

# Occupational Choice and Dynamic Incentives

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We study an overlapping generations version of the principal-agent problem, where incentive contracts are determined in general equilibrium. All individuals are workers when young, but have a choice between becoming entrepreneurs or remaining workers when old. Imperfections in the credit market give rise to rents in entrepreneurial activities involving capital. These rents motivate poor young agents to work hard and save to overcome the borrowing constraints. With a labour market that is subject to moral hazard, the increased effort raises social welfare. Policies that reduce credit market imperfections, or redistribute income, may reduce welfare by dampening this effect.

## 1. INTRODUCTION

An important insight in the development economics literature is that credit market imperfections can lead to poverty traps. As argued by Banerjee and Newman (1993, 1994), credit constrained individuals cannot enter profitable occupations with set-up costs, hence upward mobility is limited. But this gives an incomplete account of the incentives for credit constrained individuals to work and save. For example, Banerjee and Newman assume individuals live for one period only, and Aghion and Bolton (1997) assume entrepreneurs must invest at the beginning of their lives. Under these assumptions, there is no way to become an entrepreneur through hard work and thrift. In contrast, we argue that credit market imperfections give rise to rents in occupations involving set-up costs, and these rents may motivate poor individuals to work hard and save to overcome the borrowing constraints.

These conflicting views on economic mobility go well back in history. The address of Abraham Lincoln at the Wisconsin State Agricultural Society in 1859 captures well the idea of dynamic incentives to work and save: “The prudent, penniless beginner in the world labors for wages awhile, saves a surplus with which to buy tools or land for himself, then labors on his own account another while, and at length hires another new beginner to help him. This . . . is the just, and generous, and prosperous system, which opens the way for all, gives hope to all, and energy, and progress, and improvement in condition to all.” Others, such as Tocqueville, advocated a “mud-sill” theory, which is similar to the poverty trap view.

The “agricultural ladder” exemplifies the kind of economic environment we are interested in: “. . . a potential entrant began his career as a hired hand and through diligent work and wise spending, he accumulated sufficient funds to purchase a set of machinery. Subsequently the new entrant became a renter, then a part-owner of real estate, and finally the pinnacle of success was reached with full ownership of land and machinery” (Boehlje, 1973). Financial constraints are believed to be the key to this ladder phenomenon, and to the life-cycle pattern observed in the size of the farm (see Gale (1994)). Historically, a similar process was observed in small-scale crafts and manufacturing. Poor farm households, often working as hired labourers in peak seasons, saved their meager earnings to buy tools and set up small scale rural industries (see Kriedte *et al.* (1981)). In a recent study of a group of independent small-scale craft enterprises in rural Scotland from the middle of the 19th to the early 20th century, Young (1995) found that most of these craft producers were former workers who used savings from previous wage employment for start-up capital. They were unable to secure credit from banks or merchants because they could not offer any collateral. Another good example is businesses set up by immigrants in the U.S. In a recent study of nearly four hundred Korean business owners in modern day Chicago and Los Angeles, Yoon (1997) found that most of his respondents started in the U.S. as manual, service or sales workers. After accumulating capital (mainly through personal savings) they frequently bought the business from the existing owner. A survey of one hundred and fifty Asian businesses in Chicago in 1987-88 found that before starting a business most immigrants worked for others and the single most important source of capital was personal savings from wages earned in the labour market. Very few of these businesses received a bank loan, often alleging discriminatory practices by banks (see Engstrom and McCready (1990)).

The role of credit market constraints in limiting entry to entrepreneurship, and the importance of inter-class mobility, are well documented phenomena. Several empirical studies based on panel data from the U.S. and the U.K. have shown the importance of credit market constraints for potential entrepreneurs. Evans and Leighton (1989) found that men with greater assets were more likely to switch into entrepreneurship from wage-employment, other things being equal (see also Evans and Jovanovic (1989)). Liquidity constraints seem to be an important explanatory variable for entrepreneurship even after controlling for ability (see Blanchflower and Oswald (1998)). These studies also provide evidence of significant presence of individual transitions from worker to entrepreneur (see Evans and Leighton (1989) and Quadrini (1997)).

In this paper, we study an overlapping generations version of the principal-agent model that captures the type of mobility described above. Young agents are born without any wealth, and are matched with entrepreneurs (principals) who own productive assets. There is moral hazard in the principal-agent relationship, and the wage depends on a noisy measure of the effort supplied by the agent, just as in Holmström (1979). Young agents can invest in assets in order to become entrepreneurs when old. The demand and supply of labour in the future depends on the incentive and the ability of the current generation of young workers to borrow and save. The incentive to borrow and save depends on expectations of future labour contracts, and on the imperfections (transactions costs) in the credit market. The ability to save depends on current labour contracts. Since any young agent may borrow money to buy the asset, bank-financed entrepreneurs cannot earn rents in equilibrium. However, agents who earn a high wage when young can earn positive rents as *self-financed* entrepreneurs when old, which motivates young agents to work harder than they would if they were offered the same

labour contract in the standard static principal-agent model. The dynamic incentives for young agents to work hard and save in order to become self-financed entrepreneurs will be referred to as the *American Dream* effect. This captures the idea that “anybody can make it” through hard work, thrift and luck and provides a sharp contrast with the poverty trap view described earlier.<sup>1</sup> The more imperfect the credit market is, the greater are the entrepreneurial rents. A reduction of the imperfections leads to higher equilibrium wages and increased effort from old workers. But the fall in entrepreneurial rents reduces the shadow value of earnings for young workers, hence reducing their dynamic incentives to work and save. Just as suggested by the second best arguments of Lipsey and Lancaster (1956), the net effect on social welfare is ambiguous. Starting from a situation where the cost of capital is small and most (but not all) entrepreneurs are self-financed, reducing credit market imperfections may in fact reduce social welfare.

Our paper is closely related to the recent literature on occupational choice. Like Banerjee and Newman (1993, 1994), we consider the endogenous determination of returns to different occupations in the presence of imperfect credit markets and set-up costs.<sup>2</sup> In contrast to these papers, our model is driven by the *joint* presence of incentive problems in the labour market *and* imperfections in the credit market. Moreover, in our model all individuals are identical at birth, with no inherited wealth, and we focus on the *life-cycle* aspects of work incentives, savings and occupational choice. In Banerjee and Newman (1993), individuals live for one period only and the dynamics are driven by bequests. Credit markets are imperfect so unless an agent has some threshold level of wealth, he cannot enter into a profitable occupation. In Aghion and Bolton (1997), poor individuals have to borrow to invest, and debt overhang reduces entrepreneurial effort, but entrepreneurs invest at the beginning of their lives so there is no way to reduce the need for debt-financing by hard work and thrift.<sup>3</sup> Redistributing wealth, or reducing credit market imperfections, is efficiency enhancing in the Banerjee and Newman (1993) and Aghion and Bolton (1997) models.<sup>4</sup> In our model, *ex post* inequality motivates young agents to work hard, so redistributive policies will reduce the incentives to work hard and save, which may reduce social welfare. Similarly, removing the credit market imperfections reduces the *ex post* inequality, which discourages hard work. Another feature that distinguishes our paper is that in the occupational choice literature, contracting issues are under-emphasized. For example, in Banerjee and Newman (1993) workers are paid a fixed wage, and there is a monitoring technology that enables the entrepreneur to perfectly monitor the workers. In Aghion and Bolton (1997) and Piketty (1997) everyone is self-employed. In contrast, in our model, as in the standard one-period principal-agent model, optimal contracts are derived subject to incentive-compatibility and limited liability constraints. However, in contrast to the standard principal-agent model, in our model individuals make occupational choices, and the terms of labour market contracts are determined by the demand and supply of labour.

1. This is not the only possible interpretation of the *American Dream*. One interpretation would focus on heterogeneous ability, which is absent in our model.

2. For related contributions see Aghion and Bolton (1997), Galor and Zeira (1993), Legros and Newman (1996), Newman (1992), and Piketty (1997).

3. One can re-interpret our model in such a way that the cost of credit is a reduced form for the cost of entrepreneurial debt-overhang in the sense of Aghion and Bolton (1997). In this sense, our analysis is complementary to theirs.

4. A large theoretical literature explores the idea that with an imperfect credit market, the distribution of wealth across firms and investors have important consequences for real economic activity. See, for example, Holmström and Tirole (1997). But this literature does not address the issues of occupational choice and the life-cycle incentives of individuals to work hard and save.

Our paper is also related to the career concern model studied by Holmström (1999).<sup>5</sup> Holmström's workers differ in terms of ability which cannot be directly observed by the market. Since effort and ability are substitutes in production, young workers work hard because otherwise an unfavourable inference about ability will be drawn. Because of this "rat race" effect, young workers' effort levels can be inefficiently high.<sup>6</sup> Effort will be declining over time since the worker's type is revealed over time. In our model all agents are born identical. Successful young agents work hard, not with the aim of passing off as more able workers, but because they hope to become self-financed entrepreneurs. Old workers do not have these dynamic incentives because of their limited time horizon. This life-cycle pattern of effort supply in our model is similar to that in the career concerns literature. However, the mechanism leading to this feature, its efficiency properties, and the features of the resulting market equilibrium are very different. In our dynamic model, as in the standard one-period principal-agent model, the limited wealth of the agent and a limited liability constraint prevents the principal and agent from maximizing their joint surplus (and hence social surplus). The equilibrium effort levels will always be too low, and hence the fact that the American dream effect raises effort levels of young workers is always social welfare enhancing. The rents from self-financed entrepreneurship represent a net social gain from avoiding costly credit market transactions and does not signify an externality. Thus, the efficiency properties of our model are different from the career concerns model. Notice also that the fact that a young worker supplies extra effort raises the principal's profit. In equilibrium, the size of these profits determines how much extra effort the young worker will supply (since he hopes to become a principal in the future), which in turn determines the wages young workers receive in the competitive labour market. This interaction between equilibrium profit rates and wages is missing in the career concerns model where the worker does not expect to earn entrepreneurial profits in the future.

The paper is organized as follows. In Section 2 we introduce the model. In Section 3 we discuss some properties of equilibria. In Section 4, we prove existence and uniqueness of equilibria, and characterize different types of equilibria that exist for different parameter values. We focus our attention on the most interesting kind of equilibrium, where both bank- and self-financed entrepreneurs coexist. Section 5 discusses the role of economic policy. Section 6 explains how the credit market imperfection can be endogenized, and discusses the consequences of allowing bond posting. The final section contains a brief summary, and the Appendix contains technical proofs.

## 2. THE MODEL

### 2.1. *Set-up*

We consider an overlapping generations model. Risk-neutral individuals live and are productive for two periods. Thus, in each period there are two generations alive, "young" and "old". We normalize the population size of each cohort to unity. There are no bequests, so each individual is born without any wealth. Second period utility is discounted by a factor  $\delta \in (0, 1)$ .

5. See also Gibbons and Murphy (1992) and Long and Shimomura (1997).

6. Although in Holmström (1999) there is no asymmetric (as opposed to imperfect) information about worker ability, the main result is similar to the well-known fact that equilibria of signalling models are often characterized by socially inefficient over-investment in the signal.

Production requires three inputs: capital, labour and supervision. All agents are workers when young. Entrepreneurial activity involves a set-up cost,  $k$ , which an agent can incur at the end of the first period of his life. This can be thought of as the purchase of one unit of capital at a cost  $k$ . Paying this cost allows him to be a principal (or “entrepreneur”) rather than a worker when old.<sup>7</sup> Buying more than one unit of capital is not feasible, and buying a fraction of a unit is useless. The technology is fixed-coefficient type: a principal with one unit of capital can supervise at most  $n \geq 2$  identical projects, each operated by one worker. Here  $n$  is an exogenously given parameter, not a choice variable.<sup>8</sup> A principal together with his workers is called a firm. Investment is irreversible, so once installed the capital cannot be consumed. Capital perishes completely after one period. One possible interpretation is that the investment  $k$  is a cost of acquiring *human capital*, which makes it possible to perform the supervisory role of a principal.

