Judgment aggregation: a survey*

Christian List
Department of Government
London School of Economics

Clemens Puppe
Department of Economics
University of Karlsruhe

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1 Introduction

Judgment aggregation is the subject of a growing body of work in economics, political science, philosophy and related disciplines. Although the literature on judgment aggregation has been influenced by earlier work in social choice theory, the recent interest in the problem was sparked by the so-called 'doctrinal paradox' in law and economics (Kornhauser and Sager 1986). Suppose a three-member court has to reach a verdict in a breach-of-contract case. According to legal doctrine, the defendant is liable (the *conclusion*, here denoted c) if and only if he or she did a particular action *and* had a contractual obligation not to do it (the two *premises*, here denoted a and b). The doctrinal paradox consists in the fact that majority voting on the premises may support a different verdict from majority voting on the conclusion. As illustrated in Table 1, suppose the first judge holds both premises to be true; the second holds the first premise, but not the second, to be true; and the third holds the second premise, but not the first, to be true. Then a majority of judges holds each premise to be true, which seems to support a 'liable' verdict, and yet a majority of judges holds the conclusion to be false. Although the first discussions of this problem focused on the distinction between 'premise-based' and 'conclusion-based' methods of decision-making, the doctrinal paradox illustrates a more general point, which Pettit (2001) has called the 'discursive dilemma': Majority voting on multiple, interconnected propositions may lead to an inconsistent set of collective judgments. In the court example, majorities accept a, b, [c if and only if (a and b)], and the negation of c, an inconsistent set of propositions in the standard sense of logic (see also Brennan 2001).

Naturally, the observation that majority voting may fail to produce consistent collective judgments raises several questions. In particular, how general is the problem? Is

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	action done a	obligation held b	c if and only if (a and b)	liable c
Judge 1	True	True	True	True
Judge 2	True	False	True	False
Judge 3	False	True	True	False
Majority	True	True	True	False

Table 1: Doctrinal paradox / discursive dilemma

it restricted to majority voting, or does it extend to other decision methods? And does it occur only in special situations, such as the breach-of-contract case, or does it arise more generally?

In response to these questions, List and Pettit (2002, 2004) proposed a model of judgment aggregation, combining a logical representation of propositions with an axiomatic approach inspired by Arrowian social choice theory. Using this model, they proved a simple impossibility theorem showing that if judgments are to be made on at least two atomic propositions and at least one suitable composite proposition (and their negations), there exists no judgment aggregation rule satisfying four conditions: universal domain (all combinations of rational individual judgments are admissible as inputs), collective rationality (only rational collective judgments are admissible as outputs), anonymity (the aggregation is invariant under permutations of the individuals), and systematicity (the collective judgment on each proposition is the same function of individual judgments on that proposition).

This result, however, gives only a partial answer to the questions raised above. Its conditions on both the aggregation rules and the decision problems under consideration can be significantly generalized or otherwise refined (e.g., Pauly and van Hees 2006; Dietrich 2006a, 2007a; Nehring and Puppe 2005a). Moreover, instead of producing mere impossibility results, the literature has now provided several general characterizations of both aggregation rules and decision problems with salient properties (e.g., Dokow and Holzman 2005; Nehring and Puppe 2005b, 2006; Dietrich and List 2007a). Some of these draw on other branches of aggregation theory that are closely related cousins of the logic-based framework, including the aggregation of binary evaluations (Wilson 1975, Rubinstein and Fishburn 1986) and the theory of strategy-proof social choice on generalized single-peaked domains (Nehring and Puppe 2002). The interest in the problem of judgment aggregation is enhanced by the observation that the classical preference aggregation problem of social choice theory is a special case, by representing preference relations as sets of binary ranking judgments (List and Pettit 2004, Nehring 2003, Dietrich and List 2007a), as explained in Section 3.2.1. An earlier precursor is Guilbaud's (1966) logical reformulation of Arrow's theorem. More generally, by representing decision problems not in standard propositional logic but other more expressive logics, many realistic decision problems can be expressed as judgment aggregation problems (Dietrich 2007a). Judgment aggregation is also related to the theory of belief merging in computer science (Konieczny and Pino-Perez 2002).

Our aim in this survey article is to provide an accessible overview of some key results and questions in the theory of judgment aggregation. We omit proofs and technical details, focusing instead on concepts and underlying ideas. But our perspective in this survey is a social-choice-theoretic one; we do not attempt to review the related legal and philosophical literatures. After introducing and discussing the formal framework in Section 2, we devote the bulk of our discussion to propositionwise aggregation rules, i.e., ones satisfying an independence condition (Section 3). For the purpose of this survey, this focus is justified by the fact that most of the technical results in the literature pertain to aggregation rules satisfying independence. We do not unreservedly endorse the independence condition, however, and discuss its relaxation in Section 4. In Section 5, we address other themes and developments in the literature.

