EXPONENTIAL-GROWTH BIAS AND LIFECYCLE CONSUMPTION

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Abstract
Exponential-growth bias (EGB) is the tendency for individuals to partially neglect compounding of exponential growth. We develop a model wherein biased agents misperceive the intertemporal budget constraint, and derive conditions for overconsumption and dynamic inconsistency. We construct an incentivized measure of EGB in a US-representative population and find substantial bias, with approximately one-third of subjects estimated as the fully-biased type. The magnitude of the bias is negatively associated with asset accumulation, and does not respond to a simple graphical intervention. (JEL: D03, D11, D12, D14, D18, D91, E21)

Keywords: Exponential-growth bias, dynamic inconsistency, personal finance, overconsumption, lifecycle consumption.

The editor in charge of this paper was Nicola Gennaioli.

Acknowledgments: We would like to gratefully acknowledge the financial support of a Fletcher Jones Foundation Faculty Research Grant from Claremont Graduate University and Time-sharing Experiments for the Social Sciences (TESS) [NSF Grant 0818839]. We thank Masyita Crystallin, Peiran Jiao, Andrew Royal, Quinn Keefer, and Oliver Curtiss for research assistance, and Ananda Ganguly, Matthew Rabin, Paige Skiba, Justin Sydnor, Charles Thomas, Jonathan Zinman, various seminar participants, and three anonymous referees for helpful comments. Institutional Review Board approval was obtained from OHRPP at UCLA [IRB #12-001092] and CGU [IRB #1591].

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1. Introduction

Virtually all intertemporal financial decisions involve real or nominal values that change exponentially over time. Proper computation of exponential functions is thus at the heart of many economic decisions such as lifecycle consumption and portfolio choice. Many people, however, exhibit a strong systematic bias towards linear growth when estimating such functions. This can have large positive and normative consequences. Moreover, an economist who does not account for this bias may mis-interpret observed behavior, such as the pervasive borrowing through payday loans with APRs exceeding 3000%[1] as well as the apparently low retirement savings of many households.[2] We incorporate the misperception of exponential growth in a model of lifecycle consumption, measure the bias in a representative sample of the US population, and find that savings behavior is consistent with the predictions of the model. This paper suggests that exponential-growth bias (EGB)— the tendency for individuals to underestimate exponential growth due to the neglect of compounding — may be an important missing factor in lifecycle consumption puzzles, and must be considered alongside other explanations such as time-inconsistent preferences, or unobserved substitution to home production.

While there are folk stories illustrating people's underestimation of exponential growth going back millennia,[3] to our knowledge, Wagenaar and Sagaria (1975) conducted the first published experiment demonstrating this phenomenon in the psychology literature. Subsequent studies found the same pattern of underestimation (Wagenaar and Timmers, 1979; Keren, 1983; Benzion, Granot, and Yagil, 1992; Almenberg and Gerdes, 2012). Wagenaar and Sagaria (1975) wrote an early model of exponential-growth bias that used two parameters in which an exponential function of the form $x(t) = ac^t$ is perceived as $\hat{x}(t) = \alpha ac^{\beta t}$.

Stango and Zinman (2009) introduced EGB to the economics literature and extended the Wagenaar-Sagaria model to environments where people face loans with periodic payments. They show that EGB causes people to underestimate the future value of savings and the costs associated with borrowing. Using an interest-rate question as a proxy for EGB on the 1977 and 1983 Survey of Consumer Finances, they find that errors on the question are correlated with a number of important economic outcomes. Those with larger errors have higher short-term debt to income ratios, lower

1. The U.S. payday loan industry is so successful that brick-and-mortar payday loan locations exceed the number of McDonalds and Starbucks combined (Skiba and Tobacman, 2011).

2. While the adequacy of retirement savings is the subject of ongoing debate, Munnell et al. (2006) argue that more than 40% of U.S. households are saving insufficiently to maintain their standard of living into retirement. Poterba et al. (2011) find that the 30th percentile household age 65–69 had total investment savings (financial assets plus personal retirement account) of $5,500 in 2008, while the median had total investment savings of only $52,000.

3. According to legend, the ruler of an Indian kingdom granted the inventor of chess a single boon. The inventor requested a quantity of rice that doubled for every square on the chessboard, starting with a single grain. The ruler quickly accepted the request only to later discover that the sum exceeded the kingdom's entire store.

stock ownership as a percentage of portfolios, lower savings rates, lower net worth, and no difference in long-term debt to income ratios, all as predicted by their model. In a subsequent paper Stango and Zinman (2011) show that when APR disclosure was mandated by law and enforced, the interest rates on loans taken by the most biased and the least biased were compressed. This suggests that without regulation, lenders price-discriminated on borrowers' cognitive biases.

Our first contribution is the development of a model of EGB that is both empirically accurate and portable. We parameterize the model such that the agent's perception is as if an asset is divided into two accounts: a fraction $0 \leq \alpha \leq 1$ grows with compounding interest at the interest rate αi, and a fraction $1-\alpha$ grows with simple interest. The perception of the future period-T value at time $t \leq T$ is given by $p(i,t;\alpha) = \prod_{s=t}^{T-1}(1+\alpha i_s) + \sum_{s=t}^{T-1}(1-\alpha)i_s$. Thus when $\alpha = 1$ the agent has correct perceptions and when $\alpha = 0$ the agent believes an asset with compounding interest grows linearly. We derive a fully general model in Appendix A, for which all our main results extend.

Our model has several improvements over the Wagenaar-Sagaria model. First, the Wagenaar-Sagaria model implies that a biased agent will underestimate growth even after one period when interest has not yet compounded, whereas we find that three-quarters of our sample get this exactly correct. Second, our model nests full neglect of compounding (misperceiving compound interest as simple interest), which we observe in about one third of our sample. Third, our model predicts that a biased individual will underestimate the value of a depreciating asset, which is supported by the data. Fourth, our model predicts that an agent, when estimating the constant-rate equivalent of a fluctuating return, will tend to be biased toward the arithmetic mean — not realizing, for example, that they are left strictly worse off by a 10% gain followed by a 10% loss. As a result, the biased agent will exhibit *as-if* risk preferences, being insufficiently sensitive to large negative periodic returns. An economist who believed that uncertainty generated the variation in returns would infer that the agent had risk-seeking preferences. Finally, our model accounts for framing effects in a transparent way: the period-length of any asset must be stated as a primitive of the model.

Stango and Zinman (2009) develop a more general model in their online appendix for which both the Wagenaar-Sagaria model and our parametric model (with constant interest rates) are special cases. Their general model puts minimal assumptions on the perception function.[4] We extend the space under consideration by allowing for interest rates to vary over time and the framing of the problem, in the sense of period length, to be exogenously varied. Then in our general model in Appendix A, we put greater restrictions on what types of errors are counted as exponential-growth bias. For example, a person who underestimates the sum of a stream of payments because he cannot do addition could be characterized as suffering from EGB in the Stango

4. In particular: perceptions are accurate when the bias is zero, growth is never perceived as negative, perceptions are strictly decreasing in the bias parameter, perceptions are strictly increasing in both the interest rate and the time horizon, and differences in perceptions for a given interest rate and bias pair and another interest rate and bias pair increase in the time horizon.

and Zinman general model, whereas we restrict EGB to the underestimation of the cross-partial derivatives of the perception function with respect to two interest rates. Our model thus isolates the effect of EGB from other perceptual errors.

As our second contribution, we embed our portable model of EGB in a lifecycle-consumption environment and explore the behavior of a biased agent. EGB leads the agent to make two fundamental errors regarding his intertemporal budget constraint. First, the consumer misperceives the value of his income over time. With positive interest rates this causes the consumer to overestimate the value of future income. We call this the *wealth effect of exponential-growth bias*, as it operates through a perceived wealth effect on future earnings.[5] Second, the agent misperceives the relative prices of consumption over time. With positive interest rates this causes the agent to overestimate the price of future consumption relative to present consumption. We call this the *price effect of exponential-growth bias*, and it combines the standard income and substitution effects on consumption choices (albeit over a misperception rather than a real price change).

These effects yield novel predictions that diverge from both classical models and existing behavioral models. Because agents overestimate the value of future income, shifting income to later periods in a way that preserves lifetime wealth will increase consumption in the present. We derive sufficient conditions under which the consumer will overconsume in the present for any positive income vector and any smooth utility function. While commitment is generally thought to help present-biased agents, it will in fact often exacerbate overconsumption for an EGB agent who locks in his biased consumption plans. Moreover, because the perception of future prices and lifetime wealth changes each period, the agent will behave in a dynamically inconsistent manner that is distinct from the pattern generated by dynamically inconsistent time preference: he will revise his consumption plans upward when he is a net saver and downwards when he is a net borrower.

We believe that present-biased and other dynamically inconsistent preferences play an important role in intertemporal consumption decisions, but that many financial choices are more plausibly explained with the presence of EGB. For example, Skiba and Tobacman (2008) estimate that a short-run discount factor $\beta = 0.53$ and an (annualized) long-run discount factor $\delta = 0.45$ are necessary to explain payday loan take-up and default rates. A fully EGB agent could misperceive the costs of debt by orders of magnitude — especially if the loan is framed in terms of a daily or weekly interest rate — rationalizing the take-up of such loans with less extreme values of the discount factors.

An additional distinction is that a present-biased agent with access to credit and without commitment devices would choose to receive income to maximize his intertemporal budget constraint just like an exponential discounter, while an EGB agent may not. For instance, if the agent has a liquid asset with a return below

5. We treat income as exogenous in our exposition; endogenizing income does not qualitatively change the predictions of the model.

the prevailing interest rate, then the non-EGB agent, be it exponential discounter or present biased, will sell immediately and save in an un-dominated vehicle. In contrast, EGB agents may hold strictly dominated assets.

As a third contribution we measure the pervasiveness of EGB and test the validity and relevance of our model in a representative sample. Subjects answer questions about the relative value of two financial assets and are paid for accuracy. Using our model we estimate the accuracy α by subject and find that about one third of the population is fully biased with $\alpha = 0$. The median bias is 0.6 and 96% of subjects are estimated to have an $\alpha < 1$ (i.e. underestimate compound growth). This is despite the fact that subjects participated online and had access to whatever tools (e.g. financial calculators, help from friends) that they chose to use. Various questions also produce "fingerprinted" EGB responses that are predicted only by our model. Regressing log savings on α, we find that it enters positively and significantly while controlling for income, education, age, and other covariates. Moving from full bias to full accuracy is associated with a ceteris paribus 55–90% increase in accumulated assets. This augments the Stango-Zinman finding, that bias is correlated with savings and net worth, by using a direct elicitation of EGB instead of a proxy.[6]

We additionally find that α is uncorrelated with age and education, indicating that the bias does not diminish with some measures of experience. Moreover, we find that a graphical "de-biasing" intervention had no effect on subjects' perceptions of exponential growth. While other experimental de-biasing interventions have met with mixed results (MacKinnon and Wearing, 1991; Eisenstein and Hoch, 2007; McKenzie and Liersch, 2011; Soll et al., 2013; Goda et al., 2014; Song, 2012), ours is unique in assessing the additional effect on biased perceptions when subjects may already use external resources. As a "back of the envelope" calculation, our estimates imply that the median American suffers a welfare loss equivalent to 2–5% of lifetime wealth as a result of EGB-induced mis-optimization.

While we focus on EGB, we note that people likely make additional errors when trading off sums of money over time. In one task of our experiment, subjects must compute the value of an account subject to periodic contributions. On these questions one third of responses are below the gross contributions, indicating that not only do people misperceive exponential growth, they also cannot add. A recent literature has looked at other ways in which agents mis-perceive prices (Chetty, Looney, and Kroft, 2009; Gabaix, 2014), though often with a different focus. Read, Frederick, and Scholten (2013) demonstrate other framing effects, for example that describing the decision between money now and more money in one year as an interest rate

6. Although it would be possible to use their data (the 1977 and 1983 Survey of Consumer Finances) to estimate α, there are some shortcomings with this data. The question was not incentivized and there was only a single question. We find that there is considerable measurement error in α. In contrast we estimate α using 10 incentivized questions. Also, the question is not ideal for measuring EGB. It is sensitive to respondents beliefs about typical APRs, and solving requires several arithmetic operations. It is about as complex as our most complex questions, the "Periodic Savings" domain, on which we find evidence that many subjects under-estimate sums as well.

causes people to be more patient. Our analysis generally focuses on comparative static predictions as economic variables change within a given frame, although we discuss the framing of period length in Section 2.3. The framing of other aspects of the problem are beyond the scope of this paper.

The next section presents the model. In Section 3 we measure the prevalence of EGB in the population. The paper concludes in Section 4.

2. Theory

2.1. Model

We consider an agent who faces a vector of interest rates $\vec{\imath} = < i_0, i_1, \ldots, i_{T-1} > \in \mathbb{R}^T$, and a vector of cash flows (income, contributions, debits, etc.) $\vec{y} = < y_0, \ldots, y_T > \in \mathbb{R}^T$. Both the cash flows and interest rates are certain and known to the agent. The timing of cash flows, and the unit of time treated as a "period" are given exogenously, as is the length of the vector $\vec{y}$.[7] We define $p(\vec{i},t)$ to be the agent's perception of the period-T value of one dollar invested at time t. For example, the correct perception would be $p(\vec{i},t) = \prod_{\tau=t}^{T-1}(1+i_\tau)$, whereas a perception that ignored all compounding would be $p(\vec{i},t) = 1 + \sum_{\tau=t}^{T-1} i_\tau$.

