Motivational Investments and Financial Incentives*

Maitreesh Ghatak
LSE†

Zaki Wahhaj
University of Kent‡

April 2023

Abstract

If firms can invest in the motivation of workers to undertake costly effort, how does that affect the choice of explicit financial incentives? We develop a simple principal-agent model where the standard optimal contract is to offer a bonus that trades off incentive provision versus rent extraction. We allow the principal to undertake two types of motivational investments - one that increases the agent's disutility from deviating from a prescribed effort level, and another that reduces the cost of effort. We refer to these as guilt and inspiration, respectively. We characterize the conditions under which motivational investments and financial incentives are substitutes and complements, and find that it depends on the type of the investment as well as whether the worker's participation constraint is binding.

JEL classification numbers: D23, D86, D91, J33

Keywords: motivated agents, investment in worker motivation, job satisfaction, worker moral hazard

*We would like to thank George Akerlof, Gani Aldashev, Emmanuelle Auriol, Tim Besley, Micael Castanheira, Paul Collier, Matthias Dewatripont, Greg Fischer, Georg Kirchsteiger, Rachel Kranton, Rocco Macchiavello, David Martimort, Andy Newman, Debraj Ray and participants at the LSE lunch workshop, ThReD Workshop at Bocconi University 2018, ECARES internal seminar series, ERINN Workshop at the University of Oxford 2018, and the N.G.O. conference at Università di Roma La Sapienza 2019 for many helpful comments. We would also like to thank Linchuan Xu for excellent research assistance. The usual discalimer applies.

†Department of Economics, London School of Economics, Houghton Street, London WC2A 2AE, United Kingdom

‡School of Economics, University of Kent, Kennedy Building, Park Wood Road, Canterbury, Kent CT2 7FS, United Kingdom

1 Introduction

How can workers be motivated? In 1968 the *Harvard Business Review* carried an article titled 'How Do You Motivate Employees?' that aimed to reshape how firms and managers approached this question (Herzberg 1968). Its author, Frederick Herzberg, argued that getting an employee to do things was not the same as motivating the employee; that the threat of punishment and the promise of rewards could get an employee to 'move' but the only person 'motivated' in this transaction was the one threatening or making promises. ['If I kick you in the rear (physically or psychologically), who is motivated? *I* am motivated; *you* move!']. Herzberg emphasized, instead, a set of 'motivator factors' that are intrinsic to the job for creating motivated workers (e.g. 'achievement', 'recognition for achievement', 'responsibility', 'psychological growth') as opposed to factors that are extrinsic to the job which are in the nature of reward or punishment, such as supervision, working conditions, salary and status.

Herzberg's reasoning and terminology have since entered common parlance in management practice; implicit, for example, in a special issue in the same publication 35 years later giving advice to executives and managers on motivating those they lead (Nicholson 2003). Understanding whether and to what extent financial incentives can motivate workers and raise their productivity is fundamental for firms and organisations to develop effective management practices. Around this central queston, a growing body of academic work has explored how financial incentives interact with the intrinsic motivation of workers in different contexts, for example when organisations are mission-oriented (Besley and Ghatak 2005, 2018), when workers desire to appear pro-social (Benabou and Tirole 2006), seek praise from managers they approve (Ellingsen and Johannesson 2008), or have imperfect information about the work (Benabou and Tirole 2003). Relatedly, there is growing evidence from lab and field experiments on how incentives impact upon workers' performance in situations where pro-social motivation is deemed to be important (Ashraf, Bandiera and Jack, 2014; Berg et al, 2018; Deserranno, 2017; Muralidharan and Sundararaman, 2009; Rasul and Rogger, 2013; Ashraf, Bandiera, Davenport and Lee, 2020).

Besides financial incentives, firms and organizations often spend considerable time and resources in activities aimed at raising the morale, team-spirit and loyalty of the workforce. A broad range of activities may have such aims, including management and leadership training, team-based exercises, communication with workers about broader organisational goals. The financial incentives that firms and organisations provide may affect not only the intrinsic motivation of workers but also lead to (endogenous) adjustments in these types of motivational investments. A set of recent papers have theoretically investigated motivational investments in a principal-agent setting (Akerlof and Kranton 2005; Besley and Ghatak 2005, 2017; Kvaløy and Schottner 2015; Thakor and Quinn 2020).[1] Relatedly, a number of empirical studies have investigated how financial incentives interact with some forms of motivational investments (see, for example, Kvaløy, Nieken and Schottner 2015; Kosfeld, Neckermann and Yang 2017).

In this paper, we contribute to the theoretical literature on motivational investments. A key question that this literature has dealt with is whether organisations should use motivational investments as complements or substitutes of financial rewards in incentivising workers. The existing literature shows that either case could hold true (Akerlof and Kranton 2005; Kvaløy and Schottner 2015) and that the answer depends, in large part, on whether motivational investments raise or lower the marginal effect of financial incentives on workers' effort. We show that, for workers for whom both the participation constraint and the limited liability constraint binds, whether motivational investments substitute for or complement financial incentives is fully determined by how such investments affect the worker's *overall* welfare on the job. If motivational investments raise the worker's overall welfare then, under a binding participation constraint, it substitutes for financial incentives. If motivational investments *lower* the worker's overall welfare – we discuss such an example below – then it complements financial incentives.

[1] These papers deal with somewhat different but closely related concepts: Akerlof and Kranton (2005) consider an organisation's investment in 'motivational capital' to change a worker's identity; Besley and Ghatak (2005) consider an organisation choosing a 'compromise' mission, that reflects employee preferences, to motivate workers; in Kvaløy and Schottner (2015), a firm or an agent of the firm chooses motivational intensity/effort to motivate workers; whereas in Thakor and Quinn (2020) an organisation can choose, and commits resources to, a 'higher purpose' to motivate workers.

The intuition behind these results are as follows. A worker with a binding participation constraint will typically receive greater financial incentives than the first-best level. If investing in the worker's level of motivation raises the worker's overall welfare, then this allows the employer to reduce financial incentives while ensuring that the participation constraint is still satisfied. Thus motivational investments and financial incentives move in opposite directions. But if investing in the worker's level of motivation *lowers* the worker's overall welfare, then this needs to be accompanied by additional financial incentives to ensure that the participation constraint is still met. Thus, motivational investments and financial incentives move together.

To illustrate our arguments, we present and analyse two contrasting cases of our core model involving two different types of motivational investments that firms/organisations can make. The first type of investment, which we can think of as *guilt*, increases the agent's disutility from deviating from a level of effort specified in the labour contract as a norm or expected benchmark, even though the actual choice of effort is unobservable. The second type of investment, which we can think of as *inspiration*, lowers the agent's cost of effort. We can think of the first type of motivational investment as an example of a negative reinforcement mechanism - something that raises the cost of falling short of expectations. In contrast, the second type of motivational investment is an example of a positive reinforcement mechanism - something that lowers the cost of undertaking effort. Cruicially, while inspiration raises the worker's overall welfare on the job, guilt investments *lower* the worker's overall welfare on the job. The two types of motivational investments we model are not intended to capture all types of motivation relevant for real world situations (see Cassar and Meier 2018 for a recent review of the literature covering different types of non-monetary motivation). Rather, they have been chosen for expositional reasons to cover two contrasting scenarios.

For both types of motivational investments, a binding participation constraint changes the relationship between financial incentives and motivational investments. In a setting where motivational investments increase guilt, thereby reducing the overall expected payoff of workers, firms use motivational investments as a substitute of financial incentives if the worker's participation constraint is non-binding; but as a complement of financial re-

	Guilt	Inspiration
Participation Constraint Binding	Complements	Substitutes
Participation Constraint Not Binding	Substitutes	Independent

Table 1: Relationship between motivational investments and financial incentives

wards if the worker's participation constraint is binding. That is, motivational investments and financial incentives are substitutes or complements depending on the outside option of workers.

On the other hand, in a setting where motivational investments inspire workers and lower the cost of effort, thereby raising the overall expected payoff of the worker, firms choose financial incentives independently of the level of motivational investments if the participation constraint is non-binding; but financial incentives and motivational investments are used as substitutes if the participation constraint is binding. In Table 1, we summarize these results.

These results imply that how organisations incentivise workers – and specifically the combination of financial incentives and motivational investments they choose – should depend on the workers' outside options. This theoretical approach can generate testable predictions for how the use of motivational investments should vary over the business cycle, for workers with different levels of human capital, and across different industries.

We provide some additional analytical exercises of interest using this framework, such as what happens when the outside option of the worker improves, the profitability of the firm changes, and changes in any non-pecuniary benefits that the agent enjoys from working in the organisation. An improvement in the outside option, for example, would, other things equal, lead to an increase in financial incentives in settings with moral hazard and limited liability, given the standard trade-off between rent extraction and incentives. But a higher reward for success lowers the efficacy of guilt investments to induce effort.

Kvaløy and Schottner (2015) also investigates the relationship between motivational investments and financial incentives in a Principal-Agent model for a broad class of agent effort cost functions. However, in the Kvaløy-Schottner model, when the agent has limited liability, the participation constraint does not bind. Akerlof and Kranton (2003, 2005, 2008) investigate similar models in which an organisation has the possibility of making identity-related investments in a worker. They allow the agent's participation constraint to bind but do not explore the possibility that motivational investments could reduce the worker's overall welfare, as in our formulation of 'guilt' investments. Therefore, our main theoretical insight is missing from this literature.

The rest of the paper is organised as follows. We make our core arguments first in a Principal-Agent model of motivational investments using fairly general functional forms for the agent's cost of effort and the cost of motivational investments. This is presented in Section 2. In the following sections, we present two more specific cases of motivational investment. In Section 3, motivational investments increase the agent's 'guilt' in deviating from a prescribed level of effort. We formalise the notion of 'guilt' in Section 3.1 and provide an interpretation in Section 3.2. We derive the optimal contract in Sections 3.3-3.5. In Section 3.6, we investigate how the combination of financial incentives and motivational investments in the optimal contract changes with the agent's outside option, cost of motivational investments, taxation and career benefits for the worker. In Section 4, motivational investments lower the agent's cost of effort by 'inspiring' the agent, and the analysis of the second model proceeds in the same manner as for the first. Section 5 concludes.

2 A Simple Model of Motivational Investments

2.1 Setup

Consider a simple principal-agent model where the agent provides effort $e \in [0, 1]$. This produces output $A \in (0, 1)$ with probability e and output zero otherwise. The principal observes output but not effort. Prior to production, the principal can make an investment $\psi \geq 0$ which reduces the agent's cost of effort. Specifically, the agent incurs a disutility $C(e, \psi)$ from effort e where $C(.)$ is twice continuously differentiable function satisfying the conditions $C_e, C_{ee} > 0$ and $C_{e\psi} < 0$ for $e, \psi > 0$. In Sections 3 and 4 we will introduce additional structure to the function $C(.)$ and provide an interpretation of ψ as a form of 'motivational investment' by the principal to induce the agent to exert more effort.

The cost of investment ψ is described by $\mu h(\psi)$ where the constant $\mu > 0$. The cost function $h(\psi)$ has the following properties: $h(0), h'(0) = 0$ and $h'(\psi), h''(\psi) > 0$ for $\psi > 0$. We assume that there is limited liability such that the principal cannot extract payments from the agent, for example, as penalties or fines.

The agent receives a financial reward b only in the case of positive output (i.e. $A > 0$). We can represent a contract by (b, ψ) satisfying the conditions $b, \psi \geq 0$.[2] If the agent chooses not to accept the contract, he obtains a reservation utility of $\underline{u} \geq 0$.

Given a choice of effort e and contract (b, ψ), the agent's expected utility from the contract is given by

$$U(b, e, \psi) = be - C(e, \psi) \tag{1}$$

and the principal's expected profit is given by

$$\Pi(b, \psi) = \hat{e}(b, \psi)(A - b) - \mu h(\psi) \tag{2}$$

where $\hat{e}(b, \psi)$ is the agent's choice of effort given contract (b, ψ):

[2] Note that a contract of this form implies that the agent does not receive any financial payment if output equals 0. We take this approach because if $e < 1$ and the limited liability constraint is non-bindng then any contract that involves a non-contingent payment can be improved upon by simultaenously increasing the bonus and lowering the non-contingent payment such that the agent's expected utility is unchanged (Banerjee, Gertler and Ghatak 2002). This adjustment would lead to higher effort from the agent and thus higher expected profit for the principal.

$$\hat{e}(b,\psi) = \arg\max_{e} U(b,e,\psi). \tag{3}$$

2.2 Equilibrium

By assumption, the agent's cost function is strictly concave. Therefore, if the optimisation problem has an interior solution, the level of effort is fully characterised by the first-order condition:

$$b = C_e(\hat{e}(b,\psi),\psi). \tag{4}$$

Differentiating throughout (4) w.r.t. b and ψ, we obtain

$$\frac{\partial \hat{e}}{\partial b} = \frac{1}{C_{ee}} \text{ and } \frac{\partial \hat{e}}{\partial \psi} = -\frac{C_{e\psi}}{C_{ee}}. \tag{5}$$

Since, by assumption, $C_{ee} > 0$ and $C_{e\psi} < 0$, we have $\frac{\partial \hat{e}}{\partial b}, \frac{\partial \hat{e}}{\partial \psi} > 0$. Given the optimal choice of effort $\hat{e}(b,\psi)$, we can write the expected utility of the contract to the agent as follows:

$$V(b,\psi) = U(b,\hat{e}(b,\psi),\psi).$$

The principal's choice of contract is given by

$$\left(\hat{b},\hat{\psi}\right) = \arg\max_{b,\psi} \hat{e}(b,\psi)(A-b) - \mu h(\psi) \tag{6}$$

subject to

$$V(b,\psi) \geq \underline{u}. \tag{7}$$

Our key question of interest within this framework is whether the principal will treat financial rewards and motivational investments as substitutes and complements in incentivising the worker (equivalently, will the use of financial rewards go up or down when the cost of motivational investments increase/decrease). Intuition would suggest that this depends on whether financial rewards and motivation serve as substitutes and complements in the agent's choice of effort. If the agent's participation constraint is non-binding, this intuition holds in part, albeit with some ambiguity when financial rewards and motivation are complements in the agent's effort choice. This is shown by Kvaløy and Schottner (2015) and we provide a formal statement specific to our setup below (Further discussion and the proof of the proposition is provided in Appendix A).

