM, AND MOTIVATIONAL INVESTMENTS

Abstract

What are the determinants of an organization's investment in the loyalty and motivation of its workers? We develop a simple principal-agent model where the standard optimal contract is to offer a bonus that trades off incentive provision versus rent extraction. We allow the principal to undertake two types of motivational investments - one that increases the agent's disutility from deviating from a prescribed effort level, and another that increases the agent's on-the-job satisfaction. We argue that these two types of investments can capture a range of organizational practices - other than extrinsic rewards - that aim to raise worker motivation. We show that the two types of motivational investments are complements and both are substitutes to financial incentives. Our analysis implies that technological improvements in the form of improved worker productivity or greater observability of output will induce profit-maximizing firms to make greater use of financial incentives and less use of motivational investments. The reason is that while financial incentives have constant returns in terms of its effect on the worker's effort level, motivational investments by their very nature have decreasing returns.

JEL Classification: D23, D86, D91, J33

Keywords: Motivated agents, investment in worker motivation, Job Satisfaction, worker moral hazard

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Guilt, Esteem, and Motivational Investments*

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August 2020

Abstract

What are the determinants of an organization’s investment in the loyalty and motivation of its workers? We develop a simple principal-agent model where the standard optimal contract is to offer a bonus that trades off incentive provision versus rent extraction. We allow the principal to undertake two types of ‘motivational’ investments - one that increases the agent’s disutility from deviating from a prescribed effort level, and another that increases the agent’s on-the-job satisfaction. We argue that these two types of investments can capture a range of organizational practices – other than extrinsic rewards – that aim to raise worker motivation. We show that the two types of ‘motivational’ investments are complements and both are substitutes to financial incentives. Our analysis implies that technological improvements in the form of improved worker productivity or greater observability of output will induce profit-maximizing firms to make greater use of financial incentives and less use of motivational investments. The reason is that while financial incentives have constant returns in terms of its effect on the worker’s effort level, motivational investments by their very nature have decreasing returns.

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1 Introduction

How can workers be motivated? In 1968 the *Harvard Business Review* carried an article titled ‘How Do You Motivate Employees?’ that aimed to reshape how firms and managers approached this question.¹ Its author, Frederick Herzberg, argued that getting an employee to do things was not the same as motivating the employee; that the threat of punishment and the promise of rewards could get an employee to ‘move’ but the only person ‘motivated’ in this transaction was the one threatening or making promises. [‘If I kick you in the rear (physically or psychologically), who is motivated? *I* am motivated; *you* move!’]. Herzberg emphasized, instead, a set of ‘motivator factors’, intrinsic to the job, for creating motivated workers (e.g. ‘achievement’, ‘recognition for achievement’, ‘responsibility’, ‘psychological growth’) as opposed to factors that are extrinsic to the job and take the form of a reward or punishment (e.g. supervision, working conditions, salary and status). An extensive psychological literature on motivation distinguishes between intrinsic and extrinsic motivation.² The former refers to doing something because it is inherently interesting or enjoyable while the latter refers to doing something because it leads to a separable outcome, such as obtaining a reward or avoiding punishment.

Herzberg’s reasoning and terminology have since entered common parlance in management practice; implicit, for example, in a special issue in the same publication thirty-five years later giving advice to executives and managers on motivating those they lead.³ Today, firms and other types of organizations often spend considerable time and resources in activities aimed at raising the morale, team-spirit and loyalty of the workforce. A broad range of activities may have such aims, including management and leadership training, team-based exercises, communication with workers about broader organizational goals. These types of activities focus on the intrinsic motivation of workers, distinguishable both from worker training that directly impact upon the skills and productivity of workers, and from rewards and punishments, and can be termed as ‘motivational investments’.

In this paper we take up a set of questions dealing with the ‘endogenous’ creation of motivation within organizations. Under what conditions are organizations more likely to make motivational investments? Are these types of investments complements or substitutes to explicit bonuses and rewards? What governs the choice between ‘negative’ motivation – e.g. causing agents to feel guilt at under-performing – versus

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‘positive’ motivation – e.g. pride in being part of the organization? How do these outcomes depend on
the level of competition in the labour market as captured by the outside option facing the employee? How
would technological improvements in the form of improved worker productivity or greater observability of
output affect the choice of profit-maximizing firms between greater use of financial incentives and the use of
motivational investments? Using a simple principal-agent model, we show that technological improvements
in the form of improved worker productivity or greater observability of output will, in general, induce profit-
maximizing firms to make greater use of financial incentives and less use of motivational investments. The
reason is that while financial incentives have constant returns in terms of its effect on the worker’s effort
level, motivational investments by their very nature have decreasing returns.

Motivational investments are difficult to justify within a worldview of individuals driven entirely by self-
interest in their choices in the economic domain. However, this view has increasingly come under question.
Recent work in economics has moved beyond stylized models of (extrinsic) motivation based on the notion of
homo economicus - a person who cares about only money and leisure - to consider a richer set of motivations.
Broadly speaking, this has been focused on different approaches to pro-social motivation. These include:
commitment to a mission (e.g. Besley and Ghatak, 2005), the role of identity such as being a ‘good’ or
‘responsible’ employee or service provider (e.g. Akerlof and Kranton, 2005, 2011), intrinsic motivation and
reputational concerns (e.g. Benabou and Tirole, 2006, 2010), commitment to an "in-group" (e.g., family,
community, tribe), status rewards, pure altruism and adherence to social norms.4 There have also been
increasing use of lab and field experiments to explore how incentives impact upon performance in situations
where pro-social motivation is deemed to be important (Ashraf, Bandiera and Jack, 2014; Berg et al, 2018;
Deserranno, 2017; Muralidharan and Sundararaman, 2009; Rasul and Rogger, 2013).

The existing literature (e.g. Akerlof and Kranton, 2005; Besley and Ghatak, 2005) has shown that non-
economic motives can reduce the need for explicit monitoring or incentive pay and, thus, the importance of
selection of the ‘appropriate type’ of workers for specific tasks. However, there has been relatively little work
on the investment of organizations in practices that could potentially create motivation.

In organizational psychology, workplaces have been classified in terms of the extent to which they provide
workers with ‘controlled’ versus ‘autonomous’ motivation (Gagné and Deci, 2005) based on self-determination

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4See Besley and Ghatak (2014, 2017) for reviews of the empirical and theoretical literature relating to sources of non-pecuniary
motivation and how they affect incentives and organization design.
theory (Deci and Ryan, 1985). At one end of the spectrum, a worker experiences controlled motivation when her behaviour is driven by extrinsic rewards and punishments. The worker’s motivation becomes more autonomous when she is driven by internal pressures, e.g. to avoid a sense of guilt or improve self-esteem; and even more so when her own values or sense of purpose are aligned with the task. From this perspective, motivational investments include, for example, organizational changes that provide workers with more responsibility in problem-solving (Deci, Connell, Ryan 1989), managerial training that promotes a managerial style more supportive of worker autonomy (Hardré and Reeve 2009), and worker training to support peers in developing a more autonomous working style (Jungert et al. 2018).

In contrast to most of the existing literature on this topic (e.g., Besley and Ghatak, 2005; Kvaløy and Schöttner, 2015) in our approach investment in motivation reduces agency problems but has no effect on the overall surplus or the first-best level of effort choice. This distinguishes it from investment in skills. We take a standard principal-agent problem where all parties are risk neutral, effort is unobservable and non-contractible, and limited liability prevents a fixed-price contract that would achieve full efficiency. Although the principal cannot observe the agent’s choice of effort, an employment contract can specify an effort level and the agent may incur disutility (a psychological cost) for deviating from the specified effort level.

The principal can undertake investments that increases the agent’s disutility from such deviations - henceforth, we call these ‘guilt’ investments. Additionally, the principal can undertake investments that increase the agent’s utility from employment with the firm/organization - henceforth, we call these ‘esteem’ investments. The former type of investment affects the marginal cost of eliciting effort and the latter affects on-the-job utility and, thus, the effective reservation option for the agent relative to other employment opportunities. We argue that these two types of investments can capture a range of organizational practices – other than extrinsic rewards – that aim to raise worker motivation.

We show that guilt investments are substitutes of financial incentives. A binding participation constraint is a necessary condition for esteem investments to be undertaken. In this case, guilt investments and esteem investments are complements and both are substitutes of financial incentives. Specifically, a reduction in the cost of one type of motivational investment increases the use of both types and reduces financial incentives.

We also show that improvements in worker productivity lead, in general, to a decline in motivational investments and increased use of financial incentives. But, if the relative cost of guilt investments are sufficiently low, then the effects can go in the opposite direction. Improvements in output measurement will,
in general, increase financial incentives and lower motivational investments; but, here again, the effects can go in the opposite direction when the quality of information about output is already very good.

The model’s prediction of a co-movement in guilt and esteem investments, with financial rewards moving in the opposite direction, has a precise interpretation in self-determination theory: a decline in the cost of instilling guilt among workers (for failing to provide the prescribed level of effort), or a decline in the cost of raising the level of esteem associated with the job, will cause a shift towards autonomous motivation and away from controlled motivation. By contrast, improvements in worker productivity or output measurement will, in general, cause a shift away from autonomous motivation towards controlled motivation. Thus, although autonomous motivation is deemed to be superior to controlled motivation in the literature, our theoretical analysis implies that technological improvements (in the form of improved worker productivity or greater observability of output) will induce profit-maximizing firms to make greater use of financial incentives and less use of motivational investments.

The reason is that increasing the financial reward for success has constant returns in terms of its effect on the worker’s effort level while guilt investments have decreasing returns: the latter pushes the worker’s effort level closer to – but no higher than – the effort level prescribed in the contract. Therefore, motivational investments are less effective than financial rewards in incentivizing workers beyond a certain point.

