B Appendix (Online only): Details of Case 2

In this Appendix, I provide more details on Case 2 of the model. First, I formally define the equilibrium concept.

Definition B.1 *A linear Rational Expectation Equilibrium is given by the linear price functions p_1, p_2, mapping the aggregate random variables to prices and individual demands, d_1^i, d_1^j, d_2^j such that d_1^i, d_2^j, d_1^j, solve problems (9)-(11) and (41) , respectively and p_t clear the market in period $t = 1, 2$.*

Following the discussion in the main text, conjecture (13) and definitions of q_2, τ_2, and $\tau_\theta, b_2, c_2, e_2, g_2$ are used as before. I defined the conjecture of the first period price, q_1, ϕ and τ_1 in the main text.

I define b_I, c_I, e_I and b_J, c_J, e_J as the linear coefficients of the conditional expectations

$$E\left(q_2|x^i, y, q_1\right) = a_I x^i + c_I y + e_I q_1 \tag{B.1}$$

$$E\left(q_2|z^j, y, q_1\right) = b_J z^j + c_J y + e_J q_1 \tag{B.2}$$

and

$$\tau_I^2 \equiv \frac{1}{var\left(q_2|x^i, y, q_1\right)}$$

$$\tau_J^2 \equiv \frac{1}{var\left(q_2|z^j, y, q_1\right)}$$

as the corresponding precision.

The problem of each J-trader in the second period and that of each I-trader in the first period are very similar to their respective problems in case 1. The optimal demand of these traders leads to the same formulations of (20) and (21), respectively.

However, J-traders have to solve a two period problem in period 1. I show below that their demand function takes the form of

$$d_1^j = \frac{\left(\tau_\theta^2 b_2^2 + \tau_J^2\right)\left(E\left(p_2|z^j, y, q_1\right) - p_1\right)}{\left(b_2 + e_2\right)^2 \gamma} + \frac{\tau_\theta^2 b_2^2 \left(z^j - E\left(q_2|z^j, y, q_1\right)\right)}{\left(b_2 + e_2\right)\gamma}, \tag{B.3}$$

a weighted sum of the trader's expected price change between period 1 and 2 and her expected demand in period 2. I refer to the first term as the myopic component and the second term as the hedging component of demand.

Just as in Case 1, I have to find $\mathbf{b}_2, \mathbf{b}_1, \mathbf{c}_2, \mathbf{c}_1, \mathbf{e}_2, \mathbf{e}_1$ and $\mathbf{g}_2$, which ensure that the price functions coincide with their respective conjectures. The next proposition follows.

Proposition B.1 *Suppose that the system*

$$\begin{aligned}
\delta_2 \tau_\theta^2 \frac{b_2}{\gamma} &= \tau_2 \qquad &\text{(B.4)}\\
\delta_1 \frac{\mu \left(\tau_J^2 b_J + \tau_\theta^2 b_2^2\right) + (1-\mu)\, \tau_I^2 a_I}{\gamma\,(e_2 + b_2)} &= \tau_1, \qquad &\text{(B.5)}\\
\frac{\mu \left(\tau_J^2 b_J + \tau_\theta^2 b_2^2\right)}{\mu \left(\tau_J^2 b_J + \tau_\theta^2 b_2^2\right) + (1-\mu)\, \tau_I^2 a_I} &= \phi, \qquad &\text{(B.6)}
\end{aligned}$$

has a fixed point $\tau_1^*, \tau_2^*, \phi^*$. Then there is a linear REE. In this equilibrium price and demand in period 2 is given by (13) and (22) where (24)-(27) hold. In period 1, price is given by

$$p_1 = \frac{\mathbf{a}_1 (\theta_I + \theta_C) + \mathbf{b}_1 (\theta_J + \theta_C) + \mathbf{c}_1 y - u_1}{\mathbf{e}_1}$$

and demand functions are given by

$$\begin{aligned}
d_1^i &= \mathbf{a}_I x^i + \mathbf{c}_I y - \mathbf{e}_I p_1 \qquad &\text{(B.7)}\\
d_1^j &= \mathbf{b}_J x^i + \mathbf{c}_J y - \mathbf{e}_J p_1 \qquad &\text{(B.8)}
\end{aligned}$$

where

$$\begin{aligned}
(1-\mu)\, \mathbf{a}_I &= \mathbf{a}_1 \qquad &\text{(B.9)}\\
\mu \mathbf{b}_J &= \mathbf{b}_1 \qquad &\text{(B.10)}\\
(1-\mu)\, \mathbf{c}_I + \mu \mathbf{c}_J &= \mathbf{c}_1 \\
(1-\mu)\, \mathbf{e}_I + \mu \mathbf{e}_J &= \mathbf{e}_1 \qquad &\text{(B.11)}
\end{aligned}$$

and

$$\begin{aligned}
\mathbf{b}_J &= \frac{\tau_J^2 b_J + \tau_\theta^2 b_2^2}{\gamma\,(e_2 + b_2)} \qquad &\text{(B.12)}\\
\mathbf{a}_I &= \frac{\tau_I^2 a_I}{\gamma\,(e_2 + b_2)} \qquad &\text{(B.13)}
\end{aligned}$$

and $\mathbf{c}_I, \mathbf{c}_J, \mathbf{e}_I, \mathbf{e}_J$ can also be written as analytical functions of the parameters and τ_1, τ_2, ϕ only. Furthermore, all coefficients are calculated at $\tau_1 = \tau_1^*$ and $\tau_2 = \tau_2^*$ and $\phi = \phi^*$

Proof. Period 2 is equivalent to case 1. For period 1 objects, first, I derive expression (B.3). In period 1, J-traders maximize the expected utility

$$\max_{d_1^i} E_1\left(-\exp\left(-\gamma\left(p_2-p_1\right)d_1^j-\gamma\frac{E\left(\theta|q_2,q_1,y,z^j\right)-p_2}{\gamma var\left(\theta|q_2,q_1,y,\bar{z}^j\right)}\left(\theta-p_2\right)\right)|z^j,q_1,y\right)=$$
$$E_1\left(E_2\left(-\exp\left(-\gamma\left(p_2-p_1\right)d_1^j-\frac{E\left(\theta|q_2,q_1,y,z^j\right)-p_2}{var\left(\theta|q_2,q_1,y,\bar{z}^j\right)}\left(\theta-p_2\right)\right)|q_2,q_1,y,z^j\right)|z^j,q_1,y\right)$$

as $E\left(\exp\left(\theta\right)\right)=\exp\left(E\left(\theta\right)+\frac{1}{2}var\left(\theta\right)\right)$

$$E_2\left(-\exp\left(-\gamma\left(p_2-p_1\right)d_1^j-\gamma\frac{E\left(\theta|q_2,q_1,y,z^j\right)-p_2}{\gamma var\left(\theta|q_2,q_1,y,\bar{z}^j\right)}\left(\theta-p_2\right)\right)|q_2,q_1,y,z^j\right)=$$
$$=\exp\left(-\gamma\left(p_2-p_1\right)d_1^j\right)\exp\left(-\frac{\left(E\left(\theta|q_2,q_1,y,z^j\right)-p_2\right)^2}{var\left(\theta|q_2,q_1,y,\bar{z}^j\right)}+\frac{1}{2}\frac{\left(E\left(\theta|q_2,q_1,y,z^j\right)-p_2\right)^2}{var\left(\theta|q_2,q_1,y,\bar{z}^j\right)}\right)$$

thus, the trader maximizes

$$E_1\left(-\exp\left(-\gamma\left(p_2-p_1\right)d_1^j-\frac{\left(E\left(\theta|q_2,q_1,y,z^j\right)-p_2\right)^2}{var\left(\theta|q_2,q_1,y,z^j\right)}\frac{1}{2}\right)|z^j,q_1,y\right)=$$
$$=E_1\left(-\exp\left(-\gamma\left(p_2-p_1\right)d_1^j-var\left(\theta|q_2,q_1,y,z^{jj}\right)\left(\mathbf{b}_2\right)^2\left(z^j-q_2\right)^2\frac{1}{2}\right)|z^j,q_1,y\right)=$$
$$=E_1\left(-\exp\left(-\gamma\left(\frac{\mathbf{b}_2q_2+\mathbf{g}_2q_1-\mathbf{c}_2y}{\mathbf{e}_2}-p_1\right)d_1^j-\frac{\left(\mathbf{b}_2\right)^2\left(\left(z^j\right)^2-2q_2z^j+q_2^2\right)}{\tau_\theta^2}\frac{1}{2}\right)|z^j,q_1,y\right)$$

where I used that

$$\frac{E\left(\theta|q_2,q_1,y,z^j\right)-p_2}{\gamma var\left(\theta|q_2,q_1,y,z^{jj}\right)}=\mathbf{b}_2\left(z^j-q_2\right)=\tau_\theta^2\frac{b_2}{\gamma}\left(z^j-q_2\right).$$

A property of normal distributions is that if C is constant scalar, L is a $nx1$ constant vector,

N is an nxn constant matrix and M is an $nx1$ stochastic matrix and $\mathcal{I}$ is an information set, then

$$E\left(-\exp\left(C+L'M-M'NM'\right)|\mathcal{I}\right)=$$
$$-\left|W\right|^{-1/2}\left|2N+W^{-1}\right|^{-1/2}\exp\left(C+L'Q-Q'NQ+\frac{1}{2}\left(L'-2Q'N\right)\right)\left(2N+W^{-1}\right)^{-1}\left(L-2NQ\right)\Big) \tag{B.14}$$

where $Q = E(M|I)$ and $W = var(M|I)$. Let $q_2 = M$ and

$$\begin{aligned} C &= -\gamma \left(\frac{\mathbf{g}_2 q_1 - \mathbf{c}_2 y}{\mathbf{e}_2} - p_1 \right) d_1^j - \tau_\theta^2 b_2^2 \left(z^j\right)^2 \frac{1}{2} \\ L &= -\gamma \frac{a_2'}{e_2'} d_1^j + \tau_\theta^2 b_2^2 z^j \\ N &= \tau_\theta^2 b_2^2 \frac{1}{2} \end{aligned}$$

then the term in the brackets in (B.14) is

$$\begin{aligned} &-\gamma \left(\frac{\mathbf{g}_2 q_1 - \mathbf{c}_2 y}{\mathbf{e}_2} - p_1 \right) d_1^j - \tau_\theta^2 b_2^2 \left(z^j\right)^2 \frac{1}{2} + \\ &\left(-\gamma \frac{\mathbf{b}_2}{\mathbf{e}_2} d_1^j + \tau_\theta^2 b_2^2 z^j \right) E\left(q_2 | z^j, q_1, y\right) \\ &-E^2\left(q_2 | z^j, q_1, y\right) \frac{1}{2} \tau_\theta^2 b_2^2 \\ &+\frac{1}{2} \frac{\left(-\gamma \frac{\mathbf{b}_2}{\mathbf{e}_2} d_1^j + \tau_\theta^2 b_2^2 z^j - E\left(q_2 | z^j, q_1, y\right) \tau_\theta^2 b_2^2 \right)^2}{\tau_\theta^2 b_2^2 + \tau_J^2}. \end{aligned}$$