2 Modelling judgment aggregation

2.1 The logic-based framework

We use Dietrich's (2007a) model of judgment aggregation in general logics, which extends List and Pettit's (2002) original model in standard propositional logic. We consider a set of individuals $N = \{1, 2, ..., n\}$ (with $n \geq 3$). They are faced with a decision problem that requires making collective judgments on logically interconnected propositions. Propositions are represented in formal logic. The language of the logic, $\mathcal{L}$, can be any set of sentences (called *propositions*) closed under negation (i.e., $p \in \mathcal{L}$ implies $\neg p \in \mathcal{L}$). The best-known example is standard propositional logic; here $\mathcal{L}$ is the smallest set containing (i) given atomic propositions $a, b, c, ...$ and (ii) for any $p, q \in \mathcal{L}$, the composite propositions $\neg p$, $(p \wedge q)$, $(p \vee q)$, $(p \rightarrow q)$, $(p \leftrightarrow q)$ with logical connectives $\neg$ ('not'), $\wedge$ ('and'), $\vee$ ('or'), $\rightarrow$ ('if-then') and $\leftrightarrow$ ('if and only if'). Other logics have languages involving other logical connectives, which often feature in realistic judgment aggregation problems (see Section 5.2). The logic is endowed with a notion of *consistency*, which satisfies some regularity conditions.[1] In standard propositional logic, for instance, a set of propositions $S \subseteq \mathcal{L}$ is *consistent* if there exists a truth-value assignment (with standard properties) making all propositions in S true, and *inconsistent* otherwise. For example, the set $\{a, a \vee b\}$ is consistent, while $\{\neg a, a \wedge b\}$ is inconsistent.

A decision problem is given by an *agenda* $X \subseteq \mathcal{L}$, interpreted as the set of propositions on which judgments are to be made. We assume that X is finite and closed under

[1] The three conditions are: (C1) all sets $\{p, \neg p\} \subseteq \mathcal{L}$ are inconsistent; (C2) all subsets of consistent sets $S \subseteq \mathcal{L}$ are consistent; (C3) $\emptyset$ is consistent and each consistent set $S \subseteq \mathcal{L}$ has a consistent superset $T \subseteq \mathcal{L}$ containing a member of each pair $p, \neg p \in \mathcal{L}$.

negation (i.e., if $p \in X$, then $\neg p \in X$) and identify each doubly-negated proposition $\neg\neg p$ with the non-negated proposition p. We also exclude tautologies and contradictions from the agenda.[2]

An (individual or collective) *judgment set* is a subset $J \subseteq X$, interpreted as the set of accepted propositions in the agenda. For our purposes, to accept p means to believe p, but different interpretations of 'acceptance' can be given (for instance, in terms of desire). The notion of belief is very general here, applicable both to positive propositions (e.g., 'current CO_2 emissions lead to global warming') and to normative ones (e.g., 'we should reduce CO_2 emissions'). Judgment aggregation problems cannot be resolved simply by statistical information-pooling techniques, since individuals may agree to disagree on the propositions, particularly if these are normative. We call J *consistent* if it is a consistent set in $\mathcal{L}$, and *complete* if $p \in J$ or $\neg p \in J$ for any proposition $p \in X$. A *profile* is an n-tuple $(J_1, J_2, ..., J_n)$ of individual judgment sets.

A *(judgment) aggregation rule* F is a mapping which assigns to each profile $(J_1, J_2, ..., J_n)$ of individual judgment sets (in some domain) a collective judgment set $J = F(J_1, J_2, ..., J_n)$. An aggregation rule F has *universal domain* if its domain is the set of all profiles of consistent and complete judgment sets; it is *collectively rational* if it generates a consistent and complete collective judgment set $F(J_1, J_2, ..., J_n)$ for every profile $(J_1, J_2, ..., J_n)$ in its domain. Until Section 5, we focus on aggregation rules with these two properties.

2.2 An example

Let us consider an illustrative decision problem. A three-member cabinet, $N = \{1, 2, 3\}$, has to make judgments on the following propositions:

a:	Current CO_2 emissions lead to global warming.
$a \rightarrow b$:	If current CO_2 emissions lead to global warming, then we should reduce CO_2 emissions.
b:	We should reduce CO_2 emissions.

The agenda is the set $X = \{a, \neg a, a \rightarrow b, \neg(a \rightarrow b), b, \neg b\}$. The cabinet members' judgments as shown in Table 2 are given by the following individual judgment sets: $J_1 = \{a, a \rightarrow b, b\}$, $J_2 = \{a, \neg(a \rightarrow b), \neg b\}$, $J_3 = \{\neg a, a \rightarrow b, \neg b\}$.