Akerlof and Kranton (2005, 2008) also present a model in which an organization can undertake investments to increase worker motivation, in their case by changing the worker’s ‘identity’. But the present exercise departs from this work in two broad directions. First, we unbundle different motivational instruments and study how they relate with explicit incentive pay. Second, we explore how these investments are affected by various parameters representing differences in technology (e.g. higher productivity, lower cost of motivational investments), market environment (e.g. competitiveness of the labour market), and noise in the measurement of output.

Kandel and Lazear (1992) propose a model of a business partnership with moral hazard in teams - profits are equally shared among partners and efforts are non-contractible. Agents respond to ‘peer pressure’ that can take a number of forms including the experience of guilt when effort falls below a “group norm”. This is similar to the effort level prescribed in a contract in our setup. The crucial difference with our approach is that extrinsic rewards and other instruments such as guilt are endogenously chosen and their inter-relationship is the main focus in our analysis. For example, a norm which dictates a higher level of
effort is also more costly as it tightens the participation constraint; and so the use of the norm should be balanced against other mechanisms to elicit effort such as esteem and monetary incentives. By contrast, Kandel and Lazear (1992) take the equal sharing rule among partners as exogenous and provide alternative formulations of a cost function that arises from peer pressure.

The rest of the paper is organized as follows. In Section 2.1, we present a principal-agent model in which we formally define two types of motivational investments by organizations. We provide an interpretation of the model in Section 2.2 and characterize the equilibrium in Sections 2.3-2.4. In Section 2.5, we provide comparative statistics results involving the agent’s outside option and productivity and consider the implications of noise in output measurement in Section 2.6. We discuss the theoretical results in Section 3 and conclude in Section 4.

2 Theoretical Model

2.1 Setup

Consider a simple principal-agent model where the agent provides effort $e \in [0,1]$ at cost $\frac{1}{2}e^2$. This produces output $A \in (0,1)$ with probability $e$ and output zero otherwise. The principal observes output but not effort.

A contract between the principal and the agent specifies a ‘bonus’ (or ‘reward for success’) $b$ in case the agent produces positive output and an effort level $e_c \in \{e_l, e_h\}$. Although the principal does not observe effort, the agent experiences disutility in deviating from $e_c$ in his choice of $e$ at a cost $\frac{1}{2} \psi (e_c - e)^2$ which we call ‘guilt’. The agent obtains an additional utility of $\sigma$ from undertaking the task, which we call ‘esteem’.

Thus, the agent’s expected utility from the contract is given by

$$U(b,e_c,e,\psi,\sigma) = be - \frac{1}{2}e^2 - \frac{1}{2} \psi (e_c - e)^2 + \sigma$$  \hspace{1cm} (1)

If the agent chooses not to accept the contract, he obtains a reservation utility of $u$.

The principal can make investments to raise the agent’s sense of guilt when he deviates from the specified level of effort and the esteem associated with the job. More precisely, we assume that the principal has to make an investment equal to $\lambda \sigma$ so that the agent experiences sense of esteem with utility value $\sigma$ if he accepts the job; and that inducing a psychological cost $\psi$ requires an investment of $\mu \psi$. In the absence of any investments, $\psi = \sigma = 0$.

**Assumption 1** $A \in (0,1)$
Assumption 2  \( \mu \in (0, \infty) \) and \( \lambda \in (0, 1) \)

Assumption 3  \( 0 < e_l = \frac{A}{2} < e_h \leq A \)

As per Assumption 1, we fix the value of output to be less than 1 to ensure that the agent’s optimization problem always has an interior solution. We restrict \( \lambda \) to be below 1 (Assumption 2) so that the principal always prefers investing in esteem to making a non-contingent transfer to the agent to relax the agent’s participant constraint. We fix \( e_l = \frac{A}{2} \) (Assumption 3) as it is the level of effort that an agent will exert in a setting where there is no guilt and the principal chooses the profit maximizing level of bonus \( b = A \frac{4}{2} \).\(^5\)

Although the agent experiences disutility from both positive and negative deviations from \( e_c \), we will see that, in the optimal contract, the principal chooses \( b \) and \( e_c \) such that the agent’s effort level never exceeds \( e_c \).

### 2.2 Interpretation

Before proceeding with the analysis, we provide an interpretation of the variables associated with guilt and esteem in the principal-agent model.

Esteem investments \( \sigma \) increase, simply, the agent’s utility from undertaking the task. As \( \lambda < 1 \) (by Assumption 2), they provide a cheaper way of relaxing the agent’s participation constraint than a non-contingent wage payment. Although we use the term ‘esteem investments’ to refer to them in the paper, in practice they may represent any in-kind transfer from the principal that provides the agent more utility than an equivalent financial transfer.

The term \( e_c \) is an effort level tacitly or explicitly referred to in the agreement or contract between the principal and the agent. Although the actual effort level is a continuous variable in the unit interval, the choice of \( e_c \) is restricted to a discrete set. This modelling choice, made to improve tractability, may be justified by the notion that describing in the contract a precise level of effort is prohibitively costly and that the choice set includes only those effort levels that can be described using language in common usage (see Hart and Moore 1999 for a review of related concepts in the incomplete contracts literature).

For our analysis, we define the choice set of \( e_c \) to consist of two effort levels, ‘low’ \( e_l \) and ‘high’ \( e_h \). As noted in the previous subsection, \( e_l \) is the effort level that a worker with no sense of guilt would exert when

\(^5\)An alternative approach is to let \( e_l = b \) as \( b \) is the level of effort that an agent will exert in any contract in the absence of guilt. It will become evident that we obtain almost identical results with this alternative approach.
the principal chooses the profit maximizing level of financial reward for success. As this is likely to be a common occurrence in labour arrangements, it should be possible to specify such an effort level in a labour contract. By contrast, \( e_h \) represents a ‘higher standard’ of behaviour, possibly characteristic, for example, of workers with prosocial preferences (Benabou and Tirole 2007).

The notion of guilt in the model is loosely related to its formalization in the game-theoretic literature. For example, Battigalli and Dufwenberg (2007) defines ‘simple guilt’ as disutility experienced by one player due to the payoff loss (vis-a-vis some expectation) that his strategy inflicts on another (to capture the notion that "a player cares about the extent to which he lets another player down"). If the effort level specified in the contract \( e_c \) affects the principal’s beliefs about the agent’s actual choice of effort \( e \), then ‘simple guilt’, as defined by Battigalli and Dufwenberg (2007), would be a function of \( (e_c - e) \) as modeled here.\(^6\)

Charness and Dufwenberg (2006) show, in an experimental setting, that promises about actions made in pre-play communication in a principal-agent relationship indeed affects beliefs about behaviour and the level of cooperation in the relationship, findings that they account for using the notion of ‘guilt aversion’. The effort level specified in a contract, as modelled here, is a form of pre-play communication that could, plausibly, serve a similar role to that in the game used by Charness and Dufwenberg (2006)

### 2.3 First-Best Case

We first consider the effort level in the first-best case. In the first-best case, we solve

\[
\max_{e \in [0,1], e_c \in \{e_c, e_h\}} Ae - \frac{1}{2}e^2 - \frac{1}{2} \psi (e_c - e)^2
\]

We postulate that changing \( \psi \) would not change the first-best level of effort. We can see this as follows. The term \( Ae - \frac{1}{2}e^2 \) attains its maximum, by construction, at \( e = \frac{4}{7} \), while the term \(-\frac{1}{2} \psi (e_c - e)^2 \leq 0 \). Therefore, the maximand is at most \( A \left( \frac{4}{7} \right) - \frac{1}{2} \left( \frac{4}{7} \right)^2 = \frac{3}{8}A^2 \). This value is attained at \( e = e_c = e_l = \frac{4}{7} \). For any other value of \( e \), the term \( Ae - \frac{1}{2}e^2 < \frac{3}{8}A^2 \). Therefore, the maximand necessarily attains a lower value. Given \( e = \frac{4}{7} \), the maximand also (weakly) attains a lower value if \( e_c \neq e_l = \frac{4}{7} \). Thus, we have established that the unique solution is given by \( e = e_c = \frac{4}{7} \). And it yields a payoff of \( \frac{3}{8}A^2 \). As this expression is independent of \( \psi \), it follows that there are no gains from raising the level of motivation, as represented by \( \psi \), in the first-best case.

---

\(^6\) Note that, disutility from ‘simple guilt’, as modelled by Battigalli and Dufwenberg (2007) would equal zero when the actual effort level exceeds expectations. In contrast, in our setup any deviation – positive or negative – from the specified effort level generates disutility. But this modelling choice, made for notational simplicity, does not affect the analysis: as noted in the preceding subsection, actual effort never exceeds the specified level in equilibrium.
Therefore, motivational investments – as defined above – have no purpose in the absence of agency problems and, as such, are distinct from ‘skill-related’ investments that affect the worker’s productivity by, for example, raising the expected value of output or lowering the cost of effort.