We consider a simple one-parameter model of EGB, using the parameter α to denote the accuracy of the agent's perception. For ease of exposition, we will refer to an agent whose perception corresponds to a degree α of exponential-growth bias as an α-Eddie. While there are many possible functional forms for intermediate degrees of exponential-growth bias, our simple parameterization gives the perceptions of an α-Eddie as:

$$p(i,t;\alpha) = \prod_{s=t}^{T-1}(1+\alpha i_s) + \sum_{s=t}^{T-1}(1-\alpha)i_s \tag{1}$$

The α-Eddie model is both convenient and well-behaved mathematically, and also has a coherent psychological interpretation. A 1-Eddie correctly perceives the asset growing exponentially, while a 0-Eddie is fully biased and perceives the asset growing linearly according to simple interest. An α-Eddie's perception corresponds to what would result if part of the interest were siphoned away into a non-compounding account. That is, if a fraction α of the interest accumulated in an account that will grow in future periods with the interest rate αi, and the remaining fraction $1-\alpha$ of the accumulated interest were sequestered to a non-growing account (e.g. placed under the mattress). This is, of course, not meant to be taken literally. In Appendix A

7. The agent's perceptions are not neutral with respect to the framing of time. In general, finer divisions (e.g. days rather than years) will lead to more opportunity for compounding and hence a greater mistake on the part of the agent. Such non-neutrality is often a feature of models which deviate from neoclassical rationality with exponential discounting – for example, the importance of determining what constitutes the "present" in hyperbolic discounting. We discuss later the implications of this non-neutrality for firms facing biased consumers.

we present a non-parametric general model of exponential-growth bias, which does not rely on a particular functional form of the perception function.

Another way to understand Equation (1) is that it implicitly defines distorted interest rates $\hat{i}_s(\vec{i})$ for each period as a function of all other interest rates under consideration. The agent then applies these distorted rates to the correct exponential formula. If all interest rates are weakly positive, then $\hat{i}_s < i_s$. Because EGB results from an under-appreciation of compounding, $\hat{i}$ is more distorted as the number of non-zero elements in $\vec{i}$ increases. It is generally more convenient, however, to directly apply the perception function $p(\vec{i},t)$ in most cases.

Eddie's perception at time t of the period-τ value of a general asset with vector of cash flows $\vec{y}_t =< y_{t+1},...,y_T >$ is given by:

$$\hat{V}_{t,\tau}(\vec{y}_t,\vec{i};\alpha) = \overbrace{\left(\sum_{s=t}^{\tau} y_s \prod_{r=s}^{\tau-1}(1+\alpha i_r)\right)}^{\text{compounding interest and cash flows}} \overbrace{+(1-\alpha)\sum_{s=t}^{\tau}\left(\sum_{r=s}^{\tau-1} i_r\right) y_s}^{\text{simple interest of cash flows}} \tag{2}$$

for all $\tau \leq T$. Eddie will only misperceive the value of an asset at least two periods into the future since $p(i,T-1;\alpha) = 1+i$, which is correct.

The α-Eddie model has several strengths. First, it is tractable and relatively easy to estimate the single parameter. It is also sufficiently flexible to encompass complex choices. Moreover, while an α-Eddie's perception of the growth of an asset converges in the long run to exponential growth, it is at the lower growth rate αi rather than the true i.

2.2. Theoretical Results

We now explore a biased agent's behavior in a simple lifecycle-consumption environment. Suppose an agent has an instantaneous utility function over consumption $u(c_t)$ that is continuously differentiable, strictly concave, and satisfies the Inada conditions: $u'(0) = \infty$ and $\lim_{c_t \to \infty} u'(c_t) = 0$. The agent is born in period 0 and dies in period $T > 1$, and he must choose his consumption in each period in order to maximize lifetime utility subject to an intertemporal budget constraint. That consumers broadly bracket their consumption with reference to their entire lifetime wealth is perhaps unrealistic, but is a standard assumption in the literature (Friedman, 1957), as it captures the essence of the problem in a tractable way. We further assume for simplicity that the horizon to which perceptions apply, the parameter T in equation (1), is the final consumption period.[8]

In each period the agent receives a (possibly negative) cash flow y_t and he may purchase or sell shares of a risk-free asset with period-specific interest rates $i_t \geq 0$

8. The problem would not be well-defined if one used a shorter horizon; longer horizons would amplify the effects of EGB.

(with the inequality strict for at least two periods).[9] We treat income as exogenous for simplicity and to match our later experimental design, but the model easily allows endogenous sources of income.

The agent discounts future utility exponentially by the discount factor $\delta \leq 1$. We use conventional exponential discounting to focus on the effect of EGB and isolate its role in behavior. Alternative discounting models such as quasi-hyperbolic discounting can be straightforwardly incorporated. Moreover, we do not model the agent as applying his EGB to his discount function, as EGB is only relevant when numerical values are shrouded by math. Whereas the value of a future asset is often shrouded by exponential calculations, we assume that all explicit and implicit calculations regarding one's preferences are fully transparent. In other words, the agent knows his intertemporal utility function.[10]

Thus the agent's period-0 problem is:

$$\max_{\hat{\mathbf{c}}} \sum_{t=0}^{T} \delta^t u(\hat{c}_t) \tag{3}$$

subject to the budget constraint written in terms of the period-T value of money,

$$\sum_{s=0}^{T} \hat{c}_s \cdot p(\vec{i}, s; 1) \leq \sum_{s=0}^{T} y_s \cdot p(\vec{i}, s; 1) \tag{4}$$

where $p(\vec{i}, t; 1) = \prod_{j=t}^{T-1}(1 + i_j)$ is the correct interest perception. However, since the agent misperceives exponential growth, he believes that his budget constraint is instead:

$$\sum_{s=0}^{T} \hat{c}_s \cdot p(\vec{i}, s; \alpha) \leq \sum_{s=0}^{T} y_s \cdot p(\vec{i}, s; \alpha) \tag{5}$$

Equation (5) assumes that the agent treats all of his consumption and income in all periods as the cash flows from a single asset, and then applies Equation (2) with the constraint that his lifetime wealth must have non-negative value. This is the most parsimonious, but not the only possible assumption. For example, the agent could calculate the present value of each period's income and consumption in isolation, applying e.g. $1/p(<i_0, ..., i_{s-1}>, 0)$ to period s. The exact pattern of distortion depends

9. If the interest rate does not differ from zero for at least two periods, then compounding plays no role in the agent's optimization and our model of EGB does not come into play. We focus on positive interest rates for reasons of exposition and economic relevance, but the propositions are straightforwardly extended to admit negative rates.

10. As an analogy, someone who has trouble with trigonometry does not lose their depth perception even though the visual system of the brain uses trigonometric calculations to calculate depth. In the same way, the agent's inability to estimate exponential growth need not place any restrictions on his preferences. While the exponential discounting form is itself the subject of much debate (Frederick, Loewenstein, and O'Donoghue, 2002), we use it only as a simple approximation of time preference that allows us to place our emphasis on mistakes about the budget constraint.

on the procedure used, but our results are robust to any choice of procedure. We follow (5) for parsimony.

Taking first-order conditions, we derive a modified Euler condition of the form:

$$u'(c_{t+1}) = u'(c_t)\frac{p(\vec{i},t;\alpha)}{p(\vec{i},t+1;\alpha)} \tag{6}$$

This reduces to the standard Euler condition when $\alpha = 1$, but otherwise introduces a distortion into how the agent plans to allocate consumption across periods.

The agent's period-0 consumption is defined implicitly by:

$$\left[\sum_{s=0}^{T} p(\vec{i},s;\alpha)u'^{-1}\left(\frac{p(\vec{i},s;\alpha)u'(c_0)}{p(\vec{i},0;\alpha)\delta^s}\right)\right] \equiv \sum_{s=0}^{T} y_s \cdot p(\vec{i},s;\alpha) \tag{7}$$

Equation (7) specifies what the agent will do in the current period and what the agent plans to do in future periods but not necessarily what the agent actually does in future periods. The agent must satisfy his true budget constraint in the current period: $c_0 \leq \sum_{s=0}^{T} y_s \cdot p(\vec{i},s;1)/p(\vec{i},0;1)$. Whoever lends to the agent will ensure that this condition is satisfied. When it binds we will assume that the agent consumes his full wealth in period 0.

The intuition for the theoretical results of this paper lies in Equation (7). Exponential-growth bias leads to two perceptual errors: one for each side of the equation. On the left-hand side, the agent misperceives the relative prices of consumption over time. On the right-hand side, he misperceives his lifetime wealth.

Lemma 1 (Under-estimation by period) *Suppose* $0 \leq \alpha < \alpha' < 1$*, then:*

$$\frac{p(\vec{\imath},t;\alpha)}{p(\vec{\imath},t+1;\alpha)} \leq \frac{p(\vec{\imath},t;\alpha')}{p(\vec{\imath},t+1;\alpha')}$$

The inequality holds strictly if there exist $j > k > t$ *s.t.* $i_j > 0$*,* $i_k > 0$*.*

Lemma 1 implies that if interest rates are always positive then a biased agent perceives the price of future consumption to be relatively too high, and the price of present consumption relatively too low. We henceforth refer to this as the *price effect* of exponential-growth bias. As in standard consumer theory, a change in prices (albeit a misperception in this case) leads to an income effect and a substitution effect. Since future prices are perceived to be higher than they actually are ($1 today is perceived to buy less in the future than it actually can), income is perceived to be lower. This force will generally decrease planned consumption in all periods. But since the *relative* prices of early periods are perceived to be lower than they actually are, this will cause more planned consumption in early periods and less planned consumption in later periods. The net change in immediate consumption therefore depends on the elasticity of intertemporal substitution (EIS), which we turn to in Proposition 3.

On the right-hand side of Equation (7) the agent misperceives the future value of his income. If $\alpha < 1$ then $p(\vec{i},t;\alpha)$ will be too low and so he will overestimate the present value of future income. In other words, the agent underestimates his budget when income is received early in life but he overestimates income when he receives it late in life. We henceforth refer to this as the *wealth effect of exponential-growth bias*.[11]

The wealth effect leads to our first result. An agent with EGB will perceive an income stream that delays income as more valuable than an equally valuable income stream that expedites it.[12]

Lemma 2 (Income Deferment) *Given income streams $\vec{y}$ and $\vec{z}$ and interest rates $\vec{\imath}$ such that:*

$$(i) \qquad \sum_{s=0}^{T} \left(\Pi_{j=s}^{T}(1+i_j)\right) y_s = \sum_{s=0}^{T} \left(\Pi_{j=s}^{T}(1+i_j)\right) z_s > 0$$

$$(ii) \qquad \sum_{s=0}^{t} \left(\Pi_{j=s}^{t}(1+i_j)\right) y_s > \sum_{s=0}^{t} \left(\Pi_{j=s}^{t}(1+i_j)\right) z_s \quad \forall t \in \{0,...,T-2\},$$

then $\widehat{V}_{0,T}(\vec{y},\vec{\imath};\alpha) < \widehat{V}_{0,T}(\vec{z},\vec{\imath};\alpha)$ for any $\alpha < 1$.

The lemma states that (i) given two income streams of the same (actual) discounted value in which (ii) the value of the income received from $\vec{y}$ up to any point $t < T-1$ exceeds the value of the income received thus far from $\vec{z}$, the agent will perceive $\vec{z}$ as having higher value than $\vec{y}$. Thus $\vec{z}$ represents a stream of cash flows of the same actual present value as $\vec{y}$, but with the cash flows arising at relatively later dates. Hence any income distributions that receive proportionately more of their present value later in life will be perceived as more valuable from a $t = 0$ perspective.

Since income received in later periods is overvalued and income received in the present is undervalued, delaying income leads to the misperception that the value of lifetime income is greater than it actually is. The wealth effect thus leads to our first proposition.

Proposition 1 (Deferred Income Increases Consumption) *Delaying income from period $t < (T-1)$ to $\tau > t$ in a manner that keeps (true) lifetime income unchanged will cause an agent with EGB to increase consumption in period 0.*

The implication is that when the agent receives compensation stated nominally and received in the future, he will overestimate his budget and overconsume in the present. The larger the delay, the larger the error. For sufficiently large bias and sufficiently

11. Because we are treating the cash flows as exogenous, there is no possibility of substitution. Extending the model to endogenize income, for example by including a labor supply decision, would naturally lead to a similar income and substitution effect to those for consumption choices.

12. For example, suppose the interest rate is $i = 9\%$ and $T = 10$. The value of \$100 in $t = 0$ is \$237 in $t = 10$. A fully biased agent, however, will perceive the \$100 in $t = 0$ as worth \$190 in $t = 10$. An unbiased decision maker would be indifferent between an income stream in which \$100 is received in $t = 0$ and nothing in all other periods, and a second stream in which \$237 is received in $t = 10$ and nothing in all other periods. The biased agent would gladly choose the latter because he overestimates its present value.

high interest rates, a lump-sum payment to the agent late in the lifecycle could even make him worse off.