Proposition 1 *Suppose that the agent's cost function $C(.)$ is such that that the principal's expected profit function $\Pi(b,\psi)$ is globally concave. If the agent's participation constraint is non-binding, then*

(i) if financial rewards and motivational investments are substitutes in the agent's choice of effort, then they are also used as substitutes by the principal;

(ii) if financial rewards and motivational investments are complements in the agent's choice of effort, then they may be used either as complements or substitutes by the principal.

A rather different result occurs if the agent's participation constraint binds. To analyse this case, we define $\bar{b}(\psi,\underline{u})$ as the level of financial reward for which – given ψ – the agent obtains a reservation utility of $\underline{u}$; i.e.

$$V\left(\bar{b}(\psi,\underline{u}),\psi\right)=\underline{u}$$

$$\Longrightarrow \bar{b}(\psi,\underline{u})\,\hat{e}\left(\bar{b}(\psi,\underline{u}),\psi\right)-C\left(\hat{e}\left(\bar{b}(\psi,\underline{u}),\psi\right),\psi\right)=\underline{u} \tag{8}$$

Using $\bar{b}(\psi,\underline{u})$, we can rewrite the optimisation problem in (6)-(7) as

$$\hat{\psi} = \arg\max_{\psi} \hat{e}\left(\bar{b}(\psi,\underline{u}),\psi\right)\left(A-\bar{b}(\psi,\underline{u})\right)-\mu h(\psi) \tag{9}$$

$$\hat{b} = \bar{b}\left(\hat{\psi},\underline{u}\right) \tag{10}$$

Let us denote the maximand in (9) by $\widetilde{\Pi}(\psi,\mu)$. By definition,

$$\frac{\partial^2\widetilde{\Pi}}{\partial\mu\partial\psi}=-h'(\psi)<0$$

Thus the maximand is supermodular in ψ and $-\mu$. Then, we can apply Topkis' Theorem[3] to show that the level of motivational investment $\hat{\psi}$ is decreasing in the cost parameter μ.

To investigate how financial rewards change with the level of motivational investment, we differentiate throughout (8) w.r.t. μ:

$$\frac{\partial\hat{\psi}}{\partial\mu}\left[\frac{\partial\bar{b}}{\partial\psi}\hat{e}(.)+\left\{\bar{b}(\psi,\underline{u})-\frac{\partial C}{\partial e}\right\}\left(\frac{\partial\hat{e}}{\partial b}\frac{\partial\bar{b}}{\partial\psi}+\frac{\partial\hat{e}}{\partial\psi}\right)-\frac{\partial C}{\partial\psi}\right]=0$$

$$\Longrightarrow \frac{\partial\bar{b}}{\partial\psi}\hat{e}(.)+\left\{\bar{b}(\psi,\underline{u})-\frac{\partial C}{\partial e}\right\}\left(\frac{\partial\hat{e}}{\partial b}\frac{\partial\bar{b}}{\partial\psi}+\frac{\partial\hat{e}}{\partial\psi}\right)=\frac{\partial C}{\partial\psi} \tag{11}$$

[3] See Theorem 2.8.1 in Topkis (1998).

Then, using the agent's first-order conditon from (4) in (11), we obtain the following simplification:

$$\frac{\partial \bar{b}}{\partial \psi} = \frac{1}{\hat{e}\,(.)}\frac{\partial C}{\partial \psi}$$

Therefore, the level of financial reward $\bar{b}$ that exactly satisfies the agent's participation constraint increases (decreases) with motivational investment ψ if the agent's cost of effort $C\,(.)$ is increasing (decreasing) in ψ. We can summarise these results as follows:

Proposition 2 *Suppose the agent's participation constraint is binding. Then the principal will use financial rewards and motivational investments as complements (substitutes) if the level of the agent's cost of effort increases (decreases) with motivational investments.*

Proposition 2 highlights an interesting implication of a tight labour market (in which the worker's participant constraint binds) for the combination of financial rewards and motivational investments used by employers to incentivise workers. As noted above, if a worker's participation constraint is non-binding, then whether financial rewards and motivational investments will be used as substitutes or complements by the employer depends largely on whether they are substitutes or complements in determining the worker's effort level. By constrast, if the worker's participation constraint binds, then whether financial rewards and motivational investments are used as complements or substitutes depends on whether motivational investments increases or decreases the worker's disutility from a given level of effort. If motivational investments decrease the disutility from work, the employer will use financial rewards and motivational investments as substitutes (regardless of whether they are complements or substitutes in the worker's choice of effort).

However if motivational investments *increase* the disutility from work, the employer will use financial rewards and motivational investments as complements. While it may seem counterintuitive, motivational investments can lower the marginal cost of effort while at the same time *increasing* the overall disutility of work. In the next section, we present one such example which we call investing in 'guilt'.

3 Motivating Agent by Increasing Guilt

In this section, we present a particular case of the model in Section 2 in which motivational investments *increase* the disutility of work while lowering the marginal cost of effort. The purpose of this exercise is to provide a concrete example of such a scenario (that, to the best of our knowledge, has previously received little attention in the literature), provide an economic interpretation for it, and consider its implications for the optimal choice of financial rewards and motivational investments under different conditions.

3.1 Setup

We assume, as in Section 2, that an agent provides effort $e \in [0,1]$ at cost $\frac{1}{2}e^2$, producing output $A \in (0,1)$ with probability e and output zero otherwise. The principal observes output but not effort.

A contract between the principal and the agent specifies a 'bonus' (or 'reward for success') b in case the agent produces positive output and an effort level $e_c \in [e_l, e_h]$. Although the principal does not observe effort, the agent experiences disutility in deviating from e_c in his choice of e at a cost $\frac{1}{2}\psi\left(e_c - e\right)^2$ which we call 'guilt'. As before, we assume there is limited liability such that the principal cannot extract payments from the agent. We can represent a contract by the 3-tuple (b, e_c, ψ) satisfying the conditions $b, \psi \geq 0, e_c \in \{e_l, e_h\}$.[4] If the agent chooses not to accept the contract, he obtains a reservation utility of $\underline{u}$. The agent's expected utility from the contract is given by

$$U\left(b, e_c, e, \psi\right) = be - \frac{1}{2}e^2 - \frac{1}{2}\psi\left(e_c - e\right)^2 \tag{12}$$

The principal can make investments to raise the agent's 'motivation', represented by $\psi \geq 0$. Achieving a level of motivation ψ requires an investment equal to $\frac{1}{2}\mu\psi^2$. In the absence of any investments, $\psi = 0$. In the next subsection, we provide an interpretation of the variables associated with 'guilt'. For our main results to follow, the key assumption we make is that motivational investments *tighten* the agent's participation constraint (since

[4] As in the preceding section, we implicitly assume that the agent does not receive any financial payment if output equals 0. See footnote 4 for the rationale behind this assumption.

the coefficient of ψ is negative). By contrast, in Akerlof and Kranton (2003, 2005, 2008), there is a gain in a worker's 'identity utility' when a firm invests in 'motivational capital'; and Kvaløy and Schottner (2015) assume that motivational investments reduce the cost of any given level of effort. In both these models, motivational investments would *relax* the agent's participation constraint. Similarly, when an firm chooses a 'compromise mission' in Besley and Ghatak (2005) or a 'higher purpose' in Thakor and Quinn (2020), a worker finds employment with the firm more attractive for any given effort level, thus leading to a relaxation of the participation constraint.

Additionally, we make the following assumptions about the model parameters.

Assumption 1 $A \in (0, 1)$

Assumption 2 $\mu \in (0, \infty)$

Assumption 3 $0 < e_l < e_h \leq A$

Assumption 4 $e_l = \frac{A}{2}, e_h = A$

As per Assumption 1, we fix the value of output to be less than 1 to ensure that the agent's optimisation problem always has an interior solution. Assumption 2 ensures that motivational investments have a positive cost. We fix e_l at $\frac{A}{2}$ (Assumption 4) as it is the level of effort that an agent will exert in a setting where there is no guilt and the principal chooses a bonus $b = \frac{A}{2}$ (which is the profit-maximising bonus level if the agent's participation constraint is non-binding).[5] We fix e_h at A as it is the first-best effort level. Although the utility function in (12) implies that the agent experiences disutility from both positive and negative deviations from e_c, we will see that, in the optimal contract, the principal chooses b and e_c such that the agent's effort level never exceeds e_c.

To determine the optimal contract, we proceed with the analysis using backward induction. In Section 3.3, we determine the agent's choice of effort for a given contract (b, e_c, ψ)

[5] An alternative approach is to let $e_l = b$ as b is the level of effort that an agent will exert in any contract in the absence of guilt. It will become evident that we obtain almost identical results with this alternative approach.

and investigate how the agent's effort level responds to changes in financial incentives and motivational investments. Then, we investigate how changes in the contract affects the agent's expected utility and, thus, her participation constraint. In Section 3.5, we solve the Principal's profit maximisation problem to derive the optimal contract using the agent's effort function and her expected utility from a given contract.

3.2 Interpretation

The term e_c is an effort level tacitly or explicitly referred to in the agreement or contract between the principal and the agent. Although the actual effort level is a continuous variable in the unit interval, the choice of e_c is restricted to a discrete set. This modelling choice, made to improve tractability, may be justified by the notion that describing in the contract a precise level of effort is prohibitively costly and that the choice set includes only those effort levels that can be described using language in common usage (see Hart and Moore 1999 for a review of related concepts in the incomplete contracts literature).

For our analysis, we define the choice set of e_c to consist of two effort levels, 'low' e_l and 'high' e_h. As noted in the previous subsection, e_l is the effort level that a worker with no sense of guilt would exert when the principal chooses the profit maximising level of financial reward for success. As this is likely to be a common occurrence in labour arrangements, it should be possible to specify such an effort level in a labour contract. By contrast, e_h represents a 'higher standard' of behaviour, possibly characteristic, for example, of workers with prosocial preferences (Benabou and Tirole 2006).

The notion of guilt in the model is loosely related to its formalization in the game-theoretic literature. For example, Battigalli and Dufwenberg (2007) defines 'simple guilt' as disutility experienced by one player due to the payoff loss (vis-a-vis some expectation) that his strategy inflicts on another (to capture the notion that "a player cares about the extent to which he lets another player down"). If the effort level specified in the contract e_c affects the principal's beliefs about the agent's actual choice of effort e, then 'simple guilt', as defined by Battigalli and Dufwenberg (2007), would be a function of $(e_c - e)$ as modeled here.[6] Charness and Dufwenberg (2006) show, in an experimental setting, that promises about actions made in pre-play communication in a principal-agent relationship indeed affect beliefs about behaviour and the level of cooperation in the relationship, findings that they account for using the notion of 'guilt aversion'. The effort level specified in a contract, as modelled here, is a form of pre-play communication that could, plausibly, serve a similar role to that in the game used by Charness and Dufwenberg (2006).

3.3 Agent's Effort Choice

The agent solves the following optimisation problem:

$$\max_{e \in [0,1]} be - \frac{1}{2}e^2 - \frac{1}{2}\psi\left(e_c - e\right)^2 \tag{13}$$

It is clear upon inspection that the maximand in (13) is strictly concave in e. Therefore, the agent's optimisation problem has a unique solution. We obtain e from the first-order condition:

$$b - e - \psi\left(e_c - e\right)(-1) = 0$$

$$\begin{aligned} &\Longrightarrow \quad e + \psi e = b - \psi e_c \\ &\Longrightarrow \quad e = \frac{b + \psi e_c}{1 + \psi} \end{aligned} \tag{14}$$

We denote this solution by $\hat{e}\left(b, e_c, \psi\right)$. Using (14), it is straightforward to verify that the agent's effort is increasing in both b and e_c. We can rerrange the expression for the optimal effort choice as follows:

$$e = e_c - \left(\frac{e_c - b}{1 + \psi}\right) \tag{15}$$

Using (15), we obtain

$$\frac{\partial \hat{e}}{\partial \psi} = \frac{e_c - b}{\left(1 + \psi\right)^2} \tag{16}$$

$$\frac{\partial \hat{e}}{\partial b} = \frac{1}{1 + \psi} \tag{17}$$

[6] Note that, disutility from 'simple guilt', as modelled by Battigalli and Dufwenberg (2007) would equal zero when the actual effort level exceeds expectations. In contrast, in our setup any deviation – positive or negative – from the specified effort level generates disutility. But this modelling choice, made for notational simplicity, does not affect the analysis: as noted in the preceeding subsection, actual effort never exceeds the specified level in equilibrium.

From (16), we see that, for $e_c > b$, the agent's effort is increasing in ψ. Therefore, the principal will invest in guilt only if he sets $e_c > b$ at the same time. We can establish that $\frac{\partial^2 \hat{e}}{\partial \psi^2} < 0$, i.e. the efficacy of guilt in increasing the agent's effort level is decreasing in the existing level of guilt investments. Using (17), we can establish that $\frac{\partial^2 \hat{e}}{\partial \psi \partial b} < 0$; i.e. the financial rewards and guilt investments are substitutes in increasing the agent's effort level. We summarise these results as follows.

Lemma 1 *The responsiveness of the agent's optimal choice of effort to guilt investments and financial rewards are decreasing in the level of guilt investment, i.e.* $\frac{\partial^2 \hat{e}}{\partial \psi^2} < 0$ *and* $\frac{\partial^2 \hat{e}}{\partial \psi \partial b} < 0$.