Thus, the trader maximizes

$$\gamma \left(\frac{\mathbf{g}_2 q_1 - \mathbf{c}_2 y}{\mathbf{e}_2} - p_1 \right) d_1^j + \gamma \left(d_1^j \frac{\mathbf{b}_2}{\mathbf{e}_2} \right) E\left(q_2 | z^j, q_1, y\right) - \frac{1}{2} \frac{\left(-\gamma \frac{\mathbf{b}_2}{\mathbf{e}_2} d_1^j + \tau_\theta^2 b_2^2 z^j - E\left(q_2 | z^j, q_1, y\right) \tau_\theta^2 b_2^2 \right)^2}{\tau_\theta^2 b_2^2 + \tau_J^2}$$

taking the first order condition gives

$$\frac{\left(\frac{\mathbf{g}_2 q_1 - \mathbf{c}_2 y}{\mathbf{e}_2} + \frac{\mathbf{b}_2}{\mathbf{e}_2} E\left(q_2 | z^j, q_1, y\right) - p_1 \right) \left(\tau_\theta^2 b_2^2 + \tau_J^2\right) + \tau_\theta^2 b_2^2 \frac{\mathbf{b}_2}{\mathbf{e}_2} \left(z^j - E\left(q_2 | z^j, q_1, y\right)\right)}{\gamma \left(\frac{\mathbf{b}_2}{\mathbf{e}_2} \right)^2} = d_1^j \quad \text{(B.15)}$$

which is equivalent to (B.3). Collecting coefficients of z^j and using that $\frac{\mathbf{b}_2}{\mathbf{e}_2} = b_2 + e_2$, gives the expression for $\mathbf{b}_J$ as

$$\frac{\frac{\mathbf{b}_2}{\mathbf{e}_2} b_J \left(\tau_\theta^2 b_2^2 + \tau_J^2\right) + \tau_\theta^2 b_2^2 \frac{\mathbf{b}_2}{\mathbf{e}_2} \left(1 - b_J\right)}{\gamma \left(\frac{\mathbf{b}_2}{\mathbf{e}_2} \right)^2} = \frac{b_J \left(\tau_\theta^2 b_2^2 + \tau_J^2\right) + \tau_\theta^2 b_2^2 \left(1 - b_J\right)}{\gamma \left(e_2 + b_2\right)} = \frac{\tau_J^2 b_J + \tau_\theta^2 b_2^2}{\gamma \left(e_2 + b_2\right)}.$$

The demand of I-traders in the first period is

$$d_1^i = \frac{E\left(p_2|x^i, q_1, y\right) - p_1}{\gamma var\left(p_2|x^i, q_1, y\right)} = \tau_I^2 \frac{\frac{\mathbf{b}_2}{\mathbf{e}_2} E\left(q_2|x^i, q_1, y\right) + \frac{\mathbf{g}_2 q_1 - \mathbf{c}_2 y}{\mathbf{e}_2} - p_1}{\gamma\left(\frac{\mathbf{b}_2}{\mathbf{e}_2}\right)^2}. \tag{B.16}$$

Collecting terms multiplying x^i gives

$$\mathbf{a}_I = \frac{\tau_I^2 a_I}{\gamma\left(e_2 + b_2\right)}.$$

where (45)-(46) define coefficients $\mathbf{a}_I, \mathbf{c}_I, \mathbf{e}_I$ and $\mathbf{b}_J, \mathbf{c}_J, \mathbf{e}_J$. As, by market clearing, (47)-(50) have to hold, the system (B.4)-(B.6) comes from plugging (B.12)-(B.13) into (B.9) and (B.10) and the resulting formulas into (44),(17) and the definition of ϕ. Finally, I derive expressions for $\mathbf{c}_I, \mathbf{e}_I$ and $\mathbf{c}_J, \mathbf{e}_J$ and $\mathbf{c}_1, \mathbf{e}_1$ as functions of the primitives and τ_1, τ_2. First, collecting the terms multiplying y in (B.15) and (B.16), plugging in the definition of q_1 gives

$$\begin{aligned} \mathbf{c}_J &= \frac{\mathbf{e}_2}{\gamma(\mathbf{b}_2)^2}\left(\tau_J^2 \mathbf{b}_2 c_J + \left(\tau_\theta^2 b_2^2 + \tau_J^2\right)\left(\mathbf{c}_2 - \mathbf{g}_2 \frac{\mathbf{c}_1}{\mathbf{a}_1 + \mathbf{b}_1}\right)\right) && \text{(B.17)} \\ \mathbf{c}_I &= \frac{\mathbf{e}_2}{\gamma(\mathbf{b}_2)^2}\tau_I^2\left(\mathbf{c}_2 + \mathbf{b}_2 c_I - \left(\mathbf{g}_2 + \mathbf{b}_2 e_I\right)\frac{\mathbf{c}_1}{\mathbf{a}_1 + \mathbf{b}_1}\right). && \text{(B.18)} \end{aligned}$$

Using $\mathbf{c}_1 = \mu \mathbf{c}_J + (1-\mu)\,\mathbf{c}_I$ and $\mathbf{a}_1 + \mathbf{b}_1 = \mu \mathbf{a}_I + (1-\mu)\,\mathbf{b}_J$ This gives

$$\frac{\mathbf{c}_1}{\mathbf{a}_1 + \mathbf{b}_1} = \frac{(1-\mu)\,\tau_I^2\left(\mathbf{c}_2 + \mathbf{b}_2 c_I\right) + \mu\left(\tau_J^2 \mathbf{b}_2 c_J + \left(\tau_\theta^2 b_2^2 + \tau_J^2\right)\mathbf{c}_2\right)}{\mathbf{b}_2\left(\tau_I^2(1-\mu)\left(a_I + e_I\right) + \mu\left(\tau_J^2 a_J + \tau_\theta^2 b_2^2\right)\right) + \mathbf{g}_2\left((1-\mu)\,\tau_I^2 + \mu\left(\tau_\theta^2 b_2^2 + \tau_J^2\right)\right)}.$$

Plugging back this and the expressions for $\mathbf{e}_2, \mathbf{b}_2, \mathbf{g}_2$ into expressions (B.17)-(B.18) gives the result. By analogous steps I get

$$\begin{aligned} \frac{\mathbf{e}_1}{\mathbf{a}_1 + \mathbf{b}_1} &= \frac{\frac{\mathbf{e}_2}{\mathbf{b}_2}\left(\mu\left(\tau_J^2 + \tau_\theta^2 b_2^2\right) + (1-\mu)\,\tau_I^2\right)}{\tau_I^2(1-\mu)\left(a_I + e_I + \frac{\mathbf{g}_2}{\mathbf{b}_2}\right) + \tau_J^2\mu\left(a_J + \frac{\mathbf{g}_2}{\mathbf{b}_2}\right) + \mu\tau_\theta^2 b_2^2\left(1 + \frac{\mathbf{g}_2}{\mathbf{b}_2}\right)} \\ \mathbf{e}_I &= \left(\frac{\mathbf{e}_2}{\mathbf{b}_2}\right)^2 \frac{\tau_I^2}{\gamma}\left(1 - \frac{\mathbf{e}_1}{\mathbf{a}_1 + \mathbf{b}_1}\left(\mathbf{g}_2 + \mathbf{b}_2 e_I\right)\right) \\ \mathbf{e}_J &= \frac{1}{\gamma}\left(\frac{\mathbf{e}_2}{\mathbf{b}_2}\right)^2\left(1 - \frac{\mathbf{e}_1}{\mathbf{a}_1 + \mathbf{b}_1}\frac{\mathbf{g}_2}{\mathbf{b}_2}\right)\left(\tau_J^2 + \tau_\theta^2 b_2^2\right). \end{aligned}$$

■

Now I turn to the analyisis of the trading volume. In the second period, by the same analysis as in case 1, the equilibrium demand of each trader is described by (31) and the volume in the second period is described by (37). For demand and expected volume in the first period, observe that from (45)-(46) and the definition of q_1

$$
\begin{aligned}
d_1^i &= \mathbf{a}_I x^i + y\left(\mathbf{c}_I - \mathbf{e}_I \frac{\mathbf{c}_1}{\mathbf{e}_1}\right) - \mathbf{e}_I \frac{q_1\left(\mathbf{a}_1 + \mathbf{b}_1\right)}{\mathbf{e}_1} = \\
&= \mathbf{a}_I \varepsilon^i + \left[\mathbf{a}_1\left(\frac{1}{(1-\mu)} - \frac{\mathbf{e}_I}{\mathbf{e}_1}\right)(\theta_I + \theta_C) - \mathbf{e}_I \frac{\mathbf{b}_1}{\mathbf{e}_1}(\theta_J + \theta_C) + y\left(\mathbf{c}_I - \mathbf{e}_I \frac{\mathbf{c}_1}{\mathbf{e}_1}\right)\right] + \frac{\mathbf{e}_I}{\mathbf{e}_1} u_1,
\end{aligned}
\tag{B.19}
$$

$$
\begin{aligned}
d_1^j &= \mathbf{b}_J x^i + y\left(\mathbf{c}_J - \mathbf{e}_J \frac{\mathbf{c}_1}{\mathbf{e}_1}\right) - \mathbf{e}_J \frac{q_1\left(\mathbf{a}_1 + \mathbf{b}_1\right)}{\mathbf{e}_1} = \\
&= \mathbf{b}_J \varepsilon^j + \left[\mathbf{b}_1\left(\frac{1}{\mu} - \frac{\mathbf{e}_J}{\mathbf{e}_1}\right)(\theta_J + \theta_C) - \mathbf{e}_J \frac{\mathbf{a}_1}{\mathbf{e}_1}(\theta_I + \theta_C) + y\left(\mathbf{c}_J - \mathbf{e}_J \frac{\mathbf{c}_1}{\mathbf{e}_1}\right)\right] + \frac{\mathbf{e}_J}{\mathbf{e}_1} u_1
\end{aligned}
\tag{B.20}
$$