From Proposition 1, if income is sufficiently delayed, interest rates are sufficiently high, and the agent is sufficiently biased, the agent will overconsume in $t = 0$ relative to the optimum for any preferences. This provides the basis for Proposition 2.

Proposition 2 (Overconsumption From Future Wealth) *If an agent receives all of his wealth in the final period $T > 1$, then period-0 consumption is higher for more biased interest perception functions (i.e. increasing in EGB).*

Proposition 2 states that any biased agent who receives all his wealth in the last period will overconsume at the beginning of his lifecycle. This is a sufficient condition for overconsumption, though by no means a necessary one. If, for example, income tends to be received very early, the agent may overconsume if the perceived price effect dominates the wealth effect. Thus the price effect alone can also generate another sufficient condition for over-consumption.

Proposition 3 (Overconsumption) *If an agent faces a vector of weakly-positive cash flows $\vec{y}$ ($y_t \geq 0 \forall t$, $\exists s$ s.t. $y_s > 0$), then the period-0 consumption of an agent with EGB is greater than that of an unbiased agent if $-u'(c)/(u''(c)c) > 1$, i.e. if the elasticity of intertemporal substitution (EIS) is greater than one.*

Proposition 3 states that as long as the EIS>1, a biased agent will overconsume at the beginning of his lifecycle.[13] This of course also implies that a necessary condition for underconsumption is that the elasticity of intertemporal substitution is less than one. We stress this is not a sufficient condition for underconsumption, however, as even if the agent is highly inelastic (i.e. his instantaneous utility function is very concave and therefore EIS is very low), from Proposition 2 he will still overconsume in period 0 if his wealth is received as a lump-sum in the last period. Of course, less extreme income paths may also generate overconsumption. For example, we calibrate that an agent who earned the median real wage between ages 20 and 65 and then retired until the median life expectancy of 78 would overconsume at age 20 for any preferences which generate a non-decreasing consumption plan.[14] As a consequence, at later ages the agent will underconsume from the perspective of the optimal lifetime consumption path. The *wealth effect* is therefore likely to dominate decisions in many real-world settings, and the exact value of the agent's EIS is of second-order importance.

The wealth effect also does not rely on the agent using the modified Euler condition in (6), whereas the price effect does. It is possible that even EGB agents do satisfy the true Euler condition, since as usually formulated it requires balancing marginal utilities across a single period with a single interest rate: $u'(c_{t+1}) = (1 +$

13. This result generalizes the under-saving result of Goda, Manchester, and Sojourner (2014) in their working paper version. Their 2-period model, in which all income is received in period 1, generates overconsumption whenever the EIS >1.

14. We simplify this exercise by assuming the agent faces no risk and can freely borrow or save at a real interest rate of 5%.

$i_t)u'(c_t)$. The price effect is generated through the misperception of the Euler equation. In this sense, our results derived from the wealth effect (Propositions 1–2) are more robust than those derived from the price effect (Proposition 3).

The magnitude of the agent's misperception is a function of timespan (e.g. he makes no error regarding the present value of money from next period since compounding only occurs after spans greater than one), and so he will generally behave in a dynamically inconsistent manner. Conceptually, this can be distinguished from other varieties of dynamic inconsistency that are preference-based (Strotz, 1956; Loewenstein, 1987; Laibson, 1997; O'Donoghue and Rabin, 1999; Loewenstein et al., 2003; Kőszegi and Rabin, 2006), since this dynamic inconsistency is generated instead by perceptual errors regarding compounding interest. Of particular interest is the predictable pattern in which the dynamic inconsistency manifests.

Proposition 4 (Dynamic Inconsistency) *If the agent has a negative level of savings at the end of period $t < T-1$, i.e. if*

$$\sum_{s=0}^{t}(y_s - c_s)\Pi_{j=s}^{t}(1+i_j) < 0 \tag{8}$$

then the agent's period-t plan of consumption will exceed the period-(t+1) plan in all periods. If the inequality in expression (8) is reversed then planned consumption in $t+1$ will increase for all periods, and if the balance equals zero planned consumption in $t+1$ will be unaffected.

The proposition can be explained intuitively. Since the agent underestimates exponential growth, each period he will underestimate the change in his asset position. If his balance is positive then he receives an unexpected windfall and if the balance is negative he receives an unexpected loss. Since the agent's perception of the period-T value of income received at some intermediate future period τ depends only on interest rates between τ and T, the perception does not change over time. The only change in the perception of the budget is the growth of the current balance, and this surprise change to his current wealth causes him to shift his planned consumption vector in the same direction as the change.

This proposition implies that whenever a biased agent's net worth is currently negative then his projected consumption plans will always exceed his actual consumption. This can be particularly costly for the agent if he has the option to commit to lower bounds on his future consumption. This may manifest in the housing market, where housing is a consumption commitment. A homeowner will find it costly and difficult to decrease his housing consumption in the next period since this generally requires selling the home.

The agent will also underestimate the costs of debt, which can lead to a debt trap of sorts. Because the agent underestimates the speed at which a debt grows, he will underestimate both the size and the number of payments necessary to amortize a debt in a given amount of time. Let $a(L,\vec{\imath};\alpha)$ be the agent's perception of the periodic payment required to payoff a loan over T periods with principal L, and with interest rate vector $\vec{\imath}$.

Proposition 5 (Debt Repayment) *Define a repayment plan $a(L,\vec{i};\alpha)$ such that $\hat{V}(<L,-a,\ldots,-a>,\vec{i};\alpha) \equiv 0$. If $\alpha < \alpha'$, then $|a(L,\vec{i};\alpha)| < |a(L,\vec{i};\alpha')|$. Moreover, an agent with EGB (i.e. $\alpha < 1$) will underestimate the number of periodic payments of a fixed size necessary to repay a given debt.*

The proposition formally states the intuition introduced by Stango and Zinman (2009) that EGB can lead to excessive leverage. The agent believes that the periodic payment $a(L,\vec{i};\alpha)$ is the amount necessary to fully repay a debt of amount L in the specified number of periods. This perceived amount is strictly decreasing in the degree of bias. By underestimating compounding interest, the biased agent will underestimate the costs of holding debt leading to the various puzzles discussed earlier. While we do not focus on these predictions in this paper, we note that they are consistent with the findings of previous research. For example, Soll, Keeney, and Larrick (2013) find that US adults underestimate the number of payments needed to pay off a hypothetical credit card balance.

A corollary of Proposition 5 is that, given the repayment schedule for a loan, the agent will over-estimate the periodic interest rate. However, when the frequency of repayments exceeds the frequency at which interest is framed — for example monthly payments on a loan framed with an annual percentage rate — an agent may in fact under-estimate the annual interest rate. Consider the following algorithm. First, the agent computes the monthly interest rate from the repayment schedule. Here the agent will over-estimate the monthly rate. Second, the agent compounds the interest rate to compute the equivalent annual rate. Here the agent will under-estimate the annual rate given the monthly rate. The final result is ambiguous: the agent may either under- or over-estimate the equivalent annual rate. This process can explain the seemingly contradictory finding in Stango and Zinman (2009) that 98% of people underestimate the interest rate when given a debt and a monthly repayment plan over one year.[15] Our model predicts that an agent with EGB will under-estimate the equivalent annual rate when presented with the range of values they use.

2.3. Framing and Choice Architecture

This last result highlights the fact that a biased agent's valuation of a particular debt or investment product can depend on how the product is framed. There are several dimensions of a product description in which framing will matter such as the length of the period chosen to frame the rate, and presenting the simple interest rate versus the compound interest rate. Biased agents will also incorrectly combine interest rates. Hence how interest rates are framed for portfolios that have assets with multiple interest rates over time or over different accounts can affect a biased agent's choices.

15. More formally, Stango and Zinman (2009) use two questions from the Survey of Consumer finances, which we can interpret in our framework as the agent solving for $\widehat{V}(<L,0,...,0>,<i,...,i>;\alpha) = \widehat{V}(<0,m,...,m>,<i,...,i>;\alpha)$ and then $p(<i,...,i>,0;\alpha) = p(<0,...,0,R>,0;\alpha)$. While the agent will over-estimate i given L and m, he may nonetheless underestimate R.

Other aspects of the choice architecture could have large impacts on biased agents as well. A biased agent may have strong preferences for the payment timing of a loan about which an unbiased agent may be indifferent. Thus far our analysis has treated these features as exogenous. The model makes important predictions about how these frames affect behavior.

Lenders are incentivized to choose as short a time-period as possible in order to minimize the perceived repayment – for example, the market leader in the UK payday-lending industry advertised its loans as “1 percent per day”.[16] For investments, firms are incentivized to maximize the perceived return for potential investors and thus choose relatively large frames. A striking example of this effect is the multi-year frame chosen by many structured deposits products, which offer for example a 5-year return of 30% rather than annual returns of 5.4%.

Framing can also play an important role when presenting a short-term loan as simple interest or compound interest. When the term of a loan is shorter than a single period, the simple interest exceeds the periodic interest. Short-term loans with simple interest rates often imply exceedingly high periodic rates and as a consequence lenders will prefer to frame the short-term loans as simple interest.[17] For example, a lender will prefer to emphasize that a person will pay back $125 on a $100 loan over one month, instead of stating that the annualized rate on the loan is 1355%.

For time horizons longer than a single period there are a variety of payment schedules that are available to the lender. A lender can frame a loan as a principal with a fixed interest rate and loan term, e.g. “$1,000 at 10% per year for two years”, as a fixed periodic payment over a loan term, e.g. “$25.16 monthly payments for two years”, or a single payment at the end of the loan term, e.g. “$1,210 paid two years from today”. Because an agent with EGB underestimates the amount of the payments, the loan will be more attractive when framed in terms of the interest rate. When framed in terms of payments, since the magnitude of the error is increasing in time, periodic fixed payments will be viewed as more attractive than a single lump sum payment at the end of the term. Likewise periodic fixed payments will be seen as more favorable than a schedule of payments that is increasing over time.

The model also predicts how a biased agent will combine interest rates over time. An agent with accurate perceptions knows that an interest vector is equivalent to the vector of its geometric mean. A biased agent, on the other hand, will tend to overestimate the constant-rate equivalent for a given interest vector. A biased agent prefers a varying interest vector (i.e. not risky but simply changing over time) to an equivalent non-varying interest vector when saving. For example, if an asset has a 60% return in odd periods and 0% return in even periods, a 0-Eddie will believe that this is equivalent to a 30% return every period when in fact it is equivalent to a 26.5%

16. This compounds to 3778% annually, and 5853% including fees. Beginning 1 Jan, 2015, it was subject to a binding cost cap of 0.8% simple interest per day imposed by the financial regulator.

17. We thank an anonymous referee for this observation.

return.[18] The premium that the agent places on the varying interest vector is increasing in his bias. We find strong evidence for this prediction in Section 3.2.3. Thus even a risk-neutral 0-Eddie may exhibit *as-if risk preferences*. An economist who observes the agent's behavior, and believes the variation in returns is generated by risk, but does not account for EGB, may infer that the agent is risk-seeking. Ensthaler et al. (2013) incentivize subjects to determine the median of a distribution that is generated from a compounded geometric return, equal chance each period that an asset gives a 70% return or a -60% return. They find that subjects dramatically overestimate the median of the distribution, consistent with the notion that they perceive the asset growing linearly. As Ensthaler et al. (2013) explain not only will Eddie misperceive the returns, but he will also misperceive the skew of the distribution.

Finally, biased agents will make a different kind of combining error when projecting the value of a portfolio with several accounts at different interest rates. They will tend to underestimate the importance of accounts with relatively high interest rates. For example, if an agent has two accounts, one with \$100 at 12% and one with \$1,000 at 1%, a fully-biased agent would perceive the return on the total portfolio to be 2% per period, when in fact the return begins at 2% in the first period and then increases and asymptotically approaches 12% as the first account dominates. However a fully biased agent would be indifferent between this portfolio and a single account with \$1,100 and a 2% interest rate.

3. EGB in the U.S. Population

In this section, we examine whether our parameterized model of EGB is a useful measurement that has meaning outside the laboratory. We shift from a lifecycle consumption paradigm to a direct perception elicitation in order to directly estimate the EGB parameter α with the fewest potential confounds. The experiment has several purposes. First, we estimate the distribution of EGB among a representative subsample of the U.S. population, making decisions in their normal work environment. Second, the experiment explores the external relevance of EGB in economic decision making by correlating it with subjects' total savings. We interpret this not as a further direct test of the theory (Propositions 2 and 3) due to substantial unobserved heterogeneity, but rather as an indication that our measure of EGB can be usefully applied to explain some of the variation in financial decision making. Third, the experiment tests whether the bias is robust to a simple graphical intervention. Fourth, the experiment tests specific features of our model in contrast to the Wagenaar-Sagaria model, namely: (1) people correctly estimate a single period of interest, (2) people underestimate the value when there is negative interest, (3) people combine interest rates in a way that

18. The error is even more dramatic when negative interest rates are involved. For example a fully biased agent prefers an asset that has a 90% return followed by a -80% return, yielding a -38.4% net return, over one that does not grow.

biases them toward the arithmetic mean, and (4) people do not decompose financial problems into a series of single periods.