An agent whose wealth is insufficient to pay the investment cost  $k$  out of his pocket can borrow money on the credit market, which is assumed to be exogenous to this economy. The deposit interest rate is fixed at  $r = 1/\delta - 1$ . The effective borrowing interest rate is higher, since each loan is subject to a transactions cost  $\gamma > 0$ . Here  $\gamma$  is exogenously given, and does not depend on the size of the loan.<sup>9</sup> We take the size of  $\gamma$  to be our measure of the imperfections in the credit market. (In Section 6.1 we will interpret  $\gamma$  as a cost of monitoring done by banks.) The bank-financed entrepreneur must give some fraction of his profits to the bank. For the bank to break even on average, the expected repayment for a loan of size  $b$  must be  $(1+r)b + \gamma = b/\delta + \gamma$ .<sup>10</sup>

The return from a project is  $y = 1$  with probability  $e$  and  $y = 0$  with probability  $1 - e$ , where  $e \in [0, 1]$  is the amount of unobservable *effort* supplied by the worker who operates the project. We refer to  $y = 1$  as a “success” and to  $y = 0$  as a “failure” of a project. A worker whose project succeeds is a “successful worker”. The worker’s disutility of effort is  $ce^2/2$ . Returns of projects within a firm are uncorrelated, so there is no gain from making a worker’s wage depend on the success or failure of other workers in the same firm. Let  $h$  denote the *success wage* paid to the worker when the return from his project is  $y = 1$ , and  $l$  the *failure wage* paid when  $y = 0$ . Even though workers are risk neutral, the presence of a limited liability constraint implies that the incentive problem cannot be solved costlessly. The limited liability constraint requires  $l \geq 0$  and  $h \geq 0$ . It is easy to see that  $l = 0$  is optimal.<sup>11</sup> Thus, from now on we set  $l = 0$ , and a labour market contract will simply be characterized by the success wage  $h \geq 0$ . The principal’s gain from the relationship with the worker is  $1 - h$  if the project succeeds, and zero if the project fails. Since the project succeeds with probability  $e$ , the expected gain is  $e(1 - h)$ . An *unemployed* worker earns a subsistence income normalized to 0. Since the current principal is an old individual who dies at the end of the period, it is not possible to use long-term labour contracts with young workers.

7. For simplicity, all agents who did not pay  $k$  when young remain workers when old: they are not allowed to buy capital from some other old agent who did invest. This assumption is clearly justified in the case of investment in human capital.

8. See the working paper version of this paper (Ghatak, Morelli and Sjöström (2000)) for the case  $n = 1$  which has some special properties.

9. In equilibrium, individuals will either borrow  $k$  or nothing. Thus, we could as well assume that the transactions cost is a non-decreasing function  $\gamma(b)$  of the size of the loan  $b$ , but the value of  $\gamma(b)$  for  $b \neq k$  would be irrelevant and the fixed  $\gamma$  would simply be replaced by  $\gamma(k)$ .

10. See Ghatak, Morelli and Sjöström (2000) for a more detailed discussion.

11. If  $h < 1$  and  $l > 0$ , then the agent will supply less than the joint surplus-maximizing amount of effort. Therefore, it is possible to reduce  $l$  and increase  $h$  in such a way that both the agent and the principal are better off (the agent works harder, so the joint surplus is increased). So  $h < 1$  and  $l > 0$  cannot be optimal. On the other hand, if  $h \geq 1$  and  $l > 0$  then the principal cannot break even. Thus, we conclude that  $l = 0$ .

To further simplify the exposition, the following assumptions on the parameters will be maintained throughout the paper.

*Assumption 1.*  $c > 1 + \delta\gamma$ .

*Assumption 2.*  $\gamma < (2n - 1)/8c - k/\delta$ .

*Assumption 3.*  $k < \frac{1}{2}$ .

Assumption 1 says that the cost of effort is not too low. This simply rules out corner solutions (the equilibrium effort levels lie strictly between 0 and 1). Assumption 2 requires that the transactions cost  $\gamma$  should not be too large. Assumption 2 is equivalent to the statement that in a situation of unemployment, where entrepreneurs are on the short side of the labour market (and in effect have all the bargaining power), becoming a bank-financed entrepreneur is better than being unemployed (see the proof of Proposition 1). If this were not true, then there would be no bank-financed entrepreneurs, and given the fixed-coefficient technology there would be no force to ensure full employment. With the possibility of unemployment, the rents from *self-financed* entrepreneurship would be maximal and the *American Dream* effect we want to illustrate would be large. Nonetheless, we maintain Assumption 2 as we are interested in the role of the imperfections in the credit market, but not so much in situations where these imperfections are so great that the credit market shuts down completely. Assumption 3 guarantees that the cost of investment  $k$  is sufficiently small that in equilibrium all successful young workers earn at least  $k$  for sure. Thus, they can become *self-financed* entrepreneurs. If we dropped Assumption 3, the equilibrium might involve a randomization: rather than paying successful young workers a sure wage  $h < k$ , the entrepreneur would pay  $k$  with some probability  $q$  and 0 with probability  $1 - q$ . Successful workers would earn rents as self-financed entrepreneurs with probability  $q$ . The equilibrating variable on the labour market for young workers would be  $q$  rather than  $h$ . Except for that, our analysis would go through even with  $k > 1/2$ , at least as long as agents are not too risk averse.

## 2.2. Time line

Each period starts with the birth of a new generation. Then the sequence of events is the following.

**Morning:** Entrepreneurs and workers are matched. There are no frictions in the matching process in the sense that unmatched individuals can never be found on both sides of the market. Labour market contracts must be constrained efficient: they must maximize a weighted sum of the expected payoffs of the contracting parties subject to an incentive-compatibility (IC) constraint on the worker's effort choice, the limited liability constraint, and the worker's and entrepreneur's participation constraints. The weights are determined by market forces. If the number of projects does not equal the number of workers, then the party on the short side gets all the weight. Thus, if there are more workers than projects, the contract maximizes the entrepreneur's payoff subject to the IC and limited liability constraints, and the worker's participation constraint. If there are more projects than workers, the contract maximizes the worker's payoff subject to the IC and limited liability constraints, and the entrepreneur's participation constraint. The worker's participation constraint is that he should get at least zero, the entrepreneur's participation constraint is that he should get at least as much as he would get if he switched occupation to become a worker.

**Noon:** Output is realized and publicly observed. Wages are paid and loans are repaid.

**Evening:** Old individuals consume everything they have, since there are no bequests. Young agents decide whether or not to invest in order to become entrepreneurs in the next period. Those who do not invest remain workers in the next period. Income that is not used for investment can be either consumed today or saved in a bank for consumption tomorrow. These two options are equivalent since the deposit interest rate satisfies  $1 + r = 1/\delta$ .

### 2.3. The benchmark one-period model

Before analysing our dynamic model, we review the solution to the corresponding static model. Two individuals who live for one period have been exogenously assigned the roles of principal and agent. Since expected joint surplus is  $e - ce^2/2$ , the first-best effort level is  $1/c < 1$  (by Assumption 1). The agent sets effort to maximize  $eh - ce^2/2$  subject to  $0 \leq e \leq 1$ . The first order condition for an interior solution is  $e = h/c$ , which implies an expected utility of  $h^2/(2c)$  for the agent. The principal sets the success wage  $h$  to maximize  $e(1 - h) = h(1 - h)/c$  subject to the participation constraint  $h^2/(2c) \geq \bar{u}$ . If the agent's reservation payoff is  $\bar{u} < 1/(8c)$  then the solution involves

$$h = \frac{1}{2} \quad \text{and} \quad e = \frac{1}{2c}. \quad (1)$$

The principal's profit is

$$e(1 - h) = \frac{1}{4c}.$$

The participation constraint does not bind since

$$\frac{h^2}{2c} = \frac{1}{8c} > \bar{u},$$

and effort is strictly less than the *first best* level  $1/c$ . If  $\bar{u} > 1/(8c)$ , then the participation constraint binds. In this case, a fall in  $\bar{u}$  would allow the principal to increase his profit by lowering  $h$  (which however would reduce  $e$ , leading to a reduction in total social surplus).

Intuitively, the basic problem is as follows: with limited liability and the fact that agents have no wealth, the principal cannot "sell the firm" to the agent, and the only way he can extract any transfers from the agent is by taking a share of output ( $1 - h > 0$ ). As a result, the agent's effort is below the first best ( $h/c < 1/c$ ). The higher is  $1 - h$ , the lower is  $e$ . The optimal contract balances these two concerns of rent extraction and incentive provision. Two implications of this basic trade-off are worth noting. First, in order to provide good incentives the principal would never reduce the agent's share below a certain minimum level ( $\frac{1}{2}$  in this case), no matter how low the reservation payoff is. Second, an agent with a high reservation payoff has to receive a large share of the surplus. Given the

incentive problem, this is most efficiently done by giving him a large success wage, which leads to a high effort level.<sup>12</sup>

### 3. STEADY STATE EQUILIBRIUM

Young agents choose the occupation that offers them the highest expected payoff, given rational expectations about next period's wages and the profits of self-financed and bank-financed entrepreneurs. The age of a worker is public information, so young and old workers can receive different contracts. On the competitive labour market, entrepreneurs must be indifferent between hiring young and old workers, or else the wage of the more desirable type of worker would be bid up. We will focus our attention on steady state equilibria. Since all endogenous variables are constant over time we can omit time-subscripts without much loss of generality. Since capital depreciates fully after one period, there are no interesting issues of economic growth. In fact, we show that if the discount factor  $\delta$  is large, there is a unique steady state equilibrium, and no other (non steady state) equilibria.<sup>13</sup>

Let  $h^o$  and  $h^y$  denote the success wages paid to old and young workers, respectively. Let  $e^o$  and  $e^y$  denote the effort levels chosen by old and young workers, respectively. Let  $p$  denote the number of entrepreneurs. The number of available jobs (individual projects) is  $pn$ , the number of young workers is 1, the number of old workers is  $1 - p$ .

#### 3.1. Labour market equilibrium

Our first result is that in equilibrium neither workers nor entrepreneurs can be in short supply.

**Proposition 1.** *In any steady state equilibrium the number of workers equals the number of projects (full employment). The number of entrepreneurs is  $p = 2/(1 + n) < 1$ .*

In the proof, Assumption 2 is used to show that if there is unemployment among workers (so entrepreneurs in effect have all the bargaining power), then every young agent will want to become an entrepreneur when old, even if it means paying the transaction cost on a bank loan. Thus unemployment cannot persist.

#### 3.2. Occupational choice

In this section we derive the expected value of becoming entrepreneur. We show that in any steady state young agents who can self-finance the investment prefer (at least weakly) to become entrepreneurs, while other young agents can at best be indifferent between borrowing to become entrepreneurs and remaining workers.

12. The static model has been extended by Lewis and Sappington (2001) to the case where the agent has private information about *both* his ability and wealth. They allow wealthy agents to post bonds (in effect setting  $l < 0$ ) in exchange for a higher-powered incentive contract (higher  $h$ ). Interestingly, for the same level of ability, while wealthier agents will get a more high-powered incentive schemes in exchange for posting a greater bond when ability is known, it is not necessarily true when ability is unknown. Such a change in the contract that leaves the expected payoff of a low ability agent constant will generate rents for a high ability agent. As a result, the principal may be unwilling to offer higher-powered incentive schemes unless the agent has both higher wealth and higher ability.

13. If multiple steady state equilibria exist, then in the beginning of any period  $t$  there are many beliefs that are self-fulfilling but lead to different future paths. In this case there will be large numbers of non steady state equilibria. Conversely, if there are non steady state equilibria, there must be multiple steady states.



Consider a firm with  $v^o$  old and  $v^y$  young workers,  $v^o + v^y = n$ . Since the effort level equals the success probability, the entrepreneur's expected revenue minus wage payments is

$$A \equiv v^o e^o (1 - h^o) + v^y e^y (1 - h^y). \quad (2)$$

Competition between entrepreneurs ensures that in equilibrium they are indifferent between hiring young and old workers. Thus, in equilibrium  $A$  does not depend on the age-composition of the labour force

$$e^o (1 - h^o) = e^y (1 - h^y). \quad (3)$$

Since there are no bequests, the income of a young agent at the end of his first period equals his wage. If  $h^y \geq k$  then a successful young worker can invest  $k$ , consume  $h^y - k$ , and become a *self-financed* entrepreneur when old. His expected discounted payoff will be  $h^y - k + \delta A$ . The expected discounted payoff from consuming his wage today and remaining a worker when old is  $h^y + \delta u^o$ , where  $u^o$  denotes the payoff from being an old worker.<sup>14</sup> Let  $S$  denote the *rent* from becoming a self-financed entrepreneur

$$S \equiv h^y - k + \delta A - (h^y + \delta u^o) = \delta A - k - \delta u^o. \quad (4)$$

The young successful agent wants to become a self-financed entrepreneur when old if and only if  $S \geq 0$  (if  $S = 0$  he is indifferent).