and, consequently,

$$
\begin{aligned}
V_1 \equiv{} & E\left(\left|d_1^i\right|\right) = \\
={} & \sqrt{\frac{1}{2\pi}}(1-\mu)\left(\frac{(\mathbf{a}_I)^2}{\alpha} + \left[\frac{\left(\frac{\mathbf{a}_1}{(1-\mu)} + \mathbf{c}_I - \frac{\mathbf{e}_I}{\mathbf{e}_1}(\mathbf{a}_1 + \mathbf{c}_1)\right)^2 + \left(\mathbf{c}_I - \mathbf{e}_I \frac{\mathbf{c}_1 + \mathbf{b}_1}{\mathbf{e}_1}\right)^2}{\nu} + \frac{\left(\frac{\mathbf{a}_1}{(1-\mu)} - \frac{\mathbf{e}_I}{\mathbf{e}_1}(\mathbf{a}_1 + \mathbf{b}_1 + \mathbf{c}_1) + \mathbf{c}_I\right)^2}{\omega} + \frac{\left(\mathbf{c}_I - \mathbf{e}_I \frac{\mathbf{c}_1}{\mathbf{e}_1}\right)^2}{\beta}\right] + \frac{\left(\frac{\mathbf{e}_I}{\mathbf{e}_1}\right)^2}{\delta_1}\right)^{\frac{1}{2}} + \\
& + \sqrt{\frac{1}{2\pi}}\mu\left(\frac{(\mathbf{b}_J)^2}{\alpha} + \left[\frac{\left(\frac{\mathbf{b}_1}{\mu} + \mathbf{c}_J - \frac{\mathbf{e}_J}{\mathbf{e}_1}(\mathbf{b}_1 + \mathbf{c}_1)\right)^2 + \left(\mathbf{c}_J - \mathbf{e}_J \frac{\mathbf{c}_1 + \mathbf{b}_1}{\mathbf{e}_1}\right)^2}{\nu} + \frac{\left(\frac{\mathbf{b}_1}{\mu} - \frac{\mathbf{e}_J}{\mathbf{e}_1}(\mathbf{a}_1 + \mathbf{b}_1 + \mathbf{c}_1) + \mathbf{c}_J\right)^2}{\omega} + \frac{\left(\mathbf{c}_J - \mathbf{e}_J \frac{\mathbf{c}_1}{\mathbf{e}_1}\right)^2}{\beta}\right] + \frac{\left(\frac{\mathbf{e}_J}{\mathbf{e}_1}\right)^2}{\delta_1}\right)^{\frac{1}{2}}.
\end{aligned}
$$

Unlike in Case 1, equilibrium demand does depend on the realization of aggregate random variables. The reason is that in period 1, I-traders' and J-traders' demand react differently to each piece of information. This is so because both the joint distribution of signals in each trader's information set and the trading horizon differ across groups. As a consequence, apart from the risk-sharing and speculative parts of trades defined in Case 1, there is also trade across groups. This latter part of equilibrium demand and expected volume is in squared brackets in each expression. Note that the population weighted average of the terms in the squared brackets is 0.

Speculative volume separates the within-group part of trade and given by (51). Just as in Case 1, it is useful to establish the following result.

Proposition B.2 *In the limit $\nu \to \infty$, there is a unique equilibrium where*

$$\begin{aligned}
\mathbf{b}_2 &= \alpha\frac{1}{\gamma} \\
\mathbf{a}_1 &= (1-\mu)\frac{\alpha}{\gamma}\frac{\alpha\delta_2^2}{\gamma^2+\alpha\delta_2^2} \\
\mathbf{b}_1 &= \mu\frac{\alpha}{\gamma}.
\end{aligned}$$

Thus

$$\frac{\partial \mathbf{b}_t}{\partial \beta} = \frac{\partial \mathbf{a}_1}{\partial \beta} = \frac{\partial \mathbf{C}_t}{\partial \beta} = \frac{\partial \mathbf{V}_t^S}{\partial \beta} = 0$$

for $t = 1, 2$.

Proof. The result is a consequence of Proposition B.1 and the fact that

$$\tau_2^* = \alpha\frac{\delta_2}{\gamma},\ \tau_1^* = \delta_1\left((1-\mu)\frac{\alpha}{\gamma}\frac{\alpha\delta_2^2}{\gamma^2+\alpha\delta_2^2} + \mu\frac{\alpha}{\gamma}\right)$$

and

$$\phi = \frac{\mu}{\mu + (1-\mu)\frac{\alpha\delta_2^2}{\gamma^2+\alpha\delta_2^2}}$$

is the fixed point of the system (B.4)-(B.6) at $\nu \to \infty$. ■

The proposition shows that under the standard information structure, even if traders with heterogeneous horizon coexist, more public information has no effect on trading intensities, the information content of trade or speculative volume. Numerical simulations show that the effect on total expected volume also diminishes as $\nu \to \infty$.

To complement the analysis in the in the main text, I decompose the trading intensity of J-traders in the first period on Figure 6. As it was pointed out in the main text, this trading intensity increases in public information for any β. The left panel shows the term $\frac{\tau_\theta^2 b_2^2}{\gamma(e_2+b_2)}$ which is the hedging component of the trading intensity. This panel shows that this component is decreasing in β. The term $\frac{\tau_J^2 b_J}{\gamma}$ is the numerator in the myopic component of the trading intensity in (B.3). Comparing this term to equilibrium value of $\mathbf{b}_2$ shows that this term would be the trading intensity, if the true value were to realize in period 2 instead of period 3. The right panel on Figure 6 shows that this term is also decreasing in β. Thus, trading intensity $\mathbf{b}_J$ increases in β solely because of the remaining term, $\frac{1}{b_2+e_2}$, the numerator of the myopic component. Note that this term is the inverse of the sensitivity of p_2 to the fundamental. Intuitively, as public information increases, the second period price is more correlated to the fundamental, so in the first period all traders can estimate p_2 with more certainty. While, this effect is not sufficient to influence the sign of the derivative of $\mathbf{a}_I$ with respect to public information, it switches that of $\mathbf{b}_J$.

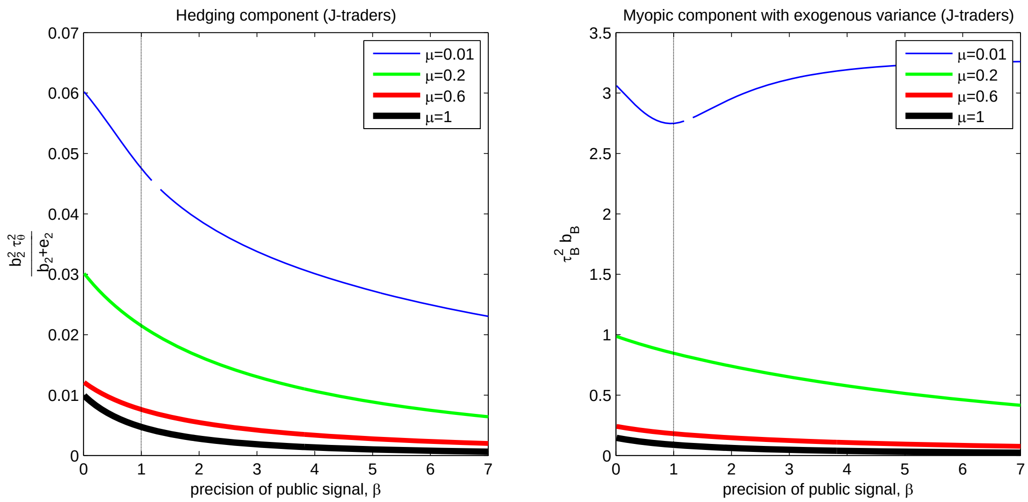

Figure 6: Decomposition of trading intensity, $\mathbf{b}_J$ of J-traders in the first period. The left and right panel show the intensity corresponding to the hedging component and myopic component respectively. In each plot, different curves correspond to different fraction of J-traders on the market,μ. The thicker the curve, the larger the fraction. The x-axis is the precision of the public signal, β. The vertical line depicts $\beta = \frac{\nu^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information. Parameter values are $\gamma = 1$, $\omega = 4.01$, $\nu = 2$, and $\alpha = \delta_1 = \delta_2 = 5$.

In the main text, numerical analysis illustrated that in a weakly correlated information structure, the larger the fraction of short-term traders, the larger the response in speculative volume. Here, I prove analytically the weaker statement that this relationship holds for the direct effect when τ_1, τ_2, ϕ are held constant.

Lemma B.1 *Holding τ_1, τ_2, ϕ fixed, the public information elasticity of speculative volume is decreasing in the fraction of long-horizon traders (J-traders) in period 1:*

$$\frac{\partial \frac{\partial V_1^S}{\partial \beta}\frac{\beta}{V_1^S}}{\partial \mu}\Big|_{\tau_1=\bar{\tau}_1,\tau_2=\bar{\tau}_2,\phi=\bar{\phi}} < 0.$$

Proof. By definition

$$\frac{\partial V_1^S}{\partial \beta}\frac{\beta}{V_1^S}\Big|_{\tau_1=\bar{\tau}_1,\tau_2=\bar{\tau}_2,\phi=\bar{\phi}} =$$

$$= (1-\mu)\frac{\frac{\partial\left|a_I\tau_I^2\right|}{\partial\beta}\Big|_{\tau_1=\bar{\tau}_1,\tau_2=\bar{\tau}_2,\phi=\bar{\phi}}}{(1-\mu)\left|a_I\tau_I^2\right| + \mu\left(b_J\tau_J^2 + b_2^2\tau_\theta^2\right)} + \mu\frac{\frac{\partial b_J\tau_J^2 + b_2^2\tau_\theta^2}{\partial\beta}\Big|_{\tau_1=\bar{\tau}_1,\tau_2=\bar{\tau}_2,\phi=\bar{\phi}}}{(1-\mu)\left|a_I\tau_I^2\right| + \mu\left(b_J\tau_J^2 + b_2^2\tau_\theta^2\right)} + \frac{\partial\frac{1}{b_2+e_2}\Big|_{\tau_1=\bar{\tau}_1,\tau_2=\bar{\tau}_2,\phi=\bar{\phi}}}{\partial\beta}.$$

By simple substitution, I can show that $\frac{\partial\left|a_I\tau_I^2\right|}{\partial\beta}|_{\tau_1=\bar{\tau}_1,\tau_2=\bar{\tau}_2,\phi=\bar{\phi}}>0$ and $\frac{\partial b_J\tau_J^2+b_2^2\tau_\theta^2}{\partial\beta}|_{\tau_1=\bar{\tau}_1,\tau_2=\bar{\tau}_2,\phi=\bar{\phi}}<0$. ■

Turning to expected total volume, the left panel on Figure 7 shows that the qualitative results are similar to speculative volume. The public information elasticity of expected volume decreases in the fraction of long-term traders, if β is sufficiently large.