3.4 Agent's Participation Constraint

We denote by $V(b, e_c, \psi)$ the agent's indirect utility from the contract (b, e_c, ψ); i.e.

$$V(b, e_c, \psi) = U(b, e_c, \hat{e}(b, e_c, \psi), \psi)$$

$$= b\left(\frac{b + \psi e_c}{1 + \psi}\right) - \frac{1}{2}\left(\frac{b + \psi e_c}{1 + \psi}\right)^2 - \frac{1}{2}\psi\left(\frac{e_c - b}{1 + \psi}\right)^2$$

$$\Longrightarrow V(b, e_c, \psi) = \frac{1}{2}\frac{(b^2 + 2\psi b e_c - \psi e_c^2)}{(1 + \psi)} \tag{18}$$

We define $\bar{b}(e_c, \psi, \underline{u})$ as the level of bonus for which – given e_c, ψ – the principal obtains a reservation utility of $\underline{u}$; i.e. $\bar{b}(e_c, \psi, \underline{u})$ is defined implicitly by the following equation.

$$\frac{1}{2}\frac{\left(\bar{b}^2 + 2\psi \bar{b} e_c - \psi e_c^2\right)}{(1 + \psi)} = \underline{u} \tag{19}$$

This is the bonus that the agent will receive for high output whenever the agent's participation constraint binds. Using (19), we can establish the following results:

Lemma 2 *When the agent's participant constraint binds, the financial reward for success* $\bar{b}(e_c, \psi, \underline{u})$ *is (i) increasing in* ψ *at a decreasing rate, i.e.* $\frac{\partial \bar{b}}{\partial \psi} > 0$ *and* $\frac{\partial^2 \bar{b}}{\partial \psi^2} < 0$*; (ii) is increasing in the agent's outside option* $\underline{u}$ *at a decreasing rate wih respect to* ψ*, i.e.* $\frac{\partial \bar{b}}{\partial \underline{u}} > 0$ *and* $\frac{\partial^2 \bar{b}}{\partial \psi \partial \underline{u}} < 0$.

The intuition behind the first part of Lemma 2 is that when the agent's participation constraint is binding, guilt is compensated through financial rewards, which translates into higher effort, which means that further increasing ψ has less effect on the agent's guilt and thus requires less financial compensation. The intuition behind the second part of the lemma is that if the agent has a strong outside option, then the financial rewards – and thus effort – are higher; therefore, raising the guilt parameter has a smaller effect on the agent's disutility, which therefore requires less compensation.

3.5 Optimal Contract

The principal's expected profits are given by

$$\Pi\left(b, e_c, \psi\right) = \hat{e}\left(b, e_c, \psi\right)\left(A - b\right) - \frac{1}{2}\mu\psi^2$$

To maximise profits, the principal solves

$$\max_{b, e_c, \psi} \hat{e}\left(b, e_c, \psi\right)\left(A - b\right) - \frac{1}{2}\mu\psi^2 \tag{20}$$

subject to

$$V\left(b, e_c, \psi\right) \geq \underline{u} \tag{21}$$

Non-Binding Participation Constraint: First, we investigate the case in which the agent's participation constraint does not bind, a situation which arises for $\underline{u}$ sufficiently low. Then, using (14), the maximisation problem in (20)-(21) can be written as

$$\max_{b, e_c, \psi} \left(\frac{b + \psi e_c}{1 + \psi}\right)\left(A - b\right) - \frac{1}{2}\mu\psi^2 \tag{22}$$

If the principal chooses $e_c = e_l$, then there is no reason to invest in guilt. Then we obtain the standard moral hazard model with the solution $b = \frac{A}{2} = e_l$.

If the principal chooses $e_c = e_h$ and μ is sufficiently small, then we have an interior solution. Then we obtain the following first-order conditions:

$$b \;:\; \frac{\partial \hat{e}}{\partial b}\left(A - b\right) - \hat{e}\left(b, e_h, \psi\right) = 0 \tag{23}$$

$$\psi \;:\; \frac{\partial \hat{e}}{\partial \psi}\left(A - b\right) - \mu\psi = 0 \tag{24}$$

It is difficult to obtain a closed-form solution using equations (23) and (24). But we can provide some comparative statics results by using the supermodularity properties of the maximand in (22). Differentiating the maximand w.r.t. b and using $e_c = e_h = A$ (Assumption 4), we obtain

$$\frac{\partial \Pi (b, e_h, \psi)}{\partial b} = \frac{A(1-\psi) - 2b}{1+\psi} \tag{25}$$

It is clear that the expression in (25) is decreasing in ψ, i.e. $\frac{\partial^2 \Pi}{\partial b \partial \psi} < 0$; i.e. $\Pi (b, e_h, \psi)$ is supermodular in b and $-\psi$. Furthermore, it is straightforward to show that $\frac{\partial^2 \Pi}{\partial \psi \partial \mu} < 0$ and $\frac{\partial^2 \Pi}{\partial b \partial \mu} = 0$. Then, using Topkis' Theorem,[7] we can show that the financial reward for success (b) is increasing, and investment in guilt (ψ) is decreasing, in the cost of motivation μ; in other words, guilt investments and financial rewards are substitutes. Formally, we state the result as follows.

Proposition 3 *If the agent's participation constraint is non-binding and the principal is making a positive level of guilt investment, then the principal will use guilt investments and financial rewards as* <u>*substitutes*</u>.

Proposition 3 echoes the result that financial rewards and guilt investments are substitutes in increasing the agent's effort level (see Lemma 1 above). However, we will see below that this parallel between the interactive effect of financial rewards and guilt investments on the agent's effort choice and on the principal's profits breaks down when the agent's participation constraint is binding.

Binding Participation Constraint: Next, we provide a partial characterisation of the case in which the agent's participation constraint is binding. Using the function $\bar{b}(e_c, \psi, \underline{u})$ (defined implicitly by (19)), we can rewrite the optimisation problem in (20)-(21) as

$$\max_{e_c, \psi} \hat{e}\left(\bar{b}, e_c, \psi\right)\left(A - \bar{b}\right) - \frac{1}{2}\mu\psi^2$$

If the principal chooses $e_c = e_l$, there are no guilt investments as in the case of the non-binding participation constraint. But b will be given by (19) to ensure that the participation

[7] Theorem 2.8.1 in Topkis (1998)

constraint is satisfied. This will mean that b is higher than in the previous case, i.e. $b > \frac{A}{2} = e_l$. Therefore, we must have $e_c = e_h$ and the maximisation problem in (20)-(21) becomes

$$\max_{\psi} \hat{e}\left(\bar{b}, e_h, \psi\right)\left(A - \bar{b}\right) - \frac{1}{2}\mu\psi^2 \tag{26}$$

If the maximisation problem has an interior solution, then ψ is given by the following first-order condition:

$$\psi : \left(\frac{\partial \hat{e}}{\partial \psi} + \frac{\partial \hat{e}}{\partial b}\frac{\partial \bar{b}}{\partial \psi}\right)\left(A - \bar{b}\right) - \hat{e}\left(b, e_c, \psi\right)\frac{\partial \bar{b}}{\partial \psi} - \mu\psi = 0$$

Intuitively, increasing motivational investment ψ increases effort. Because the participation constraint is binding, the increase in guilt has to be compensated by higher financial rewards. This compensation is captured by the term $\frac{\partial \bar{b}}{\partial \psi} > 0$. The increase in financial rewards further increases effort (captured by the term $\frac{\partial \hat{e}}{\partial b}\frac{\partial \bar{b}}{\partial \psi}$) but it also means higher payment whenever the agent generates high output (captured by the term $\hat{e}\left(b, e_c, \psi\right)\frac{\partial \bar{b}}{\partial \psi}$). Rearranging (26), we obtain

$$\frac{\partial \hat{e}}{\partial \psi}\left(A - \bar{b}\right) + \frac{\partial \bar{b}}{\partial \psi}\left\{\frac{\partial \hat{e}}{\partial b}\left(A - \bar{b}\right) - \hat{e}\left(b, e_c, \psi\right)\right\} = \mu\psi \tag{27}$$

Note that the expression within the curly brackets in (27) is identical to the left-hand side of (23). Therefore, the expression is equal to the marginal effect of increasing the financial reward on the principal's expected profits. Therefore, if the participation constraint is binding, it must be zero or negative (because if it were positive, then the principal could increase expected profits by increasing b above $\bar{b}\left(e_c, \psi, \underline{u}\right)$). As $\frac{\partial \bar{b}}{\partial \psi} > 0$, it follows that, at the optimum, we must have

$$\frac{\partial \hat{e}}{\partial \psi}\left(A - \bar{b}\right) \geq \mu\psi \tag{28}$$

Therefore, when the agent's participation constraint is binding, the marginal effect of motivational investments on the principal's expected profits is, in equilibrium, at least as large as the marginal cost of this type of investment. This property of the equilibrium is due to the fact that motivation investments tighten the agent's participation constraint. We will see in Section 4.4 that the opposite holds true when motivational investments take the form of 'inspiration' rather than guilt.

We now address the question whether the principal will use guilt investments as a complement or a substitute of financial rewards in eliciting the agent's effort. For this purpose, we consider how a change in μ, the cost of guilt investments, affects the principal's decisions. Intuitively, an increase in μ should lead to a reduction in guilt investments. This will relax the participation constraint, thus reducing the need for financial rewards (for success) to induce the agent to take up the contract. Therefore, we obtain a decline in financial rewards. Thus, guilt investments and financial rewards move together in response to a change in the cost of guilt investments; in other words, they are complements. Lower financial rewards combined with reduced guilt investments will reduce the agent's level of effort. Formally, we have the following results.

Proposition 4 *If the participation constraint binds and the principal makes positive guilt investments, then (i) the principal will use guilt investments and financial rewards as complements, and (ii) the agent's level of effort is decreasing in the cost of motivational investments.*

Proposition 4 is the equivalent of Proposition 2 when motivational investments take the form of increasing the agent's 'guilt' from deviating from a prescribed effort level. Proposition 2 implies that when motivational investments tighten the agent's participation constraint, the principal will use motivational investments and financial rewards as complements. Proposition 4 confirms this result in the case of 'guilt' investments which, as formulated above, indeed increases a worker's disutility from taking up an employment contract and thus tightens her participation constraint. Note that the principal uses guilt investments and financial rewards as complements in spite of the fact that they are substitutes in the agent's choice of effort.

3.6 Comparative Statics

Next, we consider how the optimal contract is affected by a number of other factors: the agent's outside option, taxation and career benefits.

Outside Option: Before presenting the formal results, we first provide some intuition about how changes in the agent's outside option would affect the optimal contract. An increase in $\underline{u}$ would, other things equal, lead to an increase in $\bar{b}$. A higher financial reward for success lowers the efficacy of guilt investments to induce effort (because $\frac{\partial^2 \hat{e}}{\partial \psi \partial b} < 0$ by Lemma 1).[8] The increase in $\bar{b}$ also reduces the increase in net profits due to any increment in effort (i.e. a reduction in $\left(A - \bar{b}\right)$); and the level of effort – and thus the cost of any additional financial compensation due to guilt investments ($\frac{\partial \bar{b}}{\partial \psi} \hat{e}\left(\bar{b}, e_c, \psi\right)$) – is higher. Taking all these arguments together, we must have $\frac{\partial^2 \bar{\Pi}}{\partial \psi \partial \underline{u}} < 0$. Therefore, applying Topkis' theorem, guilt investments would decline as the agent's outside option improves.

The increase in $\bar{b}$ mentioned above is a *ceteres paribus* statement. But we can show that it also holds in equilibrium. Intuitively, as guilt investments decline, financial rewards are more effective in increasing effort. Moreover, as the agent exerts lower effort when there is less guilt investment, the marginal cost of financial reward is lower. Therefore, $\frac{\partial^2 \bar{\Pi}}{\partial b \partial \underline{u}} > 0$. Applying Topkis' theorem, financial rewards increase as the agent's outside option improves. Formally, we have the following result.

Proposition 5 *If the participation constraint binds and the principal makes positive guilt investments, then an improvement in the agent's outside option (i)* *<u>decreases</u>* *guilt investments, and (ii)* *<u>increases</u>* *the financial rewards for success.*

As shown in Section 3.3, the agent's choice of effort is increasing in both the level of financial rewards and the level of motivational investments. As the former is increasing, and the latter is decreasing, in the agent's outside option, the overall effect of an improvement in the agent's outside option on the level of effort is ambiguous.

Taxation: Suppose that the principal's net profits are taxed at a rate τ. Then the principal's objective function becomes

$$\begin{aligned} \Pi\left(b, e_c, \psi; \tau, \mu\right) &= (1-\tau)\,\hat{e}\left(b, e_c, \psi\right)(A-b) - \frac{1}{2}\mu\psi^2 \\ &= (1-\tau)\left\{\hat{e}\left(b, e_c, \psi\right)(A-b) - \frac{1}{2}\left(\frac{\mu}{1-\tau}\right)\psi^2\right\} \qquad (29) \end{aligned}$$

[8] An improvement in the outside option also means that less financial compensation is needed for any guilt investments (because $\frac{\partial^2 \bar{b}}{\partial \psi \partial \underline{u}} < 0$ by Lemma 2), i.e. guilt investments are less costly, but we can show that the efficacy of guilt investments declines by even more.

Let $\tilde{\mu}(\tau) = \left(\frac{\mu}{1-\tau}\right)$. Note that the term $(1-\tau)$ outside the curly brackets in (29) does not affect the principal's optimal choice because it is a linear, monotonic transformation of the orignal objective function. Thus,

$$\arg\max_{b,e_c,\psi} \Pi\left(b, e_c, \psi; \tau, \mu\right) \text{ s.t. } (21) = \arg\max_{b,e_c,\psi} \Pi\left(b, e_c, \psi; 0, \tilde{\mu}(\tau)\right) \text{ s.t. } (21)$$

If the participation constraint is non-binding and the principal makes positive guilt investments, then an increase in μ leads to an increase in the financial reward for success, as stated in Proposition 3, in addition to a decrease in guilt investments. By construction, $\tilde{\mu}(\tau)$ is increasing in τ. Therefore, an increase in τ also leads to an increase in the financial reward for success and a decrease in guilt investments. As financial rewards and guilt investments are moving in opposite directions, the effect on effort is ambiguous.