A young agent whose first-period income is strictly smaller than  $k$  needs to use the credit market in order to become an entrepreneur. A bank-financed entrepreneur can do the same things a self-financed entrepreneur can do, but the bank loan involves a transactions cost  $\gamma > 0$  incurred in the period when production takes place. Thus, the rent from being a bank-financed entrepreneur is  $B = S - \delta\gamma$ . Becoming a bank-financed entrepreneur is desirable if and only if  $B \geq 0$  (if  $B = 0$  the individual is indifferent). Of course, in equilibrium bank-financed entrepreneurs cannot earn positive rents. Indeed, if  $B > 0$  then all agents prefer to become entrepreneurs when old, so  $p = 1$ , in contradiction of Proposition 1. Conversely, if  $S < 0$  then nobody wants to be an entrepreneur, so  $p = 0$ , again contradicting Proposition 1. Thus, we have:

**Proposition 2.** *In any steady state equilibrium,  $S \geq 0$  and  $B \leq 0$ .*

Thus, in equilibrium an agent whose first period income exceeds  $k$  at least weakly prefers to become a self-financed entrepreneur. An agent with a first-period income strictly smaller than  $k$  at least weakly prefers not to borrow to become an entrepreneur. Hence, if an agent who receives income  $w$  when young makes an optimal occupational choice, his payoff from this moment on will be  $w - k + \delta A$  if  $w \geq k$  and  $w + \delta u^o$  if  $w < k$ . Using equation (4), the value function for a young worker who receives income  $w$  can be written as

$$V(w) = \begin{cases} w + \delta u^o + S & \text{if } w \geq k \\ w + \delta u^o & \text{if } w < k \end{cases} \quad (5)$$

The value function is discontinuous at  $w = k$  if  $S > 0$ . Notice that Proposition 2 and the fact that  $B = S - \delta\gamma$  implies  $S \in [0, \delta\gamma]$ .

14. Since we always have full employment, this payoff is guaranteed.

### 3.3. Effort

In this section we show how effort levels of young and old workers depend on wages,  $h^o$  and  $h^y$ , and the rents going to self-financed entrepreneurs,  $S$ . Given that  $w = h^y$  if his project is successful and  $w = 0$  otherwise, the young worker will set  $e^y$  to maximize

$$e^y V(h^y) + (1 - e^y)V(0) - \frac{1}{2}c(e^y)^2, \quad (6)$$

subject to  $0 \leq e^y \leq 1$ . Let us assume that the solution to a worker's effort choice is interior, and consider the first order conditions. (Below we will confirm that the worker's effort choice is indeed interior). From the first order condition we obtain the young worker's IC constraint:

$$e^y = \frac{V(h^y) - V(0)}{c}. \quad (7)$$

From (5),  $V(h^y) - V(0) = h^y + S$  if  $h^y \geq k$ , otherwise  $V(h^y) - V(0) = h^y$ . Thus,

$$e^y = \begin{cases} h^y/c & \text{if } h^y < k \\ (h^y + S)/c & \text{if } h^y \geq k \end{cases} \quad (8)$$

For the principal to break even we must have  $h^y < 1$ , and we know that  $S \leq \delta\gamma$ . So a sufficient condition for the solution to the worker's effort choice to be interior is  $(1 + \delta\gamma)/c < 1$  which is ensured by Assumption 1. An *old* worker faces the same decision problem as in the static model of Section 2.3, since this is the last period of his life. His effort-choice is, therefore, determined by the IC constraint

$$e^o = \frac{h^o}{c}. \quad (9)$$

The following lemma characterizes the wages of young and old workers and the corresponding effort levels in a steady state equilibrium.

**Lemma 1.** *In any steady state equilibrium:*

- (1)  $1 > h^y \geq h^o \geq \frac{1}{2}$ , with  $h^y > h^o$  if  $S > 0$ ;
- (2)  $(1 + S)/c > e^y \geq e^o \geq 1/2c$  with  $e^y > e^o$  if  $S > 0$ .

Just as in Section 2.3, for incentive reasons we have  $h^o \geq \frac{1}{2}$  and thus  $e^o \geq 1/(2c)$ . For employers to be indifferent between hiring young and old workers, we must also have  $h^y \geq \frac{1}{2}$  which by Assumption 3 implies  $h^y > k$  so that self-financing is possible. Therefore, from (8) the young worker's IC constraint is

$$e^y = \frac{h^y + S}{c}. \quad (10)$$

In a match between a young worker and a principal, the joint gain from success is  $1 + S$ . The principal does not have to pay for  $S$ . It is a private benefit from success that the agent receives from outside the relationship. In addition,  $S$  also represents a net social gain since successful young workers are able to save on costly credit market transactions in the next period. In contrast, in a match between an old worker and a principal the joint gain from success is 1. Since  $h^y < 1$  and  $h^o < 1$  must be true for the principal to break even, the IC constraints (9) and (10) imply that both young and old workers supply strictly less than

the joint surplus maximizing effort levels (which are  $(1 + S)/c$  and  $1/c$ , respectively). In contrast with Holmström (1999) where a worker ends up supplying too much effort, in our model effort is always too low from the point of view of social surplus.<sup>15</sup>

Using (9) and (10), the condition that entrepreneurs must be indifferent between hiring young and old workers, equation (3), can be written as

$$e^y(1 - ce^y + S) = e^o(1 - ce^o). \quad (11)$$

If  $S = 0$  then the IC constraints of old and young workers, (9) and (10), are equivalent, so old and young workers work equally hard for any given wage. Equation (11) implies that they must receive the same pay ( $h^y = h^o$ ) and supply the same effort ( $e^y = e^o$ ). However, if  $S > 0$  then the IC constraints imply that for any given wage, young workers work harder than old workers, since they get an extra benefit  $S$  from succeeding. This is the *American Dream* effect. Therefore, if  $S > 0$  and  $h^y = h^o$  then young workers would be more attractive to hire than old workers, violating (11). In the competitive labour market  $h^y$  must be bid up until equation (11) is satisfied, which further raises  $e^y$ . Consequently, if the equilibrium is such that  $S > 0$  then  $h^y > h^o$  and  $e^y > e^o$ .<sup>16</sup>

The *American Dream* effect implies that young workers work extra hard in order to earn entrepreneurial rents in the future, and in a competitive spot market for labour this translates into higher wages for young workers. But if for some reason young workers cannot be paid more than old, the young worker's IC constraint still implies an *American Dream* effect. The appendix shows this formally.<sup>17</sup>

#### 4. CHARACTERIZATION OF EQUILIBRIA

From Proposition 2 and the fact that  $S - B = \gamma\delta$ , it follows that there are three possible types of steady state equilibria: (I)  $S = \delta\gamma > B = 0$ ; (II)  $S > 0 > B$ ; (III)  $S = 0 > B$ . Case I and II display the *American Dream* effect: as  $S > 0$ , young workers work extra hard in

15. If  $S$  is large enough, then the effort of a young worker can exceed  $1/c$ , which is the first-best level in the static model. However, effort is always below  $(1 + S)/c$ , which is the social surplus maximizing level in the dynamic model. The entrepreneurial rent  $S$ , which represents a saving of credit market transactions costs, must count toward social surplus. From the social point of view, it is therefore justifiable that young workers put in extra effort in order to save on these transactions costs. In contrast, in the Holmström (1999) model effort exceeds the social surplus maximizing level because of a rat-race like problem—the worker puts in extra effort in order to convince the principal that he has a high ability, but in equilibrium the principal cannot be “fooled”. Clearly, this extra effort is not justifiable from the social point of view.

16. In the benchmark one-period model with a *monopolistic* principal, if a worker receives a private benefit from success (say, pride) then the principal would *lower* his success wage (since the highly motivated worker works hard “for free”). In our model, the *competitive market* rewards young workers for their extra motivation.

17. We expect to find an *American Dream* effect on the age profile of wages in situations where credit market imperfections are important. This suggests looking at data for developing countries, or for groups of workers in developed countries that face serious credit constraints, but unfortunately there is no empirical study that directly addresses this issue. Indeed, on an aggregate level existing evidence from large data-sets covering many firms and sectors does show a positive relationship between overall labour market experience and wages in the U.S. contrary to what the model predicts, although the effect of tenure on wages is small over a given job match (see Abraham and Farber (1987)). However, there is also evidence that productivity in fact falls after a certain age in some sectors of the U.S. economy, especially in technical fields. Dalton and Thompson (1971) report that performance ratings of engineers in technology based companies fall after the age of 35, and Medoff and Abraham (1980) found that the effect of age on ordinal performance rankings was negative or zero among professional and managerial workers in two major U.S. manufacturing corporations. In our model, the labour market is a competitive spot market, so falling productivity would translate into falling wages although in models with long-term labour contracts, wages may increase over time even though productivity does not (see Lazear (1979)). There is in fact a recent study by Baker, Gibbs and Holmström (1994) that shows in some industries real wages fall for workers who stay in the same job for a long time without getting promoted.

order to capture the rents from self financed entrepreneurship. By (11) their wages must be higher than those of old workers. This further increases the difference between young and old workers' effort levels, so in equilibrium  $e^y > e^o$ . In type III equilibria there are no rents ( $S = 0$ ), so young and old workers get the same labour contracts and  $e^y = e^o$ .

In order to find the steady-state equilibria, we first solve for effort levels (and implicitly for wages from the IC constraints) as *functions of S*. The supply of entrepreneurs is a function of the effort levels of young workers, so we can then solve for  $S$  by equating the supply of entrepreneurs to  $p = 2/(1+n)$ . Once we know the steady-state level of  $S$ , we can determine all endogenous variables.

By differentiating (11), we find that (holding  $S$  fixed) an increase in  $e^o$  means  $e^y$  must increase to maintain equality in (11), using the fact that  $e^y = (h^y + S)/c \geq (1+S)/(2c)$  and  $e^o \geq 1/(2c)$  (from Lemma 1). Since the entrepreneur is indifferent between hiring young and old workers, the rent going to a self-financed entrepreneur can be evaluated as if he hired  $n$  old workers

$$S = \delta \left( ne^o(1 - ce^o) - c \frac{(e^o)^2}{2} \right) - k. \quad (12)$$

Here we used the fact that, from the IC constraint and the fact that there is full employment, the expected payoff of an old worker is

$$u^o = e^o h^o - \frac{1}{2} c (e^o)^2 = \frac{1}{2} c (e^o)^2.$$

Equation (12) implicitly determines the old worker's effort as a function of  $S$ ,  $e^o = e^o(S)$ . We can obtain  $e^y = e^y(S)$  as a function of  $S$  by substituting  $e^o = e^o(S)$  in equation (11). Lemma 2 shows that these functions are well defined.

**Lemma 2.** *For any  $S \in [0, \gamma\delta]$ , there exist real numbers*

$$e^o = e^o(S) \equiv \frac{n + \sqrt{n^2 - 4c(n + \frac{1}{2})(k + S)}/\delta}{2c(n + \frac{1}{2})} \quad (13)$$

and

$$e^y = e^y(S) \equiv \frac{1 + S + \sqrt{(1 + S)^2 - 4ce^o(S)(1 - ce^o(S))}}{2c}, \quad (14)$$

such that (11) and (12) hold. The corresponding wages for old and young workers are

$$h^o = h^o(S) \equiv \frac{n + \sqrt{n^2 - 4c(n + \frac{1}{2})(k + S)}/\delta}{2(n + \frac{1}{2})} \quad (15)$$

and

$$h^y = h^y(S) \equiv \frac{1 - S + \sqrt{(1 + S)^2 - 4ce^o(S)(1 - ce^o(S))}}{2}, \quad (16)$$

where  $h^y(S) \geq h^o(S) > \frac{1}{2}$ .