If we use (propositionwise) majority voting as the aggregation rule, we obtain the same problem as identified above: an inconsistent collective set of judgments $J = \{a, a \rightarrow b, \neg b\}$. Thus, under universal domain, majority voting on the present agenda X is not collectively rational. By contrast, a dictatorship of minister 1 – say, the prime minister – obviously guarantees a consistent collective judgment set. As we observe below, there

[2] A proposition $p \in \mathcal{L}$ is a *tautology* if $\{\neg p\}$ is inconsistent, and a *contradiction* if $\{p\}$ is inconsistent.

	global warming a	$a \to b$	reduce emissions b
Minister 1	True	True	True
Minister 2	True	False	False
Minister 3	False	True	False
Majority	True	True	False

Table 2: CO_2 emissions

are non-dictatorial and collectively rational aggregation rules with universal domain for this agenda.

2.3 The abstract aggregation framework

It is useful to relate the present logic-based framework of judgment aggregation to the framework employed in abstract aggregation theory, following Wilson (1975).[3] In abstract aggregation, individual vectors of yes/no evaluations over multiple binary issues are aggregated into a collective such vector, subject to feasibility constraints.[4] An *(abstract) aggregation rule* is a mapping $f : Z^n \to Z$, where $Z \subseteq \{0,1\}^k$ represents the set of feasible yes/no evaluation vectors over $k(\geq 1)$ binary issues. A judgment aggregation problem with agenda X can be represented in this framework by defining Z as the set of admissible truth-value assignments over the (unnegated) propositions in X, identifying binary issues with proposition-negation pairs (thus $k = \frac{|X|}{2}$).[5] To illustrate, consider the agenda of the global warming example above, $X = \{a, \neg a, a \to b, \neg(a \to b), b, \neg b\}$. The set of admissible truth-value assignments over the unnegated propositions a, $a \to b$ and b is $Z = \{(1,1,1), (1,0,0), (0,1,1), (0,1,0)\}$.[6]

There is a loss of information by moving from the logic-based framework to the abstract one. It consists in the fact that, while every agenda X uniquely defines a subset $Z \subseteq \{0,1\}^k$, the same subset Z may arise from different agendas. For example, consider the agendas $X_1 = \{a, \neg a, a \vee b, \neg(a \vee b)\}$ and $X_2 = \{a \wedge b, \neg(a \wedge b), a \to b, \neg(a \to b)\}$. As is easily verified, the set of admissible truth-value assignments corresponding to both X_1 and X_2 is $Z = \{(1,1),(0,1),(0,0)\}$. But obviously X_1 and X_2 are different in terms of

[3]The property space framework of Nehring and Puppe (2002) is informationally equivalent to the abstract aggregation framework. Any property space can be uniquely embedded into $\{0,1\}^k$ for some k up to isomorphism; conversely, any subset of $\{0,1\}^k$ (with at least two elements) uniquely defines a property space.

[4]This is generalized to non-binary issues in Rubinstein and Fishburn (1986).

[5]Under this representation, the conditions of universal domain and collective rationality, explicitly imposed on the judgment aggregation rule F, are implicitly built into the definition of the abstract aggregation rule f.

[6]Here the conditional $\to$ is interpreted as the 'material' conditional of standard propositional logic. In Section 3.2.3, we contrast this with a 'subjunctive' interpretation.

both interpretation and syntax. In particular, X_2 contains only composite propositions whereas X_1 also contains an atomic one, a fact one may wish to use in handling the two aggregation problems (see Section 5 for examples). In what follows we use the logic-based framework but make cross-references to abstract aggregation at various points.

2.4 Conditions on aggregation rules

We now turn to conditions one may wish to impose on an aggregation rule F. We begin with the uncontroversial requirement that, if all individuals unanimously submit the same judgment set, this judgment set should be the collective one.

Unanimity. For any unanimous profile $(J, ..., J)$ in the domain, $F(J, ..., J) = J$.[7]

The next condition requires that the collective judgment on each proposition p should depend only on individual judgments on p, not on individual judgments on other propositions.

Independence. For any $p \in X$ and any profiles $(J_1, ..., J_n), (J'_1, ..., J'_n)$ in the domain, if [for all $i \in N$, $p \in J_i \Leftrightarrow p \in J'_i$], then [$p \in F(J_1, ..., J_n) \Leftrightarrow p \in F(J'_1, ..., J'_n)$].

While not uncontroversial, independence has some *prima facie* appeal in that it guarantees a propositionwise approach to aggregation. A stronger condition results from combining independence with a neutrality condition, requiring in addition equal treatment of all propositions.

Systematicity. For any $p, q \in X$ and any profiles $(J_1, ..., J_n), (J'_1, ..., J'_n)$ in the domain, if [for all $i \in N$, $p \in J_i \Leftrightarrow q \in J'_i$], then [$p \in F(J_1, ..., J_n) \Leftrightarrow q \in F(J'_1, ..., J'_n)$].

The following monotonicity condition states that if one individual switches from rejecting to accepting a collectively accepted proposition (keeping fixed all other individuals' judgments), this proposition should remain collectively accepted. In the presence of independence, monotonicity seems a natural requirement. To state it formally, we call two profiles *i-variants* if they coincide for all individuals except possibly i.