2.4 Second-Best Case

In the second-best case, we can represent a contract by the 4-tuple \((b, e_c, \psi, \sigma)\). Given such a contract, the agent solves the following optimization problem:

\[
\max_{e \in [0,1]} U(b, e_c, e, \psi, \sigma) \tag{3}
\]

From (1), we see that the coefficient of \(e^2\) in \(U(b, e_c, e, \psi, \sigma)\) is negative. Therefore, the agent’s optimization problem has a unique solution. Assuming an interior solution, we obtain \(e\) from the first-order condition:

\[
b - e + \psi(e_c - e) = 0
\]

\[
\implies e = \frac{b + \psi e_c}{1 + \psi} \tag{4}
\]

We denote this solution by \(\hat{e}(b, e_c, \psi)\) and let \(\hat{e}_0(b) = \hat{e}(b, e_c, 0)\). In words, \(\hat{e}_0(b)\) is the effort level that the agent would choose if he did not experience any guilt. It is straightforward to show that \(\hat{e}(b, e_c, \psi) > \hat{e}_0(b)\) for \(e_c > \hat{e}_0(b)\), and \(\hat{e}(b, e_c, \psi)\) is increasing (decreasing) in \(\psi\) for \(e_c > \hat{e}_0(b)\) (\(e_c < \hat{e}_0(b)\)). In words, if the level of effort prescribed in the contract \(e_c\) differs from that which the agent would exert in the absence of guilt, then guilt investments bring the actual effort level closer to the level prescribed. For the subsequent analysis we make use of an indirect utility function for the agent’s expected utility from the contract, defined as follows:

\[
V(b, e_c, \psi, \sigma) = U(b, e_c, \hat{e}(b, e_c, \psi), \psi, \sigma) \tag{5}
\]

2.4.1 Principal’s Problem

We denote by \(\Pi(b, e_c, \psi, \sigma)\) the principal’s expected profits from the contract \((b, e_c, \psi, \sigma)\). Assuming that an agent who accepts the contract exerts his utility maximizing effort \(\hat{e}(b, e_c, \psi)\), the principal’s expected profit can be written as

\[
\Pi(b, e_c, \psi, \sigma) = (A - b) \hat{e}(b, e_c, \psi) - \mu \psi - \lambda \sigma \tag{6}
\]

Then the profit-maximizing contract is given by the following optimization problem:

\[
\max_{b, e_c, \psi, \sigma} \Pi(b, e_c, \psi, \sigma)
\]
subject to

\[ u \leq be - \frac{1}{2} e^2 - \frac{1}{2} \psi (e_c - e)^2 + \sigma \]  

(7)

where (7) is the agent’s participation constraint.

2.4.2 Profit-Maximizing Contract

Next, we determine which contract – bonus, specified effort and motivational investments – the principal would choose for different parameter values.

Case 1 \((e_c = e_l)\): First, consider the contract where the principal chooses \(e_c = e_l\). We can show that, in this case, the agent’s effort will be equal to or greater than \(e_l\) even in the absence of guilt (we will verify this subsequently). Therefore, there is no value in investing in guilt and \(\psi = 0\). Then, using (4), we obtain \(\hat{e}(b, \psi, e_l) = b\). Then, using (1), the agent’s expected utility is given by \(U(b, e_c, e, \psi, \sigma) = \frac{1}{2} b^2 + \sigma\). If the principal opts for a bonus \(b\) such that \(u > \frac{1}{2} b^2\), then esteem investments will be required to ensure that the agent accepts the contract, given by \(\sigma = u - \frac{1}{2} b^2\). Under these assumptions \((e_c = e_l\) and \(u > \frac{1}{2} b^2\)), the principal’s expected profit is given by \(\Pi(b, e_l, 0, u - \frac{1}{2} b^2) = b(A - b) - \lambda (u - \frac{1}{2} b^2)\), and the profit maximization problem can be written as

\[ \max_b b(A - b) - \lambda \left(u - \frac{1}{2} b^2\right) \]  

(8)

Thus, the principal’s objective function is quadratic with a coefficient of \(-1 + \frac{1}{2} \lambda\) on \(b^2\). Under Assumption 2, we have \(\lambda < 1\). Therefore, the objective function is globally concave and the optimal choice of \(b\) is given by the first-order condition with respect to \(b:\)

\[ (A - b) - b - \lambda (-b) = 0 \]

\[ \implies b = \frac{A}{2 - \lambda} \]  

(9)

\[ \implies \sigma = u - \frac{1}{2} \left(\frac{A}{2 - \lambda}\right)^2 \]  

(10)

Therefore, the contract is given by \(b = \frac{A}{2 - \lambda}\), \(e_c = e_l\), with motivational investments \(\psi = 0\) and \(\sigma = u - \frac{1}{2} \left(\frac{A}{2 - \lambda}\right)^2\). Note that, since \(\lambda \geq 0\), we have \(b \geq \frac{A}{2}\). So, indeed \(e \geq e_l\) as per our argument above and any guilt investments will lower effort. Therefore, the choice of \(\psi = 0\) is correct. We also observe that increasing \(\lambda\) increases the level of bonus chosen in the contract; i.e. if esteem investments are costly, then the principal
will offer a higher reward for success. On the other hand, if \( \lambda = 0 \), this enables the principal to relax the participation constraint costlessly, and we obtain \( b = \frac{A}{2} \). We can summarize these results as follows:

**Proposition 1** Suppose the optimal contract involves a low level of specified effort (\( e_c = e_l \)).

(i) The principal will not make any guilt-related investments (\( \psi = 0 \)).

(ii) If the agent’s participation constraint is binding, the bonus for success is given by \( b = \frac{A}{2} \) and esteem investments are given by \( \sigma = \frac{u}{2} - \frac{1}{2} \left( \frac{A}{2-\lambda} \right)^2 \).

(iii) If the agent’s participation constraint is not binding, then \( b = \frac{A}{2} \) and \( \sigma = 0 \).

**Case 2** (\( e_c = e_h \)): Next, consider the contract where the principal chooses \( e_c = e_h \). In this case, the principal can, potentially, choose a positive level of guilt investment. Using (4), we obtain \( \hat{e} (b, e_h, \psi) = \frac{b + \psi e_h}{1 + \psi} \). Then, using (1), the agent’s expected utility is given by

\[
U (b; e_c, \hat{e} (b, e_h, \psi), \psi, \sigma) = \frac{1}{2} b^2 + \psi e_h (b - \frac{1}{2} e_h) + \frac{1}{1 + \psi} u. 
\]

If \( \frac{1}{2} b^2 + \psi e_h (b - \frac{1}{2} e_h) \leq u \), then esteem investments will be required to ensure that the agent accepts the contract, given by

\[
\sigma = u - \frac{1}{2} b^2 + \psi e_h (b - \frac{1}{2} e_h). 
\]

Then, the profit maximization problem can be written as

\[
\max_{b, \psi} \left( \frac{b + \psi e_h}{1 + \psi} (A - b) - \mu \psi - \lambda \left[ u - \frac{1}{2} b^2 + \psi e_h (b - \frac{1}{2} e_h) \right] \right) \quad (11) 
\]

Assuming an interior solution, the optimal choice of \( b \) and \( \psi \) are given by the following first-order conditions:

\[
\frac{\partial \Pi}{\partial b} = \left( \frac{1}{1 + \psi} \right) (A - b) - \left( \frac{b + \psi e_h}{1 + \psi} \right) (1 - \lambda) = 0 \quad (12) 
\]

\[
\frac{\partial \Pi}{\partial \psi} = \left( \frac{e_h - b}{1 + \psi} \right)^2 (A - b) - \mu - \frac{\lambda}{2} \left( \frac{e_h - b}{1 + \psi} \right)^2 = 0 \quad (13) 
\]

We can interpret these conditions as follows. Increasing the bonus induces the agent to exert more effort, which increases expected profits but it also means higher payment to the agent for the level of effort exerted. An additional effect of increasing the bonus is that it relaxes the participation constraint, which means that less esteem investments are need to induce the worker to accept the contract. These trade-offs are represented in the condition in (12). If the principal is making large investments in guilt (\( \psi \) is large), then the agent is already exerting a high level of effort. Then any increase in bonus involves large costs in the form of additional payment to the agent, and induces only a small increase in effort. Thus, when guilt investments are high, the marginal gain to the principal from increasing the bonus is lower.

Increasing guilt investments increases the level of effort, which increases the expected profits for the principal. On the other hand, guilt investments carry a cost \( \mu \). Additionally, a higher level of guilt \( \psi \)
lowers the agent’s expected utility, thus tightens the participation constraint and raises the cost of esteem investments. These trade-offs are represented in the condition in (13). If the principal offers a large reward for success ($b$ is large), then the returns from increasing $\psi$ are lower because the principal makes less profit from output and the agent’s effort is less responsive to guilt.

The intuition provided above suggests that $b$ and $\psi$ should be substitutes in the contract. Formally, we have the following result (the proof is provided in the Appendix):

**Lemma 1** The function $\Pi(b, e_h, \psi, \sigma)$ with $\sigma = u - \frac{b^2 - \frac{1}{2}e_h(b - \frac{1}{2}e_h)}{1 + \psi}$ is supermodular in $b$ and $-\psi$.

Note that the condition in (12) is linear in $b$ and $\psi$ while the second condition in (13) is quadratic in $b$ and $\psi$. This means that the two equations can be solved simultaneously to provide closed-form solutions for $b$ and $\psi$. However, the supermodularity property of the profit function can be used to generate comparative statics results without deriving explicit solutions for $b$ and $\psi$. We adopt this approach in the next section.

### 2.5 Comparative Statics

Next, we consider how the optimal contract is affected by changes in the cost parameters for motivational investment, $\mu$ and $\lambda$, the agent’s reservation utility $u$ and the value of the output $A$.

**Agent’s Outside Option:** First, from observing the maximands in (8) and (11), we note that changes in $u$ do not affect the principal’s expected profits except to shift profits by a factor $\lambda$ (when the agent’s participation constraint is binding). In particular, for a given value of $e_c$, changes in $u$ do not affect the optimal choice of $b$ and $\psi$. Furthermore, changes in $u$ will not change the relative attractiveness of a contract with $e_c = e_l$ or $e_c = e_h$ because the principal’s expected profits change by the same amount in the two cases. Therefore, if the agent’s participation constraint is binding, an increase in $u$ will induce only a change in the level of esteem investment.