Lemma 2 characterizes the effort levels and wages of young and old workers that are consistent with any given value of the rents of self-financed entrepreneurs. For any  $S$ , effort levels and wages are uniquely determined by (11) and (12).<sup>18</sup> In the optimal contract of the benchmark one-period model, the effort level and wage can also be expressed as a function of the reservation payoff of one party. However, in the dynamic model young workers work harder due to the *American Dream* effect, and the wages of young and old workers adjust to make entrepreneurs indifferent between hiring young and old workers. Moreover, the rent  $S$  will be endogenously determined in equilibrium. Before showing precisely how  $S$  is determined, we will describe the relationship between entrepreneurial rents and workers' effort levels.

**Lemma 3.** *The function  $e^o(S)$  is always decreasing in  $S$ . If there exists  $S'$  such that  $e^y(S)$  is increasing at  $S = S'$ , then  $e^y(S)$  is increasing for all  $S \geq S'$ .*

Since the expected profit per young worker must equal the expected profit per old worker, both must increase if  $S$  increases. The principal can extract more profits from *old* workers only by lowering  $h^o$ , which however also reduces  $e^o$ . The reasoning here is the same as in the benchmark one-period model of Section 2.3, where any redistribution of surplus from agents to principals is necessarily associated with a fall in effort. Indeed, the old worker's situation is essentially the same as in the static model. Therefore,  $e^o(S)$  is decreasing in  $S$ . However, in our dynamic model a redistribution of surplus from agents to principals does not necessarily lead to lower effort from *young* agents since, from equation (10), there are two opposing effects. If  $h^y$  is reduced then there is a direct negative effect on  $e^y$ . However, a higher  $S$  has a direct positive effect on  $e^y$  since the young worker wants to succeed and become a self-financed entrepreneur when old. On balance, young workers may work harder even if their wages fall. The worse it is to be a worker instead of a principal, the harder the young worker may work today to try to become a principal tomorrow. The direct positive effect of  $S$  on effort is more important the higher is  $S$ , so that if  $e^y(S)$  is increasing at  $S = S'$ , then  $e^y(S)$  is increasing for all  $S \geq S'$ . We will now describe how the equilibrium value of  $S$  is determined.

In a type I equilibrium, the credit market is active,  $B = 0$  and  $S = \gamma\delta$ . Effort is  $e^y(\gamma\delta)$  for young workers and  $e^o(\gamma\delta)$  for old. In each cohort, on average  $e^y(\gamma\delta)$  young workers succeed, and they all go on to become entrepreneurs, as  $S > 0$ . Unsuccessful workers are indifferent between becoming bank-financed entrepreneurs and old workers, as  $B = 0$ . Thus, any fraction of them may become entrepreneurs. As we need  $p = 2/(1+n)$  entrepreneurs from Proposition 1, the *necessary and sufficient* condition for a type I equilibrium to exist is

$$\frac{2}{1+n} \geq e^y(\gamma\delta). \quad (17)$$

The number of bank-financed entrepreneurs in each generation is  $2/(1+n) - e^y(\gamma\delta)$ .

In a type II equilibrium, the credit market is inactive,  $B < 0$ , and  $0 < S < \gamma\delta$ . All successful workers become entrepreneurs but there are no bank-financed entrepreneurs, as

18. We are solving quadratic equations, but as argued in the proof of Lemma 2 only the larger root is relevant.

$B < 0$ . Thus, the number of entrepreneurs equals the number of successful young workers,  $e^y(S)$ . Given Proposition 1,  $S$  must be determined by

$$e^y(S) = \frac{2}{1+n}. \quad (18)$$

Finally, consider a type III equilibrium. Unsuccessful workers will not invest since  $B < 0$ , but any fraction of the successful workers may invest, as  $S = 0$ . Young and old workers earn the same wages and supply the same effort. Since the number of successful workers is  $e^y(0)$  and we need  $2/(1+n)$  entrepreneurs, the *necessary and sufficient* condition for a type III equilibrium to exist is

$$e^y(0) \geq \frac{2}{1+n}. \quad (19)$$

Having described the characteristics of the three types of equilibrium, we can now locate them in the parameter space. If the cost of effort is low, success is very easy to accomplish so the entrepreneurial rent will be driven to zero. If success is more difficult, then self-financed entrepreneurs will earn rents. Since  $e^y(0) = e^o(0)$ , setting  $S = 0$  in (13) and combining with (19) holding as an equality gives us the following threshold cost of effort that determines which type of equilibrium will result:

$$\bar{c}(n, k) \equiv \frac{1+n}{2n+1} \left\{ n - (1+n) \frac{k}{2\delta} \right\}.$$

**Proposition 3.** *If  $c > \bar{c}(n, k)$  then there exists a steady state equilibrium with positive rents ( $S > 0$ ). If  $c \leq \bar{c}(n, k)$  then there exists a steady state equilibrium with no rents ( $S = 0$ ).*

The proof of Proposition 3 is simple and can be done diagrammatically. Figures 1, 2 and 3 show the different ways the “supply schedule of entrepreneurs” can look, given Lemma 3. If  $S = 0$ , then there are  $e^y(0)$  successful workers who are indifferent between becoming entrepreneurs and old workers, so the supply schedule has a vertical element at  $S = 0$  of height  $e^y(0)$ . If  $0 < S < \gamma\delta$  then all successful workers strictly prefer to become entrepreneurs, but no unsuccessful workers want to do so. The supply of entrepreneurs is therefore precisely  $e^y(S)$ . Finally, if  $S = \gamma\delta$  (maximum rent for entrepreneurs) then all  $e^y(\gamma\delta)$  successful workers strictly prefer to become entrepreneurs and the  $(1 - e^y(\gamma\delta))$  unsuccessful workers are indifferent. Thus, the supply schedule has a vertical segment at  $S = \gamma\delta$ , from  $e^y(\gamma\delta)$  to 1.

From equation (14) we see that  $e^y(S)$  is a continuous function of  $S$ , and the “supply schedule” is a continuous curve from  $(0, 0)$  to  $(\gamma\delta, 1)$ . In Figure 1,  $e^y(S)$  is always increasing in  $S$ . In Figure 2,  $e^y(S)$  is decreasing in  $S$  for every interior value of  $S$ . Figure 3 illustrates the possibility that  $e^y(S)$  is first decreasing and then increasing.

The equilibrium number of entrepreneurs is  $p = 2/(1+n) < 1$  from Proposition 1. Thus,  $S \in [0, \gamma\delta]$  is an equilibrium rent if and only if the “supply schedule” crosses the horizontal line  $p = 2/(1+n)$  at  $S$ . This must happen at least once, which implies the existence of an equilibrium. An equilibrium with  $S = 0$  exists if and only if the horizontal line crosses the left vertical segment of the supply schedule, which is true if and only if (19) holds. Indeed, (19) implies that there are enough successful workers when  $S = 0$  to fill the necessary number of entrepreneurial positions. Using (13) and the fact that  $e^o(0) = e^y(0)$  we find that (19) is equivalent to  $c \leq \bar{c}(n, k)$ . If instead the cost of effort  $c$  exceeds the

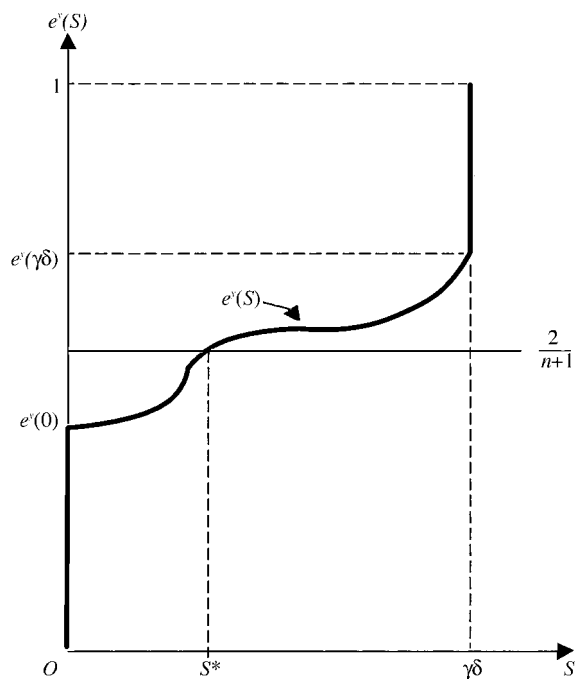


FIGURE 1

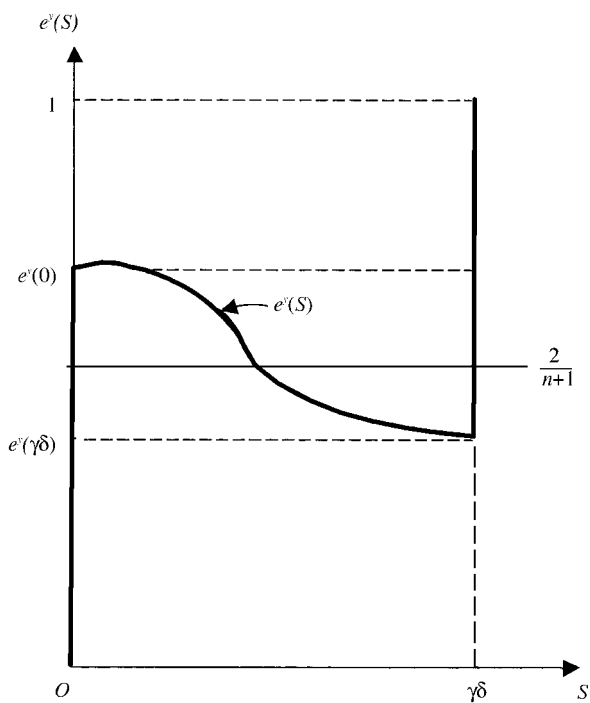


FIGURE 2

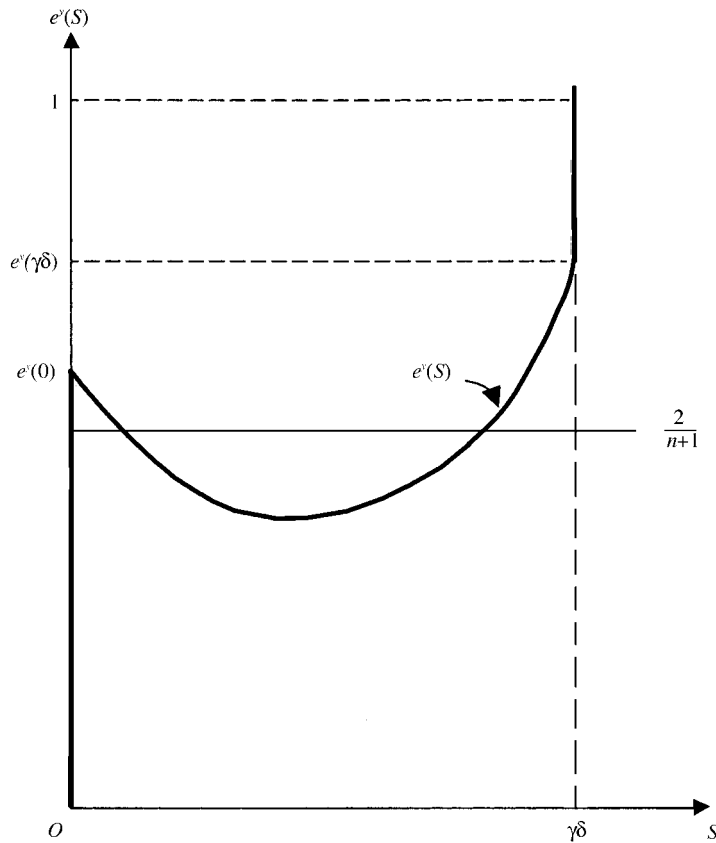


FIGURE 3

threshold  $\bar{c}(n, k)$ , then  $e^y(0) < 2/(1+n)$  and the horizontal line must cross the supply schedule at a point where  $0 < S \leq \gamma\delta$ , in which case an equilibrium with positive rents exists.

Proposition 3 can alternatively be stated as follows in terms of the *size of the firm*.

**Corollary 1.** *For any  $c$  and  $k$ , there exists a threshold size of the firm  $\bar{n}(c, k)$  such that an equilibrium with  $S = 0$  exists if  $n$  exceeds the threshold, and an equilibrium with  $S > 0$  exists otherwise.*

When  $n$  is large, workers are more scarce than entrepreneurs in the labour market and to guarantee full employment some successful agents must be indifferent between being workers and self-financed entrepreneurs. So a zero-profit equilibrium will result. The dynamic incentives emphasized in this paper are more likely to be large in a situation where firms are small, so that entrepreneurs are more scarce in the labour market. To ensure full employment, all successful young workers, and perhaps even some unsuccessful young workers must be induced to become entrepreneurs. As a result, the rents for self-financed entrepreneurs must not be driven to zero.