If the participation constraint is binding and the principal makes positive guilt investments, then an increase in μ leads to a decrease in the financial reward for success and a reduction in the agent's level of effort, as stated in Proposition 4, in addition to a decrease in guilt investments. Therefore, if the agent's participation constraint is binding, an increase in τ also leads to a decrease in the financial reward for success, agent's level of effort and guilt investments.

We summarise these results as follows.

Proposition 6 *Suppose the principal's net profits are taxed at a rate τ. If the principal makes positive guilt investments then an increase in the tax rate τ leads to a decrease in motivational investments whether or not the agent's participation constraint is binding. Furthermore, it leads to:*

(i) an <u>increase</u> in the financial reward for success when the agent's participation constraint is non-binding but the effect on effort is ambiguous.

(ii) a <u>decrease</u> in the financial reward for success and a <u>decrease</u> in the agent's choice of effort when the agent's participation constraint is binding.

Career Benefits: Suppose that, in addition to the financial rewards for success, there are benefits associated with the job that are exogenously determined and not contingent on performance. These could include career benefits that are accessible to the worker even if they have low (measured) job performance. As far as the agent's participation constraint is concerned, an increase (decrease) in non-contigent benefits associated with the job is equivalent to a decrease (increase) in the agent's outside option. Therefore, the comparative statics results involving the outside option in Proposition 5 can be applied directly. Specifically, we have the following results.

Proposition 7 *Suppose that there are exogenously defined benefits, not contingent on performance, associated with the job.*

(i) If the agent's participation constraint does not bind, then a change in these benefits has no effect on the optimal contract.

(ii) If the agent's participation constraint is binding, and the principal makes positive guilt investments, then an increase in these benefits increases guilt investments and decreases financial rewards.

4 Motivating Agent by Inspiration

In this section, we present another case of our general model in Section 2. In contrast to the model of guilt investments, in this case motivational investments will decrease the disutility of work as well as lower the marginal cost of effort. Although this formulation has previously been explored in the literature, the following exercise will allow a direct comparison of the optimal contract with the preceding case, particularly when the agent's participation constraint is binding.

4.1 Setup

As before, the agent provides effort $e \in [0, 1]$ at cost $\frac{1}{2\psi}e^2$. This produces output $A \in (0, 1)$ with probability e and output zero otherwise. The principal observes output but not effort.

A contract between the principal and the agent specifies a 'bonus' (or 'reward for success') b in case the agent produces positive output. As with the previous models, we assume there is limited liability such that the principal cannot extract payments from the agent. We can represent a contract by the 2-tuple (b, ψ) satisfying the conditions $b, \psi \geq 0$.[9] If the agent chooses not to accept the contract, he obtains a reservation utility of $\underline{u}$. The agent's expected utility from the contract is given by

$$U(b, e, \psi) = be - \frac{1}{2\psi}e^2 \tag{30}$$

The principal can make investments to raise the agent's 'motivation', represented by $\psi \geq 0$. Drawing on Kvaløy and Schottner (2015), we interpret the parameter ψ as investments by an organisation in leaders or mentors who can inspire workers in a way that lowers the agent's disutility from effort. Achieving a level of motivation ψ requires an investment equal to $\frac{1}{2}\mu\psi^2$. In the absence of any investments, $\psi = 0$. We make the following assumptions about the model parameters.

Assumption 5 $A \in (0, 1)$

Assumption 6 $\mu \in (0, \infty)$

The justification for Assumptions 5 and 6 are the same as for Assumptions 1 and 2 in Section 3.1. We proceed with analysing the model in the same manner as in Section 3. In Section 4.2, we determine the agent's choice of effort for a given contract (b, ψ) and investigate how the agent's effort level responds to changes in financial incentives and motivational investments. Then, we investigate how changes in the contract affects the agent's expected utility and, thus, her participation constraint. In Section 4.4, we solve the Principal's profit maximisation problem to derive the optimal contract using the agent's effort function and her expected utility from a given contract.

4.2 Agent's Effort Choice

The agent solves the following optimization problem:

$$\max_{e \in [0,1]} be - \frac{1}{2\psi}e^2 \tag{31}$$

[9] As in the previous model, this contractual form implies that the agent does not receive any financial payment if output equals 0. See footnote 4 for the rationale behind this approach.

The coefficient of e^2 in the maximand in (31) is negative. Therefore, the agent's optimisation problem has a unique solution. Assuming an interior solution, we obtain e from the first-order condition:

$$b - \frac{e}{\psi} = 0$$
$$\Longrightarrow e = \psi b \tag{32}$$

We denote this solution by $\hat{e}(b, \psi)$. Using (32), it is straightforward to establish the following results:

Lemma 3 *The responsiveness of the agent's optimal choice of effort to financial rewards is increasing in the level of motivational investments, i.e.* $\frac{\partial^2 \hat{e}}{\partial b \partial \psi} > 0$*; the responsiveness of the agent's optimal choice of effort to motivational investments is constant in the level of motivational investments, i.e.* $\frac{\partial^2 \hat{e}}{\partial \psi^2} = 0$

4.3 Agent's Participation Constraint

The agent's indirect utility from a contract (b, ψ) is given by

$$V(b, \psi) = U(b, \hat{e}(b, \psi), \psi)$$

$$\begin{aligned} &= b(\psi b) - \frac{1}{2\psi}(b\psi)^2 \\ &= \frac{1}{2} b^2 \psi \end{aligned}$$

We define $\bar{b}(\psi, \underline{u})$ as the level of bonus for which – given ψ – the agent obtains a reservation utility of $\underline{u}$; i.e.

$$\frac{1}{2} b^2 \psi = \underline{u} \tag{33}$$

Rearranging (33), we obtain

$$\bar{b}^2 = \frac{2\underline{u}}{\psi} \tag{34}$$

Differentiating throughout (34) w.r.t. ψ, we obtain

$$2\bar{b}\frac{\partial \bar{b}}{\partial \psi} = -\frac{2\underline{u}}{\psi^2} \tag{35}$$

$$\Longrightarrow \frac{\partial \bar{b}}{\partial \psi} = -\frac{\underline{u}}{\bar{b}\psi^2} < 0 \tag{36}$$

Thus, motivational investments reduce the need for financial rewards to satisfy the participation constraint. Differentiating throughout (34) w.r.t. $\underline{u}$, we obtain

$$2\bar{b}\frac{\partial \bar{b}}{\partial \underline{u}} = \frac{2}{\psi}$$

$$\Longrightarrow \frac{\partial \bar{b}}{\partial \underline{u}} = \frac{1}{\bar{b}\psi} > 0$$

Thus, as expected, a stronger outside option increases the financial rewards required to satisfy the participation constraint. Using (35), we can also establish the following results.

Lemma 4 *When the agent's participant constraint binds, the financial reward for success* $\bar{b}(e_c, \psi, \underline{u})$ *is (i) decreasing in motivatonal investment* ψ *at a decreasing rate, i.e.* $\frac{\partial \bar{b}}{\partial \psi} < 0$ *and* $\frac{\partial^2 \bar{b}}{\partial \psi^2} > 0$*; (ii) is increasing in the agent's outside option* $\underline{u}$ *at a decreasing rate, i.e.* $\frac{\partial \bar{b}}{\partial \underline{u}} > 0$ *and* $\frac{\partial^2 \bar{b}}{\partial \psi \partial \underline{u}} < 0$.

Thus, when the agent has a binding participation constraint, a better outside option increases the amount by which financial rewards can be reduced when there are additional motivational investments; additionally, at higher levels of motivational investments, the smaller is the amount by which financial rewards can be reduced following an increment in motivational investments.

4.4 Optimal Contract

The principal's expected profits are given by

$$\Pi(b, \psi) = \hat{e}(b, \psi)(A - b) - \frac{1}{2}\mu\psi^2$$

To maximise profits, the principal solves

$$\max_{b,\psi} \hat{e}(b, \psi)(A - b) - \frac{1}{2}\mu\psi^2 \tag{37}$$

subject to

$$V(b, \psi) \geq \underline{u} \tag{38}$$

Non-Binding Participation Constraint: First we analye the case in which the agent's participation constraint is non-binding. For $\underline{u}$ sufficiently low, the participation constraint does not bind. Then the maximisation problem in (37)-(38) becomes

$$\max_{b,\psi} \Pi(b,\psi)$$

We obtain the following first-order conditions:

$$\frac{\partial \Pi(b,\psi)}{\partial b} = \frac{\partial \hat{e}}{\partial b}(A-b) - \hat{e}(b,\psi) = 0 \quad (39)$$

$$\frac{\partial \Pi(b,\psi)}{\partial \psi} = \frac{\partial \hat{e}}{\partial \psi}(A-b) - \mu\psi \quad (40)$$

Substituting for $\frac{\partial \hat{e}}{\partial b}$ and $\hat{e}(b,\psi)$ in (39), we obtain

$$\begin{aligned} \frac{\partial \Pi(b,\psi)}{\partial b} &= \psi(A-b) - b\psi = 0 \\ &= \psi(A-2b) = 0 \quad (41) \end{aligned}$$

$$\Longrightarrow b = \frac{A}{2}$$

Substituting for $\frac{\partial \hat{e}}{\partial \psi}$ and $\hat{e}(b,\psi)$ in (40), we obtain

$$\frac{\partial \Pi(b,\psi)}{\partial \psi} = b(A-b) - \mu\psi = 0 \quad (42)$$

$$\Longrightarrow \psi = \frac{b(A-b)}{\mu} = \frac{A^2}{4\mu}$$

Therefore, motivational investments are decreasing in μ (as we would expect) while financial rewards are independent of μ. Thus motivational investments and financial rewards are neither complements, nor substitutes. It follows from (32) that the agent's effort level is also decreasing in μ. Formally, we state these results as follows.

Proposition 8 *If the agent's participation constraint is non-binding, then financial rewards are neither a substitute nor a complement of motivational investments: an increase in the cost of motivation (μ) has <u>no effect</u> on the financial reward for success although it <u>reduces</u> motivational investments and effort goes down.*

It is evident from the equation for the optimal choice of effort (32) that financial rewards and motivational investments are complements in eliciting the agent's effort. Therefore, this case is covered by Proposition 1(ii) describing the case of a slack participation constraint in Section 2. But while we obtain an ambiguous result for the general model, the additional structure we introduce in this section enables an explicit statement about how access to motivational investments affects the use of financial rewards by the principal in eliciting agent effort. For this particular model, it does not but, more significantly, we will see in the next section that this relationship changes when the agent's participation constraint binds.

Binding Participation Constraint: Next, we provide a partial characterisation of the case in which the agent's participation constraint is binding. Using the function $\bar{b}(\psi, \underline{u})$ – defined implicitly by (33) – we can rewrite the optimisation problem in (37)-(38) as

$$\max_{\psi} \hat{e}\left(\bar{b}, \psi\right)\left(A - \bar{b}\right) - \mu\psi^2 \tag{43}$$

If the maximisation problem has an interior solution, then ψ is given by the following first-order condition:

$$\psi : \left(\frac{\partial \hat{e}}{\partial \psi} + \frac{\partial \hat{e}}{\partial b}\frac{\partial \bar{b}}{\partial \psi}\right)\left(A - \bar{b}\right) - \hat{e}\left(\bar{b}, \psi\right)\frac{\partial \bar{b}}{\partial \psi} - \mu\psi = 0 \tag{44}$$

Intuitively, increasing motivational investment ψ increases effort. Because the participation constraint is binding, the increase in motivation means that the participation constraint can be satisfied for a lower level of financial reward. This reduction in financial rewards is captured by the term $\frac{\partial \bar{b}}{\partial \psi} < 0$. The decrease in financial rewards decreases effort (captured by the term $\frac{\partial \hat{e}}{\partial b}\frac{\partial \bar{b}}{\partial \psi}$) but it also means lower payment whenever the agent generates high output (captured by the term $\hat{e}(b, \psi)\frac{\partial \bar{b}}{\partial \psi}$). Rearranging (44), we obtain

$$\frac{\partial \hat{e}}{\partial \psi}\left(A - \bar{b}\right) + \frac{\partial \bar{b}}{\partial \psi}\left\{\frac{\partial \hat{e}}{\partial b}\left(A - \bar{b}\right) - \hat{e}\left(\bar{b}, \psi\right)\right\} = \mu\psi \tag{45}$$

Note that the expression within the curly brackets in (45) is identical to the right-hand side of (39). Therefore, the expression is equal to the marginal effect of increasing the financial reward on the principal's expected profits. Therefore, if the participation constraint is binding, it must be zero or negative (becaue if it were positive, then the principal could increase expected profits by increasing b above $\bar{b}(\psi, \underline{u})$). As $\frac{\partial \bar{b}}{\partial \psi} < 0$, it follows that, at the optimum, we must have

$$\frac{\partial \hat{e}}{\partial \psi}\left(A - \bar{b}\right) < \mu\psi \tag{46}$$

Therefore, when the agent's participation constraint is binding, the marginal effect of motivational investments on the principal's expected profits is, in equilibrium, smaller than the marginal cost of this type of investment. This is the opposite of the case shown in Section 3.5 where motivational investments take the form of guilt.

Next, we address the question whether the principal will use guilt investments as a complement or a substitute of financial rewards in eliciting the agent's effort. For this purpose, we consider, as in our analysis of the previous model, how a change in μ, the cost of guilt investments, affects the principal's decisions. Intuitively, at any given level of motivation ψ, an increase in μ increases the marginal cost of motivational investments ($\mu\psi$). The increase in μ has no direct effect on the marginal benefit of motivational investments. Therefore, ψ would have to adjust to ensure that the first-order condition is satisfied. We can show that the maximand in (43) ($\hat{e}\left(\bar{b}, \psi\right)\left(A - \bar{b}\right) - \frac{1}{2}\mu\psi^2$) is globally concave in ψ.[10] Therefore, if we lower ψ, this leads to an increase in $\frac{\partial \Pi}{\partial \psi}$. Therefore, when there is an increase in the marginal cost of motivational investments, the level of motivational investments will go down. Then the principal would increase financial rewards to ensure that the agent's participation constraint is still satisfied. Therefore, financial rewards and motivational investments will go in opposite directions, i.e. they are substitutes. Although financial rewards and motivational investments go in opposite directions, we can show that the agent's effort level will go down, i.e. the effect of motivational investments will dominate. Formally, we have the following result.