An exception to this rule occurs if the value of $u$ is such that the agent’s participation constraint is non-binding for the optimal contract involving $e_c = e_l$ but it is binding for the optimal contract involving $e_c = e_h$. In this situation, an increase in $u$ may cause the principal to offer the contract where $e_c = e_l$. This will cause guilt investment $\psi$ to drop to zero. Because $b$ and $\psi$ are substitutes (Lemma 1), it will also lead to an increase in the bonus $b$. We summarize these results as follows.

**Proposition 2** (i) If the agent’s participation constraint is not binding, an increase in $u$ has no effect on the profit-maximizing contract.
(ii) If the agent’s participation constraint is binding and the contract specifies a low level of effort \((e_c = e_l)\), an increase in \(u\) increases esteem-related investment \(\sigma\) with no change in \(b\) and \(\psi\).

(iii) If the agent’s participation constraint is binding and the contract specifies a high level of effort \((e_c = e_h)\), an increase in \(u\) can produce a switch to an alternative contract with \(e_c = e_l\), zero motivational investments \((\psi = \sigma = 0)\), and a higher reward for success \(b\); otherwise, the increase in \(u\) increases esteem investments, with no change in \(e_c\), \(b\) and \(\psi\).

To the extent that an agent will have a higher reservation utility when the labour market is tight, the comparative statics result involving \(u\) in Proposition 2 illustrates the effect of labour market tightness on motivational investments. The results imply that the relationship between labour market tightness and motivational investments may be non-monotonic.

To see this, consider the case in which the optimal contract specifies a high level of effort \((e_c = e_h)\) and the agent’s outside option is sufficiently strong that the participation constraint binds, but would be slack if the agent were offered a contract specifying low effort \((e_c = e_l)\) instead. Then, an increase in labour market tightness will initially lead to an increase in esteem investments (Proposition 2(iii)). If the labour market continues to tighten then, at one point, the principal will switch from a contract involving a high level of specified effort to one with a low level of specified effort. This will lead to a reduction in both esteem investments and guilt investments (the latter will fall to zero; the former will also fall to zero if the participation constraint becomes slack following the switch in contract type). Subsequent labour market tightening will eventually cause esteem investments to rise again (Proposition 2(ii)).

Cost of Investment in Esteem: Next, we consider how changes in the cost of esteem investments \(\lambda\) affect the profit-maximizing contract. If the agent’s participation constraint is not binding then, of course, changes in \(\lambda\) do not affect the contract. If the agent’s participation constraint is binding and the contract specifies a low level of effort \((e_c = e_l)\) then, as per (9)-(10), an increase in \(\lambda\) increases the reward for success \((b)\) and, consequently, leads to a reduction in esteem investments \((\sigma)\). Furthermore, there are no guilt investments \((\psi)\). If the agent’s participation constraint is binding and the contract specifies a high level of effort \((e_c = e_h)\), then we can use the supermodularity of the profit function (Lemma 1) to establish the following result (the proof is provided in the Appendix):

**Proposition 3** If the agent’s participation constraint is binding and the contract specifies a high level of
effort \( (e_c = e_h) \), then an increase in the cost of investment in esteem increases the reward for success \((b)\), lowers investment in guilt \((\psi)\) and lowers investment in esteem \((\sigma)\).

Cost of Investment in Guilt: Next, we consider how changes in the cost of guilt investments \(\mu\) affects the profit-maximizing contract. If the contract specifies a low level of effort \((e_c = e_l)\), then changes in \(\mu\) have no effect on the contract as there are no guilt investments to start with. If the contract specifies a high level of effort \((e_c = e_h)\) and positive levels of guilt investments \((\psi > 0)\), then an increase in \(\mu\) makes guilt investments less attractive. Since \(b\) and \(\psi\) are substitutes (Lemma 1), this leads to an increase in the reward for success. Increasing the reward for success and decreasing guilt makes the contract more attractive to the agent which, if the participation constraint is initially binding, allows the principal to lower esteem investments. Formally, we have the following result (the proof is provided in the Appendix):

**Proposition 4:** If the profit-maximizing contract specifies a high level of effort \((e_c = e_h)\) and positive investment in guilt \((\psi > 0)\), then an increase in the cost of investment in guilt \((\mu)\) increases the reward for success \((b)\) and lowers investment in guilt \((\psi)\). If the agent’s participation constraint is initially binding, then it also lowers investment in esteem \((\sigma)\).

Propositions 3 and 4 highlight that guilt and esteem investments are complementary in the sense that a decline in the cost of one type of motivational investment leads to increased use of both types.

Value of Output \(A\): Next, we consider how changes in the value of output \(A\) affects the contract. If the original contract specifies a low level of effort then, as per (9)-(10), an increase in \(A\) leads to a higher reward for success \((b)\), no change in guilt investment \((\psi)\) and, if the participation constraint is initially binding, a lower level of esteem investment \((\lambda)\). If the original contract specifies a high level of effort, then it seems intuitive that the principal would increase both \(b\) and \(\psi\) to incentivize higher effort. But as \(b\) and \(\psi\) are substitutes (Lemma 1), it is plausible that an increase in \(A\) leads to a decline in either the reward for success or guilt investment (but not both). A higher reward for success, coupled with a lower level of guilt, improves the agent’s expected utility from the contract, allowing the principal to lower investment in esteem (if the participation constraint was initially binding). Formally, we can establish the following result (the proof is provided in the Appendix).

**Proposition 5:** Suppose \(\mu \in \left(\frac{1}{4}, \infty\right)\). If the profit-maximzing contract specifies a high level of effort \((e_c = e_h)\) and positive investment in guilt \((\psi > 0)\), then an increase in the value of output \((A)\)
(i) lowers investment in guilt ($\psi$);
(ii) increases the reward for success ($b$);
(iii) lowers esteem investments if the agent’s participation constraint is binding.

Thus, Proposition 5 says that if the cost of guilt investment is relatively large, then the principal responds to an increase in the value of output by increasing the reward for success, lowering investment in guilt and increasing investment in esteem. If guilt investments have low cost, ($\mu$ close to 0), then the principal will respond to an increase in the value of output by increasing guilt investments. The effects of an increase in the value of output on the reward for success and esteem investments are ambiguous. But, we can show that there exist values of $\mu$ below $\frac{1}{4}$ such that the principal responds to an increase in the value of output by adjusting the contract in the opposite direction to that described in Proposition 5. We summarize the result in the following corollary (the proof is provided in the Appendix).

**Corollary 1 to Proposition 5:** If the profit-maximizing contract specifies a high level of effort ($e_c = e_h$) and positive investment in guilt ($\psi > 0$), then there exist values of $\mu$ in the interval $(0, \frac{1}{4})$ such that an increase in the value of output ($A$)

(i) raises investment in guilt ($\psi$);
(ii) lowers the reward for success ($b$);
(iii) raises esteem investments if the agent’s participation constraint is binding.

We can interpret Proposition 5 and its corollary as follows. An increase in $A$ can represent a technological improvement or an increase in the worker’s skill level. The proposition implies that if guilt investments are sufficiently costly, then improvements in worker skills or in the technology used in production will induce firms to move away from motivational investments towards greater financial rewards for success. But the corollary implies that if guilt investments are sufficiently inexpensive, then the opposite can happen: a move towards motivational investments and away from financial rewards for success as the worker skills or production technology improve.

### 2.6 Extension: Noisy Signal of Output

In this section, we extend the analysis to a setting where the principal does not directly observe output but receives only a signal of output which is imprecise. More precisely, let us suppose – following Besley and
Ghatak (2018) – that the principal receives a signal $s \in \{0, 1\}$ of output $y$ as follows:

$$\Pr(s = 1|y = A) = \rho = \Pr(s = 0|y = 0) \in \left(\frac{1}{2}, 1\right]$$

In words, the quality of the signal $s$ is improving in the value of $\rho$. If $\rho = \frac{1}{2}$, the signal is uninformative. If $\rho = 1$, we revert to our original model described in the previous sections. Additionally, we modify Assumption 3 as follows:

**Assumption 4** $0 < e_l \leq \frac{A(2\rho-1)-(1-\rho)}{2(2\rho-1)} < e_h \leq A$

The rationale for the modified assumption is that (as we will show below), in the presence of a noisy signal, the absence of guilt, a non-binding participation constraint and the optimal level choice of financial reward for success, an agent exerts effort to equal to $\frac{A(2\rho-1)-(1-\rho)}{2(2\rho-1)}$.

In the presence of a noisy signal, the agent’s remuneration is contingent not only on output but also the signal received by the principal. The agent receives the reward $b$ if the project generates positive output and the signal confirms this ($s = 1$); the agent also receives the reward if the project generates zero output but the signal is incorrect. Thus, the agent’s expected utility from a contract $(b, e_c, \psi, \sigma)$ is given by

$$\tilde{U}(b, e_c, e, \psi, \sigma; \rho) = b \{\rho e + (1 - \rho)(1 - e)\} - \frac{1}{2} e^2 - \frac{1}{2} \psi (e_c - e)^2 + \sigma \quad (14)$$

and the agent’s effort is given by the solution to the following optimization problem:

$$\max_{e \in [0, 1]} \tilde{U}(b, e_c, e, \psi, \sigma; \rho) \quad (15)$$

From (14), we see that the coefficient of $e^2$ in $U(b, e_c, e, \psi, \sigma)$ is negative. Therefore, the agent’s optimization problem has a unique solution. Assuming an interior solution, we obtain $e$ from the first-order condition as before:

$$b (2\rho - 1) - e + \psi (e_c - e) = 0 \quad (16)$$

$$\Rightarrow e = \frac{b (2\rho - 1) + \psi e_c}{1 + \psi}$$

We denote this solution by $\tilde{e}(b, e_c, \psi; \rho)$. We observe that, for any positive reward for success ($b > 0$), the agent’s effort is increasing in the quality of the signal. Also, if the signal is uninformative, then the agent’s effort is independent of the reward for success. Using the function $\tilde{e}(b, e_c, \psi; \rho)$, we can define an indirect utility function for the agent to represent his expected utility from the contract:

$$\tilde{V}(b, e_c, \psi, \sigma; \rho) = \tilde{U}(b, e_c, \tilde{e}(b, e_c, \psi; \rho), \psi, \sigma; \rho)$$
2.6.1 Principal’s Problem

The principal’s problem differs from that in the original model in two ways. First, the agent’s effort is a function of signal quality. Second, the bonus is paid out if the project is successful and the signal is accurate or the project is unsuccessful and the signal is incorrect. The principal’s expected profit can be written as

$$\bar{\Pi}(b, e_c, \psi, \sigma, \rho) = \{A - b(2\rho - 1)\} \bar{e}(b, e_c, \psi; \rho) - b(1 - \rho) - \mu \psi - \lambda \sigma$$ \hspace{1cm} (17)

Then the profit-maximizing contract is given by the following optimization problem:

$$\max_{b, e_c, \psi, \sigma} \bar{\Pi}(b, e_c, \psi, \sigma; \rho)$$

subject to

$$u \leq \bar{V}(b, e_c, \psi, \sigma; \rho)$$ \hspace{1cm} (18)

where (18) is the agent’s participation constraint.