Monotonicity. For any $p \in X$, $i \in N$ and i-variants $(J_1, ..., J_i, ..., J_n)$, $(J_1, ..., J'_i, ..., J_n)$ in the domain, if [$p \notin J_i$, $p \in J'_i$ and $p \in F(J_1, ..., J_i, ..., J_n)$] then $p \in F(J_1, ..., J'_i, ..., J_n)$.

The final two conditions are basic democratic requirements. The first requires that no single individual should always determine the collective judgment set; the second requires that all individuals should have equal weight in the aggregation.

[7] If F is also required to satisfy monotonicity as defined below, unanimity follows from the even weaker condition of *sovereignty*, whereby F has all complete and consistent judgment sets in its range.

Non-dictatorship. There exists no $i \in N$ such that, for any profile $(J_1, ..., J_n)$ in the domain, $F(J_1, ..., J_n) = J_i$.

Anonymity. For any profiles $(J_1, ..., J_n), (J'_1, ..., J'_n)$ in the domain that are permutations of each other, $F(J_1, ..., J_n) = F(J'_1, ..., J'_n)$.

3 Propositionwise aggregation

We are now ready to express the fundamental questions raised by the discursive dilemma more formally. First, is the failure to achieve collective rationality restricted to majority voting, or does it extend to other aggregation rules? And second, how large is the class of agendas for which the problem arises? Notice the difference in focus between these two questions. The first concerns the class of *aggregation rules* that guarantee collective rationality for a given agenda, whereas the second concerns the class of *agendas* for which collectively rational aggregation rules with specific additional properties (such as the ones just introduced) exist.

The original theorem by List and Pettit (2002) answers these questions for a special class of aggregation rules (those satisfying universal domain, collective rationality, systematicity and anonymity) and a special class of agendas (those containing at least two atomic propositions and at least one suitable composite proposition in standard propositional logic).

Theorem 1 *(List and Pettit 2002) If $X \supseteq \{a, b, a \wedge b\}$ (where $\wedge$ could be replaced by $\vee$ or $\rightarrow$), there exists no aggregation rule satisfying universal domain, collective rationality, systematicity and anonymity.*

A stronger version of this result, due to Pauly and van Hees (2006), weakens anonymity to non-dictatorship. These results, however, should only be seen as first 'baseline' results. They provide only sufficient, but not necessary, conditions on the agenda for an impossibility to arise, and for many agendas, the systematicity condition on the aggregation rule can be weakened to independence alone in the presence of some other conditions. Let us therefore review several more general characterization results.

3.1 Characterization results

In all results reviewed in this subsection, we consider collectively rational aggregation rules with universal domain satisfying unanimity and independence. We ask whether such rules can also be non-dictatorial and/or anonymous. We distinguish between results with and without the requirement of monotonicity, and within each category between results with and without the requirement of neutrality, i.e., independence strengthened to systematicity. For our exposition, we call an aggregation rule *regular* if it is collectively rational, has universal domain and satisfies unanimity.

3.1.1 Results with monotonicity

The main advantage of assuming monotonicity is the resulting simple characterization of the class of propositionwise aggregation rules for any given agenda X, as follows (Nehring and Puppe 2002, 2006). Let $\mathcal{W}$ denote a non-empty family of subsets of N, closed under taking supersets and interpreted as a family of 'winning coalitions' of individuals. A *structure of winning coalitions* assigns to each proposition $p \in X$ a family $\mathcal{W}_p$ with these properties such that $W \in \mathcal{W}_p \Leftrightarrow (N \setminus W) \notin \mathcal{W}_{\neg p}$. An aggregation rule F is called *voting by issues* if, for all $p \in X$ and all profiles $(J_1, ..., J_n)$ in the universal domain,

$$p \in F(J_1, ..., J_n) \Leftrightarrow \{i : p \in J_i\} \in \mathcal{W}_p.$$

Thus a proposition is collectively accepted if and only if the set of individuals accepting it is a winning coalition for that proposition. As shown in Nehring and Puppe (2006), an aggregation rule with universal domain satisfies unanimity, independence, monotonicity and always accepts exactly one member of each pair $p, \neg p \in X$ if and only if it is voting by issues.

It is important to note, however, that voting by issues does not generally guarantee collective rationality. This can be seen from the fact that majority voting is an instance of voting by issues, where the family of winning coalitions $\mathcal{W}_p$ for each proposition p consists of all subsets of N with more than $n/2$ members. The necessary and sufficient condition for collective rationality of voting by issues is the following. Call a set of propositions S *minimal inconsistent* if S is inconsistent and every proper subset of S is consistent. Examples of minimal inconsistent sets are $\{a, \neg a\}$ and $\{a, a \rightarrow b, \neg b\}$; by contrast, the set $\{\neg a, \neg b, a \wedge b\}$ is inconsistent, but not minimally so.