Figure 1 shows the case of a unique equilibrium with  $0 < S < \gamma\delta$ . Figures 2 and 3 illustrate that there can be multiple steady-state equilibria. Generically, we can have three steady states: one must be of type III and one of type II, while the third can be type II or type I. To show this, start from a type III steady state where  $S=0$  and  $e^y = e^y(0) > 2/(1+n)$ . Now suppose agents expect wages to be slightly lower (today and in the future) and  $S$  to be correspondingly higher. For young agents this means a lower immediate benefit from working hard but a larger future gain from becoming a self-financed entrepreneur. If young agents do not care much about the future, they reduce their effort and therefore the supply of (self-financed) entrepreneurs  $e^y(S)$  falls. When  $0 < S < \gamma\delta$  the supply of entrepreneurs equals the number of successful young workers, which equals the young workers' effort level  $e^y(S)$ . As long as the supply of entrepreneurs  $e^y(S)$  exceeds the equilibrium number of entrepreneurs  $2/(1+n)$ , the wages of young workers can fall, until  $e^y(S)$  falls to  $2/(1+n)$ . At this point, the balance on the labour market is restored with lower wages and effort levels than before. This is the type II equilibrium. If wages fall even further, the number of successful young workers falls below  $2/(1+n)$ . Two things can happen: profits eventually get so high that bank-financed entrepreneurs appear (type I equilibrium, as in Figure 2), or the dynamic incentives caused by higher profits push the number of successful young workers back to  $2/(1+n)$  (a second type II equilibrium, as in Figure 3). This argument suggests that the *impatience* of young agents may lead to multiplicity of steady states. If  $\delta$  is high we can indeed prove that a *unique* steady state equilibrium exists (and in this case there cannot be any non-steady state equilibria either).

**Proposition 4.** *If  $\delta > \frac{1}{2}$  then a unique steady-state equilibrium exists.*

The intuition behind this result is that the strength of the American dream effect increases with  $\delta$ . Hence for high values of  $\delta$ , the effort of young workers is always increasing in rents of self-financed entrepreneurs. As a result, the “supply schedule of entrepreneurs” of entrepreneurs is always upward sloping and a unique equilibrium results.<sup>19</sup>

## 5. ECONOMIC POLICY

In this section we consider the implications of economic policy. First, we consider a policy that lowers the transactions cost on the credit market. The effects on social welfare are ambiguous in general. However, we show that a reduction in the transactions cost  $\gamma$  is welfare-reducing if  $k$ ,  $c$ , and  $\gamma$  are small enough. Second, we consider a transfer of money from successful to unsuccessful workers. We show that a small amount of redistribution unambiguously reduces total output. While the effect on social welfare is ambiguous in general, if  $\gamma$  is small, welfare decreases as well. Due to the non-convexity inherent in this model, we restrict our analysis to the effects of local variations, which can be studied using calculus. An economic policy of sufficiently large size may shift the economy from one type of equilibrium to another, and the effects of such large shifts are difficult to evaluate analytically. Our measure of social welfare will be the expected utility of a new-born agent

19. If  $\delta < \frac{1}{2}$  the steady state equilibrium may still be unique. If  $n \geq \bar{n}(c, k)$  then not many entrepreneurs are needed, and successful agents must be indifferent between being workers and self-financed entrepreneurs. As a result, a unique zero-profit equilibrium will result. Conversely, when  $n$  is very small a unique equilibrium with an active credit market will result.

in steady state equilibrium.<sup>20</sup> Notice that the results are all second best results. With no imperfections in the labour market, a decrease in  $\gamma$  must raise social welfare. However, in a model with multiple imperfections in different markets, a reduction in one of them (here,  $\gamma$ ) is not necessarily welfare enhancing since it may exacerbate the problems in another market (here, the labour market).

### 5.1. Reduction of credit market imperfections

A natural economic policy in the context of poverty trap models such as Banerjee and Newman (1993) is to reduce the transactions cost in the credit market. For example, if the government can administer loans without transactions costs (to make the most generous assumption), it could simply provide cheap loans to unsuccessful agents. This would be equivalent to a reduction of  $\gamma$  to zero. In our model such a reduction of  $\gamma$  may be social welfare reducing. The transactions cost  $\gamma$  imposes a penalty on bank-financed entrepreneurs. This amounts to a penalty imposed on unsuccessful agents, which may be welfare enhancing if agents respond by altering their behaviour in a productive way. The fact that some unlucky agents suffer the penalty may, from a social point of view, be less important than the fact that the penalty motivates agents to succeed more often. The penalty will most likely be beneficial to society if most individuals manage to avoid it, so for social welfare to be increasing in  $\gamma$  the cost of effort  $c$  should be small enough.

In type II and type III equilibria, variations in  $\gamma$  are irrelevant since there are no bank-financed entrepreneurs. So we focus on the type I equilibrium, where  $e^y = e^y(S)$ ,  $e^o = e^o(S)$  and  $S = \delta\gamma$ . In terms of Figures 1, 2 and 3, by reducing  $\gamma$  we push the right-most vertical segment of the supply curve towards the vertical axis. Clearly, for small enough  $\gamma$  the equilibrium will be unique. The type of equilibrium will be determined by whether  $e^y(0)$  is smaller or greater than  $p = 2/(1+n)$ . If  $c > \bar{c}(n, k)$  then for sufficiently small  $\gamma$  there is a *unique* steady state equilibrium, which is of type I. The expected utility of a new-born agent in steady-state equilibrium is

$$W(S) = e^y(h^y + S + \delta u^o) + (1 - e^y)\delta u^o - \frac{1}{2}c(e^y)^2 = \frac{c}{2}[(e^y(S))^2 + \delta(e^o(S))^2]. \quad (20)$$

Here we have used the fact that  $u^o = c(e^o)^2/2$  and  $e^y = (h^y + S)/c$ . Since there is 1 young worker and  $1 - p$  old workers, the total output in this economy is

$$Y(S) = e^y(S) + (1 - p)e^o(S).$$

$Y(S)$  and  $W(S)$  differ in the way the effort levels are weighted, with  $W(S)$  taking into account also the disutility of effort. The level of output is determined by the number of firms ( $p = 2/(1+n)$ ) and the effort levels of young and old workers. Effort levels depend on the profit rate of self financed entrepreneurs. In a type I equilibrium this profit rate is  $\gamma\delta$  and a change in it induces different effects on the effort levels of young and old workers. Differentiating (20) and using  $dS/d\gamma = \delta$  we obtain

$$\frac{dW}{d\gamma} = c\delta \left[ e^y \frac{de^y}{dS} + \delta e^o \frac{de^o}{dS} \right]. \quad (21)$$

20. Specifically, we take the expected utility of a new-born agent in steady state equilibrium before the change in policy, and compare it with the expected utility of a new-born agent in the new steady state equilibrium that is reached after the (small) change in policy. This implies that we ignore the effect of the policy on welfare during the transition to the new steady state.

Thus, social welfare is increasing in  $\gamma$  if and only if

$$e^y(S) \frac{de^y(S)}{dS} + \delta e^o(S) \frac{de^o(S)}{dS} > 0, \quad (22)$$

evaluated at  $S = \gamma\delta$ . By Lemma 3,  $de^o/dS < 0$ . Thus, for social welfare to be increasing in  $\gamma$ , the positive effect on the young worker's effort must be sufficiently strong to dominate the negative effect on the old worker's effort. In particular, since  $e^y \geq e^o$  and  $\delta < 1$ , an increase in  $\gamma$  raises social welfare if  $de^y/dS > |de^o/dS|$ . Ghatak, Morelli and Sjöström (2000) present numerical examples where (22) is satisfied. What we can show analytically is that (22) holds if  $\gamma$ ,  $c$  and  $k$  are small enough.

**Proposition 5.** *Assume there exists a unique type I equilibrium. Reducing the credit market imperfection strictly lowers social welfare and total output if  $\gamma$ ,  $c$  and  $k$  are strictly positive but sufficiently small.*

Proposition 5 shows that, starting with no credit market imperfection, introducing a little bit of it may increase output and welfare. This result is in sharp contrast with the implication of models displaying poverty traps such as Banerjee and Newman (1993) where improving the functioning of the credit market always improves efficiency. Our model shares with Banerjee and Newman (1993) the presence of indivisibility of investment and credit market imperfections. In addition moral hazard in effort supply and limited liability combine to create an additional distortion—workers supply too little effort. The prospect of earning the rents created by the first distortion causes agents to work harder, thereby alleviating the second distortion.

Even if the expected utility of a new-born agent is increasing in  $\gamma$ , inequality is increasing in  $\gamma$  too, both between successful and unsuccessful workers and between young and old workers. Therefore, a social planner who puts sufficient weight on reducing inequality may prefer a lower  $\gamma$  even if this reduces average welfare. The utility differential between a successful and an unsuccessful worker is  $V(h^y) - V(0) = h^y + S$  which is increasing in  $\gamma$  (otherwise young workers wouldn't work harder when  $\gamma$  increases). Moreover, not only the relative position but also the absolute payoff of unsuccessful agents is decreasing in  $\gamma$  (this follows from Lemma 3 and the fact that  $u^o = c(e^o)^2/2$ ). Furthermore, the income inequality between young and old workers is also increasing in  $\gamma$ . The average income of old (resp. young) workers is  $e^o h^o$  (resp.  $e^y h^y$ ). Since  $h^o = ce^o$ , the average income of old workers always decreases with  $\gamma$ . For young workers,  $h^y = ce^y - S$ , so it is possible that their effort and wages move in opposite directions. But we can calculate

$$\left. \frac{d(e^y h^y)}{dS} \right|_{S=0} = \frac{e^y(0)}{2ce^y(0) - 1} (1 - 2cf(e^y(0))), \quad (23)$$

where  $f$  is defined by equation (31) in the Appendix. As shown in the proof of Proposition 5,  $\delta(1+n)f(e^y(0)) < 1$ , so (23) is non-negative if  $\delta(1+n) \geq 2c$ . So long as  $k$  is small, Assumption 2 guarantees this, so that average income of young workers increases with  $\gamma$ .

*Remark 1.* As argued above,  $c$  has to be sufficiently small to obtain the positive effect of credit market imperfections on social welfare. On the other hand, if  $c < \bar{c}(n, k)$  then there will be sufficiently many successful young workers to drive the rents from entrepreneurship to zero, and in this case credit market imperfections are irrelevant for

social welfare. Thus, for an increase in  $\gamma$  to *strictly* raise social welfare, we need  $c > \bar{c}(n, k)$ . There exists a non-empty interval where  $c$  is small enough to make sure welfare is increasing in  $\gamma$ , but large enough to guarantee the existence of a type I equilibrium. The interval is

$$\frac{n^2 + n}{2n + 1} < c < \frac{\delta n^2}{2n - 1}. \quad (24)$$

The proof of Proposition 5 shows that social welfare is strictly increasing in  $\gamma$  if  $\gamma$  and  $k$  are sufficiently small and (24) holds (the left hand side of (24) is simply  $\bar{c}(n, 0)$ ). Since the American dream effect is the strongest the more patient young agents are, let  $\delta$  be close enough to 1. Since

$$\frac{n^2}{2n - 1} > \frac{n^2 + n}{2n + 1},$$

for all  $n$ , for  $\delta$  close enough to 1 the interval indicated by (24) always exists although the size of it shrinks as  $n$  increases.

*Remark 2.* We have shown that a social planner might want to artificially create credit market transactions costs even if none were originally present. Could an individual young worker also gain by artificially creating a private borrowing cost for himself, if this were publicly observed? The answer is no. Suppose  $\gamma = 0$  and so  $S = 0$ . If one agent creates a borrowing cost for himself, then it will be commonly known that this agent will not borrow money if he fails (since  $S = 0$  it would not pay). Hence, this private cost will not change his incentives or his wage. In contrast, a social planner can increase  $S$  by increasing everybody's borrowing cost, which has an effect on incentives.