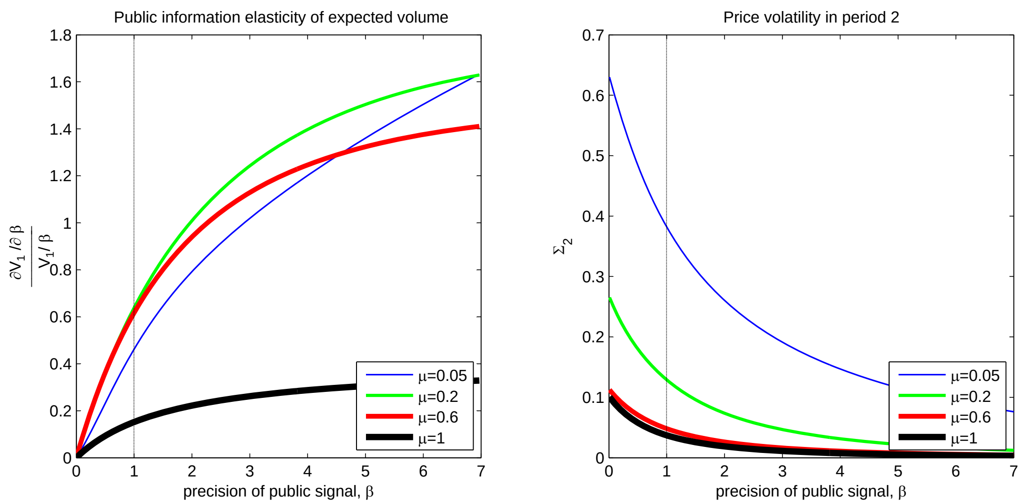

Figure 7: Expected volume in period 1 and price volatility in period 2 in Case 2. The left panel depicts the public information elasticity of expected volume in period 1 while the right panel shows price volatility in period 2. In each plot, different curves correspond to different fraction of J-traders on the market,μ. The thicker the curve, the larger the fraction. The x-axis is the precision of the public signal, β. The vertical line depicts $\beta=\frac{\nu^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information. Parameter values are $\gamma=1$, $\omega=4.01$, $\nu=2$, and $\alpha=\delta_1=\delta_2=5$.

Turning to the volatility of prices, as in Case 1, the coefficients $\frac{\mathbf{b}_2}{\mathbf{e}_2}$, $\frac{\mathbf{a}_1}{\mathbf{e}_1}$, $\frac{\mathbf{b}_1}{\mathbf{e}_1}$ show the price effects of the part of fundamentals which agents have private information on, the coefficients $\frac{\mathbf{1}}{\mathbf{e}_2}$, $\frac{\mathbf{1}}{\mathbf{e}_1}$ show the price effect of supply shocks while $\frac{\mathbf{c}_2}{\mathbf{e}_2}$, $\frac{\mathbf{c}_1}{\mathbf{e}_1}$ show price effect of public information. The definition of price volatility in the second period is still given by (40). The definition in the first period changes to

$$\Sigma_1 \equiv var\left(p_1|y\right)=\left[\left(\frac{\mathbf{a}_1}{\mathbf{e}_1}\right)^2+\left(\frac{\mathbf{b}_1}{\mathbf{e}_1}\right)^2\right]\frac{1}{\nu}+\left(\frac{\mathbf{a}_1}{\mathbf{e}_1}+\frac{\mathbf{b}_1}{\mathbf{e}_1}\right)^2\frac{1}{\omega}+\frac{1}{\left(\mathbf{e}_1\right)^2\delta_1}.$$

As in case 1, it is useful to start with the standard information structure. The next Proposition shows that the coexistence of I and J-traders does not change the conclusion that the standard information structure is inconsistent with volatility-generating public announcements.

Proposition B.3 *In Case 2, in the limit $\nu \to \infty$, price coefficients $\frac{\mathbf{b_2}}{\mathbf{e_2}}, \frac{\mathbf{b_1}}{\mathbf{e_1}}, \frac{\mathbf{c_2}}{\mathbf{e_2}}, \frac{\mathbf{c_1}}{\mathbf{e_1}}, \frac{\mathbf{1}}{\mathbf{e_2}}, \frac{\mathbf{1}}{\mathbf{e_1}}$ inherit all the properties of Case 1 described in Proposition 5. Also, price in period 1 is positively affected by the average information of I-traders and this effect decreases in the precision of public information. That is,*

$$\frac{\mathbf{a}_1}{\mathbf{e}_1} > 0, \frac{\partial \frac{\mathbf{a}_1}{\mathbf{e}_1}}{\partial \beta} < 0.$$

Thus, just as in Case 1,

$$\frac{\partial \Sigma_1}{\partial \beta}, \frac{\partial \Sigma_2}{\partial \beta} < 0.$$

Proof. The results for coefficients in period 2 are implied by the same steps as in Proposition 5. Below I provide the main steps for the rest of the results. Just as in Case 1, I use the fact that $(\tau_1^*)^2, (\tau_2^*)^2$ do not change in this limit with β, so I have to only consider the direct effects.

1.

$$= \frac{\left(\alpha+\tau_2^2\right)\mu}{\alpha+\beta+\omega+\tau_1^2+\tau_2^2} \frac{\alpha^2\tau_2^2+\alpha^3\mu+\alpha\tau_2^4+\alpha^2\mu\tau_1^2+\alpha^2\mu\tau_2^2+\alpha\tau_1^2\tau_2^2(2-\mu)+\beta\tau_1^2\tau_2^2+\tau_1^4\tau_2^2+\tau_1^2\tau_2^4(1-\mu)+\omega\tau_1^2\tau_2^2}{\left(\tau_1^2\tau_2^2+\alpha^2\mu+\alpha\tau_2^2+\beta\tau_2^2+\omega\tau_2^2\right)\left(\tau_2^2+\alpha\mu\right)} > 0$$

and

$$\frac{\partial\left(\lim_{\nu\to\infty}\frac{\mathbf{b}_1}{\mathbf{e}_1}\right)}{\partial\beta} =$$

$$= \frac{\partial\left(\frac{\left(\alpha+\tau_2^2\right)\mu}{\alpha+\beta+\omega+\tau_1^2+\tau_2^2}\right)}{\partial\beta} \frac{\alpha^2\tau_2^2+\alpha^3\mu+\alpha\tau_2^4+\alpha^2\mu\tau_1^2+\alpha^2\mu\tau_2^2+\alpha\tau_1^2\tau_2^2(2-\mu)+\beta\tau_1^2\tau_2^2+\tau_1^4\tau_2^2+\tau_1^2\tau_2^4(1-\mu)+\omega\tau_1^2\tau_2^2}{\left(\tau_1^2\tau_2^2+\alpha^2\mu+\alpha\tau_2^2+\beta\tau_2^2+\omega\tau_2^2\right)\left(\tau_2^2+\alpha\mu\right)} +$$

$$+ \frac{\left(\alpha+\tau_2^2\right)\mu}{\alpha+\beta+\omega+\tau_1^2+\tau_2^2} \frac{\partial \frac{\alpha^2\tau_2^2+\alpha^3\mu+\alpha\tau_2^4+\alpha^2\mu\tau_1^2+\alpha^2\mu\tau_2^2+\alpha\tau_1^2\tau_2^2(2-\mu)+\beta\tau_1^2\tau_2^2+\tau_1^4\tau_2^2+\tau_1^2\tau_2^4(1-\mu)+\omega\tau_1^2\tau_2^2}{\left(\tau_1^2\tau_2^2+\alpha^2\mu+\alpha\tau_2^2+\beta\tau_2^2+\omega\tau_2^2\right)\left(\tau_2^2+\alpha\mu\right)}}{\partial\beta}$$

where

$$\frac{\partial\left(\frac{\left(\alpha+\tau_2^2\right)\mu}{\alpha+\beta+\omega+\tau_1^2+\tau_2^2}\right)}{\partial\beta} = -\mu\frac{\alpha+\tau_2^2}{\left(\alpha+\beta+\omega+\tau_1^2+\tau_2^2\right)^2} < 0$$

and

$$\frac{\partial \frac{\alpha^2\tau_2^2+\alpha^3\mu+\alpha\tau_2^4+\alpha^2\mu\tau_1^2+\alpha^2\mu\tau_2^2+\alpha\tau_1^2\tau_2^2(2-\mu)+\beta\tau_1^2\tau_2^2+\tau_1^4\tau_2^2+\tau_1^2\tau_2^4(1-\mu)+\omega\tau_1^2\tau_2^2}{\left(\tau_1^2\tau_2^2+\alpha^2\mu+\alpha\tau_2^2+\beta\tau_2^2+\omega\tau_2^2\right)\left(\tau_2^2+\alpha\mu\right)}}{\partial\beta} =$$

$$= -\tau_2^2\left(\alpha+\tau_2^2\right)\frac{\tau_1^2\tau_2^2+\alpha^2\mu+\alpha\tau_2^2-\mu\tau_1^2\tau_2^2}{\left(\tau_2^2+\alpha\mu\right)\left(\tau_1^2\tau_2^2+\alpha^2\mu+\alpha\tau_2^2+\beta\tau_2^2+\omega\tau_2^2\right)^2} < 0.$$

2.