3.1. Design/Method

3.1.1. Design. Subjects faced a series of questions describing two assets and were asked to indicate the initial value for one asset which would equate the assets' final values after an indicated length of time. For example, the first question is a choice between "Asset A that has an initial value of $100 and grows at an interest rate of 10% each period" and "Asset B that has an initial value of $X and does not grow." Subjects were asked for the value of X which would make the two assets equal value after 20 periods. Appendix Table C.C.2 displays the full list of questions presented to subjects.

Questions 1–10 are our primary focus in the analysis, and the order of presentation was randomized first at the domain level (corresponding to the first 3 question categories listed in Appendix Table C.C.2) and then within domain at the question level. The "exponential" domain comprised four questions similar to the example above. The "fluctuating-interest" domain comprised three questions of the form: "Asset A has an initial value of $\$P$ and grows at an interest rate of $i\%$ in odd periods (starting with the first) and at $j\%$ in even periods; Asset B has an initial value of $\$X$ and does not grow; What value of X will cause the two assets to be of equal value after T periods?". The "catch-up savings" domain comprised three questions which varied the maturity of the assets, of the form: "Asset A has an initial value of $\$P$ and grows at an interest rate of $i\%$ each period; Asset B has an initial value of $\$X$ and grows at an interest rate of $i\%$ each period; What value of X will cause the two assets to be of equal value after Asset A grows for T periods and Asset B grows for S periods?"

Subjects received a payment based on their accuracy on each question. The payment rule was piecewise-linear in the percentage error: each answer within 10% of the truth would receive $0.80; each answer within 25% would receive $0.60; each answer within 50% would receive $0.20; and each answer less than 50% of or more than 150% than the truth would not receive a payment. In addition to the incentive payments, subjects received $5.00 for completing the entire experiment. Subjects had a week to do the experiment at their leisure. All payments were made through Knowledge Networks' internal payment mechanism, which subjects were already experienced with, and were usually paid within a week of completion.

The experimental instrument intentionally did not mention the use of tools for answering the questions. Subjects could potentially use whatever tools that they had access to: from nothing to advice from friends or financial calculators. This design neither discouraged subjects' natural tendency to use tools nor did it prime subjects to use them. Although the incentives are not nearly as large as they would be for making actual financial decisions in the marketplace, a subject that exploited tools could earn substantially more for their time.

After completing the primary experiment, subjects were randomly assigned into a control (N=384) and a treated (N=185) group to test the effect of a simple information presentation or "nudge" on a second set of questions. The intervention, shown in

Appendix Figure C.C.1, shows the growth of $100 at one or more relevant interest rates, and allowed the subject to specify the time horizon plotted. Treated subjects were shown this intervention beneath each question. Subjects answered an additional 16 questions, 10 from the original three diagnostic domains and 3 each from the domains of periodic savings – asking the final value of a series of regular contributions – and portfolio – asking the equivalent principal for a portfolio of assets at different rates. These latter two domains are qualitatively more complicated to solve.[19] We exclude these additional questions from all analysis until Sections 3.2.3, focusing just on the 10 pre-intervention questions where all subjects faced identical circumstances.

3.1.2. Sample. The experiment consists of an incentivized online experiment conducted on a nationally-representative sample. Participants were recruited through Knowledge Networks, which maintains a recruited panel of U.S. households.[20] A random sample of subjects from the Knowledge Networks panel were invited to participate in our study. Subjects logged into the Knowledge Networks portal, and were automatically transferred to an external website where our study was hosted.

Table 1 shows summary statistics for our sample. Column 1 shows the characteristics of all 990 KN panelists who were invited to participate, while Column 2 comprises the 569 subjects who chose to participate. Men were significantly more likely to complete the study (63% vs. 52% for women, $p<0.01$), so that 46% of the final sample were women. The average age of those opting to complete the study was also somewhat lower than those opting out, although this result was largely driven by a very high completion rate amongst 18–21 (i.e. college-aged) panelists. Race and education characteristics did not predict study completion, with 28% of subjects having only a high school degree, 29% some college or an associate's degree, and 37% having a bachelor's degree or more.

For some of the analysis, we merge our experimental dataset with an external dataset containing subjects' financial characteristics. Participants in the Knowledge Networks panel are regularly asked about their income and assets, and we will use this information to investigate the effect of exponential-growth bias on savings. These data are only available for a subset of subjects (the others either being ineligible or refusing to answer), and we present them in the fourth column of Table 1. Unsurprisingly, this subsample tends to be older and better educated than those for whom financial data are unavailable: the mean age is 50.02, and 53% have at least a bachelor's degree. The higher education attainment rate is also reflected in the high average household income of $90,257 among this group. This group also had significant investible assets — a mean of $241,055 — suggesting that they could overstate the degree of financial

19. We do note formalize a metric of "difficulty", but note that, for example, Catch-up Savings questions require a subject to sum T separate Exponential questions.

20. Participant households are selected randomly by Knowledge Networks based on their address, and are provided with a laptop and free internet access if necessary. Full details on the KnowledgePanel sampling methodology are available at http://www.knowledgenetworks.com/knpanel/KNPanel-Design-Summary.html

sophistication relative to a poorer, less well-educated population. However, we find that the sample with complete financial data are in fact slightly *more* biased than the sample with missing financials – the partial correlation of our α measure with the absence of financial data is 0.024 (p<0.01).

TABLE 1. Summary Statistics

	Invited Sample	Study Completers	Completers w/o Assets Data	Completers with Assets Data	US Population
	(1)	(2)	(3)	(4)	(5)
Demographics					
Female	0.51	0.46	0.47	0.46	0.52
Age	47.38	44.73	39.00**	50.02***	45.21
	(17.30)	(17.17)	(17.41)	(15.15)	
Education					
High School	0.27	0.28	0.37	0.19	0.28
Some College	0.30	0.29	0.33	0.26	0.27
Bachelor's Degree+	0.36	0.37	0.20***	0.53***	0.28
Race/Ethnicity					
Black, Non-Hispanic	0.07	0.07*	0.08	0.06	0.14
Other, Non-Hispanic	0.05*	0.05**	0.04	0.07	0.08
Hispanic	0.13**	0.13**	0.17	0.10*	0.18
2+ Races	0.03	0.03	0.03	0.03	0.02
Financial Products					
Has had credit card		0.83	0.70	0.94	
Has used payday loan		0.07	0.08	0.05	
Has had car loan		0.61	0.48	0.74	
Has had mortgage		0.60	0.39	0.79	
Has had 2^{nd} mortgage		0.16	0.08	0.23	
EGB - alpha					
Alpha		0.54	0.57	0.52	
		(0.55)	(0.57)	(0.54)	
Assets/Income					
Non-Housing Assets				241055.74***	132822
				(375913.20)	
Household Income		79945.08	68763.74***	90257.60***	69759
		(50224.95)	(49887.68)	(48371.71)	
Observations	990	569	273	296	

Notes: Initial sample comprises all subjects invited to participate in the study. Study completers answered or skipped all questions. Assets data merged from external dataset provided by Knowledge Networks. Financial products indicate whether subject has ever had or used the product in the past. US Population is based on authors calculation from 2014 Current Population Survey (and SCF for asset data). Table entries are unweighted means; standard deviations for non-binary variables given in parentheses. Stars denote significant difference from US population average, using study weights; $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

3.2. Results

We begin this section by showing evidence that subjects in the experiment were systematically affected by exponential-growth bias despite the availability of financial tools outside the laboratory. We then estimate an individual-level bias parameter based on the model presented in Section 2, and investigate its distribution and correlation with household finances.

3.2.1. Bias. For each subject i and exponential-growth question j, we first calculate the natural logarithm of the ratio of the given answer to the correct answer. Let a subject's responses on question $j \in \{1,...,J\} = \mathbb{J}$ be denoted by r_{ij}, and the correct response be given by c_j. We calculate $e_{ij} = \ln(r_{ij}/c_j)$. This provides a consistent measure across questions that may have answers that differ by several orders of magnitude. Were a subjects to answer exactly correctly, this statistic would be exactly zero. As subjects were not prohibited from using calculators, spreadsheets, and online tools to help them answer the questions, such an outcome would not strain credulity. If subjects' answers are unbiased but noisy such that errors on an absolute or percentage basis are symmetrically distributed around zero, then the median of $\ln(r_{ij}/c_j)$ should also be zero. Moreover, if subjects' answers are a power of the correct answer, $r_{ij} = c_j^{1+\varepsilon_{ij}}$ where $\varepsilon \sim \mathcal{N}(0, \sigma^s)$, then the log-ratio should be normally distributed about zero.

We also calculate subject-level averages of the above log-ratio:

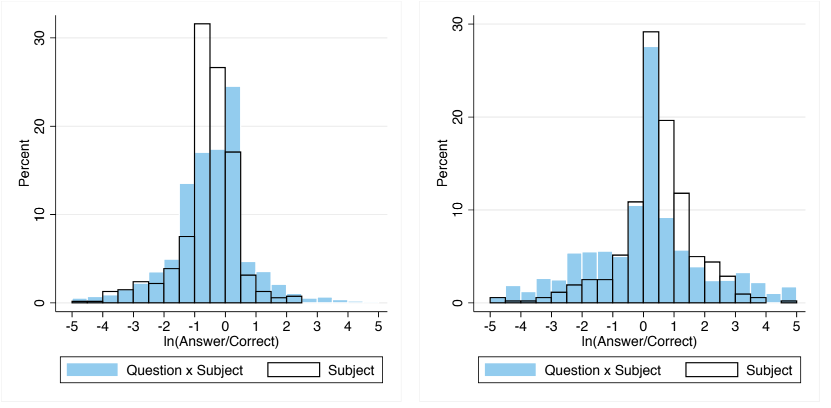
$$\bar{e}_i = \frac{1}{10}\sum_{j=1}^{10} \ln(answer_{ij}/correct_j)$$

If subjects are making unbiased errors, then the above results for the means and medians hold. Moreover, if r_{ij}/c_j is i.i.d. lognormal, then the averaging should cause the distributions to collapse towards zero.

Instead, we find a systematic bias in the error, the sign of which depends on whether exponential-growth bias predicts that subjects should over- or under-predict on that question. Figure 1 plots the distribution of log errors at the question×subject and subject level. As expected, the modal question×subject error is zero – the likeliest interpretation is that a large mass of subjects are able to use calculators to get the answer exactly correct. However, where under-estimation is predicted, both the median (-0.349) and mean (-0.554) of the question×subject error distribution are significantly negative (p<0.01). At the subject level, both the median (-0.507) and mean (-0.602) are more negative than before. The pattern is reversed where exponential-growth bias predicts over-estimation: the question×subject distribution is shifted sharply to the right and the mean error (0.209) is now significantly positive (p<0.01), although the median error in this case is zero.[21]

21. These results imply that pooling all questions by reversing the scale on the underestimation-prone items would also produce a significant result. At the subject level, both the mean (0.400) and the median (0.405) are significantly positive.

FIGURE 1. Subject Mistakes



(a) Under-Estimation (b) Over-Estimation

Notes: Underestimation based on the questions for which EGB predicts a downward-biased answer; overestimation from those where an upward bias is predicted by the theory. Question-level plots the distribution of errors in predicted asset growth, and should be symmetric about zero if subjects' errors on a percentage basis are symmetric about zero; subject-level computes the mean of $\ln(answer/correct)$ at the subject level, and should converge to a point mass at zero in the absence of systematic bias. The means of all four distributions are significantly different from zero (significant at $p < 0.01$).

3.2.2. Estimating Alpha. Let $\vec{a}(\alpha) : \mathbb{R} \to \mathbb{R}^J_+$ be a function that generates the answers consistent with a given level of α on a set of questions $\mathbb{J}$. Thus $\vec{a}(1)$ is a vector containing the J correct answers. For every subject, we calculate the value of α which minimizes the mean squared error of the model against their actual answers, with each question normalized by the correct answer. This normalization avoids having those questions which contain large values for the solution arbitrarily dominate the estimation procedure. That is, we estimate:

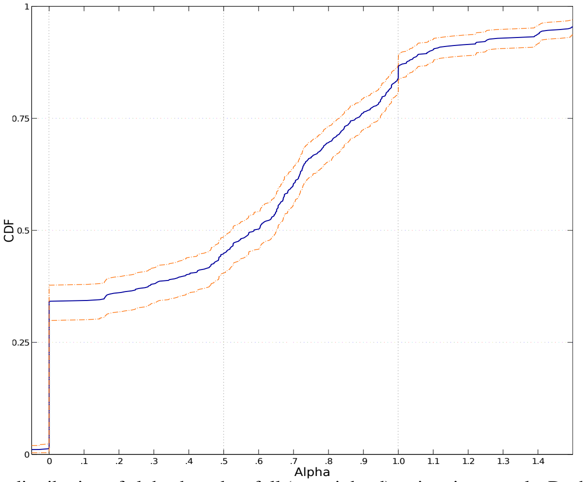
$$\widehat{\alpha}_i = \arg\min_{\alpha} \frac{1}{J} \sum_{j \in \mathbb{J}} \left(\frac{r_{ij} - a_j(\alpha)}{a_j(1)} \right)^2 \tag{9}$$

The estimator described by (9) is not constrained to values lying within the unit interval. Values of α greater than one are simply interpreted as an individual who overestimates the rapidity of exponential growth. We perform an unconstrained numerical optimization to estimate an $\widehat{\alpha}_i$ for each subject.