[10] We can see this as follows. Substituting for $\hat{e}\left(\bar{b}, \psi\right)$ and $\bar{b}$ in the expression, we obtain

$$\begin{aligned} & \psi\bar{b}\left(A - \bar{b}\right) - \frac{1}{2}\mu\psi^2 \\ = \; & A\psi\left(\frac{2\underline{u}}{\psi}\right)^{\frac{1}{2}} - \psi\left(\frac{2\underline{u}}{\psi}\right) - \frac{1}{2}\mu\psi^2 \\ = \; & A\psi^{\frac{1}{2}}\left(2\underline{u}\right)^{\frac{1}{2}} - 2\underline{u} - \frac{1}{2}\mu\psi^2 \end{aligned}$$

It is clear that this last expression is globally concave in ψ.

Proposition 9 *If the participation constraint binds and the principal makes positive motivational investments, then (i) the principal will use motivational investments and financial rewards as substitutes, (ii) the agent's level of effort is* <u>*decreasing*</u> *in the cost of motivational investments.*

Proposition 9 is the equivalent of Proposition 2 when motivational investments involve 'inspiring' workers, thus lowering their cost of effort. Proposition 2 implies that when motivational investments relax the agent's participation constraint, the principal will use motivational investments and financial rewards as substitutes. Proposition 9 confirms this result in the case of motivation through 'inspiration' which, as formulated above, indeed decreases a worker's disutility from taking up an employment contract and thus relaxes her participation constraint.

4.5 Comparative Statics

In this section we present a number of other comparative statics results; specifically, how the optimal contract is affected by the agent's outside option, taxation and career benefits.

Outside Option: We start by providing some intuition for how an improvement in the agent's outside option affects the optimal contract. An increase in $\underline{u}$ would, other things equal, lead to an increase in $\bar{b}$. A higher financial reward for success increases the efficacy of motivational investments to induce effort (since $\frac{\partial^2 \hat{e}}{\partial \psi \partial b} > 0$ by Lemma 3). In addition, an improvement in the outside option means that financial rewards can be reduced by even more when there is an increase in motivational investments (since $\frac{\partial^2 \bar{b}}{\partial \psi \partial \underline{u}} < 0$ by Lemma 4). Taking these arguments together, we must have $\frac{\partial^2 \bar{\Pi}}{\partial \psi \partial \underline{u}} > 0$. Therefore, applying Topkis' theorem, motivational investments are increasing in the agent's outside option. An increase in the outside option will also mean that the agent is provided higher financial rewards for success to induce her to take up the contract. Higher motivational investments and financial rewards will increase the agent's effort level. Formally, we have the following result.

Proposition 10 *If the participation constraint binds and the principal makes positive motivational investments, then an improvement in the agent's outside option (i)* <u>*increases*</u> *mo-*

tivational investments, (ii) <u>increases</u> the financial reward for success, and (iii) <u>increases</u> the agent's level of effort.

Taxation: As before, suppose that the principal's net profits are taxed at a rate τ. Then the principal's objective function becomes

$$\begin{aligned} \Pi\left(b, e_c, \psi; \tau, \mu\right) &= (1-\tau)\, \hat{e}\left(b, \psi\right)(A-b) - \frac{1}{2}\mu\psi^2 \\ &= (1-\tau)\left\{ \hat{e}\left(b, \psi\right)(A-b) - \frac{1}{2}\left(\frac{\mu}{1-\tau}\right)\psi^2 \right\} \end{aligned} \tag{47}$$

Let $\tilde{\mu}(\tau) = \left(\frac{\mu}{1-\tau}\right)$. Note that the term $(1-\tau)$ outside the curly brackets in (29) does not affect the principal's optimal choice because it is a linear, monotonic transformation of the orignal objective function. Thus,

$$\arg\max_{b,\psi} \Pi\left(b, \psi\right) \text{ s.t. } (38) = \arg\max_{b,e_c,\psi} \Pi\left(b, \psi; 0, \tilde{\mu}\left(\tau\right)\right) \text{ s.t. } (38)$$

If the participation constraint is non-binding and the principal makes motivational investments, then an increase in μ has no effect on the financial reward for success, as stated in Proposition 8, and leads to a reduction in motivational investments and in the agent's level of effort. Since $\tilde{\mu}(\tau)$ is increasing in τ, an increase in τ is equivalent to an increase in μ in the original model. Therefore, an increase in τ also has no effect on the financial reward for success but lowers motivational investments and the agent's level of effort.

By Proposition 9, if the participation constraint is binding and the principal makes motivational investments, then an increase in μ leads to an increase in the financial reward for success and a decrease in the agent's choice of effort. Therefore, if the agent's participation constraint is binding, an increase in τ also leads to an increase in the financial reward for success and a decrease in the agent's level of effort.

Proposition 11 *Suppose the principal's net profits are taxed at a rate τ.*

(i) If the principal makes positive motivational investments and the agent's participation constraint is non-binding, then an increase in the tax rate τ has <u>no effect</u> on the financial reward for success but <u>decreases</u> motivational investments and the agent's level of effort.

(ii) If the principal makes positive motivational investments and the agent's participation constraint is binding, then an increase in the tax rate τ leads to an <u>increase</u> in the financial reward for success, a <u>decrease</u> in motivational investments and a <u>decrease</u> in the agent's level of effort .

Career Benefits: Next, we consider again the effects of a change in (exogenous and non-contingent) benefits associated with the job. As before, an increase (decrease) in non-contigent benefits associated with the job is equivalent to a decrease (increase) in the agent's outside option. Therefore, the comparative statics results involving the outside option can be applied directly. We have the following results.

Proposition 12 *Suppose that there are exogenously defined benefits, not contingent on performance, associated with the job.*

(i) If the agent's participation constraint does not bind, then a change in these benefits has no effect on the optimal contract.

(ii) If the agent's participation constraint is binding, and the principal makes positive motivational investments, then an increase in these benefits decreases motivational investments, financial rewards for success and the agent's level of effort.

5 Conclusion

In this paper, we use the term "motivational investments" to describe a broad range of activities that firms and other types of organisations can undertake to incentivise workers, including management and leadership training, team-based exercises, communication with workers about broader organizational goals aimed at raising the morale, team-spirit and loyalty of the workforce. It is well-known from the existing literature that organisations may use motivational investments either as a substitute or a complement of financial incentives to induce workers to exert more effort. Our focus in the paper has been on how the worker's outside opportunities affect an organisation's choice of motivational investments and financial incentives.

We model two types of motivational investments to explore this question. In the first model, motivational investments increase the agent's disutility from deviating from a level of effort specified in the labour contract (which we call 'investing in guilt'). In the second model, motivational investments lower the agent's cost of effort (which we call 'investing in inspiration')

The key insight to emerge from our analysis is that the worker's outside option is a key determinant of whether motivational investments and financial incentives are used as complements or substitutes in the optimal employment contract. The reason is that motivational investments affect not only the worker's effort level but also overall job satisfaction. Some forms of motivational investments can make the work seem more enjoyable and thus increase job satisfaction. Other forms may elicit effort by exerting 'pressure' on the worker and thus lower job satisfaction. We are agnostic about the type of motivational investment that an employer would choose: this choice ultimately depends on the availability and cost of different technologies for motivational investments. But, in both cases, the fact that motivational investments affect job satisfaction means that financial incentives play a dual role: to elicit the agent's effort and to ensure that the agent's participation constraint is satisfied. If the worker's outside option is sufficiently strong such that her participation constraint is binding, an increase in guilt investments (due, for example, to a decrease in the cost of such investments) is accompanied by a compensatory increase in financial incentives; while motivational investments that lower the agent's cost of effort are accompanied by a reduction in financial incentives.

Our theoretical results imply that the tightness of the labour market is an important factor in determining whether organsations use motivational investments as a substitute or complement of financial incentives. In particular, our approach can yield testable predictions about the motivational investments and financial incentives over the business cycle, for workers with different levels of human capital, and across different industries.

6 Appendix

6.1 Appendix A: General Model

In this subsection of the Appendix, we present results relating to the general model presented in Section 2. First, we consider the case where the agent's participation constraint is non-binding. If the function $\Pi(b, \psi)$ is concave in (b, ψ)[11] and there is an interior solution to the optimisation problem, the contract is fully characterised by the first-order conditions:

$$\frac{\partial \hat{e}}{\partial b}(A-b) - \hat{e}(b, \psi) = 0 \tag{48}$$
$$\frac{\partial \hat{e}}{\partial \psi}(A-b) - \mu h'(\psi) = 0. \tag{49}$$

We are interested in whether financial rewards and motivational investments are used as substitutes or complements by the primcipal (alternatively, whether the level of financial rewards go up or down when the cost of motivational investments increase). Propositon 1, stated in Section 2, describes this property. The proof of the proposition is provided below.

Proof. of Proposition 1: First, we establish the conditions under which the function $\Pi(b, \psi)$ is strictly concave. The Hessian of $\Pi(b, \psi)$ is given by

$$D^2\Pi(b, \psi) = \begin{bmatrix} \left(\frac{\partial^2 \hat{e}}{\partial b^2}\right)(A-b) - 2\frac{\partial \hat{e}}{\partial b} & \left(\frac{\partial^2 \hat{e}}{\partial b \partial \psi}\right)(A-b) - \frac{\partial \hat{e}}{\partial \psi} \\ \left(\frac{\partial^2 \hat{e}}{\partial \psi \partial b}\right)(A-b) - \frac{\partial \hat{e}}{\partial \psi} & \left(\frac{\partial^2 \hat{e}}{\partial \psi^2}\right)(A-b) - \mu h''(\psi) \end{bmatrix}$$

Strict concavity requires that the Hessian is negative definite. Therefore, we need

$$\frac{\partial^2 \Pi}{\partial b^2} = \left(\frac{\partial^2 \hat{e}}{\partial b^2}\right)(A-b) - 2\frac{\partial \hat{e}}{\partial b} < 0 \tag{50}$$

$$K = \left\{\left(\frac{\partial^2 \hat{e}}{\partial b^2}\right)(A-b) - 2\frac{\partial \hat{e}}{\partial b}\right\}\left\{\left(\frac{\partial^2 \hat{e}}{\partial \psi^2}\right)(A-b) - \mu h''(\psi)\right\} - \left\{\left(\frac{\partial^2 \hat{e}}{\partial \psi \partial b}\right)(A-b) - \frac{\partial \hat{e}}{\partial \psi}\right\}^2 > 0 \tag{51}$$

[11] The condition we need to ensure strict concavity is that the Hessian of the function $\Pi(b, \psi)$ is negative definite. The precise condition in terms of the model primitives are provided in the proof of Proposition 1.

for $b \in [0, A]$. Using the expressions for $\frac{\partial \hat{e}}{\partial b}$ and $\frac{\partial \hat{e}}{\partial \psi}$ in (5), we obtain

$$
\begin{aligned}
\frac{\partial^2 \hat{e}}{\partial b^2} &= -\frac{C_{eee}}{(C_{ee})^3} && (52)\\
\frac{\partial^2 \hat{e}}{\partial b \partial \psi} &= -\frac{1}{(C_{ee})^2}\left(C_{ee\psi} - \frac{C_{eee}C_{e\psi}}{C_{ee}}\right) && (53)\\
\frac{\partial^2 \hat{e}}{\partial \psi^2} &= -\frac{1}{C_{ee}}\left\{C_{e\psi\psi} - \frac{C_{e\psi e}C_{e\psi}}{C_{ee}} + C_{eee}\left(\frac{C_{e\psi}}{C_{ee}}\right)^2\right\}. && (54)
\end{aligned}
$$

Substituting in (50) and (51) using (5) and (52)-(54), we obtain sufficient conditions on the model primitives to ensure strict concavity of the function $\Pi(b, \psi)$.[12]

Using the Implicit Function Theorem,[13] we obtain

$$
\begin{aligned}
\begin{bmatrix} \frac{\partial \hat{b}}{\partial \mu} \\ \frac{\partial \hat{\psi}}{\partial \mu} \end{bmatrix} &= -\begin{bmatrix} \left(\frac{\partial^2 \hat{e}}{\partial b^2}\right)(A-b) - 2\frac{\partial \hat{e}}{\partial b} & \left(\frac{\partial^2 \hat{e}}{\partial b \partial \psi}\right)(A-b) - \frac{\partial \hat{e}}{\partial \psi} \\ \left(\frac{\partial^2 \hat{e}}{\partial \psi \partial b}\right)(A-b) - \frac{\partial \hat{e}}{\partial \psi} & \left(\frac{\partial^2 \hat{e}}{\partial \psi^2}\right)(A-b) - \mu h''(\psi) \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -h'(\psi) \end{bmatrix} \\
&= -\frac{1}{K}\begin{bmatrix} \left(\frac{\partial^2 \hat{e}}{\partial \psi^2}\right)(A-b) - \mu h''(\psi) & -\left(\frac{\partial^2 \hat{e}}{\partial b \partial \psi}\right)(A-b) + \frac{\partial \hat{e}}{\partial \psi} \\ -\left(\frac{\partial^2 \hat{e}}{\partial \psi \partial b}\right)(A-b) + \frac{\partial \hat{e}}{\partial \psi} & \left(\frac{\partial^2 \hat{e}}{\partial b^2}\right)(A-b) - 2\frac{\partial \hat{e}}{\partial b} \end{bmatrix} \begin{bmatrix} 0 \\ -h'(\psi) \end{bmatrix}
\end{aligned}
$$

Therefore, we have

$$
\begin{aligned}
\frac{\partial \hat{b}}{\partial \mu} &= \frac{1}{K}h'(\psi)\left\{-\left(\frac{\partial^2 \hat{e}}{\partial b \partial \psi}\right)(A-b) + \frac{\partial \hat{e}}{\partial \psi}\right\} && (55)\\
\frac{\partial \hat{\psi}}{\partial \mu} &= \frac{1}{K}h'(\psi)\left\{\left(\frac{\partial^2 \hat{e}}{\partial b^2}\right)(A-b) - 2\frac{\partial \hat{e}}{\partial b}\right\} && (56)
\end{aligned}
$$

By assumption, the function $\Pi(b, \psi)$ is strictly concave. Therefore, as shown above, we must have $K > 0$ and $\frac{\partial^2 \Pi}{\partial b^2} < 0$.