$$\lim_{\nu\to\infty}\frac{\mathbf{a}_1}{\mathbf{e}_1} = \tau_2^2\frac{1-\mu}{\alpha+\beta+\omega+\tau_1^2+\tau_2^2}\frac{\alpha^2\tau_2^2+\tau_1^2\tau_2^4(1-\mu)+\tau_1^4\tau_2^2+\alpha^3\mu+\alpha\tau_2^4+\alpha^2\mu\tau_1^2+\alpha^2\mu\tau_2^2+\alpha\tau_1^2\tau_2^2(2-\mu)+\beta\tau_1^2\tau_2^2+\omega\tau_1^2\tau_2^2}{\left(\tau_1^2\tau_2^2+\alpha^2\mu+\alpha\tau_2^2+\beta\tau_2^2+\omega\tau_2^2\right)\left(\tau_2^2+\alpha\mu\right)} > 0$$

and

$$\frac{\partial \lim_{\nu\to\infty}\frac{\mathbf{a}_1}{\mathbf{e}_1}}{\partial\beta} = \frac{\partial\tau_2^2\frac{1-\mu}{\alpha+\beta+\omega+\tau_1^2+\tau_2^2}}{\partial\beta}\frac{\alpha^2\tau_2^2+\tau_1^2\tau_2^4(1-\mu)+\tau_1^4\tau_2^2+\alpha^3\mu+\alpha\tau_2^4+\alpha^2\mu\tau_1^2+\alpha^2\mu\tau_2^2+\alpha\tau_1^2\tau_2^2(2-\mu)+\beta\tau_1^2\tau_2^2+\omega\tau_1^2\tau_2^2}{\left(\tau_1^2\tau_2^2+\alpha^2\mu+\alpha\tau_2^2+\beta\tau_2^2+\omega\tau_2^2\right)\left(\tau_2^2+\alpha\mu\right)}+$$

$$+\tau_2^2\frac{1-\mu}{\alpha+\beta+\omega+\tau_1^2+\tau_2^2}\frac{\partial\frac{\alpha^2\tau_2^2+\tau_1^2\tau_2^4(1-\mu)+\tau_1^4\tau_2^2+\alpha^3\mu+\alpha\tau_2^4+\alpha^2\mu\tau_1^2+\alpha^2\mu\tau_2^2+\alpha\tau_1^2\tau_2^2(2-\mu)+\beta\tau_1^2\tau_2^2+\omega\tau_1^2\tau_2^2}{\left(\tau_1^2\tau_2^2+\alpha^2\mu+\alpha\tau_2^2+\beta\tau_2^2+\omega\tau_2^2\right)\left(\tau_2^2+\alpha\mu\right)}}{\partial\beta} < 0$$

3.

$$\lim_{\nu\to\infty}\frac{\mathbf{c}_1}{\mathbf{e}_1} = \beta\frac{\tau_2^4+\tau_1^2\tau_2^2+\alpha^2\mu+2\alpha\tau_2^2+\beta\tau_2^2+\omega\tau_2^2}{\left(\alpha+\beta+\omega+\tau_1^2+\tau_2^2\right)\left(\tau_1^2\tau_2^2+\alpha^2\mu+\alpha\tau_2^2+\beta\tau_2^2+\omega\tau_2^2\right)}$$

and

$$\frac{\partial\beta\frac{\tau_2^4+\tau_1^2\tau_2^2+\alpha^2\mu+2\alpha\tau_2^2+\beta\tau_2^2+\omega\tau_2^2}{\left(\alpha+\beta+\omega+\tau_1^2+\tau_2^2\right)\left(\tau_1^2\tau_2^2+\alpha^2\mu+\alpha\tau_2^2+\beta\tau_2^2+\omega\tau_2^2\right)}}{\partial\beta} > 0.$$

$$\lim_{\nu\to\infty}\frac{\mathbf{1}}{\mathbf{e}_1} = \frac{\alpha\gamma\left(\alpha+\tau_2^2\right)}{\left(\alpha+\tau_2^2-\alpha\mu\right)\left(\alpha+\beta+\omega+\tau_1^2+\tau_2^2\right)}\frac{\alpha^2\tau_2^2+\tau_1^2\tau_2^4(1-\mu)+\tau_1^4\tau_2^2+\alpha^3\mu+\alpha\tau_2^4+\alpha^2\mu\tau_1^2+\alpha^2\mu\tau_2^2+\alpha\tau_1^2\tau_2^2(2-\mu)+\beta\tau_1^2\tau_2^2+\omega\tau_1^2\tau_2^2}{\left(\tau_1^2\tau_2^2+\alpha^2\mu+\alpha\tau_2^2+\beta\tau_2^2+\omega\tau_2^2\right)}$$

$$\frac{\partial\left(\frac{\alpha^2\tau_2^2+\tau_1^2\tau_2^4+\tau_1^4\tau_2^2+\alpha^3\mu+\alpha\tau_2^4+\alpha^2\mu\tau_1^2+\alpha^2\mu\tau_2^2+2\alpha\tau_1^2\tau_2^2+\beta\tau_1^2\tau_2^2-\mu\tau_1^2\tau_2^4+\omega\tau_1^2\tau_2^2-\alpha\mu\tau_1^2\tau_2^2}{\left(\tau_1^2\tau_2^2+\alpha^2\mu+\alpha\tau_2^2+\beta\tau_2^2+\omega\tau_2^2\right)}\right)}{\partial\beta} =$$

$$-\tau_2^2\left(\alpha+\tau_2^2\right)\frac{\alpha^2\mu+\alpha\tau_2^2+(1-\mu)\,\tau_1^2\tau_2^2}{\left(\tau_1^2\tau_2^2+\alpha^2\mu+\alpha\tau_2^2+\beta\tau_2^2+\omega\tau_2^2\right)^2} < 0$$

$$\frac{\partial\left(\frac{\alpha\gamma\left(\alpha+\tau_2^2\right)}{\left(\alpha+\tau_2^2-\alpha\mu\right)\left(\alpha+\beta+\omega+\tau_1^2+\tau_2^2\right)}\right)}{\partial\beta} = - \alpha\gamma\frac{\alpha+\tau_2^2}{\left(\alpha+\tau_2^2-\alpha\mu\right)\left(\alpha+\beta+\omega+\tau_1^2+\tau_2^2\right)^2} < 0.$$

4. it is a direct consequence of statements 1,2 and 4.

■

Figures 8-9 and the right panel in Figure 7 complement the analysis of the general case in the main text showing the relevant equilibrium coefficients, and volatility in period 2. Just as I highlighted in the main text only when the structure is sufficiently close to Case 1, does volatility in period 1 increases with the amount of public information in any range of the parameter space.

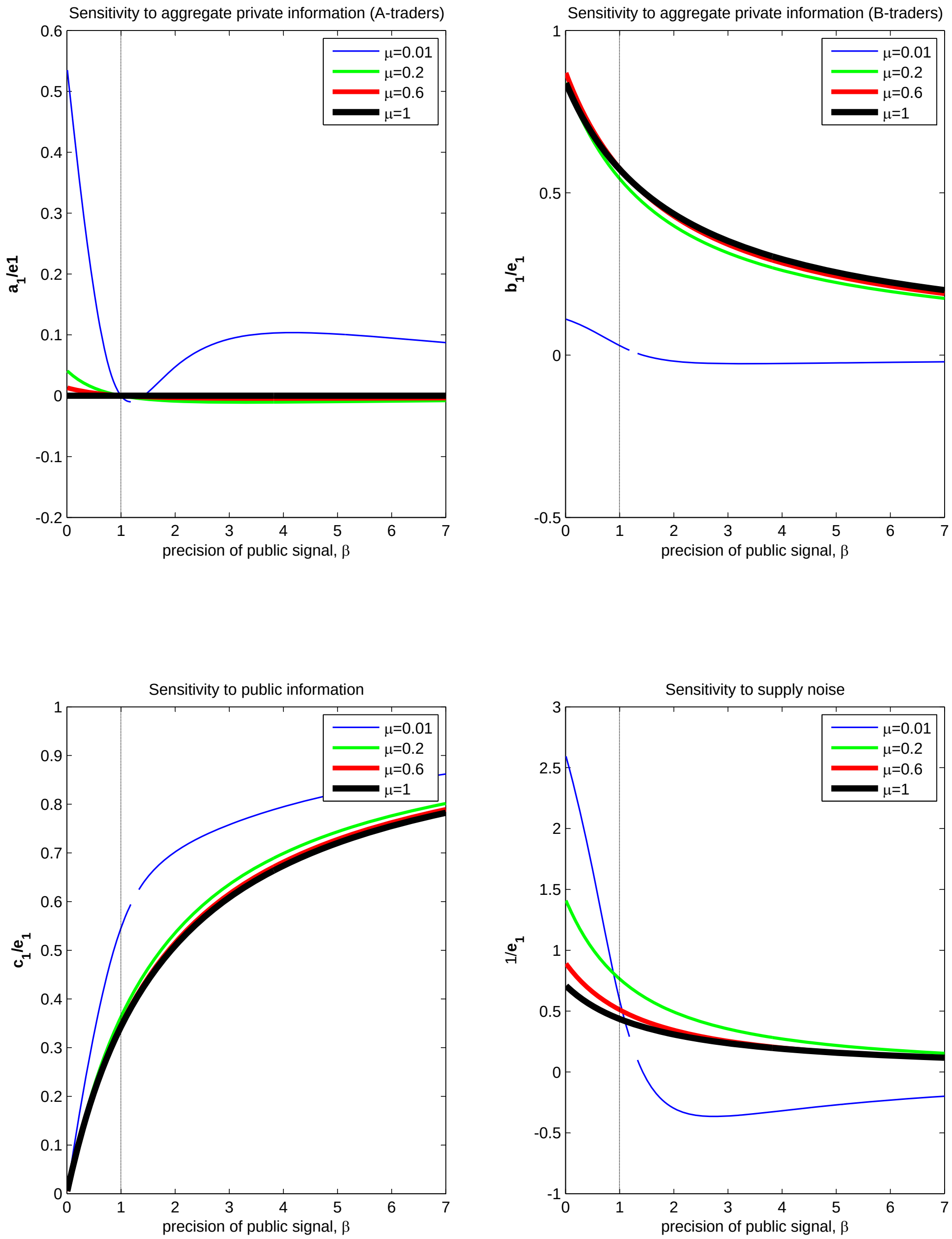

Figure 8: Coefficients of the price function in period 1 in Case 2. Each panel shows a given coefficient of the price function. In each plot, different curves correspond to different fraction of B-traders on the market, μ. The thicker the curve, the larger the fraction. The x-axis is the precision of the public signal, β. The vertical line depicts $\beta = \frac{\nu^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information. Parameter values are $\gamma = 1$, $\omega = 4.01$, $\nu = 2$, and $\alpha = \delta_1 = \delta_2 = 5$.