Figure 2 plots the cumulative distribution of our estimates of α, using our full sample of completers. We characterize 85% of the population with an α between $[0,1]$. The median α is 0.53, and the mean is 0.60. Moreover, we have a large number of people who are completely, or nearly completely, fully biased: 33% of subjects (184/561) have an alpha of "exactly" zero (i.e. within $[-0.001, 0.001]$). In contrast, only 4% (23/561) are completely correct (even using a more generous interval of [0.99,1.01]). Based on our bootstrapping procedure, we can reject that the 80th percentile has $\alpha = 1$ at 95% confidence. Similarly, we cannot reject that the 37th percentile has $\alpha = 0$.

Although EGB appears to be pervasive we qualify this with two major caveats. The stakes in our experiment are much lower than in many financial decisions. Second, when horizons are short and interest rates are in the single digits, people in the upper quartile of α will not make very large errors. For instance with a horizon as long as 20 periods and a 5% periodic interest rate, a 0-Eddie will underestimate the value of an asset by 25% but a 0.9-Eddie will only underestimate by 5%. On the other hand, a 0.9-Eddie will make substantial errors in some situations, for instance the 0.9-Eddie will underestimate the annual equivalent of a loan that charges 1% a day by 30 percent.

FIGURE 2. Population Distribution of Alpha



Notes: Cumulative distribution of alpha, based on full (unweighted) estimation sample. Dashed lines indicate 95% confidence interval for percentiles of the distribution, based on 5000 bootstrap replications. Confidence intervals should be read horizontally, e.g. the median alpha is estimated to lie between 0.520 and 0.651 while the 25th percentile is estimated to be exactly zero.

Question 3 gives us a test of basic interest-rate numeracy. The question asks for the value of an asset after it grows for only one period ($P_0 = 100$, $i = 4\%$), and a correct answer would result from any degree of bias. A wrong answer can thus be interpreted as innumeracy. About 74% of our subjects answered this question correctly and 26% did not. Dropping subjects who fail to answer this question correctly does not substantively change any of the remaining analysis. Moreover, mistakes on this question are uncorrelated with our measure of α ($r = 0.04$, p=0.29), which provides reassurance that we are estimating a systematic bias rather than understanding of the task. Almenberg and Gerdes (2012) find that EGB and financial literacy are negatively correlated in contrast to what we find here. These results are not inconsistent, however, given that their measure of financial literacy is quite different from our Question 3.

Our measure of α is uncorrelated with education, age, race, and sex. Unsurprisingly, the 6% of subjects who reported an online calculator perform substantially better than the rest of the population. The mean α in this group is 0.84 (0.32 higher than those who do not use financial calculators) and with a median α of 0.96 (relative to a median amongst those who do not use financial calculators of 0.56).

There may be both a causal and self-selection effect in this population, which our research design does not distinguish. The use of calculators may also help to explain why our graphical treatment had no effect, as we explain later.

With estimates of individuals' α we can explore the relationship between EGB and long-run financial outcomes. Proposition 3 states that biased agents will systematically overconsume in early periods relative to the optimal consumption path when the elasticity of intertemporal substitution is greater than unity. This posits exponential-growth bias as a partial explanation for the high degree of variation in retirement savings within income and education categories found by Bernheim et al. (2001) and supported by the correlation found in Stango and Zinman (2009).While we do not have exogenous variation in α we can correlate α with total accumulated assets.

We are able to match financial records from an external survey to 296 of our 569 experimental subjects. As the unmatched cases correspond to refusals or ineligible cases (often college students), this leaves us with a slightly older and better-educated subsample. We then estimate the relationship between our estimate of α and investible assets. We perform regressions of the form:

$$assets_i = \theta_1 + \theta_2 \cdot \widehat{\alpha}_i + \theta_3 \cdot income_i + \theta_4 \cdot age_i + \theta_5 \cdot male_i + \theta_6 \cdot hhsize_i + \theta_7 educ_i + \varepsilon_i \tag{10}$$

where assets and income are measured either as the level or natural logarithm.

The results of this regression are shown in Table 2. Columns 1 and 2 present the results where $\ln(assets_i)$ is the dependent variable, and columns 3 and 4 present the results where $assets_i$ is the dependent variable. In columns 1 and 3 subjects are uniformly weighted whereas in columns 2 and 4 subjects are weighted by study-specific post-stratification survey weights that re-balance the sample to national representativeness. All models have state fixed effects. Unsurprisingly, older people have accumulated more assets, and a 1% increase in income tends to be associated with slightly more than 1% higher level of savings. Although education is positively associated with higher total savings, it is only marginally significant.

Our coefficient of interest, Alpha, is positive and substantial throughout all the models. The estimated coefficient in logs is 0.438 and 0.640 in the two models and both are highly significant. The magnitude of the effect is large: all else equal, an unbiased agent will accumulate from 55% (column 1) to 90% (column 2) more assets than a fully biased agent. We caution that this association is not causally identified, but note that are several plausible channels including the over-consumption result of Proposition 3 and the mis-use of credit of Proposition 5 , as well as possible reverse-causation. Translated into levels in columns 3 and 4, this effect is equivalent to $\$87,877.81$ and $\$93,499.75$ respectively, or approximately one third of the median household's non-annuitized wealth at retirement (Poterba, Venti, and Wise, 2011). We thus find that EGB can help to predict the large differences in otherwise equivalent households' wealth.

3.2.3. Domain-Specific Predictions and Fingerprints and Stability. The domains were carefully designed not only for aggregate estimation of α, but also to test

TABLE 2. Alpha and Savings

	Logs		Levels	
	(1)	(2)	(3)	(4)
Alpha	0.438**	0.640***	87,877.807**	93,499.748***
	(0.178)	(0.215)	(39,269.207)	(31,057.441)
Ln(Income)	1.114***	1.201***		
	(0.159)	(0.149)		
Education	0.078	0.023	21,507.573*	16,636.850
	(0.061)	(0.058)	(12,683.178)	(13,485.775)
Age	0.059***	0.058***	10,639.556***	10,720.404***
	(0.006)	(0.006)	(1,385.909)	(1,407.761)
Female	-0.000	-0.018	-34,768.528	-48,270.127
	(0.208)	(0.212)	(40,560.958)	(33,933.839)
Household Size	-0.104	-0.160*	-24,140.997	-28,202.743**
	(0.094)	(0.091)	(14,654.991)	(13,251.836)
Income			2.715***	2.791***
			(0.567)	(0.516)
N	296	296	296	296
R^2	0.47	0.55	0.42	0.47

Notes: Dependent variable in columns (1) and (2) is ln(investible assets), and in columns (3) and (4) is investible assets. Columns (2) and (4) weight observations using study-specific post-stratification survey weights; columns (1) and (3) are unweighted. All specifications include state fixed effects. Robust standard errors in parentheses. $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

subtle and specific predictions of the model. The "exponential" domain is the simplest and allows for direct tests of our model to the Wagenaar and Sagaria (1975) model described in the introduction. Question 3 has no compounding and thus our model predicts that subjects should not make any error in contrast to the Wagenaar-Sagaria model. In our sample, 75.3% of subjects correctly stated the correct value of $104, and a further 4.0% answered with the interest-only value of $4. Another novel prediction of our model is that subjects will *underestimate* the value of a compounding asset if the interest rates are negative. Amongst the questions with negative interest rates (questions 4, 13, and 14), twice as many underestimate the asset as overestimate – another strong test favoring our model. Furthermore, 33% of all responses are exactly what a 0-Eddie would choose.

The fluctuating-interest domain and the portfolio domain both demonstrate people's tendency to take the arithmetic mean when combining multiple interest rates.[22] A fully biased agent will use the arithmetic mean on both of these problems. Our model predicts that subjects will overweight the impact of the higher interest rate on the fluctuating-interest domain, but we also predict that they will underweight

22. The fluctuating-interest domain is the simplest example of an asset that has known but varying returns over time. The portfolio domain is the simplest example of a portfolio with two assets that have different returns.

the impact of the higher interest rate in the portfolio domain. We find exactly this pattern amongst our subjects. For example, in question 7 of the fluctuating-interest domain, the mean growth is exactly zero, but the arithmetic mean of the interest rates is positive. On this question, 58% of subjects believed the asset would increase while only 22% believed it would decrease (and 20% got it exactly right). Over the domain as a whole 17% of responses left a full-bias fingerprint. In the portfolio domain, on question 24 for example, 74% underestimated the impact of the high interest rate as predicted, and 15% left a full-bias fingerprint. Over the domain as a whole 9% left a full-bias fingerprint.

These two domains show how Eddie will incorrectly combine interest rates and as a consequence exhibit *as-if risk preferences*. Keep in mind that there is no uncertainty in these problems. But an economist who thinks that uncertainty generates the fluctuating-interest and believes Eddie to be unbiased, would infer that Eddie is risk-seeking. In contrast an economist who thinks that Eddie's portfolio faces risk would observe that Eddie is heavily invested in low-return assets. If the economist assumed a risk-return tradeoff, she would infer that Eddie is quite risk averse.[23]

The catch-up savings and periodic savings domains agree with the directional predictions of the model, but the complexity of these questions implicates additional mathematical errors. Indeed, 33% of responses in the periodic savings domain are below the sum of the contributions!

Fundamentally, the model presumes that Eddie treats assets separately rather than canceling common terms. For example, Question 8 (20 vs. 15 years at 13%) can be reduced to solving for a principal of \$100 growing for five periods at 13% interest. More generally, since Eddie gets one round of interest exactly right, if he were to break down a problem into a sequence of iterated one-period problems, he would make no mistake. Question 10 in the catch-up savings domain was designed to directly address this issue; it asks what principal is needed for a one period delay in savings (9 periods vs 10 periods; the answer is $[1/(1+i)]y_0$). Subjects do not seem to simplify this into a one period problem: 19% got the answer correct which is about their accuracy on other problems, and 67% respond with a principal in the biased direction.

To address the stability of our α parameter estimates, we re-estimate equation (9) using only subjects' responses to the second set of 10 questions and then compare our two estimates of α within subjects. We are most interested in whether subjects identified as the "extreme" types – that is, with $\alpha \in \{0,1\}$ are consistent. This does appear to be the case. Of 126 control subjects identified as having $\alpha = 0$ on the first set of questions, 78 (61.9%) yielded an estimate of $\alpha = 0$ on the second set.[24] In a linear probability model, we find that having $\alpha = 0$ in the first set of questions raises the probability of having $\alpha = 0$ in the second set by 22.28 percentage points (s.e. 4.41),

23. Eddie's as-if risk preferences and behavior under uncertainty is the subject of a sister paper in progress.

24. We focus on control subjects to separate out any effect of the graphical intervention. It is not surprising, however, that a comparable 34 of 58 (58.6%) treated subjects identified as having zero in the first set of questions were also identified as having zero in the second set.

while having initially had $\alpha = 1$ lowers the probability by 41.61 percentage points (s.e. 9.21).

We also conclude that the bias is robust to the provision of information. This may be surprising, as the intervention made calculating the correct answer in the "exponential" domain all but trivial, but is not unreasonable – subjects were already free to use whatever tools they wanted to help them, including ones far more sophisticated than a simple graph. It is likely that subjects for whom the extra information would have been helpful were already using tools on their own. We find that subjects in the control and treated groups were statistically indistinguishable both in the pre-intervention and post-intervention phase. A Kolmogorov-Smirnov test of equal distributions in α values calculated from pre-intervention data fails to reject at a significance of p=0.802. The same test on α values calculated from post-intervention data fails to reject at p=0.618. Thus exponential-growth bias is unlikely to be eliminated by simple "nudges": those who would benefit from the intervention may already be using tools of their own accord. More involved interventions may have some effect, however. For example, Song (2012) finds that face-to-face explanation of compounding along with financial advising has a large effect on savings in rural China.

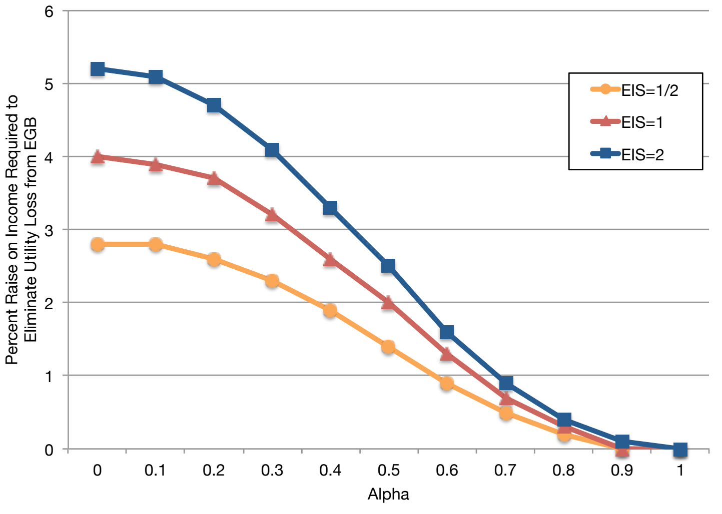
4. Conclusion

While the un-intuitively rapid growth of exponential functions has been observed for ages, the economic implications have only been considered recently. We find that consumers will make very specific — and very large — errors in their consumption plans. Moreover, since the bias is fundamentally about the budget constraint, the model is modular and can thus be easily married to other economic settings or extensions. Moreover, the bias seems to prevail in the population as a whole and is a strong predictor of saving behavior even after controlling for the standard explanatory factors. The bias was robust to an intervention designed to make exponential growth more salient, and which could be used to obtain the correct response in some domains.