Therefore, the expression on the right-hand side of (56) is negative. Thus $\hat{\psi}$ is decreasing in μ. Furthermore, if $\frac{\partial^2 \hat{e}}{\partial b \partial \psi} \leq 0$, then $\hat{b}$ is increasing in μ; but if $\frac{\partial^2 \hat{e}}{\partial b \partial \psi} < 0$, then $\hat{b}$ is potentially decreasing in μ. ■

6.2 Appendix B: Model of Motivation through Guilt

In this subsection of the Appendix, we present proofs of results stated in Section 3.

[12] We can show that for any function $C(e, \psi)$ satisfying $\frac{\partial^3 C}{\partial e^3} = 0$, the condition for strict concavity is always satisfied if $\mu h''(\psi)$ is sufficiently large.

[13] See Mas-Colell, Whinston and Green (1995), Theorem M.E.1 .

Proof. of Lemma 2: (i) Rearranging (19), we obtain

$$\bar{b}^2 + \psi e_c \left(2\bar{b} - e_c\right) = 2\underline{u}\left(1 + \psi\right) \tag{57}$$

Differentiating throughout (57) w.r.t. $\underline{u}$, we obain

$$2\bar{b}\frac{\partial \bar{b}}{\partial \underline{u}} + 2\psi e_c \frac{\partial \bar{b}}{\partial \underline{u}} = 2\left(1 + \psi\right)$$

$$\implies \frac{\partial \bar{b}}{\partial \underline{u}} = \frac{1 + \psi}{\bar{b} + \psi e_c} > 0 \tag{58}$$

Differentiating throughout (57) w.r.t. ψ, we obtain

$$2\bar{b}\frac{\partial \bar{b}}{\partial \psi} + e_c\left(2\bar{b} - e_c\right) + 2\psi e_c \frac{\partial \bar{b}}{\partial \psi} = 2\underline{u} \tag{59}$$

$$\implies \frac{\partial \bar{b}}{\partial \psi} = \frac{2\underline{u} + e_c\left(e_c - 2\bar{b}\right)}{2\left(\bar{b} + \psi e_c\right)} \tag{60}$$

Then, substituting for $\underline{u}$ in (60) using (19), we obtain[14]

$$\frac{\partial \bar{b}}{\partial \psi} = \frac{\left(\bar{b} - e_c\right)^2}{2\left(\bar{b} + \psi e_c\right)\left(1 + \psi\right)} \tag{61}$$

If $\psi > 0$, we have must $\bar{b} < e_c$ (otherwise, guilt either lowers the agent's effort or has no effect on effort; and so the principal is better-off setting $\psi = 0$). Hence, we have $\frac{\partial \bar{b}}{\partial \psi} > 0$.Note that the right-hand side of (61) is decreasing in ψ. Therefore, $\frac{\partial \bar{b}}{\partial \psi}$ is decreasing in ψ, i.e. $\frac{\partial^2 \bar{b}}{\partial \psi^2} < 0$.

[14] The intermediary steps are as follow. Substituting for $\underline{u}$ in (60), we obtain

$$\begin{aligned}
\frac{\partial \bar{b}}{\partial \psi} &= \frac{1}{2\left(\bar{b} + \psi e_c\right)}\left\{\frac{\left(\bar{b}^2 + 2\psi\bar{b}e_c - \psi e_c^2\right)}{\left(1 + \psi\right)} + e_c\left(e_c - 2\bar{b}\right)\right\} \\
&= \frac{1}{2\left(\bar{b} + \psi e_c\right)\left(1 + \psi\right)}\left\{\left(\bar{b}^2 + 2\psi\bar{b}e_c - \psi e_c^2\right) + e_c\left(e_c - 2\bar{b}\right)\left(1 + \psi\right)\right\} \\
&= \frac{1}{2\left(\bar{b} + \psi e_c\right)\left(1 + \psi\right)}\left\{\left(\bar{b}^2 + 2\psi\bar{b}e_c - \psi e_c^2\right) + e_c\left(e_c + e_c\psi - 2\bar{b} - 2\bar{b}\psi\right)\right\}
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2\left(\bar{b} + \psi e_c\right)\left(1 + \psi\right)}\left\{\left(\bar{b}^2 + 2\psi\bar{b}e_c - \psi e_c^2\right) + e_c^2 + e_c^2\psi - 2\bar{b}e_c - 2\bar{b}\psi e_c\right\} \\
&= \frac{\left(\bar{b}^2 + e_c^2 - 2\bar{b}e_c\right)}{2\left(\bar{b} + \psi e_c\right)\left(1 + \psi\right)} = \frac{\left(\bar{b} - e_c\right)^2}{2\left(\bar{b} + \psi e_c\right)\left(1 + \psi\right)}
\end{aligned}$$

(ii) We have shown above (58) that $\frac{\partial\bar{b}}{\partial\underline{u}} > 0$. Differentiating throughout (61) w.r.t. $\underline{u}$, we obtain

$$\frac{\partial^2\bar{b}}{\partial\psi\partial\underline{u}} = \frac{\partial\bar{b}}{\partial\underline{u}}\frac{d}{db}\left\{\frac{\left(\bar{b}-e_c\right)^2}{2\left(\bar{b}+\psi e_c\right)(1+\psi)}\right\}$$

$$= \frac{\partial\bar{b}}{\partial\underline{u}}\left[\frac{2\left(\bar{b}-e_c\right)}{2\left(\bar{b}+\psi e_c\right)(1+\psi)} - \frac{\left(\bar{b}-e_c\right)^2}{2(1+\psi)\left(\bar{b}+\psi e_c\right)^2}\right] \tag{62}$$

Simplifying the expression within the square brackets in (62) and substituting for $\frac{\partial\bar{b}}{\partial\underline{u}}$ using (58), we obtain[15]

$$\frac{\partial^2\bar{b}}{\partial\psi\partial\underline{u}} = \frac{1}{2}\frac{\left(\bar{b}-e_c\right)\left\{\bar{b}+2\psi e_c+e_c\right\}}{\left(\bar{b}+\psi e_c\right)^3}$$

Therefore, if $e_c > \bar{b}$ (as reasoned in the proof of part (i)), then $\frac{\partial^2\bar{b}}{\partial\psi\partial\underline{u}} < 0$. ■

Proof. of Proposition 5: Let us denote by $\bar{\Pi}\left(e_c,\psi,\underline{u},\mu\right)$ the maximand in (26). Then, we have

$$\frac{\partial\bar{\Pi}}{\partial\psi} = \frac{\partial\hat{e}}{\partial\psi}\left(A-\bar{b}\right) + \frac{\partial\bar{b}}{\partial\psi}\left\{\frac{\partial\hat{e}}{\partial b}\left(A-\bar{b}\right) - \hat{e}\left(\bar{b},e_c,\psi\right)\right\} - \mu\psi \tag{63}$$

(i) Substituting for $\frac{\partial\hat{e}}{\partial\psi}$ using (16), for $\frac{\partial\bar{b}}{\partial\psi}$ using (61), for $\frac{\partial\hat{e}}{\partial b}$ using (17), and for $\hat{e}\left(\bar{b},e_c,\psi\right)$ using (14) in (63), we obtain

$$\frac{\partial\bar{\Pi}}{\partial\psi} = \frac{\left(A-\bar{b}\right)\left(e_c-\bar{b}\right)}{(1+\psi)^2} + \frac{\left(A-\bar{b}\right)}{\left(\bar{b}+\psi e_c\right)}\left\{\frac{\left(\bar{b}-e_c\right)^2}{2(1+\psi)^2}\right\} - \mu\psi - \frac{\left(\bar{b}-e_c\right)^2}{2(1+\psi)^2} \tag{64}$$

Then, using $e_h = A$ (Assumption (4)) in the expression above and simplifying and rearrang-

[15] We can simplify the expression in (62) as follows:

$$\frac{\partial^2\bar{b}}{\partial\psi\partial\underline{u}} = \frac{\partial\bar{b}}{\partial\underline{u}}\left\{\frac{\left(\bar{b}-e_c\right)}{\left(\bar{b}+\psi e_c\right)(1+\psi)}\right\}\left\{1-\frac{\left(\bar{b}-e_c\right)}{2\left(\bar{b}+\psi e_c\right)}\right\}$$

$$= \frac{\partial\bar{b}}{\partial\underline{u}}\left\{\frac{\left(\bar{b}-e_c\right)}{\left(\bar{b}+\psi e_c\right)(1+\psi)}\right\}\left\{\frac{2\left(\bar{b}+\psi e_c\right)-\left(\bar{b}-e_c\right)}{2\left(\bar{b}+\psi e_c\right)}\right\}$$

$$= \frac{\partial\bar{b}}{\partial\underline{u}}\left\{\frac{\left(\bar{b}-e_c\right)}{\left(\bar{b}+\psi e_c\right)(1+\psi)}\right\}\left\{\frac{\bar{b}+2\psi e_c+e_c}{2\left(\bar{b}+\psi e_c\right)}\right\}$$

$$= \frac{1}{2}\frac{\left(\bar{b}-e_c\right)\left\{\bar{b}+2\psi e_c+e_c\right\}}{\left(\bar{b}+\psi e_c\right)^3}$$

ing terms, we obtain

$$
\begin{aligned}
\frac{\partial \bar{\Pi}}{\partial \psi} &= \frac{\left(e_c - \bar{b}\right)^3}{2\left(1+\psi\right)^2\left(\bar{b}+\psi e_c\right)} + \frac{\left(e_c - \bar{b}\right)^2}{2\left(1+\psi\right)^2} - \mu\psi \\
&= \frac{\left(e_c - \bar{b}\right)^2}{2\left(1+\psi\right)^2}\left\{\frac{\left(e_c - \bar{b}\right)}{\left(\bar{b}+\psi e_c\right)} + 1\right\} - \mu\psi \qquad (65)
\end{aligned}
$$

It is clear that, if $e_c > \bar{b}$, then this last expression is decreasing in $\bar{b}$. Therefore, $\frac{\partial^2 \bar{\Pi}}{\partial \psi \partial \underline{u}} < 0$. Since $\frac{\partial^2 \bar{\Pi}}{\partial \psi \partial \underline{u}} < 0$, we can apply Topkis' Theorem (Theorem 2.8.1 in Topkis 1998) to show that ψ is decreasing in $\underline{u}$.

(ii) To investigate the effect of increasing $\underline{u}$ on b, we define $\overline{\psi}\left(b, e_c, \underline{u}\right)$ as the level of motivational investment that – given $b, e_c, \underline{u}$ – cause the participation constraint to hold with equality. Using (18) and Assumption 4, we can write

$$
\frac{1}{2}\frac{\left(b^2 + 2\overline{\psi}bA - \overline{\psi}A^2\right)}{\left(1+\overline{\psi}\right)} = \underline{u}
$$

Rearranging terms, we obtain

$$
\overline{\psi} = \frac{b^2 - 2\underline{u}}{A^2 + 2\underline{u} - 2bA} \qquad (66)
$$

By assumption, $\overline{\psi} \geq 0$. Therefore,

$$
\frac{b^2 - 2\underline{u}}{A^2 + 2\underline{u} - 2bA} \geq 0 \qquad (67)
$$

Then, we can show that $b^2 - 2\underline{u} \geq 0$ and $A^2 + 2\underline{u} - 2bA > 0$.[16] Differentiating throughout (66) with respect to b and rearranging terms, we obtain

$$
\frac{\partial \overline{\psi}}{\partial b} = \frac{4b\underline{u}}{\left(A^2 + 2\underline{u} - 2bA\right)^2} \qquad (68)
$$

Differentiating throughout (68) with respect to $\underline{u}$ and rearranging terms, we obtain

$$
\frac{\partial^2 \overline{\psi}}{\partial b \partial \underline{u}} = \frac{4b\left(A^2 + 6\underline{u} - 2bA\right)}{\left(A^2 + 2\underline{u} - 2bA\right)^3} \qquad (69)
$$

[16] A proof-by-contradiction for this last statement is as follows. The only other way in which (67) can be satisfied is if $b^2 - 2\underline{u} \leq 0$ and $A^2 + 2\underline{u} - 2bA < 0$. Combining these two inequalities, we obtain

$$
A^2 + b^2 - 2bA < 0
$$

$$
\Longrightarrow (A-b)^2 < 0
$$

But $A - b \geq 0$ under profit maximisation. This contradicts the last inequality above.