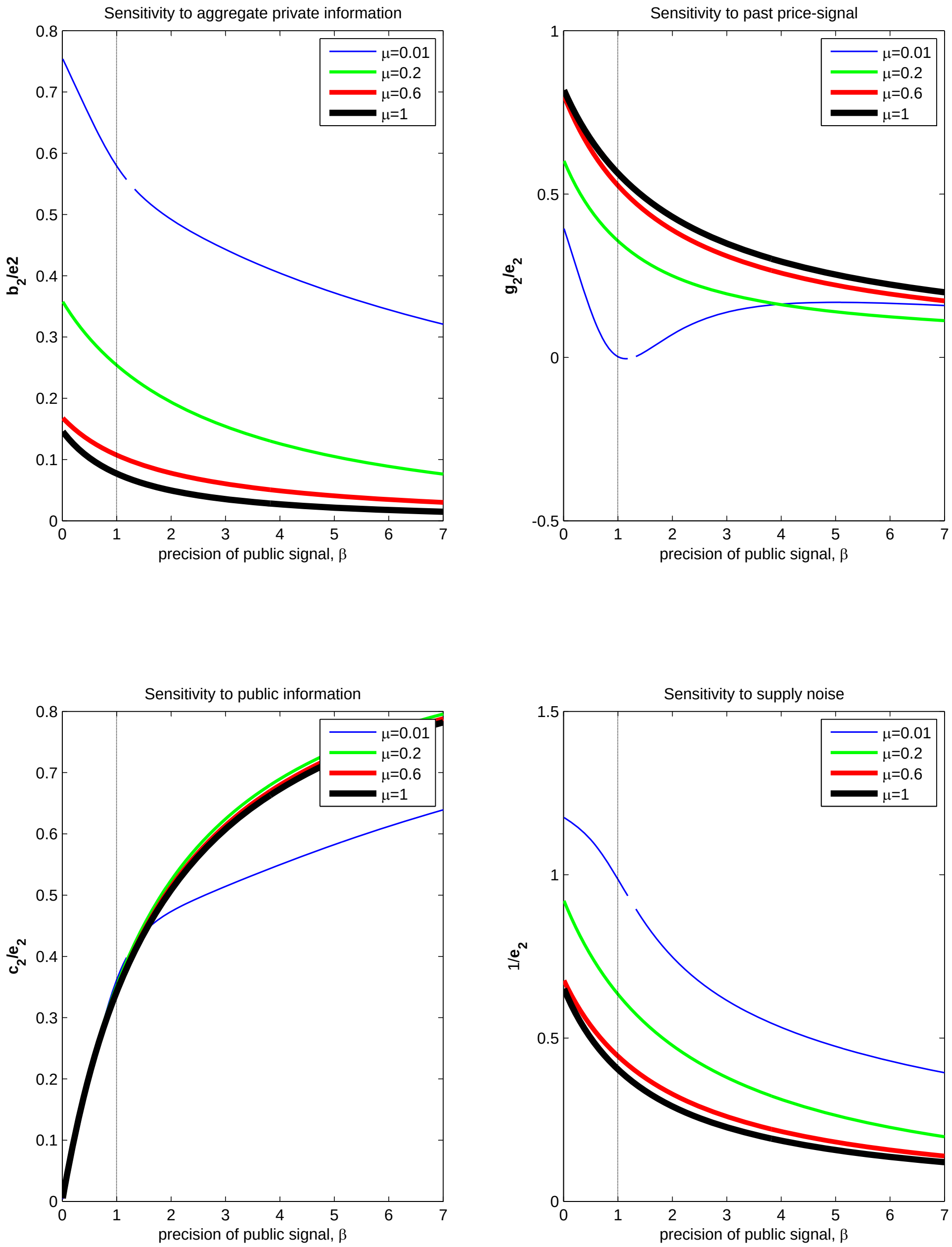

Figure 9: Coefficients of the price function in period 2 in Case 2. Each panel shows a given coefficient of the price function. In each plot, different curves correspond to different fraction of B-traders on the market, μ. The thicker the curve, the larger the fraction. The x-axis is the precision of the public signal, β. The vertical line depicts $\beta = \frac{\nu^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information. Parameter values are $\gamma = 1$, $\omega = 4.01$, $\nu = 2$, and $\alpha = \delta_1 = \delta_2 = 5$.

where D_t is the vector of strategies of all traders active in period t.

In this Appendix, I consider only Case 1, where I-traders trade in period 1 and sell their portfolio to J-traders in period 2 and consume the proceeds, while J-traders liquidate their portfolio in period 3 for the fundamental value θ and consume the proceeds. In period 1, only I-traders trade. J-traders arrive and trade in period 2 only. Thus the utility of traders and the market clearing conditions are determined as follows.

Case 1 *Each I-trader solves*

$$\begin{aligned} & \max_{d_1^i\left(p_1,\mathcal{I}_1^i\right)} E\left[-e^{-\gamma W_I^i}|\mathcal{I}_1^i\right] \\ W_I^i &= d_1^i\left(p_1,\mathcal{I}_1^i\right)\left(p_2-p_1\right) \end{aligned} \tag{C.2}$$

and each J-trader solves

$$\begin{aligned} & \max_{d_2^i\left(p_2,\mathcal{I}_2^i\right)} E\left[-e^{-\gamma W_J^i}|\mathcal{I}_2^j\right] \\ W_J^j &= d_2^j\left(p_2,\mathcal{I}_2^j\right)\left(\theta-p_2\right). \end{aligned} \tag{C.3}$$

Components of the random supply, u_1 and u_2 are drawn independently from the distributions

$$u_1 \sim \boldsymbol{N}\left(0,\frac{N^2}{\delta_1^2}\right), u_2 \sim \boldsymbol{N}\left(0,\frac{N^2}{\delta_2^2}\right).$$

The information structure is defined in the main text in equations (1)-(5). The information sets of agents are

$$\begin{aligned} \mathcal{I}_1^i &= \left\{x^i, y\right\} \\ \mathcal{I}_1^j &= \left\{z^j, y\right\} \\ \mathcal{I}_2^j &= \left\{z^j, y, p_1\right\}. \end{aligned}$$

We are looking for a Perfect Bayesian Equilibrium in demand schedules defined as follows.

Definition C.1 *A Perfect Bayesian Equilibrium in demand schedules is given by the strategy profiles D_1, D_2 in which individual strategies are best responses given the equilibrium strategies of all other players and expectations are formed according to Bayes' rule. That is, for any given i and j, and any realization of the information sets $\mathcal{I}_t^i$ or $\mathcal{I}_t^j$, $d_1^i\left(p_1,\mathcal{I}_1^i\right)$ and $d_2^j\left(p_2,\mathcal{I}_2^j\right)$ solve problems (C.2)-(C.3) subject to (C.1), respectively.*

Note that in this equilibrium, each trader not only takes into account her direct impact on prices due to the market clearing mechanism, but also her indirect impact through the information content of prices.

C.1.1 Equilibrium

In the conjectured equilibrium strategies are

$$d_2^j = \mathbf{b}_2 z^j + \mathbf{c}_2 y + \mathbf{g}_2 q_1 - \mathbf{e}_2 p_2 \tag{C.4}$$

$$d_1^i = \mathbf{a}_1 x^i + \mathbf{c}_1 y - \mathbf{e}_1 p_1 \tag{C.5}$$

for J and I-traders, respectively, where q_1 is the price signal corresponding to period 1

$$q_1 \equiv \frac{\mathbf{e}_1 p_1 - \mathbf{c}_1 y}{\mathbf{a}_1} = \bar{x} - \frac{u_1}{\mathbf{a}_1 N} \tag{C.6}$$

with $\bar{x} \equiv \Sigma_i x^i$ and its conditional precision is

$$\tau_1^2 \equiv \frac{1}{var\left(q_1|\bar{x}\right)} = \frac{1}{\delta_1^2 \mathbf{a}_1^2}. \tag{C.7}$$

It is easy to see that p_1 and y are informationally equivalent to y and the price signal q_1. For the definition of q_1 I used the market clearing condition in the first period. Also, from the market clearing condition for the second period, we define the price signal q_2 as

$$q_2 \equiv \frac{\mathbf{e}_2 p - \mathbf{c}_2 y - \mathbf{g}_2 q_1}{\mathbf{b}_2} = \bar{x} - \frac{u_2}{\mathbf{b}_2 N}$$

with a conditional precision

$$\tau_2^2 \equiv \frac{1}{var\left(q_2|\bar{z}_2\right)} = \frac{1}{\delta_2^2 \mathbf{b}_2^2 N^2}. \tag{C.8}$$

Finally, we define b_2, c_2, e_2, g_2 and a_1, c_1, e_1as the linear coefficients of the conditional expectations

$$E\left(\theta|z^j, y, q_1, q_2\right) = b_2 z^j + c_2 y + e_2 q_2 + g_2 q_1 \tag{C.9}$$

$$E\left(q_2|x^i, y, q_1\right) = a_1 x^i + c_1 y + e_1 q_1$$

and

$$\tau_\theta^2 \equiv \frac{1}{var\left(\theta | z^j, y, q_1, q_2\right)}$$
$$\tau_q^2 \equiv \frac{1}{var\left(q_1 | x^i, y, q_1\right)}$$

as the corresponding precisions. Note that all the expectational coefficients and precisions are functions of the primitive parameters and the equilibrium values of τ_1, τ_2.

I will prove the following proposition.