2.6.2 Profit Maximizing Contract

As before, we determine which contract – bonus, specified effort and motivational investments – the principal would choose for different parameter values.

Case 1 \((e_c = e_l)\): If the principal chooses \(e_c = e_l\) then, as before (we will show that) \(b \geq e_c\) and there is no value derived from guilt investments. Thus, \(\psi = 0\). Then, the agent’s effort is given by

$$\bar{e}(b, e_l, 0; \rho) = b(2\rho - 1)$$ \hspace{1cm} (19)

and expected utility is given by

$$\bar{V}(b, e_l, 0, \sigma; \rho) = \frac{1}{2} b^2 (2\rho - 1)^2 + b(1 - \rho) + \sigma$$

To satisfy the participation constraint, the esteem investment will be given by \(\sigma = u - \bar{V}(b, e_l, 0, \sigma; \rho)\).

Therefore, the principal’s optimization problem becomes

$$\max_{b} b(2\rho - 1) \{A - b(2\rho - 1)\} - b(1 - \rho) - \lambda \left\{u - \frac{1}{2} b^2 (2\rho - 1)^2 - b(1 - \rho)\right\}$$ \hspace{1cm} (20)

Note that the coefficient on \(b^2\) in the maximand in (20) is \(- (2\rho - 1)^2 + \frac{1}{2} \lambda (2\rho - 1)^2\). Therefore, the maximand is globally concave in \(b\) iff \(\lambda < 2\). By assumption 2, \(\lambda < 1\). Therefore, this condition is satisfied. Then, the unique solution is given by the first-order condition:

$$(2\rho - 1) \{A - b(2\rho - 1)\} - b(2\rho - 1)^2 - (1 - \rho) - \lambda \left(b(2\rho - 1)^2 - (1 - \rho)\right) = 0$$
\[ b = \frac{A (2\rho - 1) + (\lambda - 1) (1 - \rho)}{2 (2\rho - 1)^2 - \lambda (2\rho - 1)^2} = \frac{A - (1 - \lambda) \left(\frac{1-\rho}{2\rho-1}\right)}{(2 - \lambda) (2\rho - 1)} \tag{21} \]

Therefore, the contract is given by \( b = A \left(\frac{1}{2} - \left(\frac{1-\rho}{2\rho-1}\right)\right) \), \( e_c = e_t \), with motivational investments \( \psi = 0 \) and \( \sigma = u - \tilde{V}(b, e_t, 0, \sigma; \rho) \). Note that if \( \lambda = 0 \) (in which case the principal can costlessly relax the participation constraint), we obtain \( b = \frac{A(2\rho-1)-(1-\rho)}{2(2\rho-1)} \). Then, using (19), the agent’s effort level equals \( \frac{A(2\rho-1)-(1-\rho)}{2(2\rho-1)} \) which, by Assumption 4, is equal to \( e_t \). By observation, we can verify from (21) that \( b \) increasing in \( \lambda \).

Therefore, \( e = \frac{b}{(2\rho-1)} \geq e_t \) for \( \lambda \in (0, 1] \). Therefore, the choice of \( \psi = 0 \) is indeed profit-maximizing when \( e_c = e_t \).

Differentiating throughout (21) with respect to \( \rho \), we obtain

\[ \frac{\partial b}{\partial \rho} = \frac{(3 - 2\rho) (1 - \lambda) - 2A (2\rho - 1)}{(2 - \lambda) (2\rho - 1)^3} \]

The denominator is positive because \( \lambda < 1 \) and \( \rho > \frac{1}{2} \). The numerator is positive if and only if \( \rho < \frac{1}{2} + \frac{(1-\lambda)}{2A+1(1-\lambda)} \). Thus, if the quality of information is very low, then the bonus increases with the quality of information. But when the quality of information reaches the threshold \( \rho = \frac{1}{2} + \frac{(1-\lambda)}{2A+1(1-\lambda)} \), the bonus begins to decline with the quality of information.

The intuition behind the non-monotonic relationship between the financial reward for success and information quality is as follows. When the quality of information about output is poor, financial rewards are not very effective in eliciting effort from the worker; consequently, the principal offers a low level for reward for success. As the quality of information improves, the financial reward increases and the agent exerts more effort. But, when the agent’s effort level is high, increments in financial rewards are very costly. Therefore, when the quality of information is high, the principal lowers financial rewards in response to further improvements in information quality to save on labour costs.

**Case 2** \( (e_c = e_h) \): If the principal chooses \( e_c = e_h \) then, as per our previous reasoning, the level of guilt investment is potentially positive. The agent’s effort level is given by

\[ \hat{e}(b, e_h; \rho) = \frac{b (2\rho - 1) + \psi e_h}{1 + \psi} \]

Plugging this level of effort into the agent’s utility function, we obtain the agent’s expected utility from the contract \( (b, e_h, \psi, \sigma) \):

\[ \tilde{V}(b, e_h, \psi, \sigma; \rho) = \frac{1}{2} b^2 (2\rho - 1)^2 + \psi e_h \left(\frac{b (2\rho - 1) - \frac{1}{2} e_h}{1 + \psi}\right) + b (1 - \rho) + \sigma \]
To satisfy the participation constraint, the esteem investment will be given by \( \sigma = \underline{u} - \bar{V}(b, e_h, \psi, \sigma; \rho) \).

Therefore, the principal’s optimization problem becomes

\[
\max_{b, \psi} \{ A - b(2\rho - 1) \} - b(1 - \rho) - \lambda \{ \underline{u} - \bar{V}(b, e_h, \psi, \sigma; \rho) \}
\]

(22)

We define \( \tilde{\Pi}(b, e_h, \psi, \sigma; \rho) \) as the maximand in the optimization problem in (22). Assuming an interior solution, the optimal choice of \( b \) and \( \psi \) are given by the following first-order conditions:

\[
\frac{\partial \tilde{\Pi}}{\partial b} = \left( \frac{2\rho - 1}{1 + \psi} \right) \{ A - (2 - \lambda) b(2\rho - 1) - \psi e_h (1 - \lambda) \} + (\lambda - 1) (1 - \rho) = 0
\]

(23)

\[
\frac{\partial \tilde{\Pi}}{\partial \psi} = \frac{e_h - b(2\rho - 1)}{(1 + \psi)^2} \{ A - b(2\rho - 1) \} - \frac{1}{2} \lambda \{ b^2 (2\rho - 1)^2 + (e_h)^2 \} = 0
\]

(24)

As in the case where output is observable, \( b \) and \( \psi \) are substitutes. Formally, we have the following result:

**Lemma 2** The function \( \tilde{\Pi}(b, e_h, \psi, \sigma; \rho) \) with \( \sigma = \underline{u} - \bar{V}(b, e_h, \psi, \sigma; \rho) \) is supermodular in \( b \) and \( -\psi \).

Also, we can show that \( \frac{\partial^2 \tilde{\Pi}}{\partial \psi \partial \rho} < 0 \) and, if \( \rho \leq \frac{5 - 3\lambda}{6 - 2\lambda} \), then \( \frac{\partial^2 \tilde{\Pi}}{\partial \psi \partial \rho} \geq 0 \). Then, applying Topkis’ Theorem, we obtain the following result:

**Proposition 6**: Suppose information about output is noisy and \( \rho \leq \frac{5 - 3\lambda}{6 - 2\lambda} \). If the profit-maximizing contract specifies a high level of effort \( e_c = e_h \) and positive investment in guilt \( \psi > 0 \), then an increase in the quality of information increases the reward for success \( b \) and lowers investment in guilt \( \psi \) and esteem \( \sigma \).

Proposition 6 implies that when the quality of information about output is below a certain threshold, improvements in quality cause the principal to lower motivational investments and increase the financial rewards for success. The intuition behind this result is that if the quality of information is very poor, then financial rewards are largely ineffective in motivating the agent; however, guilt investments are still effective because they bring the agent’s effort level closer to that prescribed independently of whether the principal can observe output. And, as per our previous reasoning, the principal will invest in esteem alongside guilt as doing so relaxes the agent’s participation constraint.

However, this reasoning may break down if the information quality is very high, specifically if \( \rho > \frac{5 - 3\lambda}{6 - 2\lambda} \). The intuition is similar as that provided above for the case in which the agent’s participation constraint is not binding. As per equation (19), the agent’s effort level increases with information quality, and increments
in financial rewards are very costly for the principal when the agent’s effort level is already high. In these circumstances, the principal may lower financial rewards in response to further increases in information quality, and allocate more resources to guilt and esteem investments instead.