## 5.2. Redistribution of income

The advantage of a reduction in  $\gamma$  is that it does not directly tax successful agents (although it does reduce the penalty for failure). Now we consider an alternative redistributive taxation that taxes successful agents and subsidizes unsuccessful agents. Such a policy introduces a penalty for succeeding (while simultaneously reducing the penalty for failing). This must reduce the incentives for young agents to work hard and succeed, hence it must reduce *total output*. However, the effect on *social welfare* is ambiguous, if the tax revenue collected from successful agents is redistributed to unsuccessful agents in the form of a lottery that enables some unsuccessful agents to become self-financed entrepreneurs. If self-financed entrepreneurs earn rents, such a redistribution is not completely zero sum from the point of view of a young agent who does not know the outcome of his project yet.<sup>21</sup>

Suppose we impose a small tax  $t$  on each successful young agent. This tax is small enough so that all successful young agents can still become self-financed entrepreneurs. The total tax revenue,  $e^y t$ , is redistributed to the  $1 - e^y$  unsuccessful young agents via a lottery where each winner gets  $k$ . Since the lucky winners may become self-financed entrepreneurs and earn rents  $S$ , the gain from winning the lottery is  $k + S$ . Of this gain,  $k$

21. Even though agents have risk-neutral preferences, because of the investment indivisibility it is as if young workers are risk loving. It is well known that in such a model, some amount of inequality may be desirable. Still, if effort were fixed, there would be no negative effect of redistributive taxation as long as the tax would be small enough that the successful workers could still all become self-financed entrepreneurs. In our model, however, even a very small amount of taxation has a negative effect: it reduces the incentive to work hard, hence it reduces the number of successful workers.

represents a pure transfer from successful to unsuccessful agents. Such pure transfers reduce social welfare since they reduce the dynamic incentives to work hard. But  $S$  represents a non-zero sum component, a net social gain from allowing some unsuccessful agents to become self-financed entrepreneurs and earn rents. It is clear, however, that for  $S/k$  small enough, the social loss due to lower effort will dominate the social gain from allowing more agents to earn rents, and the tax must reduce social welfare. The type I equilibrium is the one where the redistributive policy has the greatest hope of raising social welfare, since the entrepreneurial rents are maximal ( $S = \delta\gamma$ ). If  $\gamma$  is small, however, then rents are also small and the above argument shows that the tax reduces social welfare.

**Proposition 6.** *Assume there exists a unique type I equilibrium. A small amount of redistribution from successful to unsuccessful agents always reduces total output. It also reduces social welfare if  $\gamma$  is sufficiently small.*

In poverty trap models such as Banerjee and Newman (1993), redistribution of income can increase the number of firms operating in the economy. This effect is absent in our model, since we always have full employment. If we drop Assumption 2, then there can be unemployment in equilibrium and policies that enable some unemployed workers to set up new firms, such as redistribution of income or reducing  $\gamma$ , would increase employment. Instead, redistribution has a negative effect on output in our model because of its negative incentive effects on effort and saving. In the models displaying the poverty trap feature, these effects are absent. While redistribution cannot increase total output in our model, it can allow some unsuccessful agents to avoid the transactions costs related to borrowing and receive the rents enjoyed by self-financed entrepreneurs. But if  $\gamma$  is not too big, the negative incentive effect dominates the “transactions cost saving” effect, and redistribution reduces social welfare.

## 6. EXTENSIONS

### 6.1. Endogenous cost of credit

This section shows that if the transactions cost  $\gamma$  is endogenized using the model of Holmström and Tirole (1997), the results derived above still remain valid. Suppose the probability that a project yields a high output depends on actions taken by the worker *and* by the entrepreneur. The entrepreneur can either *shirk* or *work hard*. Shirking yields him a *private benefit*  $M > 0$ . If the entrepreneur works hard, the probability of success is  $e$  as before, where  $e$  is the worker’s effort. But if the entrepreneur shirks, then all his projects fail with probability one. The entrepreneur’s effort can only be observed by the bank if it monitors the entrepreneur. Monitoring costs the bank  $\gamma$ , and makes it impossible for the entrepreneur to shirk.

If  $M$  is very large, even self financed entrepreneurs will shirk, and if  $M$  is very small, bank-financed entrepreneurs will work hard even if they are not monitored. We now show that if  $M$  lies in an intermediate range, the analysis of the previous sections goes through unchanged. Notice that on the competitive credit market the borrower has to bear the full cost of the monitoring. Thus, a borrower with a loan of size  $k$  has to expect to make a repayment of  $k/\delta + \mu$ , where  $\mu = \gamma$  if the bank monitors, and  $\mu = 0$  otherwise. If we force banks to monitor each loan, then we know from Section 4 that a steady state equilibrium exists. Let  $A$  denote the entrepreneur’s expected revenue minus wage payments in that

equilibrium (as in equation (2)). To show that this remains an equilibrium in this section, we need to show that no bank wants to deviate by offering credit contracts with no monitoring. Suppose in fact a bank deviates in this way. To be profitable, the new contract must induce the bank-financed entrepreneur to work hard even without monitoring (since otherwise the loan cannot be repaid). Without loss of generality we can assume the deviation is such that the bank still makes zero profit. If the entrepreneur works hard he earns  $A$ , and his expected repayment of the loan is  $k/\delta$  (since there is no monitoring,  $\mu = 0$ ). If he shirks, all his projects fail, and he will not pay anything to the bank; his payoff will be  $M$ . Thus, the entrepreneur will shirk if  $M > A - k/\delta$ , so the new contract is *not* profitable for the bank if  $M > A - k/\delta$ . The *self-financed* entrepreneur will work hard if  $M \leq A$ . Therefore, we have:

**Proposition 7.** *The equilibrium found in Section 4 is still an equilibrium with endogenous cost of credit, as long as the private benefit  $M$  satisfies  $A - k/\delta < M \leq A$ .*

If the benefit from shirking is lower,  $M < A - k/\delta$ , then the following situation may occur. Banks do not monitor their debtors, and make zero profit. Bank-financed entrepreneurs are indifferent between shirking and not shirking. There is full employment. Bank-financed and self-financed entrepreneurs both earn strictly positive rents, so all unsuccessful young agents want a loan. However, despite the excess demand for credit, banks will not raise interest rates, since an agent paying a higher interest on the loan will be expected to shirk (with the original interest rate, they were just indifferent). Instead there is *equilibrium rationing on the credit market*. There is still an *American Dream* effect: the young agent has an incentive to work extra hard, since successful young workers can avoid being rationed on the credit market and will earn entrepreneurial rents for sure (see Ghatak, Morelli and Sjöström (2000) for details).

## 6.2. Posting of bonds

In the static model, the main cause of inefficiency is the limited wealth of agents and the limited liability constraint. Asking workers to post a bond, to be forfeited in case their project fails, would allow the failure wage to be negative and lead to an efficiency gain (more effort). We have ruled this out by the assumption  $l \geq 0$ . Many authors have provided theoretical arguments that support the assumption that workers cannot post bonds.<sup>22</sup> Dickens, Katz, Lang and Summers (1989) provide an excellent discussion of this literature, and in addition, argue convincingly that both legal restrictions and social norms rule out the use of bonds in practice. Nevertheless, we may comment on the consequences of allowing workers to save and post bonds in our model.

When bonds are allowed there are potentially two kinds of old workers: “rich” old workers who can post bonds and “poor” old workers who cannot. Entrepreneurs will compete to hire rich workers, so these workers will reap the efficiency gain from the bond. Let  $\Delta > 0$  denote the size of this gain. As before, the fact that successful agents do not need credit makes them more willing to become entrepreneurs, but now the fact that they have to give up the efficiency gain  $\Delta$  by not becoming a rich old worker adds an effect that makes them *less* willing to become entrepreneurs. The question is which of these effects

22. For example, there could be commitment problems on the part of the firm not to usurp the bond using false pretenses, or more subtly, not to increase the intensity of monitoring the worker from the level that is specified in the contract in the hope of catching him shirking (see Carmichael (1985)). Ritter and Taylor (1994) argue that if firms have private information, then workers may believe that any firm that asks for a bond knows that it is likely to go bankrupt, which discourages the use of bonds.

dominates. If we again let  $S$  (resp.  $B$ ) denote the rent received by self-financed entrepreneurs (resp. bank-financed entrepreneurs), then we now have  $S - B = \delta(\gamma - \Delta)$ . If  $\gamma > \Delta$  then  $S > B$  and the previous analysis would be essentially unchanged. Suppose instead that  $\gamma < \Delta$  so that  $S < B$ . The value of being a worker who can post a bond is so high that *successful* agents are *less* likely to want to become entrepreneurs than unsuccessful agents. There are three possibilities. If  $S < 0 < B$  then the supply of entrepreneurs equals the number of unsuccessful young workers. If  $S < B = 0$  then the unsuccessful young workers are indifferent between becoming entrepreneurs and old workers, while all successful young workers go on to become rich old workers. If  $S = 0 < B$  then the successful young workers are indifferent between becoming entrepreneurs and rich old workers, while all unsuccessful young workers become entrepreneurs. Corresponding to these three different cases, there would be three kinds of equilibria, and the analysis would mirror the one in Section 4. The main difference is that now there is an *American Dream* effect in all equilibria. Young agents work strictly harder than they would for the same wage in the benchmark one-period model, since the money they earn can be posted as bonds when they are old, which raises its shadow value.

## 7. CONCLUDING REMARKS

In a dynamic market economy, current hard work is motivated by expected future rewards. If the credit market is imperfect, rents will be available in occupations involving capital. When there is moral hazard in the labour market, these rents perform a socially useful function by encouraging individuals to work hard and save. As a result, policies that reduce these rents will dampen the incentives to work hard and save, which may reduce social welfare. This is the main point of this paper. We now briefly discuss how this conclusion depends on the assumptions of the model. The roles of Assumptions 1, 2 and 3 have already been discussed in Section 2.1. As we noted there, there is a non-convexity associated with the transactions cost in the credit market which in general may make randomized contracts valuable, but Assumption 3 guarantees that the set-up cost  $k$  is low enough to make randomization unnecessary. Removing Assumption 3 would have a substantial effect only if agents were risk averse.<sup>23</sup> In that case, randomized contracts would impose a cost on the agent, and for sufficiently high  $k$  the agent would prefer a non-random success wage rather than getting  $k$  with some very small probability. In this case an *American Dream* effect would only exist if the transactions cost on a loan were proportional to the size of the loan (since the failure wage would be lower than the success wage even with risk averse agents, as in Holmström (1979), successful agents would need a smaller loan than unsuccessful agents).

While risk aversion does limit the operation of the *American Dream* effect in the current model, they need not conflict in general. Consider an alternative model where workers have differing unobservable disutilities of effort, and there is a deterministic relation between effort and output. If output is observable but not effort, the wage would in general be a function of output and there would be some efficiency loss due to the presence of private information. If however lumpy investments and credit market imperfections generate rents from self-financing as in the current model, there would be an *American Dream* effect for young workers with lower disutility of effort.

23. In general, our main results depend on risk aversion being not too important, because very risk averse individuals do not respond to incentives, static, or dynamic (of the kind emphasized in this paper).

We now turn to the basic role of the market imperfections in this model. Our argument relies on the existence of incentive problems in the labour market. Without moral hazard, the entrepreneur and the worker would maximize their joint surplus; the old worker's effort would equal  $1/c$  and the young worker's effort would equal  $(1 + S)/c$ . This would be socially efficient as well as joint surplus maximizing since the rents from self-financed entrepreneurship represent a net social gain from economizing on costly credit market transactions. We still would get a kind of *American Dream* effect, but, with no imperfections in the labour market, abolishing the credit market imperfections would always be socially optimal. Alternatively, suppose there is moral hazard but we relax the assumption of limited liability. This can be done by assuming there exists non-monetary ways of transferring utility between the principal and the agent (e.g. free labour). This case, however, is just like the case of no moral hazard, since with perfectly transferable utility and risk-neutral agents the moral hazard problem can be solved costlessly. The limited liability would also be relaxed if young workers are born with enough wealth (say, due to bequests) so that they can post bonds and/or self-finance their enterprise. If enough agents are born wealthy, the entrepreneurial rents will be driven to zero and the American dream effect will disappear. The limited liability is also relaxed if old workers can post bonds as discussed in Section 6.2.