Theorem 2 *(Nehring and Puppe 2002, 2006) Voting by issues on X with winning coalitions $(\mathcal{W}_p)_{p \in X}$ is collectively rational if and only if, for all minimally inconsistent subsets $\{p_1, ..., p_l\} \subseteq X$ and all selections $W_j \in \mathcal{W}_{p_j}$, $\bigcap_{j=1}^{l} W_j \neq \emptyset$.*

The characterizing condition in Theorem 2 is called the *intersection property*.[8] It provides a powerful tool for determining both the class of regular, independent and monotonic aggregation rules for a given agenda and the class of agendas admitting such rules with additional properties. Theorems 3, 4 and 5 in this section are instances of the second category of results; we illustrate the first category of results in Section 3.2.2 below.

We distinguish classes of agendas in terms of their logical complexity. The first result uses the following condition:

Median Property. All minimally inconsistent subsets of the agenda X contain exactly two propositions.

[8] Dietrich and List (2007b) show that the intersection property can be generalized to one for collective consistency and one for collective deductive closure (each are weakenings of collective rationality).

The median property says that the agenda is 'simple' in the sense that direct interconnections between propositions are confined to pairs.[9]

Theorem 3 *(Nehring and Puppe 2006) There exist regular, monotonic, systematic and non-dictatorial aggregation rules on the agenda X if and only if X has the median property.*

Using Theorem 3 it can easily be shown that, if the number of individuals is odd, an agenda with the median property even admits regular, monotonic and systematic rules that are *anonymous*. In other words, with an odd number of individuals the median property is also necessary and sufficient for majority voting to be collectively rational (see Nehring and Puppe 2006, Dietrich and List 2007b).

The next result uses a stronger condition on the agenda. Say that p *conditionally entails* q if $p \neq \neg q$ and there exists a minimally inconsistent subset $Y \subseteq X$ such that $p, \neg q \in Y$. Intuitively, this means that q can be deduced from p, using other propositions in the agenda. We write $p \triangleright q$ if there exists a sequence $p_1, p_2, ..., p_m$ with $p = p_1$ and $q = p_m$ such that p_1 conditionally entails p_2, p_2 conditionally entails p_3, ..., and p_{m-1} conditionally entails p_m.

Total Blockedness. The agenda X is *totally blocked* if for any pair of propositions $p, q \in X$, $p \triangleright q$.

Total blockedness says that any proposition in the agenda can be reached from any other proposition in it via a sequence of conditional entailments. One can show that if an agenda satisfies the median property it cannot be totally blocked.

Theorem 4 *(Nehring and Puppe 2005b) There exist regular, monotonic, independent and non-dictatorial aggregation rules on the agenda X if and only if X is not totally blocked.*

Viewed as a possibility result, Theorem 4 is not completely satisfactory, since it admits quite degenerate possibilities such as local dictatorships, i.e., dictatorships on particular propositions. Agendas admitting regular, monotonic, independent and locally non-dictactorial aggregation rules can be characterized as well using the Intersection Property (Nehring and Puppe 2005b). They coincide with the class of agendas admitting regular, monotonic, independent and anonymous rules with an odd number of individuals, but the characterizing condition ('quasi-blockedness') is somewhat complicated. A much simpler characterization is obtained by requiring anonymity for an arbitrary number of individuals.

Blockedness. The agenda X is *blocked* if for some proposition $p \in X$, $p \triangleright \neg p$ and $\neg p \triangleright p$.

[9] The terminology stems from the fact that agendas satisfying the median property correspond to so-called 'median spaces' when embedded into the property space framework.

Theorem 5 *(Nehring and Puppe 2005b) There exist, for any number of individuals n, regular, monotonic, independent and anonymous aggregation rules on the agenda X if and only if X is not blocked.*

For many agendas, all anonymous rules are unanimity rules (with certain default judgments in the absence of unanimity), and, without anonymity, all admissible rules are 'oligarchic' (with certain default judgments when the oligarchs disagree), as defined in Section 3.2.2 below. In particular, as shown in Nehring (2006), under regularity, monotonicity and independence, a non-trivial agenda[10] admits only oligarchic rules in this sense if and only if, for all $p, q \in X$, $[p \vartriangleright q$ and $q \vartriangleright p]$ or $[p \vartriangleright \neg q$ and $\neg q \vartriangleright p]$ ('semi-blockedness'). In Section 5.1, we review some characterizations of another kind of oligarchic rules (where disagreements between the oligarchs lead to incomplete judgments).

3.1.2 Results without monotonicity

While monotonicity is arguably an appealing condition in the presence of independence, it is not used in many classic results in standard social choice theory, notably Arrow's theorem, and in several early results on judgment aggregation, including Theorem 1. We may therefore ask whether it is needed to arrive at general characterization results of the above kind. Without requiring monotonicity, a characterization of aggregation rules in terms of structures of winning coalitions, along the lines of the Intersection Property, is not known. However, characterization results can be obtained by introducing an additional agenda complexity condition.