Ignoring the presence of EGB can potentially lead to substantially mis-specified econometric models. As a proof of concept, we generate simulated lifecycle consumption data assuming a constant interest rate of 5%, CRRA utility with EIS=2, full bias, no discounting, and mean-zero normally distributed error added to optimal consumption each period. We simulate a sample of 1000 agents randomly drawn between the ages of 19 and 66, observing their current and initial (age 18) consumption. This is as if the economist has two observations of each agent's annual consumption, which can be used to infer the agent's consumption growth over time. We assume the economist knows that agents are homogeneous with CRRA utility functions. We simulate the economist's non-linear least squares estimates of EIS, δ, and α. In Monte Carlo simulations, this unrestricted estimator performs well, estimating $\widehat{EIS}_U = 2.05$ (s.e. 0.104), a discount factor $\widehat{\delta}_U = 0.97$ (s.e. 0.03), and level of EGB $\widehat{\alpha}_U = -0.014$ (s.e. 0.18). An economist who did not account for EGB will implicitly restrict $\alpha = 1$. The restricted estimator halves the EIS estimate to

$\widehat{EIS}_R = 0.89$ (s.e. 0.026) and imposes significant impatience in the discount factor $\widehat{\delta}_R = 0.68$ (s.e. 0.025) to account for the biased agent's lack of consumption growth over the lifecycle. While a simple exercise, this shows that EGB can have a large effect on estimates of important economic objects.

FIGURE 3. Welfare Effects of EGB



Notes: Simulated welfare loss due to EGB. The agent earns the median US personal income between ages 20 and 65, then retires and later dies at 78. We assume discounting is already reflected in interest rates, so the risk-free rate (post-retirement) is zero and risky rate (pre-retirement) is 5%.

In principle, the re-framing of assets in the presence exponential-growth bias could actually be used to identify the EIS.[25] By reframing the period length at which interest compounds, one changes a biased agent's perception of intertemporal prices. This affords the exogenous variation in intertemporal prices necessary to identify the EIS. The degree of additional structural assumptions required for identification depends on the quality of the data.

Ultimately field experiments with large-stake financial decisions are needed for ecological validity — since ultimately these are the target applications of interest.[26] Additional research on the efficacy of interventions to combat EGB in the field is also necessary. While EGB was robust to the intervention in our experiment, it is possible that other "nudges" designed around the predictions of our model could help improve welfare. The potential welfare consequences are quite large. Figure 3 plots the welfare costs of EGB as a function of α in a back-of-the-envelope calculation. For this exercise, we again consider CRRA utility and consumption that is dynamically

25. We thank an anonymous referee for this suggestion.

26. That is, while our experiment used a nationally representative sample, the data are still "artefactual".

re-optimized every year when the agent observes his true savings balance or debt. A representative agent earns the median US personal income between ages 20 and 65, and then retires until death at 78. We assume discounting is already reflected in interest rates, so the risk-free rate (post-retirement) is zero and risky rate (pre-retirement) is 5%. The welfare loss is measured as the fraction of additional income required to eliminate the loss of utility from EGB. As the figure shows, for a 0-Eddie the welfare effects are at least 2.8% of income, and can be as high as 5.2%.

Additional market implications of EGB also remain to be explored. Our results suggest that it is unlikely that competition would eliminate the bias; firms may find it more profitable to maximally exploit the bias instead (Gabaix and Laibson, 2006; Heidhues and Kőszegi, 2010; Stango and Zinman, 2011). Consider a consumer's choice of mutual funds as an illustration. Not only will managers have an incentive to present the arithmetic mean return of the fund (rather than the geometric mean), they may also take a riskier position in order to achieve a higher arithmetic mean. We expect that firms' responses to exponential-growth-biased consumers will leave fingerprints in many such markets.

References

Almenberg, Johan and Christer Gerdes (2012). "Exponential Growth Bias and Financial Literacy." *Applied Economics Letters*, 19, 1693–1696.

Benzion, Uri, Alon Granot, and Joseph Yagil (1992). "The Valuation of the Exponential Function and Implications for Derived Interest Rates." *Economic Letters*, 38, 299–303.

Bernheim, B. Douglas, Jonathan Skinner, and Steven Weinberg (2001). "What Accounts for the Variation in Retirement Wealth among U.S. Households?" *American Economic Review*, 91(4).

Chetty, Raj, Adam Looney, and Kory Kroft (2009). "Salience and Taxation: Theory and Evidence." *American Economic Review*, 99(4), 1145–1177.

Cronqvist, Henrik and Stephan Siegel (2014). "The Genetics of Investment Biases." *Journal of Financial Economics*, 113(2), 215–234.

Eisenstein, Eric M. and Stephen J. Hoch (2007). "Intuitive Compounding: Framing, Temporal Perspective, and Expertise." Working Paper.

Ensthaler, Ludwig, Olga Nottmeyer, Georg Weizsäcker, and Christian Zankiewicz (2013). "Hidden skewness: On the difficulty of multiplicative compounding under random shocks." DIW Berlin Discussion Paper No. 1337.

Frederick, Shane, George Loewenstein, and Ted O'Donoghue (2002). "Time Discounting and Time Preference: A Critical Review." *Journal of Economic Literature*, 40(2), 351–401.

Friedman, Milton (1957). *A Theory of the Consumption Function*. National Bureau of Economic Research, Inc.

Gabaix, Xavier (2014). "A Sparsity-Based Model of Bounded Rationality." *Quarterly Journal of Economics*, 129(4), 1661–1710.

Gabaix, Xavier and David Laibson (2006). "Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets." *Quarterly Journal of Economics*, 121(2), 505–540.

Goda, Gopi Shah, Colleen Flaherty Manchester, and Aaron Sojourner (2014). "What Will My Account Be Worth? Experimental Evidence on How Retirement Income Projections Affect Saving." *Journal of Public Economics*, 119, 80–92.

Heidhues, Paul and Botond Kőszegi (2010). "Exploiting Naïvete about Self-Control in the Credit Market." *American Economic Review*, 100(5), 2279–2303.

Keren, Gideon (1983). "Cultural Differences in the Misperception of Exponential Growth." *Perception and Psychophysics*, 34(3), 289–293.

Kőszegi, Botond and Matthew Rabin (2006). "A Model of Reference-Dependent Preferences." *Quarterly Journal of Economics*, 121(4), 1133–1166.

Laibson, David (1997). "Golden Eggs and Hyperbolic Discounting." *Quarterly Journal of Economics*, 112(2), 443–477.

Loewenstein, George (1987). "Anticipation and the Valuation of Delayed Consumption." *The Economic Journal*, 97(387), 666–684.

Loewenstein, George, Ted O'Donoghue, and Matthew Rabin (2003). "Projection Bias in Predicting Future Utility." *The Quarterly Journal of Economics*, pp. 1209–1248.

MacKinnon, Andrew J. and Alexander J. Wearing (1991). "Feedback and the Forecasting of Exponential Change." *Acta Psychologica*, 76, 177–191.

McKenzie, Craig R.M. and Michael J. Liersch (2011). "Misunderstanding Savings Growth: Implications for Retirement Savings Behavior." *Journal of Marketing Research*, 48, S1–S13.

Munnell, Alicia, Anthony Webb, and Luke Delorme (2006). "A New National Retirement Index." Working Paper.

O'Donoghue, Ted and Matthew Rabin (1999). "Doing It Now or Later." *The American Economic Review*, 89(1), 103–124.

Poterba, James, Steven Venti, and David Wise (2011). "The Composition and Drawdown of Wealth in Retirement." *The Journal of Economic Perspectives*, pp. 95–117.

Read, Daniel, Shane Frederick, and Marc Scholten (2013). "DRIFT: An Analysis of Outcome Framing in Intertemporal Choice." *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 39(2), 573—588.

Skiba, Paige Marta and Jeremy Tobacman (2008). "Payday Loans, Uncertainty and Discounting: Explaining Patterns of Borrowing, Repayment, and Default." Vanderbilt Law and Economics Research Paper No. 08-33.

Skiba, Paige Marta and Jeremy Tobacman (2011). "Do Payday Loans Cause Bankruptcy?" Vanderbilt Law School Working Paper 11-13.

Soll, Jack B, Ralph L Keeney, and Richard P Larrick (2013). "Consumer Misunderstanding of Credit Card Use, Payments, and Debt: Causes and Solutions." *Journal of Public Policy & Marketing*, 32(1), 66–81.

Song, Changcheng (2012). "Financial Illiteracy and Pension Contributions: A Field Experiment on Compound Interest in China." Working Paper.

Stango, Victor and Jonathan Zinman (2009). "Exponential Growth Bias and Household Finance." *Journal of Finance*, 64(6), 2807–2849.

Stango, Victor and Jonathan Zinman (2011). "Fuzzy Math, Disclosure Regulation, and Market Outcomes: Evidence from Truth-in-Lending Reform." *Review of Financial Studies*, 24(2), 506–534.

Strotz, Robert H. (1956). "Myopia and Inconsistency in Dynamic Utility Maximization." *Review of Economic Studies*, 23, 165–180.

Wagenaar, Willem A. and Sabato D. Sagaria (1975). "Misperception of Exponential Growth." *Perception and Psychophysics*, 18(6), 416–422.

Wagenaar, Willem A. and Han Timmers (1979). "The Pond-and-Duckweed Problem: Three Experiments on the Misperception of Exponential Growth." *Acta Psychologica*, 43, 239–251.

Appendix A: General Model of Exponential-Growth Bias

We define a general *interest perception function* as the following:

Interest Perception Function: *An interest perception function is a continuously differentiable function* $p : \mathbb{R}^T \times \{0,1,...,T\} \to \mathbb{R}$ *such that:*

(A1) (boundary conditions) $p(\vec{0},t) = 1$ for all t and $p(\vec{\imath},T) = 1$ for all $\vec{\imath}$
(A2) (compounding) $\partial p(\vec{i},t)/\partial i_k \geq 0$ and $\partial^2 p(\vec{i},t)/\partial i_j \partial i_k \geq 0$. For $k \geq t$, if $i_\tau = 0$ for all $\tau \neq k$, $\tau \geq t$, then $\partial p(\vec{i},t)/\partial i_k = 1$
(A3) (irrelevancy of history) $p(<i_1,\ldots,i_t,i_{t+1},\ldots,i_{T-1}>,t) = p(<0,\ldots,0,i_t,i_{t+1},\ldots,i_{T-1}>,0)$
(A4) (irrelevancy of order) If for some $k,l \geq t$, $i_k = j_l$, $i_l = j_k$, and for any $\tau \notin \{k,l\}$, $i_\tau = j_\tau$, then $\partial p(\vec{\imath},t)/\partial i_k = \partial p(\vec{j},t)/\partial j_l$

These assumptions define the minimum requirements for a well-behaved perception of interest rates. Assumption A1 simply states that an agent does not introduce discounting into a world with zero interest rates. Similarly, A2 states that agents correctly perceive that a single non-zero interest rate will discount period-T consumption on a one-for-one basis. This isolates the psychological phenomenon of neglecting compound growth from simple arithmetic mistakes. It also states that the cross-partials are weakly positive implying that the perception function is quasiconcave. A3 states both that historical rates do not directly affect beliefs about future growth, and that perceptions are not affected by prepending an arbitrary number of zero interest rate periods. This rules out, for instance, that a high interest rate experienced in an early period would raise the perceived growth rate in all subsequent periods. It also means that perceptions depend on the number of times interest compounds rather than the number of periods *per se*. Finally, assumption A4 states that any two future interest rates are interchangeable from the perspective of period t. This is akin to a stationarity assumption, in as much as it implies all future periods are treated symmetrically.

Exponential-Growth Bias: An interest perception function $p(\vec{\imath},t)$ exhibits exponential-growth bias if:

$$\frac{\partial}{\partial i_k}\left[\frac{\frac{\partial p(\vec{\imath},t)}{\partial i_j}}{\frac{\partial e(\vec{\imath},t)}{\partial i_j}}\right] \leq 0 \tag{A.1}$$

where $e(\vec{\imath},t) = \prod_{j=t}^{T-1}(1+i_j)$ is the correct interest perception, $T > 1$, and the inequality is strict if $j \neq k$. An interest perception function p exhibits greater exponential-growth bias than q if:

$$\frac{\partial}{\partial i_k}\left[\frac{\frac{\partial p(\vec{\imath},t)}{\partial i_j}}{\frac{\partial q(\vec{\imath},t)}{\partial i_j}}\right] \leq 0 \tag{A.2}$$

where $T > 1$, and the inequality is strict if $j \neq k$.