Proposition C.1 *1. For any $\hat{N} > 2$, there are $\hat{\delta}_1, \hat{\delta}_2$ thresholds that for every $N > \hat{N}, \delta_1 < \hat{\delta}_1$ and $\delta_2 < \hat{\delta}_2$ there is a symmetric linear equilibrium,where*

$$\mathbf{b}_2 = \tau_\theta^2 \frac{b_2 (N-2) - e_2}{\gamma (N-1)} \tag{C.10}$$
$$\mathbf{c}_2 = \tau_\theta^2 \frac{(N-2) b_2 - e_2}{\gamma (N-1)} \frac{c_2}{b_2 + e_2} \tag{C.11}$$
$$\mathbf{e}_2 = \tau_\theta^2 \frac{b_2 (N-2) - e_2}{\gamma (N-1) (b_2 + e_2)}. \tag{C.12}$$
$$\mathbf{g}_2 = \tau_\theta^2 \frac{(N-2) b_2 - e_2}{\gamma (N-1)} \frac{g_2}{e_2 + b_2} \tag{C.13}$$

and

$$\mathbf{a}_1 = \frac{\tau_q^2}{\gamma} \frac{a_1}{(e_2 + b_2)\left(1 + \frac{(a_1 + e_1)}{((N-2)a_1 - e_1)}\right)} \tag{C.14}$$
$$\mathbf{c}_1 = \frac{\tau_q^2}{\gamma} \frac{((b_2 + e_2) c_1 + c_2) \frac{a_1}{(e_2 + b_2)(a_1 + e_1) + g_2}}{(e_2 + b_2)\left(1 + \frac{(a_1 + e_1)}{((N-2)a_1 - e_1)}\right)} \tag{C.15}$$
$$\mathbf{e}_1 = \frac{\tau_q^2}{\gamma} \frac{(N-2) a_1 - e_1}{(N-1)(e_2 + b_2)((e_2 + b_2)(a_1 + e_1) + g_2)} \tag{C.16}$$

Furthermore, all coefficients and equilibrium constants are calculated at $\tau_1 = \tau_1^* (N)$ and $\tau_2 = \tau_2^* (N)$ where τ_2^*, τ_1^* are the fixed point of

$$\delta_2 \tau_\theta^2 \frac{b_2 (N-2) - e_2}{\gamma (N-1)} = \tau_2 \tag{C.17}$$
$$\delta_1 \frac{\tau_q^2}{\gamma} \frac{a_1}{(e_2 + b_2)\left(1 + \frac{(a_1 + e_1)}{((N-2)a_1 - e_1)}\right)} = \tau_1 \tag{C.18}$$

respectively.

2. *For any parameters, as $N \to \infty$, the equilibrium converges to a symmetric linear equilibrium where the equilibrium objects converges to their counterparts in Proposition 2 of the main text.*

First I derive the equilibrium objects for given τ_1 and τ_2.

The problem of J-traders is very similar to the corresponding problem in Kyle (1989). The main difference is that now they use the price of the first period as as an additional, endogenous public signal on the fundamental value. From conjecture (C.4) and the market clearing condition, agent j faces the residual demand curve

$$p_2 = \tilde{p}_2 - \lambda_2 d_2^j$$

where

$$\lambda_2 \equiv \frac{1}{(N-1)\,\mathbf{e}_2}. \tag{C.19}$$

Thus, a J trader maximizes

$$\begin{aligned}\left(E\left(\theta|z^j, y, \tilde{p}_2, q_1\right) - \tilde{p}_2 - \lambda_2 d_2^j\right) d_2^j - \left(d_2^j\right)^2 \left(\frac{\gamma}{2\tau_\theta^2}\right) = \\ = \left(E\left(\theta|z^j, y, \tilde{p}_2, q_1\right) - \tilde{p}_2\right)\left(d_2^j\right) - \left(d_2^j\right)^2 \left(\lambda_2 + \frac{\gamma}{2\tau_\theta^2}\right)\end{aligned}$$

which gives

$$\left(E\left(\theta|z^j, y, \tilde{p}_2, q_1\right) - \tilde{p}_2\right) - d_2^j \left(2\lambda_2 + \frac{\gamma}{\tau_\theta^2}\right) = 0$$

implying

$$d_2^j = \frac{E\left(\theta|z^j, y, \tilde{p}_2, q_1\right) - p_2 + \lambda_2 d_2^j}{2\lambda_2 + \frac{\gamma}{\tau_\theta^2}}$$

and

$$d_2^j = \frac{E\left(\theta|z^j, y, q_2, q_1\right) - p_2}{\lambda_2 + \frac{\gamma}{\tau_\theta^2}}. \tag{C.20}$$

Using (C.9) and the definition of q_2, rewrite this as

$$d_2^j = \frac{b_2 z^j + c_2 y + e_2 \frac{\mathbf{e}_2 p_2 - \mathbf{c}_2 y - \mathbf{g}_2 q_1}{\mathbf{b}_2} + g_2 q_1 - p_2}{\lambda_2 + \frac{\gamma}{\tau_\theta^2}}.$$

In a symmetric equilibrium, the coefficient of z^j, y, q_1 and p_2 has to be equal $\mathbf{b}_2, \mathbf{c}_2, \mathbf{g}_2$ and $\mathbf{e}_2$ respectively. Using also (C.19) gives (C.10)-(C.13).

The problem of each I trader is more subtle. Each I trader has to liquidate her assets before the fundamental value, θ, realizes. Thus, she has to form expectations about the price in period 2. As (C.20) shows for this she have to form expectations about the expectations of J-traders. That is, second order expectations matter. Furthermore, when she forms expectations on p_2, she has to take into account the effect of her own trades on period 2 prices through period 1 prices which J-traders use as signals.

In particular, market clearing in period 1 implies that given the conjectured strategies of J-traders, I-traders face the residual demand curve

$$p_1 = \tilde{p}_1 - \lambda_1 d_1^j$$

in period 1 where

$$\lambda_1 \equiv \frac{1}{(N-1)\,\mathbf{e}_1}.$$

Market clearing in period 2 implies that

$$p_2 = \frac{\mathbf{b}_2 \bar{z} + \mathbf{c}_2 y + \mathbf{g}_2 q_1 - \frac{u_2}{N}}{\mathbf{e}_2}.$$

By the definition of q_1 we can rewrite this as

$$p_2 = \tilde{p}_I - \lambda_I d_1^j \tag{C.21}$$

where

$$\begin{aligned} \lambda_I &\equiv \frac{\mathbf{g}_2}{\mathbf{e}_2}\frac{\mathbf{e}_1}{\mathbf{a}_1}\lambda_1 \\ \tilde{p}_I &\equiv \frac{\mathbf{b}_2 \bar{z} + \left(\mathbf{c}_2 - \mathbf{g}_2 \frac{\mathbf{c}_1}{\mathbf{a}_1}\right) y - \frac{u_2}{N} + \mathbf{g}_2 \frac{\mathbf{e}_1}{\mathbf{a}_1}\tilde{p}_1}{\mathbf{e}_2}. \end{aligned}$$

Thus, problem (C.2) is equivalent to maximizing

$$\begin{aligned} &\left(E\left(\tilde{p}_I | \mathcal{I}_1^j, p_1\right) - \lambda_I d_1^i - \left(\tilde{p}_1 - \lambda_1 d_1^i\right)\right) d_1^i - \left(d_1^i\right)^2 \left(\frac{\gamma}{2} var\left(p_2 | x^i, y, p_1\right)\right) = \\ &\qquad = \left(E\left(\tilde{p}_I | x^i, y, \tilde{p}_I\right) - \tilde{p}_1\right) d_1^i - \left(d_1^i\right)^2 \left(\left(\lambda_1 - \lambda_I\right) + \frac{\gamma}{2} var\left(p_2 | x^i, y, p_1\right)\right) \end{aligned}$$

with the second order condition

$$\left(\lambda_1 - \lambda_I\right) + \frac{\gamma}{2} var\left(p_2 | x^i, y, p_1\right) > 0.$$

From the first order condition of I-traders this gives

$$d_1^i = \frac{E\left(\tilde{p}_I|\mathcal{I}_1^j, q_1\right) - \tilde{p}_1}{2\left(\lambda_1 - \lambda_I\right) + \gamma var\left(p_2|x^i, y, p_1\right)}$$

or, equivalently,

$$d_1^i = \frac{E\left(p_2|x^i, y, q_1\right) - p_1}{\lambda_1 - \lambda_I + \gamma var\left(p_2|x^i, y, q_1\right)} = \frac{E\left(\frac{1}{N}\sum_j E\left(\theta|z^j, y, q_2, q_1\right) - \left(\lambda_2 + \frac{\gamma}{\tau_\theta^2}\right)\frac{u_2}{N}|x^i, y, q_1\right) - p_1}{\lambda_1 - \lambda_I + \gamma var\left(p_2|x^i, y, q_1\right)}. \tag{C.22}$$

For the second equation, I used the market clearing condition in period 2 in order to emphasize the role of second-order expectations in I-traders demand functions. Rewrite (C.5) as

$$d_1^i = \frac{\frac{\mathbf{b}_2}{\mathbf{e}_2}\left(a_1 x^i + e_1\left(\frac{\mathbf{e}_1}{\mathbf{a}_1}p_1 - y\frac{\mathbf{c}_1}{\mathbf{a}_1}\right) + c_1 y\right) + \frac{\mathbf{g}_2\left(\frac{\mathbf{e}_1}{\mathbf{a}_1}p_1 - y\frac{\mathbf{c}_1}{\mathbf{a}_1}\right) + \mathbf{c}_2 y}{\mathbf{e}_2} - p_1}{\lambda_1 - \lambda_I + \left(\frac{\mathbf{b}_2}{\mathbf{e}_2}\right)^2\frac{\gamma}{\tau_q^2}.}$$

For an equilibrium, we have to find $\mathbf{a}_1, \mathbf{c}_1, \mathbf{e}_1$, such that (C.5) equals to (C.5) for any realizations of the random variables. Equating the coefficients in the two equations give (C.14)-(C.16). Finally, substituting (C.10) and (C.14) into (C.7) and (C.8) give (C.17) and (C.18).