Even-Number-Negation Property. The agenda X has a minimal inconsistent subset Y such that $(Y \backslash Z) \cup \{\neg p : p \in Z\}$ is consistent for some subset $Z \subseteq Y$ of even size.

This condition says that the agenda has a minimal inconsistent subset that can be made consistent by negating an even number of propositions in it (Dietrich 2007a, Dietrich and List 2007a). An equivalent algebraic condition is *non-affineness* (Dokow and Holzman 2005), which in turn is equivalent to the requirement that the agenda is not structurally equivalent to a set of propositions whose only logical connectives are $\neg$ and $\leftrightarrow$. The agendas in the discursive dilemma and global warming examples above, for instance, satisfy the even-number-negation property; we give further examples in Section 3.2. The following theorem generalizes the earlier results on systematicity by List and Pettit (2002) and Pauly and van Hees (2006) (i.e., Theorem 1 above and its strengthening, respectively).

[10] An agenda is called *non-trivial* if it contains at least two propositions p and q such that p is neither logically equivalent to q nor to $\neg q$.

Theorem 6 *(Dietrich and List 2007a) There exist regular, systematic and non-dictatorial aggregation rules on the agenda X if and only if X satisfies the median property or violates the even-number-negation property.*

By contraposition, Theorem 6 says that the class of regular and systematic aggregation rules are precisely the dictatorships if and only if the agenda satisfies the even-number-negation property and does not have the median property. Variants of this result continue to hold even when, as in the first theorems on systematicity just cited, no unanimity requirement is imposed on the aggregation rule. If an agenda satisfies the even-number-negation property and does not have the median property, the class of admissible aggregation rules then grows to contain all dictatorial and inverse dictatorial rules, provided the latter are consistent (Dietrich and List 2007a). If the agenda in addition has an inconsistent subset $Y \subseteq X$ such that $\{\neg p : p \in Y\}$ is consistent, then systematicity alone (under universal domain and collective rationality) suffices to characterize dictatorships (Dietrich 2007a).

Pauly and van Hees (2006) derived the first impossibility theorem on judgment aggregation with systematicity weakened to independence, followed by Dietrich (2006a). These results made strong assumptions on the agenda, but – like the systematicity results just reviewed – imposed no unanimity requirement on the aggregation rule. Instead, aggregation rules were merely required to be non-constant.

With unanimity Dokow and Holzman (2005) showed that if an agenda is totally blocked, then regular, independent and non-dictatorial rules exist if and only if the agenda violates the even-number-negation property. The aggregation rules needed for this result are derived from the *parity rule*, under which a proposition is collectively accepted if and only if it is accepted by an odd number of individuals (assuming, for simplicity, that n is odd); clearly, this defines a regular, systematic, anonymous (and hence, non-dictatorial) but non-monotonic aggregation rule.[11] Combined with Theorem 4 one thus obtains the following result. Its 'only if' direction is also contained in Dietrich and List (2007a).

Theorem 7 *(Dokow and Holzman 2005) There exist regular, independent and non-dictatorial aggregation rules on the agenda X if and only if X is not totally blocked or violates the even-number-negation property.*

A simple example of a totally blocked agenda that violates the even-number-negation property (and for which parity rules are collectively rational) is the agenda $X = \{a, \neg a, b, \neg b, a \leftrightarrow b, \neg(a \leftrightarrow b)\}$, which shows that the even-number-negation property is essential in Theorems 6 and 7.

[11] The general class of 'parity rules' emerges by fixing any subset $M \subseteq N$ with an odd number of individuals and applying the rule stated in the text to subprofiles restricted to M.

The following table summarizes the main results just presented. Each cell gives a necessary and sufficient condition for an agenda to admit regular and independent aggregation rules with the additional properties stated in the respective row and column; the even-number-negation property is abbreviated by 'e-n-n'. The results corresponding to anonymous but possibly non-monotonic, respectively non-oligarchic and possibly non-monotonic rules, have recently been obtained by Dietrich and List in an unpublished note. A characterization of the agendas admitting regular, independent and locally non-dictatorial aggregation rules without monotonicity is not yet known.[12]

	monotonic	possibly non-monotonic
neutral and non-dictatorial	median	median or (not e-n-n)
anonymous (for every n)	not blocked	not blocked
locally non-dictatorial	not quasi-blocked	?
non-oligarchic	(not semi-blocked) or trivial	(not semi-blocked) or (not e-n-n)
non-dictatorial	not totally blocked	(not totally blocked) or (not e-n-n)

Table 3: Characterization of agendas for possibility results

3.2 Specific agendas

To illustrate the applicability of the results just reviewed, we turn to agendas with specific additional structure.