3 Discussion

We have developed a framework within which the standard classification of different types of worker motivation in organizational psychology can be reproduced. The theoretical analysis also provides further insights about what, under different circumstances, would be the exact composition of motivational investments, and the combination of worker motivation versus financial rewards that an organization would employ. In the first instance, we show that (i) guilt and esteem investments are complements; i.e. an exogenous decline in the cost of one type of motivational investment leads to an increase in the use of both types and (ii) both types of motivational investments are substitutes of financial rewards; i.e. an exogenous decline in the cost of either type of motivational investment decreases the use of financial rewards.

We also show that improvements in worker productivity lead, in general, to a decline in motivational investments and increase in financial rewards for success. But, if the relative cost of guilt investments is sufficiently low, then the effects can go in the opposite direction. Similarly, improvements in output measurement will, in general, increase financial rewards and lower motivational investments; but the effects can go in the opposite direction when information quality is already very good.

The model’s prediction of a co-movement in guilt and esteem investments, with financial rewards moving in the opposite direction, has a precise interpretation in self-determination theory (Deci and Ryan 1985): a decline in the cost of instilling guilt among workers (for failing to provide the prescribed level of effort), or a decline in the cost of raising the level of esteem associated with the job, will cause a shift towards autonomous motivation and away from controlled motivation. By contrast, improvements in worker productivity or output measurement will, in general, cause a shift away from autonomous motivation towards controlled motivation. Thus, although autonomous motivation is deemed to be superior to controlled motivation in the literature, our theoretical analysis implies that technological improvements (in the form of improved worker productivity or greater observability of output) will induce profit-maximizing firms to make greater use of financial rewards and less use of motivational investments.

The reason is that increasing the financial reward for success has constant returns in terms of its effect on
the worker’s effort level while guilt investments have decreasing returns: the latter pushes the worker’s effort level closer to – but no higher than – the effort level prescribed in the contract. Therefore, motivational investments are less effective than financial rewards in incentivizing workers beyond a certain point.

4 Conclusion

In this paper, we developed a theoretical model of ‘motivational’ investments within organizations. In relation to the existing literature, our approach is distinct in two ways: first, we include within our definition of motivational investments only those types of investments that reduce agency problems within the organization, but has no effect on the first-best outcome - this helps us separate out motivational investments from investments that raise productivity of workers; second, we unbundle two different dimensions of motivational investments: those that increase a worker’s on-the-job utility (thus relaxing the agent’s participation constraint; we call these ‘esteem investments’) and those that lower the worker’s marginal cost of effort (thus relaxing the agent’s incentive compatibility constraint – we call these ‘guilt investments’).

There are some broader questions that our paper throws up. If intrinsic motivation is important then what organizational forms make the best use of it; for example, the use of motivational investments in non-profits versus for-profit organizations. Also, the role of multi-tasking considerations in these settings seem like a natural extension to study, as is the role of heterogeneity among workers and self-selection.
5 Appendix

Proof. of Lemma 1: Using (12), we obtain
\[
\frac{\partial^2 \Pi}{\partial b\partial\psi} = (A - b) \frac{\partial}{\partial\psi} \left( \frac{1}{1 + \psi} \right) - \frac{\partial e}{\partial\psi} (1 - \lambda) \\
= -(A - b) \frac{1}{(1 + \psi)^2} - (1 - \lambda) (e_h - b) \frac{1}{(1 + \psi)^2}
\] (25)

By Assumption 2, \( \lambda < 1 \). If \( \psi > 0 \), we must have \((e_h - b) \geq 0\) (if not, the principal can induce higher effort and thus increase profits by choosing \( \psi = 0 \)), Then, using (25), we obtain \( \frac{\partial^2 \Pi}{\partial b\partial\psi} < 0 \). Thus, we have supermodularity in \( \psi \) and \( -b \).

Proof. of Proposition 3: Using (12) and (13), we obtain
\[
\frac{\partial^2 \Pi}{\partial b\partial\lambda} = \frac{b + \psi e_h}{1 + \psi} > 0 \\
\frac{\partial^2 \Pi}{\partial\psi\partial\lambda} = -\frac{1}{2} \left( \frac{e_h - b}{1 + \psi} \right)^2 < 0
\]

Then, using Lemma (1), and Topkis’ theorem, we obtain the result that, in the solution to the optimization problem in (11), the reward for success \( (b) \) is increasing in \( \lambda \) and guilt investment \( (\psi) \) is decreasing in \( \lambda \). As per (1), the agent’s expected utility is increasing in \( b \) and decreasing in \( \psi \). Therefore, if the agent’s participation constraint is binding, esteem investment \( (\sigma) \) is decreasing in \( \lambda \).

Proof. of Proposition 4: Using (12) and (13), we obtain
\[
\frac{\partial^2 \Pi}{\partial b\partial\mu} = 0 \\
\frac{\partial^2 \Pi}{\partial\psi\partial\mu} = -1 < 0
\]

Then, using Lemma (1), and Topkis’ theorem, we obtain the result that, in the solution to the optimization problem in (11), the reward for success \( (b) \) is increasing in \( \mu \) and guilt investment \( (\psi) \) is decreasing in \( \mu \). As per (1), the agent’s expected utility is increasing in \( b \) and decreasing in \( \psi \). Therefore, if the agent’s participation constraint is binding, esteem investment \( (\sigma) \) is decreasing in \( \mu \).

Proof. of Proposition 5: Using (12) and (13), we obtain
\[
A = b + (b + \psi e_h) (1 - \lambda) \\
\mu (1 + \psi)^2 = (e_h - b) \left\{ A - \frac{\lambda}{2} e_h - \left( 1 - \frac{\lambda}{2} \right) b \right\}
\]
Then, using the Implicit Function Theorem, we have

\[
A = (2 - \lambda) b(A) + (1 - \lambda) \psi(A) e_h \tag{26}
\]

\[
\mu (1 + \psi(A))^2 = (e_h - b(A)) \left\{ A - \frac{\lambda}{2} e_h - \left( 1 - \frac{\lambda}{2} \right) b(A) \right\} \tag{27}
\]

Differentiating throughout (26) and (27) with respect to \( A \), we obtain

\[
1 = (2 - \lambda) b'(A) + (1 - \lambda) \psi'(A) e_h \tag{28}
\]

\[
2\mu (1 + \psi(A)) \psi'(A) = -b'(A) \left\{ A - \frac{\lambda}{2} e_h - \left( 1 - \frac{\lambda}{2} \right) b(A) \right\} + (e_h - b(A)) \left\{ 1 - \left( 1 - \frac{\lambda}{2} \right) b'(A) \right\} \tag{29}
\]

Substituting for \( \psi'(A) \) in (29) using (28), we obtain

\[
2\mu (1 + \psi(A)) \left\{ \frac{1 - (2 - \lambda) b'(A)}{(1 - \lambda) e_h} \right\} = -b'(A) \left\{ A - \frac{\lambda}{2} e_h - \left( 1 - \frac{\lambda}{2} \right) b(A) \right\} + (e_h - b(A)) \left\{ 1 - \left( 1 - \frac{\lambda}{2} \right) b'(A) \right\}
\]

\[
\Rightarrow \quad \frac{2\mu (1 + \psi(A))}{(1 - \lambda) e_h} - \frac{2\mu (1 + \psi(A)) (2 - \lambda)}{(1 - \lambda) e_h} b'(A) \equiv -b'(A) \left( A - \frac{\lambda}{2} e_h \right) + b'(A) b(A) \left( 1 - \frac{\lambda}{2} \right) + (e_h - b(A)) \left\{ 1 - \left( 1 - \frac{\lambda}{2} \right) b'(A) \right\}
\]

\[
\Rightarrow \quad \frac{2\mu (1 + \psi(A))}{(1 - \lambda) e_h} - (e_h - b(A)) \equiv \frac{2\mu (1 + \psi(A)) (2 - \lambda)}{(1 - \lambda) e_h} b'(A) - \left( A - \frac{\lambda}{2} e_h \right) + (e_h - b(A)) \left\{ 1 - \left( 1 - \frac{\lambda}{2} \right) b'(A) \right\}
\]

\[
\Rightarrow \quad \frac{2\mu (1 + \psi(A))}{(1 - \lambda) e_h} - (e_h - b(A)) \equiv \frac{2\mu (1 + \psi(A)) (2 - \lambda)}{(1 - \lambda) e_h} b'(A) - \left( A + e_h (1 - \lambda) \right) b'(A) + (2 - \lambda) b(A) b'(A)
\]

\[
\Rightarrow \quad \frac{2\mu (1 + \psi(A))}{(1 - \lambda) e_h} - (e_h - b(A)) \equiv \frac{2\mu (1 + \psi(A)) (2 - \lambda)}{(1 - \lambda) e_h} b'(A) + \left( (2 - \lambda) b(A) - A - (1 - \lambda) e_h \right) b'(A)
\]

\[
\Rightarrow \quad 2\mu (1 + \psi(A)) - (e_h - b(A)) (1 - \lambda) e_h \equiv \\
\left\{ 2\mu (1 + \psi(A)) (2 - \lambda) \right\} b'(A) + (1 - \lambda) e_h \left\{ (2 - \lambda) b(A) - (1 - \lambda) e_h \right\} b'(A)
\]

\[
\Rightarrow b'(A) \equiv \frac{2\mu (1 + \psi(A)) - (e_h - b(A)) (1 - \lambda) e_h}{2\mu (1 + \psi(A)) (2 - \lambda) + (1 - \lambda) e_h \left\{ (2 - \lambda) b(A) - A - (1 - \lambda) e_h \right\}} \tag{30}
\]