Equation (A.1) highlights the centrality of compounding to EGB. Combined with (A1)-(A2), it is clear that a biased agent will only misperceive the value of an asset with at least two periods of non-zero interest for it to grow. There is nothing to act on without any nonzero interest rates, and because there is no compounding with a single nonzero rate, (A2) implies correct perception even for a biased agent. Beginning with the addition of a second nonzero rate, however, the neglect of compounding implied by (A.1) implies that biased agents will begin to under-estimate exponential growth.

Moreover, because (A1) bounds the level of the perception function to be one when there is no interest, and (A2) requires that the partial derivative of the perception function with respect to an interest rate to be at least one and weakly increasing in all the other interest rates, (A1)-(A4) and (A.1) jointly imply that the interest perception function exhibiting the greatest extent of exponential-growth bias is one which fully linearizes growth: $p(\vec{\imath},t) = 1 + \sum_{j=t}^{T-1} i_j$. This "fully biased" agent corresponds to the 0-Eddie in our parametric model.

Below, we present the generalized form of Lemmas 1 and 2, and Proposition 5. All other results hold for the general model as stated in the text. The main text explains the intuition and implications of these results.

Lemma A.1 (Generalization of Lemma 1) *Suppose p and q both satisfy (A1)-(A4), and p exhibits greater exponential-growth bias than q. Then:*

$$\frac{p(\vec{\imath},t)}{p(\vec{\imath},s)} \leq \frac{q(\vec{\imath},t)}{q(\vec{\imath},s)} \text{ for all } s > t.$$

The inequality holds strictly if there exist $j > k > t$ s.t. $i_j > 0$, $i_k > 0$.

For notational convenience, we define $\widehat{V}_{t,\tau}(\vec{y},\vec{\imath}) \equiv \sum_{s=t}^{\tau} y_s \cdot p(\vec{i},s)$ as the perceived future value of an income stream and vector of interest rates.

Lemma A.2 (Generalization of Lemma 2) *Given income streams $\vec{y}$ and $\vec{z}$ and interest rates $\vec{\imath}$ such that:*

$$(i) \qquad \sum_{s=0}^{T} \left(\Pi_{j=s}^{T}(1+i_j)\right) y_s = \sum_{s=0}^{T} \left(\Pi_{j=s}^{T}(1+i_j)\right) z_s > 0$$

$$(ii) \qquad \sum_{s=0}^{t} \left(\Pi_{j=s}^{t}(1+i_j)\right) y_s > \sum_{s=0}^{t} \left(\Pi_{j=s}^{t}(1+i_j)\right) z_s \quad \forall t \in \{0,\ldots,T-2\},$$

then $\widehat{V}_{0,T}(\vec{y},\vec{\imath}) < \widehat{V}_{0,T}(\vec{z},\vec{\imath})$ for any interest perception function p that exhibits EGB.

Appendix B: Proofs

The proofs below use the notation of the general form of the model from Appendix A rather than the special case α-Eddie model presented in the main text. Because the α-Eddie model satisfies assumptions (A1)–(A4), these proofs imply the results in the main text directly.

Proof of Lemma A.1 (Generalization of Lemma 1) First, define $\vec{\imath}^{(s)} = < x, 0, ..., 0, i_{s+1}, ..., i_{T-1} >$. By (A3), we can write

$$p(\vec{i},t) = p(\vec{i},t+1) + \int_0^{i_t} \frac{\partial p(\vec{\imath}^{(t)},0)}{\partial x} dx$$

by using the fundamental theorem of calculus. Continuing further:

$$p(\vec{i},t) = p(\vec{i},T) + \sum_{j=0}^{T-t} \int_0^{i_{T-j}} \frac{\partial p(\vec{\imath}^{(j)},0)}{\partial x} dx$$

Thus

$$\frac{p(\vec{i},t)}{q(\vec{i},t)} = \frac{1 + \sum_{j=0}^{T-t} \int_0^{i_{T-j}} \frac{\partial p(\vec{\imath}^{(j)},0)}{\partial x} dx}{1 + \sum_{j=0}^{T-t} \int_0^{i_{T-j}} \frac{\partial q(\vec{\imath}^{(j)},0)}{\partial x} dx} \tag{B.1}$$

Because $\vec{\imath}^{(t)}$ and $\vec{\imath}^{(t+1)}$ differ only on a single element, namely the (t+2)-th element i_t, we have by the definition of p is more EGB than q that

$$\frac{\partial p(\vec{\imath}^{(t)},0)/\partial x}{\partial q(\vec{\imath}^{(t)},0)/\partial x} \leq \frac{\partial p(\vec{\imath}^{(t+1)},0)/\partial x}{\partial q(\vec{\imath}^{(t+1)},0)/\partial x}.$$

The ratio between the term in the numerator and the denominator in the two summations in (B.1) is decreasing in j. Reducing the summations an additional step will therefore lower the overall ratio, and thus we have:

$$\frac{p(\vec{i},t)}{q(\vec{i},t)} = \frac{1 + \sum_{j=0}^{T-t} \int_0^{i_{T-j}} \frac{\partial p(\vec{\imath}^{(j)},0)}{\partial x} dx}{1 + \sum_{j=0}^{T-t} \int_0^{i_{T-j}} \frac{\partial q(\vec{\imath}^{(j)},0)}{\partial x} dx} \leq \frac{1 + \sum_{j=0}^{T-t-1} \int_0^{i_{T-j}} \frac{\partial p(\vec{\imath}^{(j)},0)}{\partial x} dx}{1 + \sum_{j=0}^{T-t-1} \int_0^{i_{T-j}} \frac{\partial q(\vec{\imath}^{(j)},0)}{\partial x} dx} = \frac{p(\vec{i},t+1)}{q(\vec{i},t+1)} \tag{B.2}$$

Rearranging (B.2) yields $p(\vec{i},t)/p(\vec{i},t+1) \leq q(\vec{i},t)/q(\vec{i},t+1)$, which can be iterated as many times as needed to yield $p(\vec{i},t)/p(\vec{i},t+n) \leq q(\vec{i},t)/q(\vec{i},t+n)$ for any positive integer n.

Proof of Lemma A.2 (Generalization of Lemma 2) In this and all subsequent proofs, let the period-t perception of final wealth net of obligations incurred in periods $\{0,\ldots,t-1\}$ be given by $\widehat{W}_{t,T}(\vec{y}) = \left[\sum_{s=0}^{t-1}(y_s - c_s)\prod_{j=s}^{t-1}(1+i_j)\right] p(i,t) + \sum_{s=t}^{T} p(i,s)y_s$, and note that $\widehat{W}_{0,T}(\vec{y}) = \hat{V}_{0,T}(\vec{y},\vec{\imath})$. By (ii), $y_0 > z_0$. Define a wealth-preserving shift of income from period zero to period one $\vec{y}^{(1)} = \vec{y} + < (z_0 - y_0), (y_0 - z_0)(1+i_0), 0, \ldots, 0 >$. Note that $(1+i_s)p(i,s+1) > p(i,s)$ by Lemma A.1, since p exhibits EGB.

Then $\widehat{W}_{0,T}(\vec{y}^{(1)}) - \widehat{W}_{0,T}(\vec{y}) = ((1+i_0)p(\vec{i},1) - p(\vec{i},0))(y_0 - z_0) > 0$.

Similarly, for $s = 2,\ldots,T$, recursively define $\vec{y}^{(s)} = \vec{y}^{(s-1)} + < 0, \ldots, (b_s - a_s^{(s-1)}), (a_s^{(s-1)} - b_s)(1+i_{s-1}), \ldots, 0 >$, that is, by shifting $(y_s^{(s-1)} - z_s)$ from period $s-1$ to period s, at the interest rate i_{s-1}. By (ii), $(y_s^{(s-1)} - z_s) > 0$, and so $\widehat{W}_{0,T}(\vec{y}^{(s-1)}) - \widehat{W}_{0,T}(\vec{y}^{(s)}) = \left[p(\vec{i},s)(1+i_{s-1}) - p(\vec{i},s-1)\right](y_{s-1} - z_{s-1})$ for all $s < T$, and equal to zero for $s = T$. From (i), however, we have that $\vec{y}^{(T)} = \vec{z}$. Thus $\widehat{W}_{0,T}(\vec{y}) < \widehat{W}_{0,T}(\vec{y}^{(1)}) < \ldots < \widehat{W}_{0,T}(\vec{y}^{(T-1)}) = \widehat{W}_{0,T}(\vec{y}^{(T)}) = \widehat{W}_{0,T}(\vec{z})$.

Proof of Proposition 1 We can rewrite equation (7), using the general perception function and separating period-0 consumption, as $c_0 p(\vec{i},0) + g(c_0) = \widehat{W}_{0,T}(\vec{y})$, where

$$g(c_0) = \left[\sum_{s=1}^{T} p(\vec{i},s) u'^{-1}\left(\frac{p(\vec{i},s)u'(c_0)}{p(\vec{i},0)\delta^s}\right)\right]$$

$\partial g(c_0)/\partial c_0 > 0$ because utility is concave and increasing. Now suppose we reduce period-s income by ε and increase period (s+1) income by $(1+i_s)\varepsilon$. By Lemma 2, $\Delta\widehat{W}_{0,T}(\vec{y}) > 0$ if $s < T-1$, and $\Delta\widehat{W}_{0,T}(\vec{y}) = 0$ if $s = T-1$. Thus c_0 strictly increases for $s < T-1$ and is unchanged for $s = T-1$.

Proof of Proposition 2 Consider a perception function $p(\vec{i},0)$ and a less biased perception function $q(\vec{i},0)$. We apply equation (7) to both perception functions. The RHS is equal for both because all wealth is received lump sum at T and so there is no misperception about the wealth available in period T units.

$$c_0^p p(\vec{i},0) + \left[\sum_{s=1}^{T} p(\vec{i},s) u'^{-1}\left(\frac{p(\vec{i},s)u'(c_0^p)}{p(\vec{i},0)\delta^s}\right)\right] = c_0^q q(\vec{i},0) + \left[\sum_{s=1}^{T} q(\vec{i},s) u'^{-1}\left(\frac{r(\vec{i},s)u'(c_0^q)}{q(\vec{i},0)\delta^s}\right)\right]$$

where c_0^j represents consumption with the perception function j. By Lemma A.1 $p(\vec{i},s)/(p(\vec{i},0)\delta^s) > q(\vec{i},s)/(q(\vec{i},0)\delta^s)$. By (A1), (A2), and the definition of EGB $p(\vec{i},t) \leq q(\vec{i},t)$. If $c_0^p \leq c_0^q$ then the inverse-utility term on the LHS is less than the inverse utility term on the RHS since $u'(\cdot)$ and $u'^{-1}(\cdot)$ are decreasing functions. But this is a contradiction since every term on the LHS is now less than every term on the RHS. Therefore $c_0^p > c_0^q$.

Proof of Proposition 3 By Proposition 1 it is sufficient to show that an agent will over-consume when all their income is included in their period-0 endowment; any deferment will exacerbate the over-consumption so long as all cash flows are weakly positive. Let $e(\vec{\imath},t)$ denote exponential perception. We prove by contradiction.

Suppose $c_0 \leq c_0^*$ and $-u'(c)/(u''(c)c) > 1$. Then from the agent's Euler condition,

$$u'(\hat{c}_s)\frac{p(\vec{i},0)}{p(\vec{i},s)}\delta^s = u'(c_0) \geq u'(c_0^*) = u'(c_s^*)\frac{e(\vec{\imath},0)}{e(\vec{\imath},s)}\delta^s \tag{B.3}$$

$$\text{By Lemma A.1,}\quad \frac{p(\vec{i},0)}{p(\vec{i},s)} < \frac{e(\vec{\imath},0)}{e(\vec{\imath},s)} \Rightarrow \hat{c}_s < c_s^*$$

Now we consider two cases. Case 1: $\hat{c}_s[p(\vec{i},s)/p(\vec{i},0)] \geq c_s^*[e(\vec{\imath},s)/e(\vec{\imath},0)]$. Multiply the LHS of this inequality with the LHS of (B.3) and the RHS of the inequality with the RHS of (B.3). This yields $\hat{c}_s u'(\hat{c}_s) \geq c_s^* u'(c_s^*)$. Observe $c \cdot u'(c)$ is an increasing function iff $d/dc[c \cdot u'(c)] > 0 \Leftrightarrow -u'(c)/(u''(c)c) > 1$ which is the assumed EIS condition. Therefore $\hat{c}_s \geq c_s^*$

This is a contradiction, as $\hat{c}_s < c_s^*$.

Case 2: $\hat{c}_s[p(\vec{i},s)/p(\vec{i},0)] < c_s^*[p(\vec{i},s)/p(\vec{i},0)]$.
The budget constraint then implies:

$$c_0 + \sum_{s=1}^{T} \frac{p(\vec{i},s)}{p(\vec{i},0)} \hat{c}_s < c_0^* + \sum_{s=1}^{T} \frac{e(\vec{i},s)}{e(\vec{i},0)} c_s^* = y_0$$

Which is a violation of Walras' law (not expending the full budget), and therefore c_0 cannot be optimal. Thus $c_0 > c_0^*$.