3.2.1 Preference agendas

An important class of agendas are the *preference agendas*, in terms of which preference aggregation problems can be represented in the judgment aggregation model.[13] Here the additional structure stems from the rationality conditions on preferences. To represent preference relations, we use a simple language of predicate logic, with a two-place predicate symbol P representing strict preference and a set of constant symbols $K = \{x, y, z, ...\}$ representing social alternatives. A preference agenda is of the form $X = \{xPy, \neg xPy : x, y \in K \text{ with } x \neq y\}$. To capture rationality conditions on preferences, one has to define consistency appropriately: a set of propositions $S \subseteq X$ is deemed

[12]Note that the characterizing condition of the agendas admitting regular, independent and anonymous rules (for all n) is the same regardless of whether monotonicity is required or not.

[13]There are various ways of representing preference aggregation problems in judgment aggregation or abstract aggregation. The present construction using predicate logic is based on Dietrich and List (2007a), extending List and Pettit (2004). For other related approaches, see Wilson (1975), Nehring (2003), Dokow and Holzman (2005) and Nehring and Puppe (2005a). An early construction was given by Guilbaud (1966).

consistent if $S \cup H$ is consistent in the standard logical sense, where H is the appropriate set of rationality conditions on preference relations. In the case of strict preference orderings, these are asymmetry, transitivity and connectedness.[14] It is easily seen that the judgment aggregation problem on X represents a classical preference aggregation problem, with each consistent and complete judgment set representing a fully rational preference relation.

There has been a sequence of contributions on how the results on judgment aggregation apply to preference aggregation. In a companion paper to their original paper, List and Pettit (2004) adapted their proof of Theorem 1 above to the preference agenda, showing that there is no anonymous, systematic and collectively rational aggregation rule with universal domain here. Nehring (2003) proved that the preference agenda is totally blocked and hence, applying Theorem 4 above, showed that all regular, independent and monotonic aggregation rules are dictatorial. Dietrich and List (2007a) and Dokow and Holzman (2005) showed that the preference agenda in addition satisfies the even-number-negation property (equivalently, non-affineness) and, by applying the 'only if' part of Theorem 7, showed that all regular and independent aggregation rules are dictatorial. The latter result is Arrow's theorem for strict preferences.[15]

3.2.2 Truth-functional agendas

An important feature of the agenda in the original doctrinal paradox is that there is a conclusion (e.g., liability of the defendant) whose truth-value is uniquely determined by the truth-values of several premises (e.g., action and obligation). An agenda is called *truth-functional* if it can be partitioned into a subagenda of premises and a subagenda of conclusions such that each conclusion is truth-functionally determined by the premises. Nehring and Puppe (2005a) and Dokow and Holzman (2005) characterized classes of regular aggregation rules satisfying certain conditions on truth-functional agendas.

The bottom line is that all regular, independent and monotonic rules on such agendas are oligarchic.[16] An *oligarchic rule* with default $J^0 \subseteq X$ specifies a non-empty set $M \subseteq N$ (the 'oligarchs') such that, for all $p \in X$ and all profiles $(J_1, ..., J_n)$ in the universal domain,

$$p \in F(J_1, ..., J_n) \Leftrightarrow \begin{cases} & p \in J_i \text{ for all } i \in M \\ \text{or} & p \in J^0 \text{ and } [p \in J_i \text{ for some } i \in M] \end{cases}$$

[14] For example, transitivity is represented by the proposition $(\forall v_1)(\forall v_2)(\forall v_3)((v_1 P v_2 \wedge v_2 P v_3) \rightarrow v_1 P v_3)$.

[15] Dokow and Holzman (2006) and Dietrich (2007b) provided derivations of Arrow's theorem for weak preferences in judgment aggregation, each using different constructions.

[16] Dokow and Holzman (2005) did not assume monotonicity and weakened the unanimity requirement to surjectivity of the aggregation rule; therefore these authors obtained slightly different characterizations depending on the complexity of the (truth-functional) agenda.

Clearly, dictatorships are special cases (with M singleton). Nehring and Puppe (2005a) identified the truth-functional agendas admitting non-dictatorial oligarchic rules ensuring collective rationality. For instance, in the global warming example above with agenda $X = \{a, \neg a, a \to b, \neg(a \to b), b, \neg b\}$, any oligarchic rule with default $J^0 = \{\neg a, a \to b, b\}$ is collectively rational.

3.2.3 Agendas with subjunctive implications

Dietrich (2006b) argued that, in many contexts, the material interpretation of the implication operator is not natural. To illustrate, consider again the global warming example above. Under a material interpretation of implication the set of propositions $\{\neg a, \neg(a \to b), \neg b\}$ is inconsistent, since negating the antecedent a makes the material implication $a \to b$ true by definition. In everyday language, however, negating a (i.e., negating the proposition that current emissions lead to global warming) and negating b (i.e., negating the proposition that one should reduce emissions) seems perfectly consistent with the negation of any implication between a and b. Accordingly, a 'subjunctive' interpretation of the implication operator (Lewis 1973) renders the set $\{\neg a, \neg(a \to b), \neg b\}$ consistent. Dietrich (2006b) showed that under this interpretation, the agenda in the global warming example admits collectively rational supermajority ('quota') rules (which are anonymous, monotonic and independent). Generalizing the anonymous version of the intersection property, Dietrich (2006b) characterized the admissible quota rules on a large class of agendas with subjunctive implications.