Since $A^2 + 2\underline{u} - 2bA > 0$, it follows that $\frac{\partial \overline{\psi}}{\partial b} > 0$ and $\frac{\partial^2 \overline{\psi}}{\partial b \partial \underline{u}} > 0$. Differentiating (68) w.r.t. b, we obtain

$$\begin{aligned} \frac{\partial^2 \overline{\psi}}{\partial b^2} &= \frac{4\underline{u}}{\left(A^2 + 2\underline{u} - 2bA\right)^2} + \frac{(-2)\, 4b\underline{u}\,(-2A)}{\left(A^2 + 2\underline{u} - 2bA\right)^3} \\ &= \frac{4\underline{u}\left(A^2 + 2\underline{u} + 2bA\right)}{\left(A^2 + 2\underline{u} - 2bA\right)^3} > 0 \end{aligned}$$

Using $\overline{\psi}(b, e_c, \underline{u})$, we can rewrite the principal's optimisation problem as follows:

$$\max \hat{e}\left(b, e_c, \overline{\psi}\right)(A - b) - \frac{1}{2}\mu\overline{\psi}^2 \tag{70}$$

We denote the maximand in (70) as $\widetilde{\Pi}(b, e_c, \underline{u})$. The first-order condition to (70) can be written as

$$\frac{\partial \widetilde{\Pi}}{\partial b} = \left(\frac{\partial \hat{e}}{\partial b} + \frac{\partial \hat{e}}{\partial \psi}\frac{\partial \overline{\psi}}{\partial b}\right)(A - b) - \hat{e}\left(b, e_c, \overline{\psi}\right) - \mu\overline{\psi}\frac{\partial \overline{\psi}}{\partial b} = 0 \tag{71}$$

Let us denote by $b^*(\underline{u})$ the solution to (71) when the agent's outside option is $\underline{u}$. Then, using the Implicit Function Theorem, we obtain

$$\frac{\partial b^*(\underline{u})}{\partial \underline{u}} = -\frac{\partial^2 \widetilde{\Pi}(b, e_h, \underline{u})}{\partial b \partial \underline{u}} / \frac{\partial^2 \widetilde{\Pi}(b, e_h, \underline{u})}{\partial b^2} \tag{72}$$

Differentiating throughout (71) with respect to $\underline{u}$, we obtain

$$\begin{aligned} \frac{\partial^2 \widetilde{\Pi}(b, e_h, \underline{u})}{\partial b \partial \underline{u}} &= \left(\frac{\partial^2 \hat{e}}{\partial b \partial \psi}\frac{\partial \overline{\psi}}{\partial \underline{u}} + \frac{\partial^2 \hat{e}}{\partial \psi^2}\frac{\partial \overline{\psi}}{\partial \underline{u}}\frac{\partial \overline{\psi}}{\partial b} + \frac{\partial \hat{e}}{\partial \psi}\frac{\partial^2 \overline{\psi}}{\partial b \partial \underline{u}}\right)(A - b) - \frac{\partial \hat{e}}{\partial \psi}\frac{\partial \overline{\psi}}{\partial \underline{u}} - \mu\left(\frac{\partial \overline{\psi}}{\partial \underline{u}}\frac{\partial \overline{\psi}}{\partial b} + \overline{\psi}\frac{\partial^2 \overline{\psi}}{\partial b \partial \underline{u}}\right) \\ &= \left(\frac{\partial^2 \hat{e}}{\partial b \partial \psi}\frac{\partial \overline{\psi}}{\partial \underline{u}} + \frac{\partial^2 \hat{e}}{\partial \psi^2}\frac{\partial \overline{\psi}}{\partial \underline{u}}\frac{\partial \overline{\psi}}{\partial b}\right)(A - b) - \frac{\partial \hat{e}}{\partial \psi}\frac{\partial \overline{\psi}}{\partial \underline{u}} - \mu\frac{\partial \overline{\psi}}{\partial \underline{u}}\frac{\partial \overline{\psi}}{\partial b} + \left\{\frac{\partial \hat{e}}{\partial \psi}(A - b) - \mu\overline{\psi}\right\}\frac{\partial^2 \overline{\psi}}{\partial b \partial \underline{u}} \end{aligned}$$

Since $\frac{\partial \overline{\psi}}{\partial \underline{u}} < 0$, $\frac{\partial^2 \hat{e}}{\partial b \partial \psi} < 0$ and $\frac{\partial^2 \hat{e}}{\partial \psi^2} < 0$ (Lemma 1), $\frac{\partial \hat{e}}{\partial \psi} > 0$ and $\frac{\partial \overline{\psi}}{\partial b} > 0$ (see Section 3.3), all the terms within the first parentheses, and the terms $-\frac{\partial \hat{e}}{\partial \psi}\frac{\partial \overline{\psi}}{\partial \underline{u}}$ and $-\mu\frac{\partial \overline{\psi}}{\partial \underline{u}}\frac{\partial \overline{\psi}}{\partial b}$ are positive. Furthermore, $\frac{\partial^2 \overline{\psi}}{\partial b \partial \underline{u}} > 0$ (shown above). The only remaining term is that within the curly brackets. Suppose it is negative, i.e.

$$\frac{\partial \hat{e}}{\partial \psi}(A - b) - \mu\overline{\psi} < 0$$

Then the principal can increase expected profits by lowering guilt investments. Doing so would relax the participation constraint. This contradicts the original premise that the participation constraint is binding. Therefore, we must have

$$\frac{\partial^2 \widetilde{\Pi}(b, e_h, \underline{u})}{\partial b \partial \underline{u}}\Big|_{b = b^*(\underline{u})} > 0$$

Next, considering the denominator of (72), we must have local concavity at the optimum. If not, the principal can increase profits by increasing the financial reward, which would imply that the choice of financial reward $b = b^* (\underline{u})$ is not optimal.[17] Therefore,

$$\frac{\partial^2 \widetilde{\Pi} (b, e_h, \underline{u})}{\partial b^2}|_{b=b^*(\underline{u})} < 0$$

Then, it follows from (72) that $\frac{\partial b^*(\underline{u})}{\partial \underline{u}} > 0$, i.e. the level of financial reward is increasing in the agent's outside option. ■

Proof. of Proposition 4: Using (65), we can write

$$\frac{\partial \bar{\Pi}}{\partial \psi} = h\left(\psi, \bar{b}\left(e_c, \psi, \underline{u}\right)\right)$$

where the function $h(.)$ is defined implicitly by the right-hand side of (65). The, using the Chain Rule, we obtain

$$\frac{\partial^2 \bar{\Pi}}{\partial \psi^2} = \frac{\partial h}{\partial \psi} + \frac{\partial h}{\partial b}\frac{\partial \bar{b}}{\partial \psi} \tag{73}$$

It is evident from (65) that, if $e_c > \bar{b}$, then $\frac{\partial h}{\partial \psi} < 0$ and $\frac{\partial h}{\partial b} < 0$. Also, we have shown previously that $\frac{\partial \bar{b}}{\partial \psi} > 0$. It follows from (73) that $\frac{\partial^2 \bar{\Pi}}{\partial \psi^2} < 0$.[18] Using (63), the first-order condition w.r.t. ψ can be written as

$$\frac{\partial \hat{e}}{\partial \psi}\left(A - \bar{b}\right) + \frac{\partial \bar{b}}{\partial \psi}\left\{\frac{\partial \hat{e}}{\partial b}\left(A - \bar{b}\right) - \hat{e}\left(\bar{b}, e_h, \psi\right)\right\} - \mu\psi = 0$$

Then, the Implicit Function Theorem implies that

$$\frac{\partial \hat{e}\left(\bar{b}, e_h, \psi(\mu)\right)}{\partial \psi}\left(A - \bar{b}\right) + \frac{\partial \bar{b}}{\partial \psi}\left\{\frac{\partial \hat{e}\left(\bar{b}, e_h, \psi(\mu)\right)}{\partial b}\left(A - \bar{b}\right) - \hat{e}\left(\bar{b}, e_h, \psi(\mu)\right)\right\} - \mu\psi(\mu) \equiv 0$$

Differentiating throughout the equation above w.r.t. μ, we obtain

$$\frac{\partial \psi}{\partial \mu}\left[\begin{array}{c}\left\{\frac{\partial^2 \hat{e}}{\partial \psi^2}\left(A - \bar{b}\right) - \frac{\partial \hat{e}}{\partial \psi}\frac{\partial \bar{b}}{\partial \psi} + \frac{\partial^2 \bar{b}}{\partial \psi^2}\left(\frac{\partial \hat{e}}{\partial b}\left(A - \bar{b}\right) - \hat{e}\left(\bar{b}, e_c, \psi\right)\right)\right\} \\ +\frac{\partial \bar{b}}{\partial \psi}\left\{\frac{\partial^2 \hat{e}}{\partial b \partial \psi}\left(A - \bar{b}\right) - \frac{\partial \hat{e}}{\partial b}\frac{\partial \bar{b}}{\partial \psi} - \frac{\partial \hat{e}}{\partial b}\frac{\partial \bar{b}}{\partial \psi} - \frac{\partial \hat{e}}{\partial \psi}\right\}\end{array}\right] - \mu\frac{\partial \psi}{\partial \mu} - \psi(\mu) \equiv 0 \tag{75}$$

[17] Additionally, if $b^* (\underline{u}) > 0$, we must have an interior solution because the principal makes zero or negative profits for $b \geq A$.

[18] Alternative proof: Differentiating throughout (63) w.r.t. ψ, we obtain

$$\begin{aligned}\frac{\partial^2 \bar{\Pi}}{\partial \psi^2} &= \left[\frac{\partial^2 \hat{e}}{\partial \psi^2}\left(A - \bar{b}\right) - \frac{\partial \hat{e}}{\partial \psi}\frac{\partial \bar{b}}{\partial \psi} + \frac{\partial^2 \bar{b}}{\partial \psi^2}\left\{\frac{\partial \hat{e}}{\partial b}\left(A - \bar{b}\right) - \hat{e}\left(\bar{b}, e_c, \psi\right)\right\}\right] \\ &\quad + \frac{\partial \bar{b}}{\partial \psi}\left[\frac{\partial^2 \hat{e}}{\partial b \partial \psi}\left(A - \bar{b}\right) - \frac{\partial \hat{e}}{\partial b}\frac{\partial \bar{b}}{\partial \psi} - \frac{\partial \hat{e}}{\partial b}\frac{\partial \bar{b}}{\partial \psi} - \frac{\partial \hat{e}}{\partial \psi}\right] - \mu \end{aligned} \tag{74}$$

Since $\frac{\partial^2 \hat{e}}{\partial \psi^2} < 0$, $\frac{\partial \bar{b}}{\partial \psi} > 0$, $\frac{\partial^2 \bar{b}}{\partial \psi^2} < 0$, $\frac{\partial \hat{e}}{\partial \psi} > 0$, $\frac{\partial \hat{e}}{\partial b} > 0$ and $\frac{\partial^2 \hat{e}}{\partial b \partial \psi} < 0$, it follows that $\frac{\partial^2 \bar{\Pi}}{\partial \psi^2} < 0$.

Note that the term within the square brackets on the left-hand side of (75) is identical to the right-hand side of (74). Therefore, based on the reasoning above, the term within the square brackets is negative. Therefore, we must have $\frac{\partial\psi}{\partial\mu} < 0$ to satisfy (75), i.e. ψ is decreasing in μ. Since $\bar{b}$ is increasing in ψ, it follows that $\bar{b}$ is decreasing in μ. Since $\hat{e}\left(\bar{b}, e_c, \psi\right)$ is increasing in $\bar{b}$ and ψ, it follows that effort is decreasing in μ. ■

6.3 Appendix C: Model of Motivation through Inspiration

In this subset of the Appendix, we present proofs of results stated in Section 4.

Proof. of Lemma 4: (i) Differentiating throughout (35) w.r.t. $\underline{u}$, we obtain

$$2\frac{\partial\bar{b}}{\partial\underline{u}}\frac{\partial\bar{b}}{\partial\psi} + 2\bar{b}\frac{\partial^2\bar{b}}{\partial\psi\partial\underline{u}} = -\frac{2}{\psi^2}$$

$$\Longrightarrow \bar{b}\frac{\partial^2\bar{b}}{\partial\psi\partial\underline{u}} = -\left(\frac{1}{\psi^2} + \frac{\partial\bar{b}}{\partial\underline{u}}\frac{\partial\bar{b}}{\partial\psi}\right)$$

$$\begin{aligned}
\Longrightarrow \quad \frac{\partial^2\bar{b}}{\partial\psi\partial\underline{u}} &= -\left(\frac{1}{\psi^2} + \frac{\partial\bar{b}}{\partial\underline{u}}\frac{\partial\bar{b}}{\partial\psi}\right)/\bar{b} \\
&= -\left(\frac{1}{\psi^2} - \frac{1}{\bar{b}\psi}\frac{\underline{u}}{\bar{b}\psi^2}\right)/\bar{b} = -\left\{\frac{1}{\psi^2} - \frac{\underline{u}}{\left(\bar{b}\right)^2\psi^3}\right\}/\bar{b} \\
&= -\left\{\frac{1}{\psi^2} - \frac{\underline{u}}{\left(\frac{2\underline{u}}{\psi}\right)\psi^3}\right\}/\bar{b} = -\left(\frac{1}{\psi^2} - \frac{1}{2\psi^2}\right)/\bar{b} \\
&= -\frac{1}{2\psi^2\bar{b}} < 0
\end{aligned}$$

(ii) Differentiating throughout (35) w.r.t. ψ, we obtain

$$\frac{\partial\bar{b}}{\partial\psi}\frac{\partial\bar{b}}{\partial\psi} + \bar{b}\frac{\partial^2\bar{b}}{\partial\psi^2} = \frac{2\underline{u}}{\psi^3}$$

$$\begin{aligned}
\Longrightarrow \quad \frac{\partial^2\bar{b}}{\partial\psi^2} &= \frac{1}{\bar{b}}\left\{\frac{2\underline{u}}{\psi^3} - \left(\frac{\partial\bar{b}}{\partial\psi}\right)^2\right\} \\
&= \frac{1}{\bar{b}}\left\{\frac{2\underline{u}}{\psi^3} - \left(\frac{\underline{u}}{\bar{b}\psi^2}\right)^2\right\} = \frac{1}{\bar{b}}\left\{\frac{2\underline{u}}{\psi^3} - \frac{\underline{u}^2}{\left(\frac{2\underline{u}}{\psi}\right)\psi^4}\right\} \\
&= \frac{1}{\bar{b}}\left\{\frac{2\underline{u}}{\psi^3} - \frac{\underline{u}}{2\psi^3}\right\} = \frac{\underline{u}}{\bar{b}\psi^3}\left(2 - \frac{1}{2}\right) > 0
\end{aligned}$$

■

Proof. of Proposition 10: We denote by $\bar{\Pi}(b,\psi)$ the maximand in (43). Using (44), when the agent's participation constraint is binding, the marginal effect of increasing motivational investments can be written as

$$\frac{\partial\bar{\Pi}}{\partial\psi} = \frac{\partial\hat{e}}{\partial\psi}\left(A-\bar{b}\right) + \frac{\partial\bar{b}}{\partial\psi}\left\{\frac{\partial\hat{e}}{\partial b}\left(A-\bar{b}\right) - \hat{e}\left(\bar{b},\psi\right)\right\} - \mu\psi \tag{76}$$