Proof of Proposition 4 Let $\hat{c}_{t,\tau}$ denote the agent's period-t expectation of consumption in period $\tau > t$, and let the period-t perception of final wealth net of obligations incurred in periods $\{0,\ldots,t-1\}$ be given by $\widehat{W}_{t,T}(\vec{y})$ as in the proof of Lemma 2.

We note that consumption in every period is a normal good, and from the perceived budget constraint $\widehat{c}_{t,t}p(\vec{i},t) + \sum_{s=t+1}^{T} p(\vec{i},s)\widehat{c}_{t,s} = \widehat{W}_{t,T}$, it is clear that if $\widehat{W}_{t,T}$ increases, $\widehat{c}_{t,s}$ must also increase for all $s \in \{t,\ldots T\}$.

At time t, the budget constraint yields $\sum_{s=t+1}^{T} \widehat{c}_{t,s}p(\vec{i},s) = \widehat{W}_{t,T} - c_t p(\vec{i},t)$

At time $t+1$,

$$\widehat{W}_{t+1,T} = \left[\sum_{s=0}^{t-1}(y_s - c_s)\prod_{j=s}^{t-1}(1+i_j)\right] p(\vec{i},t+1)(1+i_t) + \left(\sum_{s=t+1}^{T} p(\vec{i},s)y_s\right) + (y_t - c_t)p(\vec{i},t+1)(1+i_t)$$

and the budget constraint yields $\sum_{s=t+1}^{T} \widehat{c}_{t+1,s}p(\vec{i},s) = \widehat{W}_{t+1,T}$. Thus the perceived budget will decrease only if $\widehat{W}_{t,T} - c_t p(\vec{i},t) > \widehat{W}_{t+1,T}$, i.e.:

$$\left[\sum_{s=0}^{t-1}(y_s - c_s)\prod_{j=s}^{t-1}(1+i_j) + (y_t - c_t)\right]\left[p(\vec{i},t) - (1+i_t)p(\vec{i},t+1)\right] > 0$$

$$\sum_{s=0}^{t}(y_s - c_s)\Pi_{j=s}^{t-1}(1+i_j) < 0$$

since $p(\vec{i},t) < (1+i_t)p(\vec{i},t+1)$ by Lemma 1 since p exhibits EGB. Thus $\sum_{s=t+1}^{T} \widehat{c}_{t+1,s}p(\vec{i},s) < \sum_{s=t+1}^{T} \widehat{c}_{t,s}p(\vec{i},s)$, and from the Euler equation each term in the sequence $c_{t+1,s} < c_{t,s}$ giving the desired result.

Proof of Proposition 5 We generalize the repayment plan to $A(L,\vec{\imath};p) : \mathbb{R} \times \mathbb{R}^T \times \mathscr{P} \to \mathbb{R}$, where $\mathscr{P}$ is the set of interest perception functions. Suppose p exhibits greater EGB than q. By definition of $A(\cdot)$,

$$A(L,\vec{\imath};p)\sum_{k=1}^{T} \frac{p(\vec{\imath},k)}{p(\vec{\imath},0)} = L = A(L,\vec{\imath};q)\sum_{k=1}^{T} \frac{q(\vec{\imath},k)}{q(\vec{\imath},0)}$$

By Lemma A.1, $p(\vec{\imath},k)/p(\vec{\imath},0) > q(\vec{\imath},k)/q(\vec{\imath},0)$ for all $k \leq T-2$ (and equal for $k = T-1$). Thus the summation in the leftmost term above is strictly greater than the summation in the rightmost term. For the equality to hold, therefore, we require

$A(L,\vec{\imath};p) < A(L,\vec{\imath};q)$. Restricting p and q to the form given in (1) immediately yields the desired result in terms of α.

Similarly, letting $e(\cdot)$ denote correct exponential perception,

$$L = A(L,\vec{\imath};e)\sum_{k=1}^{T}\frac{e(\vec{\imath},k)}{e(\vec{\imath},0)} > A(L,\vec{\imath};p)\sum_{k=1}^{T}\frac{e(\vec{\imath},k)}{e(\vec{\imath},0)}.$$

An agent expecting under p to pay off a loan of L in T periods will thus have a strictly negative balance in period T.

Appendix C: Additional Figures and Tables

Table C.C.1 displays the results of a linear regression of α on several covariates. The model predicts that α should be positively associated with bankruptcy, payday loan use, balance on credit cards, no financial advice, and negatively associated with the use of tools and advice. The theory is silent on the other covariates.

We find that the use of a financial calculator is substantially and significantly associated with greater accuracy. We find no evidence for other associations. This certainly in part reflects the limited sample size. Our sample only has 36 subjects who use payday loans and only 38 people who have gone bankrupt. Future research should explore these relationships where high-cost debt users are oversampled. We find no association with education and α. Other model specifications that use categories for education (< HS, HS, some college, college) decade-length age categories, and a dummy to separate larger households from smaller household generate similar non-signficant results. This is consistent with the findings of Cronqvist and Siegel (2014) who find that general education explains a minuscule fraction of the variation in financial biases. It is also consistent with a casual observation of the first table Almenberg and Gerdes (2012), in which there is no clear pattern between α as a function of education or income. In contrast Stango and Zinman (2009) find that bias is negatively correlated with income, education, and being male. Their sample size is about six times larger so this could explain the discrepancy between our results.

TABLE C.C.1. Correlates of Alpha

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ln(Income)	-0.030	-0.032	-0.027					-0.035
	(0.029)	(0.030)	(0.030)					(0.030)
Education	-0.003	-0.002	-0.005	-0.006	-0.006	-0.015	-0.011	-0.011
	(0.014)	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.016)	(0.017)
Age	0.000	0.001	0.001					0.001
	(0.001)	(0.001)	(0.001)					(0.002)
Female	-0.013	0.003	0.004					-0.001
	(0.047)	(0.047)	(0.047)					(0.046)
Household Size	0.001	0.010	0.010					0.014
	(0.020)	(0.020)	(0.020)					(0.020)
Primary financial decision maker			0.075					0.087
			(0.070)					(0.071)
Unemployed / Laid off			0.065					0.068
			(0.091)					(0.090)
Knows which math needed				-0.044	-0.039			-0.073
				(0.059)	(0.060)			(0.063)
Got Q3 Correct				-0.016	-0.010			-0.031
				(0.059)	(0.059)			(0.061)
Sometimes carries balance on credit cards					0.015			0.024
					(0.058)			(0.059)
Declared bankruptcy					0.018			0.063
					(0.095)			(0.094)
Uses payday loans					0.137			0.146
					(0.123)			(0.113)
No financial advice					-0.031			-0.024
					(0.050)			(0.050)
Used pen and paper						-0.026		-0.060
						(0.108)		(0.101)
Used a simple calculator						-0.062		-0.036
						(0.049)		(0.049)
Used a graphing calculator						0.111		0.203*
						(0.108)		(0.119)
Used a spreadsheet						0.120		0.106
						(0.095)		(0.091)
Used an online financial calculator						0.352***		0.387***
						(0.104)		(0.113)
Political conservativism							-0.006	-0.003
							(0.016)	(0.016)
Catholic							-0.066	-0.051
							(0.062)	(0.060)
Jewish							0.356**	0.310*
							(0.154)	(0.159)
No religion							-0.069	-0.071
							(0.074)	(0.076)
Other religion							0.078	0.106
							(0.122)	(0.119)
Attends church more than once/year							-0.038	-0.043
							(0.058)	(0.058)
Black							-0.131*	-0.147*
							(0.076)	(0.084)
Hispanic							0.023	0.006
							(0.082)	(0.078)
Other Race							-0.017	-0.003
							(0.107)	(0.103)
Constant	0.902***	0.839***	0.756**	0.621***	0.610***	0.684***	0.714***	0.964***
	(0.308)	(0.324)	(0.308)	(0.160)	(0.163)	(0.157)	(0.200)	(0.345)
N_Clust	560	560	560	560	559	560	555	554

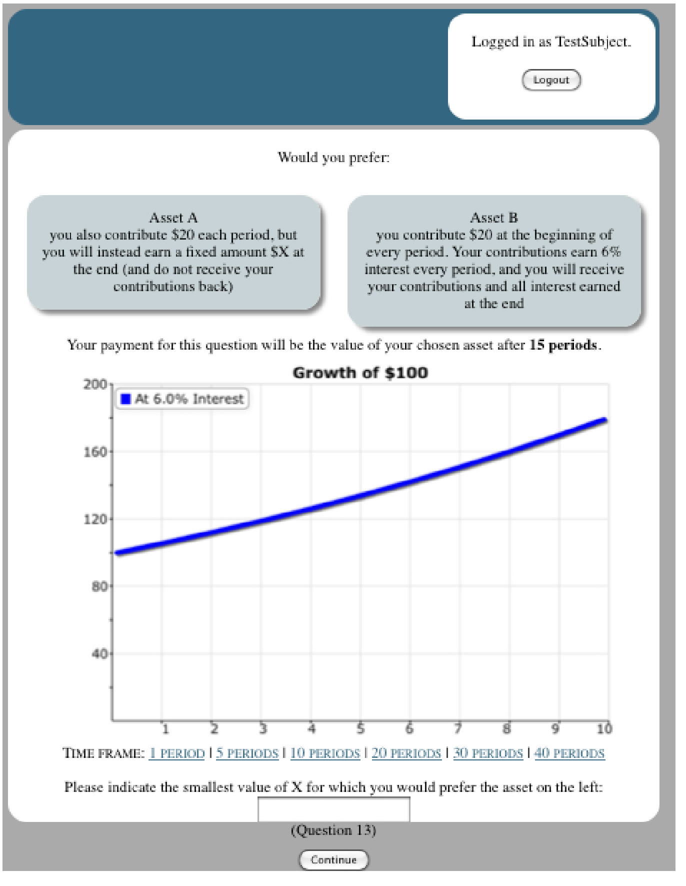
Notes: Ordinary least squares, the dependent variable is α. The omitted religious group is protestant. Specifications (2)–(8) include State fixed effects.* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

TABLE C.C.2. Experiment 2 Questions

Domain	Question	Asset A: T	Asset B: T	Asset A: P_0	Asset B: P_0	Asset A: i	Asset B: i	Correct X	Full-bias X	Q25	Q50	Q75
		Question Parameters						Answers		Responses by Quartile		
Exponential	1	20	20	100	X	10%	0%	672.75	300	20	300	800
	2	50	50	100	X	5%	0%	1146.74	350	250	350	1146.74
	3	1	1	100	X	4%	0%	104	104	104	104	104
	4	10	10	500	X	-8%	0%	217.19	100	100	217.13	420
	11	2	2	100	X	50%	0%	225	200	200	225	225
	12	35	35	100	X	7%	0%	1067.66	345	275	350	1067.66
	13	20	20	1000	X	-4%	0%	442.00	200	200	442	700
	14	10	10	1000	X	-10%	0%	348.68	0	10	348.68	800
Fluctuating i	5	20	20	100	X	0% in odd; 21% in even	0%	672.75	310	210	310	672.75
	6	24	24	100	X	4% in odd; 22% in even	0%	1740.68	412	312	412	1500
	7	24	24	500	X	-20% in odd; 25% in even	0%	500	800	500	625	2600
	15	16	16	100	X	0% in odd; 40% in even	0%	1475.79	420	420	500	1475.79
	16	20	20	500	X	-30% in odd; 30% in even	0%	194.71	500	195	500	2000
	17	36	36	100	X	-28% in odd; 59% in even	0%	1140.60	658	334	700	1500
Catch-up Savings	8	20	15	100	X	13%	13%	184.24	122.03	115	133.5	184.24
	9	10	24	500	X	8%	8%	170.23	308.22	170	208	310
	10	10	9	500	X	25%	25%	625	538.46	500	550	625
	18	40	20	100	X	7%	7%	386.97	158.33	150	200	387
	19	20	10	100	X	13%	13%	339.46	156.52	150	200	339.46
	20	14	40	100	X	5%	5%	28.12	56.67	28.12	50	104
Periodic Savings	21	40	40	+10/period	X	7%	0%	2136.10	974	149.74	428	1500
	22	20	20	+10/period	X	10%	0%	630.02	410	100	230	624.19
	23	30	30	+10/period	X	12%	0%	2702.93	692	150	350	2000
Portfolio	24	40	40	100	(50,X)	10%	(5%,15%)	15.58	50	19.105	50	200
	25	36	36	100	(50,X)	4%	(6%,1%)	2.12	100	10	63	200
	26	30	30	100	(50,X)	12%	(8%,4%)	768.60	200	75	200	765

Journal of the European Economic Association
Preprint prepared on 15 April 2015 using jeea.cls *v1.0.*

FIGURE C.C.1. Example of Task for Experiment with Graphical Intervention



Notes: Subjects estimate the value of X to make both assets equal. For the control group and the first 10 questions asked of the treated group, there was no graph, but the presentation was otherwise identical. The latter 16 questions asked of the treated group were accompanied by the graph. The graph was interactive allowing users to observe the growth over different time horizons.