For the existence and convergence to the REE equilibrium, first we have to find the coefficients $b_2, e_2, a_1, e_1, \tau_q^2, \tau_\theta^2$ by Projection Theorem and substitute in to (C.17) and (C.18). This procedure gives the following fixed point problem for any fixed N

$$\begin{aligned}
\tau_2 &= F_2\left(\tau_2, \tau_1\right) \\
\tau_1 &= F_1\left(\tau_2, \tau_1, Y\right) \\
Y &= F_Y\left(\tau_2, \tau_1\right)
\end{aligned}$$

where

$$F_2 =$$

$$= \frac{\delta_2\left(\nu + \omega\right)N\alpha\left(N\alpha\left(N-2\right) - \tau_2^2\left(N-1\right)\right)\left(\nu\tau_1^2 + N\alpha\nu + N\alpha\tau_1^2\right)}{\gamma\left(N-1\right)\begin{pmatrix} N\alpha\left(N-1\right)\left(N\alpha\left(\tau_1^2 + \nu + \omega\right) + \left(\nu^2 + 2\nu\tau_1^2 + 2\omega\nu + 2\omega\tau_1^2\right)\right) + \\ \nu\tau_1^2\left(\nu + 2\omega\right)\left(N-1\right)\tau_2^2 + \\ +N^2\alpha\left(N\alpha\left(\nu^2 + \alpha\nu + \alpha\omega + 2\nu\omega + \alpha\tau_1^2 + \nu\tau_1^2 + \omega\tau_1^2\right) + \tau_1^2\left(\nu^2 + \alpha\nu + \alpha\omega + 2\nu\omega\right)\right)\end{pmatrix}}$$

and

$$\begin{aligned} F_1 &= \delta_1 \frac{1}{\gamma} \frac{N^3\alpha^3\tau_2^2\left(\nu^2-\beta\omega\right)}{N\alpha\left(N\alpha+\left(N-1\right)\tau_2^2\right)\left(\nu\tau_1^2+N\alpha\nu+N\alpha\tau_1^2\right)\left(\nu+\omega\right)} Y \\ F_Y &= \frac{k_1}{k_2}\frac{N\alpha\frac{(N-2)}{(N-1)}-\tau_1^2}{N\alpha} \end{aligned}$$

where

$$\begin{aligned} k_1 &= \left(N-1\right)\left(\begin{array}{c} \left(\nu+N\alpha\right)\left(\nu\beta+\nu\omega+2\beta\omega+2N\alpha\nu+N\alpha\beta+N\alpha\omega\right)\tau_1^2+ \\ N\alpha\left(\nu^2\beta+\nu^2\omega+2\nu\beta\omega+N\alpha\nu^2+N\alpha\nu\beta+N\alpha\nu\omega+N\alpha\beta\omega\right) \end{array}\right)\tau_2^2+ \\ & N^2\alpha\left(\begin{array}{c} \left(\begin{array}{c} \alpha\nu^2+\nu^2\beta+\nu^2\omega+\alpha\nu\beta+\alpha\nu\omega+\alpha\beta\omega+2\nu\beta\omega+N\alpha\nu^2+2N\alpha^2\nu \\ +N\alpha^2\beta+N\alpha^2\omega+N\alpha\nu\beta+N\alpha\nu\omega+N\alpha\beta\omega \end{array}\right)\tau_1^2 \\ +N\alpha\left(\alpha\nu^2+\nu^2\beta+\nu^2\omega+\alpha\nu\beta+\alpha\nu\omega+\alpha\beta\omega+2\nu\beta\omega\right) \end{array}\right) \end{aligned}$$

and

$$\begin{aligned} k_2 &= \tau_1^2\left(N-1\right)\left(\begin{array}{c} \left(\nu+N\alpha\right)\left(\nu\beta+\nu\omega+2\beta\omega+2N\alpha\nu+N\alpha\beta+N\alpha\omega\right)\tau_2^2 \\ +N\alpha\left(\nu^2\beta+\nu^2\omega+2\nu\beta\omega+N\alpha\nu^2+N\alpha\nu\beta+N\alpha\nu\omega+N\alpha\beta\omega\right) \end{array}\right)+ \\ & N^2\alpha\left(\begin{array}{c} \alpha\nu^2+\nu^2\beta+\nu^2\omega+\alpha\nu\beta+\alpha\nu\omega+\alpha\beta\omega+2\nu\beta\omega+N\alpha\nu^2+2N\alpha^2\nu+ \\ N\alpha^2\beta+N\alpha^2\omega+N\alpha\nu\beta+N\alpha\nu\omega+N\alpha\beta\omega \end{array}\right)\tau_2^2+ \\ & N^3\alpha^2\left(\alpha\nu^2+\nu^2\beta+\nu^2\omega+\alpha\nu\beta+\alpha\nu\omega+\alpha\beta\omega+2\nu\beta\omega\right) \end{aligned}$$

Observe first, that $\tau_2 = F_2\left(\tau_2,\tau_1\right)$ gives a unique solution for any fixed τ_1 and this root is always positive. This is so, because it can be rewritten as a third-order polynomial in τ_2, with positive coefficients but negative intercept. It is easy to check that the root is finite for any $\tau_1 \in (-\infty,\infty)$. Second, $F_Y\left(\tau_1,\tau_2\right)$ is positive as long as $N\alpha\frac{(N-2)}{(N-1)} > \tau_1^2$. Finally, for any positive Y and τ_2, $\tau_1 = F_1\left(\tau_2,\tau_1,Y\right)$ gives a unique solution with the same sign as $\left(\nu^2-\beta\omega\right)$.

Then, the proof is constructed by the following procedure.

1. Let us fix a $\bar{N}^{(1)}$. Define $\tau_2^{\max}\left(N\right)$ as

$$\begin{aligned} \tau_2^{\max}\left(N\right) &= \max_{\tau_1\in(-\infty,\infty)} \tau_2 \\ s.t. F_2\left(\tau_2,\tau_1\right) &= \tau_2. \end{aligned}$$

and define $\tau_2^{\max(1)} = \max_{N>\bar{N}^{(1)}} \tau_2^{\max}\left(N\right)$.

2. Consider $F_Y\left(\tau_1^2,\tau_2^2\right)$ for any τ_1^2,τ_2^2, for any $N>\bar{N}^{(1)}$, we define $Y^{\max}\left(N\right),Y^{\min}\left(N\right)$ as

$$\begin{aligned} Y^{\max}\left(N\right) &= \max_{\tau_2^2\in\left[0,\tau_2^{\max(1))}\right],\tau_1^2\in\left[0,N\alpha\frac{(N-2)}{(N-1)}\right]} F_Y\left(\tau_1,\tau_2\right) \\ Y^{\min}\left(N\right) &= \min_{\tau_2^2\in\left[0,\tau^{\max(1))}\right],\tau_1^2\in\left[0,N\alpha\frac{(N-2)}{(N-1)}\right]} F_Y\left(\tau_1,\tau_2\right) \end{aligned}$$

also define

$$Y^{\max(1)}=\max_{N>\bar{N}^{(1)}} Y^{\max}\left(N\right)$$

3. Consider $F_1\left(\tau_1,\tau_2,Y\right)=\tau_1$ equation. Define $\tau_1^{\max}\left(N\right)$ as

$$\begin{aligned} \tau_1^{\max}\left(N\right) &= \max_{\tau_2\in\left[0,\tau_2^{\max}\right],Y\in\left[Y^{\min(1)},Y^{\max(1)}\right]} \left|\tau_1\right| \\ s.t. F_1\left(\tau_2,\tau_1,Y\right) &= \tau_1. \end{aligned}$$

Then define $\tau_1^{\max(1)}=\max_{N>\bar{N}^{(1)}}\left|\tau_1^{\max}\left(N\right)\right|.$

4. Check that whether the above steps give

$$\begin{aligned} \left(\tau_1^{\max(1)}\right)^2 &< \bar{N}^{(1)}\alpha\frac{\left(\bar{N}^{(1)}-2\right)}{\left(\bar{N}^{(1)}-1\right)} \\ \left(\tau_2^{\max(1)}\right)^2 &< \bar{N}^{(1)}\alpha\frac{\left(\bar{N}^{(1)}-2\right)}{\left(\bar{N}^{(1)}-1\right)}. \end{aligned}$$

If not, decrease δ_1 and δ_2 until the point that repeating steps 1-3 ensures that these conditions hold. As $\delta_1\to 0$ implies $\tau_1\to 0$ for any τ_2 and Y, and $\delta_2\to 0$ implies $\tau_2\to 0$ for any τ_1, for any $N^{(1)}$ there will be a $\hat{\delta}_1^{(1)}$ and $\hat{\delta}_2^{(1)}$ that this condition holds for any $\delta_1<\hat{\delta}_1^{(1)}$ and $\delta_2<\hat{\delta}_2^{(1)}$ and $N\geq N^{(1)}$.

5. Thus, we can define the compact space $\left[0,\tau_1^{\max(1)}\right] X \left[0,\tau_2^{\max(1)}\right] X \left[Y^{\min(1)},Y^{\max(1)}\right]$ for $\nu^2>\beta\omega$ and $\left[\tau_1^{\max(1)},0\right] X \left[0,\tau_2^{\max(1)}\right] X \left[Y^{\min(1)},Y^{\max(1)}\right]$ otherwise on which are fixed point problem is a continuos self-map for every $N>\bar{N}^{(1)}$, so we will have an equilibrium for each of these points. This proves existence with $\hat{\delta}_1=\hat{\delta}_1^{(1)}$ and $\hat{\delta}_2=\hat{\delta}_2^{(1)}$.

6. .Now define $\psi^{(1)}=\left[\tau_1^{(1)},\tau_2^{(1)},Y^{(1)}\right]$ as (one of) the fixed point of the system

$$\begin{aligned} \tau_2 &= F_2\left(\tau_2,\tau_1\right) \\ \tau_1 &= F_1\left(\tau_2,\tau_1,Y\right) \\ Y &= F_Y\left(\tau_2,\tau_1\right) \end{aligned}$$

for $N=\bar{N}^{(1)}$. Following the steps above, after choosing $\bar{N}^{(n+1)}=\bar{N}^{(n)}+1$, we can construct a series of fixed points,$\psi^{(n)}$, compact spaces, $\tau_1^{\max(n)}, \tau_2^{\max(n)}, Y^{\min(n)}, Y^{\max(n)}$and thresholds $\bar{N}^{(n)}, \hat{\delta}_1^{(n)}, \hat{\delta}_2^{(n)}$. If there are more than one fixed point in step n, let us choose the one which is closest to the fixed point $[\tau_1^*, \tau_2^*, Y^*]$ given in Proposition 2 in the Euclidean sense. By construction, the compact space is non increasing, the thresholds $\hat{\delta}_1^{(n)}, \hat{\delta}_2^{(n)}$ are non-decreasing, and $\lim_{n\to\infty} \hat{\delta}_1^{(n)}, \hat{\delta}_2^{(n)}=\infty$. As all the coefficients in polynomials defined by $F_1=\tau_1$ and $F_2=\tau_2$ and the coefficients in polynomials defining Y are converging to their equivalents in the limit problem, the functions must also converge to the ones in the limit problem point by point. Thus, for every ε, there is a sufficiently large n that $\left|\psi^{(n)}-\psi^{(n-1)}\right|<\varepsilon$. Thus, it is a convergent series and its limit point must be $\psi^{(\infty)}=[\tau_1^*, \tau_2^*, Y^*]$.