Our argument also relies on the existence of imperfections in the credit market. This imperfection could be modelled in a more general way, as could the production technology, without significantly changing any of our conclusions. In a more general model, the production technology need not be fixed coefficients type, the transactions cost for a small loan could be smaller than for a large loan, and projects could have more than two possible outcomes. As long as the credit market is imperfect, entrepreneurial profits will in general not be driven to zero. Even if the technology has constant returns to scale and each entrepreneur can buy more than one unit of capital, each individual's finite wealth will put an upper bound on how much capital he can buy without using the credit market. An agent whose first period income is low must either choose a smaller size firm (and hence make less profits) than an agent whose income is high, or he must borrow more money and so pay a higher transactions cost. In any case, the shadow value of an extra dollar is raised by the credit market imperfections, which provides the dynamic incentive to work hard.

Since we have assumed individuals are identical *ex ante*, there is *equality of opportunity*, and the *ex post* inequality of earnings may be socially acceptable. Social and occupational mobility are affected in reality by both effort and ability. We have stressed the importance of effort. In the opposite case where mobility is only due to ability, credit market imperfections may prevent the most talented agents from competing for entrepreneurial rents, which reduces output.<sup>24</sup> In the real world, the relative importance of effort and ability may vary. In our model the American dream effect is most important when  $n$  is small, *i.e.* in a world of small firms. In such a world heterogeneous ability may play a limited role given that the necessary human and physical capital may be rather standardized. Still, further research allowing for heterogeneous ability in a similar framework is clearly needed.

24. Adverse selection could cause credit market imperfections. If successful workers are more likely to become successful entrepreneurs, then banks will be reluctant to give loans to unsuccessful workers. This will give rise to dynamic incentives to generate earnings when young.



## 8. APPENDIX

## 8.1. Proofs

*Proof of Proposition 1.* If workers are in short supply then (by assumption) labour market contracts will give each entrepreneur his reservation payoff, *i.e.* what he could get by becoming an old worker. Then the entrepreneur does not recoup the sunk cost  $k$ , however, so he must have made the wrong occupational choice, which is impossible in equilibrium.

Suppose instead entrepreneurs are on the short side of the market. Then labour contracts maximize the entrepreneur's payoff subject to the worker receiving his reservation utility of zero, and subject to the IC constraint. Since the old worker's problem is a static one, the entrepreneur would offer him the same contract as in the static model of Section 2.3 with  $\bar{u} = 0$ . Thus,  $h^o = \frac{1}{2}$ , and the principal's expected profit from a single project operated by an old worker is  $1/(4c)$ . The old worker's expected utility is  $1/(8c)$ . The discounted expected profit from becoming a bank-financed entrepreneur with  $n$  old workers, net of the discounted opportunity cost of not being an old worker,  $\delta/(8c)$ , the cost of capital,  $k$ , and the discounted transactions cost,  $\delta\gamma$ , is

$$\delta\left(n\frac{1}{4c} - \frac{1}{8c}\right) - (k + \gamma\delta).$$

Assumption 2 guarantees that  $\gamma$  is small enough to make this expression strictly positive. Since the gain from hiring a young worker can never be lower than the gain from hiring an old worker, becoming a bank-financed entrepreneur is strictly profitable. So every old individual will be an entrepreneur, *i.e.*  $p = 1$ . But each entrepreneur can hire  $n \geq 2$  workers, and the number of young agents is 1. This contradicts the assumption that entrepreneurs are on the short-side of the market. Since neither workers nor entrepreneurs can be in short supply, the number of projects,  $pn$ , must equal the number of (young and old) workers,  $2 - p$ . Hence  $p = 2/(1+n) < 1$ .  $\parallel$

*Proof of Lemma 1.* For  $S \geq 0$  to be true we must have  $h^o < 1$  and  $h^y < 1$ . We now find lower bounds for  $h^o$  and  $h^y$ . Since  $h^o = \frac{1}{2}$  maximizes the entrepreneur's expected gain from the old worker,  $e^o(1 - h^o)$ , subject to  $h^o \geq 0$  and the IC constraint  $e^o = h^o/c$ , we clearly must have  $h^o \geq \frac{1}{2}$  in any steady state. Similarly, fix some  $S$  and consider choosing  $h^y$  to maximize the entrepreneur's expected gain from the young worker subject to  $h^y \geq 0$  and the IC constraint (8). By Assumption 3,  $k < \frac{1}{2}$ . Since the expression  $h^y(1 - h^y)/c$  is maximized at  $h^y = \frac{1}{2} > k$ , the solution to this programme cannot involve  $h^y < k$ . Therefore,  $h^y$  must maximize  $(h^y + S)(1 - h^y)/c$  subject to  $h^y \geq k$ . The solution is  $h^y = \max\{k, (1 - S)/2\}$ . This is clearly a lower bound on  $h^y$ . Thus, in any steady state we must have  $h^y \geq \max\{k, (1 - S)/2\}$ .

Since  $h^y \geq k$ , using (8) and (9) we can write equation (3) as

$$\frac{h^y}{c}(1 - h^y) + \frac{S}{c}(1 - h^y) = \frac{h^o}{c}(1 - h^o). \quad (25)$$

If  $S = 0$  then  $h^y = h^o$ . Suppose  $S > 0$ , and fix any  $h^o \in [1/2, 1)$ . If  $h^y = h^o$  then the left hand side of (25) is strictly greater than the right hand side. But the left-hand side of equation (25) decreases monotonically for all  $h^y \in [(1 - S)/2, 1)$ . Therefore, equation (25) forces  $h^y$  to be strictly greater than  $h^o$ .

Part 2 follows from inspection of (8) and (9), using part 1, and the fact that  $S \geq 0$ .  $\parallel$

*Proof of Lemma 2.* For a given  $S$ , the old worker's effort  $e^o$  must solve (12), which is a quadratic equation in  $e^o$ . It can be easily verified that one root of the equation is given by (13). Notice that

$$\frac{n^2 - \frac{1}{4}}{4c(n + \frac{1}{2})} = \frac{n}{4c} - \frac{1}{8c},$$

so from Assumption 2 and the fact that  $S \leq \delta\gamma$  we can conclude that

$$\frac{k + S}{\delta} < \frac{n^2 - \frac{1}{4}}{4c(n + \frac{1}{2})}. \quad (26)$$

Therefore, in (13) we take the square root of a strictly positive number, so  $e^o(S)$  is real. There is in fact another root to the quadratic equation (12), which is smaller. However, the expected payoff of an old worker is  $u^o = e^o h^o - c(e^o)^2/2 = c(e^o)^2/2$  which is increasing in  $e^o$ . Hence, for the same profit for the entrepreneur the larger root will give a higher expected payoff to the worker. So only the larger root can be relevant for an efficient contract.

Next, from (11),  $e^y(S)$  solves the quadratic equation

$$e^y(S)(1 - ce^y(S) + S) = e^o(S)(1 - ce^o(S)).$$

The explicit solution is given by (14). Again, we have selected the larger root because a young worker's expected payoff

$$u^y = e^y h^y - \frac{1}{2}c(e^y)^2 = \frac{1}{2}c(e^y)^2 - Se^y,$$

is strictly increasing in  $e^y$  for  $e^y \geq S/c$ , and from the IC constraint  $e^y = (h^y + S)/c > S/c$ . Note that the maximum value of the expression  $4ce^o(S)(1 - ce^o(S))$  is 1, which is attained at  $e^o(S) = 1/(2c)$ . Therefore  $e^y(S)$  is a real number if  $(1 + S)^2 \geq 1$ , which is satisfied for any  $S \geq 0$ .

We obtain (15) and (16) by substitution of  $e^o(S)$  and  $e^y(S)$  into the IC constraints. But when  $h^o(S)$  is defined by (15), the condition  $h^o(S) > 1/2$  is equivalent to (26). Therefore,  $h^o(S) > \frac{1}{2}$ . The same argument as in the proof of Lemma 1 establishes  $h^y(S) \geq h^o(S)$ .  $\parallel$

*Proof of Lemma 3.* The function  $e^o(S)$  satisfies

$$\delta \left[ ne^o(S)(1 - ce^o(S)) - c \frac{(e^o(S))^2}{2} \right] \equiv k + S. \quad (27)$$

Differentiating totally with respect to  $S$  we have

$$\frac{de^o(S)}{dS} = \frac{1}{\delta[n(1 - 2ce^o(S)) - ce^o(S)]} < 0, \quad (28)$$

where the inequality follows from

$$e^o(S) \geq \frac{1}{2c} > \frac{n}{n + \frac{1}{2}} \frac{1}{2c}.$$

The function  $e^y(S)$  is defined by the relationship

$$e^o(S)(1 - ce^o(S)) \equiv e^y(S)(1 - ce^y(S) + S). \quad (29)$$

Differentiating totally with respect to  $S$ , and using (28) we get

$$\frac{de^y(S)}{dS} = [e^y(S) - f(e^o(S))] \frac{1}{2ce^y(S) - (1 + S)}, \quad (30)$$

where

$$f(e) \equiv \frac{2ce - 1}{\delta n(ce(2n + 1)/n - 1)}. \quad (31)$$

From Lemma 2,  $h^y(S) > \frac{1}{2}$ . Therefore,

$$e^y(S) = \frac{h^y(S) + S}{c} > \frac{1 + S}{2c}$$

so the denominator in (30) is positive. Hence,  $de^y(S)/dS > 0$  if and only if  $e^y(S) - f(e^o(S)) > 0$ . It is easy to verify that  $f'(e) > 0$  for the relevant range (*i.e.*  $e \geq 1/(2c)$ ) so (28) implies  $f(e^o(S))$  is monotonically *decreasing* in  $S$ . This completes the proof.  $\parallel$

*Proof of Proposition 4.* If the "supply schedule"  $e^y(S)$  does not cross the horizontal line  $p = 2/(1 + n)$  at any  $S > 0$ , then clearly there is a unique equilibrium, which is of type III. Similarly, if it does not cross at any

$S < \gamma\delta$  there is a unique equilibrium, which is of type I. Suppose, instead, that a crossing occurs at some point  $0 < S < \gamma\delta$ . Let  $S^* = \min \{S : S > 0 \text{ and } e^y(S) = 2/(1+n)\}$ . We claim

$$\left. \frac{de^y(S)}{dS} \right|_{S=S^*} > 0.$$

From (30), it suffices to show

$$e^y(S^*) - f(e^o(S^*)) > 0, \quad (32)$$

where  $f$  is defined by (31). Now  $e^o(S^*) \leq 1/c$ , and  $f$  is increasing in  $e$ , so

$$f(e^o(S^*)) \leq f\left(\frac{1}{c}\right) = \frac{1}{\delta(1+n)} < \frac{2}{1+n}$$

since  $\delta > \frac{1}{2}$ . But  $e^y(S^*) = 2/(1+n)$  by assumption, which proves (32). Thus, as we raise  $S$  from zero, the first time  $e^y(S)$  crosses the horizontal line  $p = 2/(1+n)$ ,  $e^y(S)$  is upward sloping. By Lemma 3,  $e^y(S) > 2/(1+n)$  for all  $S > S^*$ .  $\parallel$

*Proof of Proposition 5.* Using (28), (30), (21) and the fact that  $e^y(0) = e^o(0)$  we obtain,

$$\left. \frac{dW}{d\gamma} \right|_{\gamma=0} = \delta c e^y(0) \frac{e^y(0) - (1+\delta)f(e^y(0))}{2c e^y(0) - 1}. \quad (33)$$

Since  $2c e^y(0) > 1$  this expression is strictly positive if and only if  $e^y(0) > (1+\delta)f(e^y(0))$ . (The proof of Lemma 3 showed that  $e^y(S)$  is upward sloping if and only if  $e^y(0) > f(e^y(0))$ ). Here, a strengthening of that condition is needed for social welfare to be increasing in  $\gamma$ . Turning to total output, the corresponding necessary and sufficient condition is  $e^y(0) > (2-p)f(e^y(0))$ . Thus, a sufficient condition for total output and social welfare to be strictly increasing in  $\gamma$  in a type I equilibrium is

$$e^y(0) > 2f(e^y(0)),$$

which is equivalent to

$$\frac{1}{1-\delta(n/4c)} \frac{(n+\frac{1}{2})(1-k)}{n} > 1 + \sqrt{1 - \frac{4cn + \frac{1}{2}k}{n} \frac{1}{\delta}}. \quad (34)$$

If  $c < \delta n^2/(2n-1)$ , then (34) holds for small enough  $k$ .  $\parallel$