(i) We can write (76) as

$$\begin{aligned}\frac{\partial\bar{\Pi}}{\partial\psi} &= \bar{b}\left(A-\bar{b}\right) + \left(-\frac{\underline{u}}{\bar{b}\psi^2}\right)\left\{\psi\left(A-\bar{b}\right) - \bar{b}\psi\right\} - \mu\psi \\ &= \bar{b}\left(A-\bar{b}\right) - \left(\frac{\underline{u}}{\bar{b}\psi}\right)\left(A-2\bar{b}\right) - \mu\psi\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{\partial^2\bar{\Pi}}{\partial\psi\partial\underline{u}} &= \frac{\partial\bar{b}}{\partial\underline{u}}\left(A-2\bar{b}\right) - \frac{1}{\psi}\left[\left\{\frac{1}{\bar{b}} - \frac{\underline{u}}{\left(\bar{b}\right)^2}\frac{\partial\bar{b}}{\partial\underline{u}}\right\}\left(A-2\bar{b}\right) + \left(\frac{\underline{u}}{\bar{b}}\right)\left(-2\frac{1}{\bar{b}\psi}\right)\right] \\ &= \frac{\partial\bar{b}}{\partial\underline{u}}\left(A-2\bar{b}\right) - \frac{1}{\psi}\left[\frac{1}{\bar{b}}\left(1-\frac{\underline{u}}{\bar{b}^2\psi}\right)\left(A-2\bar{b}\right) - 2\left(\frac{\underline{u}}{\bar{b}^2\psi}\right)\right] \\ &= \frac{1}{\bar{b}\psi}\left(A-2\bar{b}\right) - \frac{1}{\psi}\left[\frac{1}{2\bar{b}}\left(A-2\bar{b}\right) - 1\right] = \frac{1}{\psi}\left[\frac{1}{\bar{b}}\left(A-2\bar{b}\right) - \frac{1}{2\bar{b}}\left(A-2\bar{b}\right) + 1\right] \\ &= \frac{1}{\psi}\left[\frac{1}{2\bar{b}}\left(A-2\bar{b}\right) + 1\right] = \frac{1}{\psi}\left(\frac{A}{2\bar{b}} - 1 + 1\right) \\ &= \frac{A}{2\bar{b}\psi} > 0\end{aligned}$$

Since $\frac{\partial^2\bar{\Pi}}{\partial\psi\partial\underline{u}} > 0$, we can apply Topkis' Theorem (Theorem 2.8.1 in Topkis 1998) to show that ψ is increasing in $\underline{u}$.

(ii) To investigate the effect of increasing $\underline{u}$ on b, we define $\overline{\psi}(b,\underline{u})$ as the level of motivational investment that – given $b,\underline{u}$ – cause the participation constraint to hold with equality. Using (33), we can write

$$\overline{\psi}(b,\underline{u}) = \frac{2\underline{u}}{b^2} \tag{77}$$

Using $\overline{\psi}(b,\underline{u})$, we can rewrite the principal's optimisation problem as follows:

$$\max \hat{e}\left(b,\overline{\psi}\right)(A-b) - \frac{1}{2}\mu\overline{\psi}^2 \tag{78}$$

We denote the maximand in (78) as $\widetilde{\Pi}(b, \underline{u})$. Therefore, we have

$$\begin{aligned} \frac{\partial \widetilde{\Pi}}{\partial b} &= \left(\frac{\partial \hat{e}}{\partial b} + \frac{\partial \hat{e}}{\partial \psi}\frac{\partial \overline{\psi}}{\partial b}\right)(A - b) - \hat{e}\left(b, \overline{\psi}\right) - \mu\overline{\psi}\left(-\frac{4\underline{u}}{b^3}\right) \\ &= \left(\frac{\partial \hat{e}}{\partial b} + \frac{\partial \hat{e}}{\partial \psi}\frac{\partial \overline{\psi}}{\partial b}\right)(A - b) - \hat{e}\left(b, \overline{\psi}\right) + 4\frac{\mu\overline{\psi}\underline{u}}{b^3} \end{aligned} \tag{79}$$

Recall from (32) that $\hat{e}\left(b, \overline{\psi}\right) = b\psi \implies \frac{\partial \hat{e}}{\partial b} = \psi$, $\frac{\partial \hat{e}}{\partial \psi} = b$. Also, from (77), $\frac{\partial \overline{\psi}(b, \underline{u})}{\partial b} = -\frac{4\underline{u}}{b^3}$. Substituting using these expressions in (79), we have

$$\begin{aligned} \frac{\partial \widetilde{\Pi}}{\partial b} &= \left(\overline{\psi} - \frac{4\underline{u}}{b^2}\right)(A - b) - b\overline{\psi} + 8\frac{\mu\underline{u}^2}{b^5} \\ &= \left(\frac{2\underline{u}}{b^2} - \frac{4\underline{u}}{b^2}\right)(A - b) - b\frac{2\underline{u}}{b^2} + 8\frac{\mu\underline{u}^2}{b^5} \\ &= -\frac{2\underline{u}}{b^2}(A - b) - b\frac{2\underline{u}}{b^2} + 8\frac{\mu\underline{u}^2}{b^5} \\ &= -\frac{2\underline{u}A}{b^2} + \frac{8\mu\underline{u}^2}{b^5} \end{aligned} \tag{80}$$

Using (15), the first-order condition for the optimisation problem in (78) can be written as

$$-\frac{2\underline{u}A}{b^2} + \frac{8\mu\underline{u}^2}{b^5} = 0$$

$$\begin{aligned} &\implies \frac{1}{b^2}\left(-2\underline{u}A + \frac{8\mu\underline{u}^2}{b^3}\right) = 0 \\ &\implies \frac{8\mu\underline{u}^2}{b^3} = 2\underline{u}A \\ &\implies b = \left(\frac{4\mu\underline{u}}{A}\right)^{\frac{1}{3}} \end{aligned} \tag{81}$$

Therefore, b is increasing in the outside option $\underline{u}$.

(iii) Since ψ and b are both increasing in $\underline{u}$ and the agent's optimal level of effort is increasing in ψ and b, it follows that the level of effort is also increasing in the outside option. ■

Proof. of Proposition 9: Substituting for $\hat{e}\left(\bar{b}, \psi\right)$ and $\bar{b}$ in the maximand, we obtain

$$A\psi^{\frac{1}{2}}\left(2\underline{u}\right)^{\frac{1}{2}} - 2\underline{u} - \frac{1}{2}\mu\psi^2$$

Then, the first-order condition w.r.t. ψ can be written as

$$\frac{1}{2}A\psi^{-\frac{1}{2}}\left(2\underline{u}\right)^{\frac{1}{2}} - \mu\psi = 0$$

Using the Implicit Function Theorem, we obtain

$$\frac{1}{2}A\left\{\psi\left(\mu\right)\right\}^{-\frac{1}{2}}\left(2\underline{u}\right)^{\frac{1}{2}}-\mu\psi\left(\mu\right)=0$$

Differentiating throughout w.r.t. μ, we obtain

$$\frac{1}{2}A\left(2\underline{u}\right)^{\frac{1}{2}}\left(-\frac{1}{2}\right)\left\{\psi\left(\mu\right)\right\}^{-\frac{3}{2}}\frac{\partial\psi}{\partial\mu}-\psi\left(\mu\right)-\mu\frac{\partial\psi}{\partial\mu}=0$$

$$\Longrightarrow\quad \frac{\partial\psi}{\partial\mu}\left[-\frac{1}{4}A\left(2\underline{u}\right)^{\frac{1}{2}}\left\{\psi\left(\mu\right)\right\}^{-\frac{3}{2}}-\mu\right]=\psi\left(\mu\right)$$

$$\Longrightarrow\quad \frac{\partial\psi}{\partial\mu}=-\psi\left(\mu\right)\left[\frac{1}{4}A\left(2\underline{u}\right)^{\frac{1}{2}}\left\{\psi\left(\mu\right)\right\}^{-\frac{3}{2}}+\mu\right]^{-1}<0$$

Since $\bar{b}$ is decreasing in ψ, it follows that $\bar{b}$ will increase, i.e. the principal substitutes away from motivational investments towards financial rewards to ensure that the agent's participation constraint is satisfied.

The overall effect on the agent's effort will be given by

$$\frac{d}{d\psi}\hat{e}\left(\bar{b},\psi\right)$$

$$\begin{aligned}
&= \frac{\partial\hat{e}}{\partial b}\frac{\partial\bar{b}}{\partial\psi}+\frac{\partial\hat{e}}{\partial\psi}\\
&= \psi\left(-\frac{\underline{u}}{\bar{b}\psi^{2}}\right)+\bar{b}\\
&= -\frac{\underline{u}}{\bar{b}\psi}+\bar{b}=\frac{-\underline{u}+\left(\bar{b}\right)^{2}\psi}{\bar{b}\psi}\\
&= \frac{-\underline{u}+\frac{2\underline{u}}{\psi}\psi}{\bar{b}\psi}=\frac{-\underline{u}+2\underline{u}}{\bar{b}\psi}\\
&= \frac{\underline{u}}{\bar{b}\psi}>0
\end{aligned}$$

Therefore, the decrease in motivational investments will lead to decreased effort by the agent.

■

References

[1] Akerlof, George and Rachel Kranton (2005): "Identity and the Economics of Organizations", *Journal of Economic Perspectives*, 19(1), Winter.

[2] Akerlof, George A., and Rachel E. Kranton (2008). "Identity, Supervision, and Work Groups", *American Economic Review*, 98(2).

[3] Akerlof, George and Rachel Kranton (2011): *Identity Economics - How Our Identities Shape Our Work, Wages, and Well-Being*, Princeton University Press.

[4] Ashraf, Nava, Oriana Bandiera, and B. Kelsey Jack. "No margin, no mission? A field experiment on incentives for public service delivery" (2014). *Journal of Public Economics,* 120.

[5] Ashraf, N., Bandiera, O., Davenport, E. and Lee, S.S., 2020. Losing prosociality in the quest for talent? Sorting, selection, and productivity in the delivery of public services. *American Economic Review*, 110(5), pp.1355-94.

[6] Banerjee, A.V., Gertler, P.J. and Ghatak, M. (2002): "Empowerment and efficiency: Tenancy reform in West Bengal", *Journal of Political Economy*, 110(2), 239-280.

[7] Battigalli, P. and M. Dufwenberg (2007): "Guilt in Games", *American Economic Review, Papers and Proceedings*, 97(2), 170-176.

[8] Bénabou, R. and Tirole, J. (2003). Intrinsic and extrinsic motivation. *The Review of Economic Studies*, 70(3), pp.489-520.

[9] Benabou, Roland and Jean Tirole (2006): “Incentives and Prosocial Behavior”, *American Economic Review*, 96(5).

[10] Berg, E., Ghatak, M., Manjula, R., Rajasekhar, D. and Roy, S (2017): "Motivating knowledge agents: Can incentive pay overcome social distance?". *The Economic Journal*, 129(617).

[11] Besley, Timothy, and Maitreesh Ghatak (2005): "Competition and Incentives with Motivated Agents." *American Economic Review* 95(3), 616-636.

[12] Besley, Timothy, and Maitreesh Ghatak (2017): "Profit with Purpose? A Theory of Social Enterprise", *American Economic Journal - Economic Policy*, 9(3), 19-58.

[13] Besley, Timothy and Maitreesh Ghatak (2018): “Pro-social Motivation and Incentives”, Forthcoming, *Annual Review of Economics*, Volume 10.

[14] Charness, G. and M. Dufwenberg (2006): "Promises and Partnerships", *Econometrica*, 74(6), 1579-1601.

[15] Cassar, L. and S. Meier (2018). "Nonmonetary Incentives and the Implications of Work as a Source of Meaning", *Journal of Economic Perspectives*, 32, 215-238.

[16] Deserranno, Erika (2019). "Financial incentives as signals: experimental evidence from the recruitment of village promoters in Uganda." *American Economic Journal: Applied Economics* 11(1).

[17] Ellingsen, T. and Johannesson, M., (2008). "Pride and prejudice: The human side of incentive theory", *American Economic Review*, 98(3), pp.990-1008.

[18] Hart, O. and J. Moore (1999). "Foundations of Incomplete Contracts", *Review of Economic Studies*, 66, pp. 115-138.

[19] Herzberg, F 1968, ‘One more time : how do you motivate employees’, *Harvard Business Review*, vol. 46, no. 1, pp. 53-62.

[20] Kosfeld, M., Neckermann, S. and Yang, X. (2017). "The effects of financial and recognition incentives across work contexts: The role of meaning", *Economic Inquiry*, 55(1), pp.237-247.

[21] Kvaløy, O., Nieken, P. and Schöttner, A. (2015). "Hidden benefits of reward: A field experiment on motivation and monetary incentives", *European Economic Review*, 76, pp.188-199.

[22] Kvaløy, Ola and Anja Schöttner (2015): “Incentives to motivate”, *Journal of Economic Behavior & Organization*, 116(C), 26-42.

[23] Mas-Colell, A., Whinston, M.D. and Green, J.R. (1995). *Microeconomic Theory*. New York: Oxford University Press.

[24] Muralidharan K, Sundararaman V. 2011: "Teacher performance pay: experimental evidence from India". *Journal of Political Economy*. 119(1).

[25] Nicholson, N (2003), 'How to Motivate your Problem People', *Harvard Business Review*, vol. 81, no. 1, pp. 56-65.

[26] Rasul, I. and Rogger, D., (2018). "Management of bureaucrats and public service delivery: Evidence from the Nigerian civil service". *The Economic Journal*, 128(608), pp.413-446.

[27] Thakor, A.V. and Quinn, R.E. (2013). "The economics of higher purpose", *ECGI-Finance Working Paper*, No. 395.

[28] Topkis, D.M. (1998). *Supermodularity and Complementarity*, Princeton University Press.