Using (26), we can write

\[
b(A) \equiv \frac{A - (1 - \lambda) \psi(A) e_h}{(2 - \lambda)} \tag{31}
\]
Using this expression in the denominator of (30), we obtain
\[ 2\mu \{ 1 + \psi (A) \} (2 - \lambda) + (1 - \lambda) e_h \left[ (2 - \lambda) \frac{A - (1 - \lambda) \psi (A) e_h}{(2 - \lambda)} - A - (1 - \lambda) e_h \right] \]
\[ = 2\mu \{ 1 + \psi (A) \} (2 - \lambda) + (1 - \lambda) e_h \left[ A - (1 - \lambda) \psi (A) e_h - A - (1 - \lambda) e_h \right] \]
\[ = 2\mu \{ 1 + \psi (A) \} (2 - \lambda) + (1 - \lambda) e_h \{- (1 - \lambda) \psi (A) e_h - (1 - \lambda) e_h \} \]
\[ = 2\mu \{ 1 + \psi (A) \} (2 - \lambda) + (1 - \lambda) e_h \{- (1 - \lambda) e_h \{ \psi (A) + 1 \} \} \]
\[ = 2\mu (2 - \lambda) \{ 1 + \psi (A) \} + (1 - \lambda) e_h \{- (1 - \lambda) e_h \{ 1 + \psi (A) \} \} \]
\[ = \left[ 2\mu (2 - \lambda) - \{(1 - \lambda) e_h \}^2 \right] \{ 1 + \psi (A) \} \]  
(32)

Next, consider the expression in the numerator on the right-hand side of (30). Using (31) to substitute for \( b (A) \), we obtain
\[ 2\mu \{ 1 + \psi (A) \} - \left\{ e_h - \frac{A - (1 - \lambda) \psi (A) e_h}{(2 - \lambda)} \right\} (1 - \lambda) e_h \]
\[ = 2\mu \{ 1 + \psi (A) \} - \left\{ \frac{(2 - \lambda) e_h - A + (1 - \lambda) \psi (A) e_h}{(2 - \lambda)} \right\} (1 - \lambda) e_h \]
\[ = 2\mu \{ 1 + \psi (A) \} - \left\{ \frac{e_h + (1 - \lambda) e_h - A + (1 - \lambda) \psi (A) e_h}{(2 - \lambda)} \right\} (1 - \lambda) e_h \]
\[ = 2\mu \{ 1 + \psi (A) \} - \left\{ \frac{e_h - A + (1 - \lambda) e_h \{ 1 + \psi (A) \}}{(2 - \lambda)} \right\} (1 - \lambda) e_h \]
\[ = 2\mu (2 - \lambda) \{ 1 + \psi (A) \} + (A - e_h) (1 - \lambda) e_h - \{(1 - \lambda) e_h \}^2 \{ 1 + \psi (A) \} \]
\[ = \frac{2\mu (2 - \lambda) - \{(1 - \lambda) e_h \}^2}{(2 - \lambda)} \{ 1 + \psi (A) \} + (A - e_h) (1 - \lambda) e_h \]  
(33)

Combining (32) and (33), we obtain
\[ b' (A) \equiv \frac{2\mu (2 - \lambda) - \{(1 - \lambda) e_h \}^2}{(2 - \lambda)} \{ 1 + \psi (A) \} + (A - e_h) (1 - \lambda) e_h \]
\[ = \frac{2\mu (2 - \lambda) - \{(1 - \lambda) e_h \}^2}{(2 - \lambda)} \{ 1 + \psi (A) \} \]  
(34)

Rearranging (28), we obtain
\[ \psi' (A) \equiv \frac{1 - (2 - \lambda) b' (A)}{(1 - \lambda) e_h} \]
Substituting for $b'(A)$ using the expression in (34), we obtain

$$\psi'(A) = \frac{1}{(1 - \lambda) e_h} \left[ \frac{2\mu (2 - \lambda) - ((1 - \lambda) e_h)^2}{(1 - \lambda) e_h \left[ 2\mu (2 - \lambda) - ((1 - \lambda) e_h)^2 \right]} \{1 + \psi(A)\} + (A - e_h) (1 - \lambda) e_h \right]$$

$$= \frac{1}{(1 - \lambda) e_h} \left[ \frac{2\mu (2 - \lambda) - ((1 - \lambda) e_h)^2}{(1 - \lambda) e_h \left[ 2\mu (2 - \lambda) - ((1 - \lambda) e_h)^2 \right]} \{1 + \psi(A)\} \right] - \frac{(A - e_h)}{1 + \psi(A)}$$

$$= \frac{1}{(1 - \lambda) e_h} \left[ \frac{2\mu (2 - \lambda) - ((1 - \lambda) e_h)^2}{(1 - \lambda) e_h \left[ 2\mu (2 - \lambda) - ((1 - \lambda) e_h)^2 \right]} \{1 + \psi(A)\} \right]$$

$$\implies \psi'(A) = - \frac{(A - e_h)}{2\mu (2 - \lambda) - ((1 - \lambda) e_h)^2 \{1 + \psi(A)\}}$$

By Assumption 3, $A - e_h > 0$. Therefore, $\psi'(A) < 0$ iff $2\mu (2 - \lambda) > ((1 - \lambda) e_h)^2$. Rearranging this inequality, we obtain

$$\mu > \frac{1}{2} \frac{(1 - \lambda)^2}{(2 - \lambda)} (e_h)^2 \quad (35)$$

Note that the expression $\frac{(1 - \lambda)^2}{(2 - \lambda)}$ is decreasing in $\lambda$ for $\lambda \in (0, 1)$. Therefore, under Assumptions 1-3, the expression on the right-hand side of (35) attains its maximum value at $e_h = 1$ and $\lambda = 0$. Plugging these expressions into the inequality in (35), we obtain $\mu > \frac{1}{4}$. Therefore, $\mu > \frac{1}{4} \implies \psi'(A) < 0$.

Using (34), we can also write

$$b'(A) = \frac{1}{2 - \lambda} \left[ 1 + \frac{(A - e_h) (1 - \lambda) e_h}{2\mu (2 - \lambda) - ((1 - \lambda) e_h)^2 \{1 + \psi(A)\}} \right]$$

Therefore, if $2\mu (2 - \lambda) > ((1 - \lambda) e_h)^2$, we obtain $b'(A) > 0$. Therefore, $\mu > \frac{1}{4} \implies b'(A) > 0$.

Next we consider how an increase in $A$ affects esteem investments $\sigma$. If $\mu > \frac{1}{4}$ and the agent’s participation constraint is binding, an increase in $A$ leads to an increase in $b$ and decrease in $\psi$, as shown above. Using the Envelope Theorem, we can show that the agent’s expected utility from the contract $V(b, e_h, \psi, \sigma)$ is increasing in $b$ and decreasing in $\psi$. Furthermore, recall that $\sigma = \bar{u} - V(b, e_h, \psi, \sigma)$. Therefore, if $\mu > \frac{1}{4}$, $\sigma$ is decreasing in $A$.

**Proof.** of Corollary to Proposition 5: From the proof of Proposition 5, we obtain

$$\psi'(A) = - \frac{(A - e_h)}{2\mu (2 - \lambda) - ((1 - \lambda) e_h)^2 \{1 + \psi(A)\}}$$

7To see this, note that

$$\frac{d}{d\lambda} \left\{ \frac{(1 - \lambda)^2}{(2 - \lambda)} \right\} = \frac{(1 - \lambda)(\lambda - 3)}{(2 - \lambda)^2}$$

which is negative for $\lambda \in (0, 1)$.
\[ b'(A) \equiv \frac{1}{2 - \lambda} \left[ 1 + \frac{(A - e_h)(1 - \lambda)e_h}{2\mu(2 - \lambda) - \{(1 - \lambda)e_h\}^2} \right] \{1 + \psi(A)\} \]

Let \( \mu = \frac{1}{2} \frac{(1-\lambda)^2(e_h)^2}{(2-\lambda)} - \varepsilon \). From the proof of Proposition 5, we know that, under Assumptions 1-3, we have \( \frac{1}{2} \frac{(1-\lambda)^2(e_h)^2}{(2-\lambda)} \in (0, \frac{1}{4}) \). Therefore, for \( \varepsilon \) positive and sufficiently close to zero, we obtain \( \mu \in (0, \frac{1}{4}) \), \( 2\mu(2 - \lambda) - \{(1 - \lambda)e_h\}^2 < 0 \) and \( \frac{(A - e_h)(1 - \lambda)e_h}{2\mu(2 - \lambda) - \{(1 - \lambda)e_h\}^2} < 0 \). Therefore, \( \psi'(A) > 0 \) and \( b'(A) < 0 \).

Recall that, if the agent’s participation constraint is binding, then \( \sigma = u - V(b, e_h, \psi, \sigma) \), and \( V(b, e_h, \psi, \sigma) \) is increasing in \( b \) and decreasing in \( \psi \). Therefore, \( \sigma \) is increasing in \( A \).