*Proof of Proposition 6.* Under the redistribution scheme, the successful young worker gets a payoff of  $V(h^y) = h^y - t + S + \delta u^o$ . Each of the  $1 - e^y$  unsuccessful workers has a probability of winning the lottery equal to

$$\varepsilon = \frac{e^y t}{(1 - e^y)k}. \quad (35)$$

So the average unsuccessful young worker gets a payoff  $V(0) = \varepsilon(k+S) + \delta u^o$ . The effort supply functions are

$$e^y = \frac{h^y - t + S - \varepsilon(k+S)}{c}, \quad (36)$$

for the young worker, and  $e^o = h^o/c$  for old workers as before. Since the effort supply of old workers is not affected by the tax, the function  $e^o(S)$  is given by (13) as before. At the type I equilibrium,  $S = \delta\gamma$  and  $e^o = e^o(\delta\gamma)$  are independent of  $t$ . Now, (3) must hold, which using (36) implies

$$e^o(1 - h^o) = e^y(1 - c e^y - t + S - \varepsilon(k+S)). \quad (37)$$

We can use (37) to calculate

$$\left. \frac{de^y}{dt} \right|_{t=0} = e^y \frac{1 + ((e^y/(1 - e^y)) + ((k+S)/k))}{1 - 2c e^y + S}.$$

Since  $1 - 2c e^y + S < 0$  (see the proof of Lemma 3), and total output is  $Y = e^y + e^o(1-p)$  we conclude that  $dY/dt < 0$  when  $t = 0$ . The expected welfare of a new-born worker is

$$W = e^y(h^y - t + S + \delta u^o) + (1 - e^y)(\varepsilon(k+S) + \delta u^o) - \frac{1}{2}c(e^y)^2 = \frac{\varepsilon}{2}[(e^y)^2 + \delta(e^o)^2] + \varepsilon(k+S).$$

Here we used (36) to substitute for  $h^y$ , as well as the fact that  $u^o = c(e^o)^2/2$ . Using this result and (35) we obtain

$$\left. \frac{dW}{dt} \right|_{t=0} = \frac{e^y}{(1 - 2ce^y + S)(1 - e^y)} \left( (1 + S) \left( 1 + \frac{S}{k} \right) - ce^y \left( 1 + (2 - e^y) \frac{S}{k} \right) \right) \quad (38)$$

In general the sign of the expression within parentheses is ambiguous: while  $ce^y \leq 1 \leq 1 + S$  (by Lemma 1),  $1 + (2 - e^y)S/k > 1 + S/k$  since  $e^y < 1$ . However, for small enough  $\gamma$  the expression within parentheses will be positive since  $S$  will be very small and  $1 - ce^y > 0$  by Lemma 1. Hence the expression in (38) is strictly negative for  $\gamma$  close enough to 0.  $\parallel$

## 8.2. Equal wages for young and old

Suppose age is not observed by the employer, so old workers cannot be paid less than young. Equivalently, age could be observed but "age discrimination" ruled out by law or social norm. Then  $h^y = h^o = h$ . As before,  $p = 2/(1 + n)$  and  $h \geq k$ . If  $S > 0$  then young workers work harder than old at the same wage (the *American Dream* effect), and young workers are strictly more attractive to employers. The only difference now is that since the relative wage of young workers cannot be bid up, the greater incentives for young workers come only from the entrepreneurial rents, not from a higher success wage. As a result, for any  $S \in [0, \gamma\delta]$  the young workers work less hard, and old workers harder, than they would do if the firm could discriminate between them (and pay different wages). Forcing the two wages to be equal therefore yields ambiguous welfare results.

We now sketch how to solve for the equilibrium wage  $h$ . Consider a firm with  $v^o$  old and  $v^y$  young workers,  $v^o + v^y = n$ . Let  $\alpha = v^o/n$  denote the proportion of old workers in the firm. Each firm would prefer to hire only young workers, but since they cannot be distinguished (or cannot be selectively attracted due to legal restrictions), let us assume that firms hire young and old workers in (expected) proportion to their frequencies in the overall population. The expected fraction of old workers in a firm is

$$\hat{\alpha} = \frac{1 - p}{2 - p} = \frac{n - 1}{2n},$$

since  $p = 2/(1 + n)$ . Now (11) no longer holds. Instead, since  $h^y = h^o = h$ , the IC conditions (9) and (10) imply  $e^y = e^o + S/c$ . Using this, the entrepreneur's expected profit, gross of the cost of capital but net of wage payments, can be expressed as

$$\begin{aligned} A &= n(\hat{\alpha}e^o + (1 - \hat{\alpha})e^y)(1 - h) = \left( \frac{n-1}{2}e^o + \left( n - \frac{n-1}{2} \right)e^y \right)(1 - h) \\ &= \left( ne^o + \frac{n+1}{2} \frac{S}{c} \right)(1 - ce^o). \end{aligned}$$

We have

$$S = \delta A - k - \frac{1}{2}\delta c(e^o)^2 = \delta \left( ne^o + \frac{n+1}{2} \frac{S}{c} \right)(1 - ce^o) - k - \frac{1}{2}\delta c(e^o)^2. \quad (39)$$

Equation (39) implicitly determines  $e^o = e^o(S)$  as a function of  $S$ . Since (39) is a quadratic function in  $e^o$ , we can solve explicitly for  $e^o(S)$ , and we obtain  $e^y = e^y(S) = e^o(S) + S/c$ . Since  $e^y(S)$  is a continuous function of  $S$ , the same kind of graphic method as in section 4 can be used, and qualitatively the results are the same as in Section 4. Quantitatively, however, the effort levels and wages are different. For example, rewrite (39) as follows

$$S\beta(e^o) = \delta(ne^o(1 - ce^o) - \frac{1}{2}c(e^o)^2) - k, \quad (40)$$

where  $\beta(e^o) \equiv 1 - \delta(n+1)(1 - ce^o)/2c < 1$ . The right hand side of (40) is identical to the right hand side of equation (12). Notice that for  $S = 0$ , the equations (40) and (12) will yield identical values of  $e^o$ . Given that  $\beta(e^o) > 0$  and since the lowest value  $e^o$  can take is  $1/2c$ , if  $\beta(e^o) < 0$  for some feasible value of  $e^o$  it must be that  $\beta(1/2c) > 1$ , or,  $4c/(n+1) < \delta$ . But that implies  $4c/(n+1) < 1$  which is equivalent to  $2/(n+1) < 1/2c$ , so that  $S$  must be 0 in equilibrium. Hence the equations (40) and (12) will yield different values of  $e^o$  only when  $S > 0$  and  $0 < \beta(e^o) < 1$ . But that implies, for any  $S > 0$  the effort level  $e^o$  of old workers must now be greater than if  $h^y > h^o$  were allowed, and hence  $e^y$  will be higher as well, although the *equilibrium* value of  $S$  will now be naturally different.

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## REFERENCES

- ABRAHAM, K. and FARBER, H. (1987), "Job Duration, Seniority and Earnings", *American Economic Review*, **77**, 278–297.
- AGHION, P. and BOLTON, P. (1997), "A Trickle-down Theory of Growth and Development", *Review of Economic Studies*, **64**, 151–172.
- BAKER, G., GIBBS, M. and HOLMSTRÖM, B. (1994), "The Wage Policy of a Firm", *Quarterly Journal of Economics*, **109**, 921–956.
- BANERJEE, A. V. and NEWMAN, A. (1993), "Occupational Choice and the Process of Development", *Journal of Political Economy*, **101**, 274–298.
- BANERJEE, A. V. and NEWMAN, A. (1994), "Poverty, Incentives and Development", *American Economic Review*, **84**, 211–215.
- BLANCHFLOWER, D. G. and OSWALD, A. J. (1998), "What Makes an Entrepreneur?", *Journal of Labor Economics*, **16**, 26–60.
- BOEHLJE, M., (1973), "The Entry-Growth-Exit Processes in Agriculture", *Southern Journal of Agricultural Economics*, **5**, 23–36.
- CARMICHAEL, H. (1985), "Can Unemployment be Involuntary? Comment", *American Economic Review*, **75**, 1213–1214.
- DALTON, G. W. and THOMPSON, P. H. (1971), "Accelerating Obsolescence of Older Engineers", *Harvard Business Review*, **49**, 57–67.
- DICKENS, W., KATZ, L., LANG, K. and SUMMERS, L. (1989), "Employee Crime and the Monitoring Puzzle", *Journal of Labor Economics*, **7**, 331–347.
- ENGSTROM, D. W. and McCREADY, W. C. (1990) *Asian Immigrant Entrepreneurs in Chicago* (Chicago: Center for Urban Research and Policy Studies, University of Chicago).
- EVANS, D. and JOVANOVIĆ, B. (1989), "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints", *Journal of Political Economy*, **97**, 808–827.
- EVANS, D. and LEIGHTON, L. (1989), "Some Empirical Aspects About Entrepreneurship", *American Economic Review*, **79**, 519–535.
- GALE, H. F., JR. (1994), "Longitudinal Analysis of Farm Size over the Farmer's Life Cycle", *Review of Agricultural Economics*, **16**, 113–123.
- GALOR, O. and ZEIRA, J. (1993), "Income Distribution and Macroeconomics", *Review of Economic Studies*, **60**, 35–52.
- GHATAK, M., MORELLI, M. and SJÖSTRÖM T. (2000), "Dynamic Incentives, Occupational Choice, and the American Dream" (Penn State University Working Paper # 1-00-1).
- GIBBONS, R. and MURPHY, K. (1992), "Optimal Incentive Contracts in the Presence of Career Concerns", *Journal of Political Economy*, **100**, 468–505.
- HOLMSTRÖM, B. (1979), "Moral Hazard and Observability", *Bell Journal of Economics*, **10**, 74–91.
- HOLMSTRÖM, B. (1999), "Managerial Incentive Problems: A Dynamic Perspective," *Review of Economic Studies*, **66**, 169–182.
- HOLMSTRÖM, B. and TIROLE, J. (1997), "Financial Intermediation, Loanable Funds, and the Real Sector", *Quarterly Journal of Economics*, **112**, 663–691.
- KRIEDTE, P., MEDICK, H. and SCHLUMBOHM, J. (1981), *Industrialization before Industrialization: Rural Industry in the Genesis of Capitalism* (Cambridge: Cambridge University Press).
- LAFFONT, J.-J. and TIROLE, J. (1986), "Using Cost Observation to Regulate Firms", *Journal of Political Economy*, **94**, 614–641.
- LAZEAR, E. (1979), "Why is there Mandatory Retirement?", *Journal of Political Economy*, **87**, 1261–1284.
- LEGROS, P. and NEWMAN, A. (1996), "Wealth Effects, Distribution and the Theory of the Firm", *Journal of Economic Theory*, **70**, 312–341.
- LEWIS, T. and SAPPINGTON, D. (2001) "Optimal Contracting with Private Knowledge of Wealth and Ability", *Review of Economic Studies*, **68**, 21–44.
- LIPSEY, R. and LANCASTER, K. (1956), "The General Theory of the Second Best", *Review of Economic Studies*, **24**, 11–32.
- LONG, N.V. and SHIMOMURA, K. (1997), "Education, Moral Hazard, and Endogenous Growth" (Mimeo, McGill University).
- MEDOFF, J. and ABRAHAM, K. (1980), "Experience, Performance and Earnings", *Quarterly Journal of Economics*, **95**, 703–736.

- NEWMAN, A. (1992), "The Capital Market, Inequality, and the Employment Relation" (Mimeo, Columbia University).
- PIKETTY, T. (1997), "The Dynamics of Wealth Distribution and the Interest Rate with Credit Rationing", *Review of Economic Studies*, **64**, 173–189.
- QUADRINI, V. (1997), "Entrepreneurship, saving and social mobility" (Discussion Paper, Federal Reserve Board, Minneapolis).
- RITTER, J. A. and TAYLOR, L. J. (1994), "Workers as Creditors: Performance Bonds and Efficiency Wages" *American Economic Review*, **84**, 694–704.
- YOON, I.-J. (1997) *On My Own: Korean Businesses and Race Relations in America* (Chicago: University of Chicago Press).
- YOUNG, C. (1995), "Financing the Micro-Scale Enterprise: Rural Craft Producers in Scotland, 1840–1914", *Business History Review*, **69**, 398–421.