EC220 classes

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DF and ADF tests for non-stationarity

Our starting point in the "basic" Dickey-Fuller test of your book (formula 13.32 p.393). In a time-series, (X_t), you may suspect two types of non-stationarity problems: due to the fact that the series is **integrated** and incorporates the sum of some shocks (which implies that the variance will not be constant, eg. random walks), or due to a **deterministic trend**. To check these two assumptions, we can focus on a simple model:

(1) $X_t = \beta_1 + \beta_2.X_{t-1} + \gamma.t + u_t$

You want to test two assumptions: $\beta_2 = 1$ (vs $\beta_2 < 1$) and $\gamma \neq 0$. Under each of them (or both) the time-series, (X_t), will be non stationary.

To check if $H_0 : \beta_2 = 1$ or $H_1 : \beta_2 < 1$, we could perform a Dickey-Fuller test, but this test has two problems: it has low power and it is invalid if u_t is autocorrelated. To make the test more robust to this last problem we can perform an AR(1) transformation. Let's assume that the disturbance term of the original model, u_t, is AR(1):

(2) $u_t = \rho.u_{t-1} + \varepsilon_t$.

Where by assumptions ε_t is a white noise and $0 < \rho < 1$.

Then, we can "easily" transform the initial model to avoid the autocorrelation of u_t. Write as "usual":

(3) $X_t - \rho.X_{t-1} = \beta_1.(1-\rho) + \rho.\gamma + \beta_2.X_{t-1} - \rho.\beta_2.X_{t-2} + (1-\rho).\gamma.t + \varepsilon_t$

So that,

(4) $X_t = \beta_1.(1-\rho) + \rho.\gamma + (\beta_2 + \rho).X_{t-1} - \rho.\beta_2.X_{t-2} + (1-\rho).\gamma.t + \varepsilon_t$

You should still have in mind that you want to test $H_0 : \beta_2 = 1$ vs $H_1 : \beta_2 < 1$.

To do so, re-write the last model which is no longer subject to autocorrelation as:

(5) $\Delta X_t = \beta_1.(1-\rho) + \rho.\gamma + (\beta_2 + \rho - 1 - \rho.\beta_2).X_{t-1} + \rho.\beta_2.\Delta X_{t-2} + (1-\rho).\gamma.t + \varepsilon_t$

Or,

(6) $\Delta X_t = \beta_1.(1-\rho) + \rho.\gamma + (\beta_2 - 1).(1-\rho).X_{t-1} + \rho.\beta_2.\Delta X_{t-2} + (1-\rho).\gamma.t + \varepsilon_t$

By assumption, $0 < \rho < 1$, so a one sided test of the nullity of the parameter of X_{t-1} in the last model, $(\beta_2 - 1).(1-\rho)$, is a test of $H_0 : \beta_2 = 1$ vs $H_1 : \beta_2 < 1$.

Note that equation (4) does not contain any constraint on the initial parameters $(\rho, \beta_1, \beta_2, \gamma)$, so we can choose to re-parametrize the model (4) as:

(4') $X_t = \lambda_1 + \lambda_2.X_{t-1} + \lambda_3.X_{t-2} + \lambda_4.t + \varepsilon_t$

Which is one of the usual forms of the **ADF** test (formula 13.34, p.394). Then equation (6) may be rewritten as:

(6') $\Delta X_t = \lambda_1 + (\lambda_2 + \lambda_3 - 1).X_{t-1} - \lambda_3.\Delta X_{t-2} + \lambda_4.t + \varepsilon_t$

In this last equation (6'), we know now that the initial hypotheses: $H_0 : \beta_2 = 1$ vs $H_1 : \beta_2 < 1$ are equivalent to $H_0 : (\lambda_2 + \lambda_3 - 1) = 0$ vs $H_1 : (\lambda_2 + \lambda_3 - 1) < 0$.

There are at least three points worth noticing. First, under $H_0 : \beta_2 = 1$ the time series X_t is non stationary and so is X_{t-1}. Hence you can not test H_0 using a standard t-test. You have to use the standard t-statistic but compare its value to appropriate critical values.
Second, we have seen that X_t may be non stationary if $\beta_2 = 1$ or $\gamma \neq 0$, but we have never tested the second hypothesis. Again it may not be possible to use a standard t-test to test if $\gamma = 0$. The main focus of the ADF test and equation (1) is on β_2.
Third, under $H_0 : \beta_2 = 1$, the time series X_t is non stationary and it is likely to be non "**weakly persistent**". As a consequence, in any model that contains X_t either as a dependent or explanatory variable the usual statistical inference is very likely to be invalid. We have seen in PS17 and in the Granger-Newbold experiment that this could imply a phenomenon called **spurious regression**. Two unrelated non-stationary variables could appear to have a relationship if we use standard t-test, but this conclusion is invalid because the assumptions for the standard t-test to be valid are violated.