**Proof.** of Lemma 2: Using (23), we obtain

\[
\frac{\partial^2 \Pi}{\partial b \partial \psi} = \{A - b(2 - \lambda)(2\rho - 1) - \psi e_h(1 - \lambda)\} \frac{\partial}{\partial \psi} \left( \frac{2\rho - 1}{1 + \psi} \right) - e_h(1 - \lambda) \left( \frac{2\rho - 1}{1 + \psi} \right)
\]

\[ = - \{A - b(2 - \lambda)(2\rho - 1) - \psi e_h(1 - \lambda)\} \frac{(2\rho - 1)}{(1 + \psi)^2} - e_h(1 - \lambda) \left( \frac{2\rho - 1}{1 + \psi} \right)
\]

\[ = - \{A - b(2 - \lambda)(2\rho - 1) - \psi e_h(1 - \lambda) + e_h(1 - \lambda)(1 + \psi)\} \frac{(2\rho - 1)}{(1 + \psi)^2}
\]

\[ = - \{A - b(2 - \lambda)(2\rho - 1) + e_h(1 - \lambda)\} \frac{(2\rho - 1)}{(1 + \psi)^2}
\]

\[ = - \{A - b(1 + 1 - \lambda)(2\rho - 1) + e_h(1 - \lambda)\} \frac{(2\rho - 1)}{(1 + \psi)^2}
\]

\[ = - \{A - b(2\rho - 1) - b(1 - \lambda)(2\rho - 1) + e_h(1 - \lambda)(1 + \psi)\} \frac{(2\rho - 1)}{(1 + \psi)^2}
\]

\[ = - \{A - b(2\rho - 1) + (1 - \lambda)\{e_h - b(2\rho - 1)\}\} \frac{(2\rho - 1)}{(1 + \psi)^2}
\]

We must have \( e_h - b(2\rho - 1) > 0 \). If not, guilt investments would equal zero. Also, we must have \( A - b(2\rho - 1) > 0 \). If not, the principal would make negative profits and would do better by setting \( b = 0 \).

Therefore, \( \frac{\partial^2 \Pi}{\partial b \partial \psi} < 0 \). Thus, we have supermodularity in \( \psi \) and \(-b\).
Proof. of Proposition 6: Using (24), we obtain
\[
\frac{\partial^2 \tilde{\Pi}}{\partial \psi \partial \rho} = \frac{(-2b) \{ A - b (2 \rho - 1) \} + (-2b) \{ e_h - b (2 \rho - 1) \} - \frac{1}{2} \lambda \{ 4 b^2 (2 \rho - 1) \}}{(1 + \psi)^2}
\]
\[
= \frac{(-2b) \{ A + e_h - 2 b (2 \rho - 1) \} + \frac{1}{2} \lambda \{ 4 b^2 (2 \rho - 1) \}}{(1 + \psi)^2}
\]
\[
= -2b (A + e_h + 4 b^2 (2 \rho - 1) + 2 b^2 \lambda (2 \rho - 1))
\]
\[
= -2b (A + e_h + 4 b^2 (2 \rho - 1) + 2 b^2 \lambda (2 \rho - 1))
\]
\[
= \frac{(-2b)}{(1 + \psi)^2} \{ (A + e_h) + (-2b - b \lambda) (2 \rho - 1) \}
\]
\[
= -\frac{2b}{(1 + \psi)^2} \{ (A + e_h) - b (2 - \lambda) (2 \rho - 1) \}
\]
\[
= -\frac{2b}{(1 + \psi)^2} \{ (A + e_h) - b (1 + 1 - \lambda) (2 \rho - 1) \}
\]
\[
= -\frac{2b}{(1 + \psi)^2} \{ A - b (2 \rho - 1) + e_h - b (1 - \lambda) (2 \rho - 1) \}
\]

Based on the reasoning above, we have \( A - b (2 \rho - 1) > 0 \) and \( e_h - b (2 \rho - 1) > 0 \). Since \( \lambda \in (0, 1) \), it follows that \( e_h - b (1 - \lambda) (2 \rho - 1) > 0 \). Therefore, \( \frac{\partial^2 \tilde{\Pi}}{\partial \psi \partial \rho} < 0 \). Using (23), we obtain
\[
\frac{\partial \tilde{\Pi}}{\partial \rho} = \left( \frac{2 \rho - 1}{1 + \psi} \right) \{ A - (2 - \lambda) b (2 \rho - 1) - \psi e_h (1 - \lambda) \} + (\lambda - 1) (1 - \rho)
\]
\[
\frac{\partial^2 \tilde{\Pi}}{\partial \rho \partial \rho} = \left( \frac{2}{1 + \psi} \right) \{ A - (2 - \lambda) b (2 \rho - 1) - \psi e_h (1 - \lambda) \} + \left( \frac{2 \rho - 1}{1 + \psi} \right) \{-2 (2 - \lambda) b \} - \lambda + 1
\]
\[
= \frac{2}{1 + \psi} \{ A - (2 - \lambda) b (2 \rho - 1) - \psi e_h (1 - \lambda) \} - 2 (2 - \lambda) b \left( \frac{2 \rho - 1}{1 + \psi} \right) - \lambda + 1
\]
\[
= \frac{2}{1 + \psi} \{ A - (2 - \lambda) b (2 \rho - 1) - \psi e_h (1 - \lambda) - (2 - \lambda) b (2 \rho - 1) \} - \lambda + 1
\]
\[
= \frac{2}{1 + \psi} \{ A - 2b (2 - \lambda) (2 \rho - 1) - \psi e_h (1 - \lambda) \} - \lambda + 1
\]
(36)

For \( e_h \leq \frac{1}{2} \) and \( \rho \) close to \( \frac{1}{2} \), we obtain \( \frac{\partial^2 \tilde{\Pi}}{\partial \rho \partial \rho} > \left( \frac{2}{1 + \psi} \right) A > 0 \). Using (23) and rearranging, we obtain
\[
(2 - \lambda) b (2 \rho - 1) = \{ A - \psi e_h (1 - \lambda) \} + (\lambda - 1) (1 + \psi) \left( \frac{1 - \rho}{2 \rho - 1} \right)
\]
(37)

Substituting for the term \( (2 - \lambda) b (2 \rho - 1) \) in (36) using (37), we obtain
\[
\frac{\partial^2 \tilde{\Pi}}{\partial \rho \partial \rho} = \frac{2}{1 + \psi} \left[ A - 2 \left\{ \{ A - \psi e_h (1 - \lambda) \} + (\lambda - 1) (1 + \psi) \left( \frac{1 - \rho}{2 \rho - 1} \right) \right\} - \psi e_h (1 - \lambda) \right] + (1 - \lambda)
\]
\[
= \frac{2}{1 + \psi} \left[ A - 2 A + 2 \psi e_h (1 - \lambda) - 2 (\lambda - 1) (1 + \psi) \left( \frac{1 - \rho}{2 \rho - 1} \right) - \psi e_h (1 - \lambda) \right] + (1 - \lambda)
\]
\[
= \frac{2}{1 + \psi} \left[ -A + \psi e_h (1 - \lambda) + 2 (1 - \lambda) (1 + \psi) \left( \frac{1 - \rho}{2 \rho - 1} \right) + (1 - \lambda)
\]
\[
= \frac{2}{1 + \psi} \left[ -A + \psi e_h (1 - \lambda) + 2 (1 - \lambda) (1 + \psi) \left( \frac{1 - \rho}{2 \rho - 1} \right) + \frac{1}{2} (1 - \lambda) (1 + \psi) \right]
\]
The expression within the square brackets is increasing in $\psi$ and decreasing in $A$. By assumption, $A < 1$ and $\psi \geq 0$. Therefore

$$-A + \psi e_h (1 - \lambda) + 2 (1 - \lambda) (1 + \psi) \left( \frac{1 - \rho}{2\rho - 1} \right) + \frac{1}{2} (1 - \lambda) (1 + \psi) \geq 0$$

$$\iff -1 + 2 (1 - \lambda) \left( \frac{1 - \rho}{2\rho - 1} \right) + \frac{1}{2} (1 - \lambda) \geq 0$$

$$\iff 2 (1 - \lambda) \left( \frac{1 - \rho}{2\rho - 1} \right) - \frac{1}{2} \lambda \geq \frac{1}{2}$$

$$\iff 2 (1 - \lambda) (1 - \rho) - \frac{1}{2} \lambda (2\rho - 1) \geq \frac{1}{2} (2\rho - 1)$$

$$\iff \frac{1}{2} + 2 (1 - \lambda) (1 - \rho) - \frac{1}{2} \lambda (2\rho - 1) \geq \rho$$

$$\iff \frac{1}{2} + 2 (1 - \lambda) - \frac{1}{2} \lambda (2\rho - 1) \geq \rho + 2\rho (1 - \lambda)$$

$$\iff \frac{1}{2} + 2 (1 - \lambda) + \frac{1}{2} \lambda \geq \rho + 2\rho (1 - \lambda) + \rho\lambda$$

$$\iff \frac{1}{2} + 2 (1 - \lambda) + \frac{1}{2} \lambda \geq \rho + 2\rho - 2\rho\lambda + \rho\lambda$$

$$\iff \frac{1}{2} + 2 (1 - \lambda) + \frac{1}{2} \lambda \geq 3\rho - \rho\lambda$$

$$\iff \frac{1}{2} + 2 (1 - \lambda) + \frac{1}{2} \lambda \geq (3 - \lambda) \rho$$

$$\iff 1 + 4 (1 - \lambda) + \lambda \geq 2 (3 - \lambda) \rho$$

$$\iff 5 - 3\lambda \geq 2 (3 - \lambda) \rho$$

$$\iff \frac{5 - 3\lambda}{6 - 2\lambda} \geq \rho$$

Therefore, if $\rho \leq \frac{5 - 3\lambda}{6 - 2\lambda}$, we have $\frac{\partial^2 \bar{n}}{\partial b \partial \rho} \geq 0$. Additionally, we have $\frac{\partial^2 \bar{n}}{\partial \psi \partial \rho} > 0$ and $\frac{\partial^2 \bar{n}}{\partial \sigma \partial \rho} < 0$. Therefore, the profit function $\bar{n}(b, e_h, \psi, \sigma; \rho)$ is supermodular in $b$, $-\psi$ and $\rho$. Therefore, applying Topkis’ theorem, we obtain that as $\rho$ increases, this leads to an increase in $b$ and a decrease in $\psi$. ■

References