3.2.4 Non-truth-functional agendas with a premise/conclusion structure

Agendas with subjunctive implications are usually not truth-functional. For instance, in the global warming example affirming a and negating $a \to b$ is consistent with either affirming or negating b under a subjunctive interpretation of the implication. Nehring and Puppe (2007) studied agendas containing a 'conclusion' (a 'decision') that depends in a general, not necessarily truth-functional way on some 'premises' (the 'decision criteria') from the viewpoint of *justifying* the collective decision. They provided several characterization results, including a characterization of the logical interrelations between the premises and the conclusion that enable independent and monotonic aggregation rules with majority voting on the conclusion. While such rules cannot exist in the truth-functional case, they do exist under reasonable circumstances in the non-truth-functional one. For instance, in the global warming example, the rule according to which b is decided by majority voting while a and $a \to b$ are affirmed if and only if each reaches a quota of at least 3/4 is consistent under the subjunctive interpretation.

3.2.5 The group identification problem

In the group identification problem introduced by Kasher and Rubinstein (1997), each individual makes a judgment on which individuals belong to a particular social group subject to the constraint that the social group is neither empty nor universal. List (2006) formalized this problem in the judgment aggregation model and showed that the corresponding agenda is totally blocked and satisfies the even-number-negation property; therefore, by the 'only if' part of Theorem 7 above, all regular and independent aggregation rules for the group identification problem are dictatorial. Dietrich and List (2006a) investigated the group identification problem in the case where the membership status of some individuals can be left undecided and showed that all regular and independent aggregation rules are oligarchies with empty default (see Section 5.1 below). A. Miller (2007) developed a model in which individuals make judgments about their membership in several social groups simultaneously.

3.3 Why independence?

The independence condition in judgment aggregation is often challenged on the grounds that it fails to do justice to the fact that propositions are logically interconnected, which is the essence of the judgment aggregation problem (e.g., Chapman 2002, Mongin 2005). In this subsection, we put forward a possible 'instrumental' justification of independence on the basis of strategy-proofness.[17] In fact, this justification also supports monotonicity.

The simplest way to implement the idea of strategy-proofness is in terms of the following non-manipulability condition. Say that one judgment set J *agrees* with another J' on some proposition p if $[p \in J \Leftrightarrow p \in J']$. An aggregation rule F is *non-manipulable* if there is no individual $i \in N$, no proposition $p \in X$, and no profile $(J_1, ..., J_n)$ in the domain such that, for some i-variant $(J_1, ..., J_i', ..., J_n)$ in the domain, $F(J_1, ..., J_n)$ does not agree with J_i on p and $F(J_1, ..., J_i', ..., J_n)$ agrees with J_i on p. Dietrich and List (2007c) showed that, under universal domain, an aggregation rule is non-manipulable if and only if it is independent and monotonic, which allows the application of Theorems 2 to 5 above.

In fact, non-manipulability corresponds to a standard social-choice-theoretic notion of strategy-proofness as follows. Assume that each individual i has a (reflexive and transitive) preference relation $\succsim_i$ over consistent and complete judgment sets such that, for some (unique) 'ideal' judgment set J_i, we have $[J \cap J_i \supsetneq J' \cap J_i] \Rightarrow J \succ_i J'$ for any pair of judgment sets J, J'. Call such preferences *generalized single-peaked* (Nehring and Puppe 2002). A social choice function $\mathcal{F}$ mapping profiles of such preference relations to collective judgment sets is *strategy-proof* if, for all individuals i and all i-variants

[17] A closely related argument could be based on the absence of manipulations by the agenda setter, see List (2004) and Dietrich (2006a).

$(\succeq_1, ..., \succeq_n)$, $(\succeq_1, ..., \succeq_i', ..., \succeq_n)$ in the domain,

$$\mathcal{F}(\succeq_1, ..., \succeq_n) \succeq_i \mathcal{F}(\succeq_1, ..., \succeq_i', ..., \succeq_n).$$

It can be shown that any such strategy-proof social choice function $\mathcal{F}$ depends only on the ideal judgment sets and thus induces a judgment aggregation rule F defined by $F(J_1, ..., J_n) := \mathcal{F}(\succeq_1, ..., \succeq_n)$, where, for all i, $\succeq_i$ is some generalized single-peaked preference relation with ideal judgment set J_i. The induced judgment aggregation rule is independent and monotonic; conversely, any independent and monotonic judgment aggregation rule F satisfying universal domain and collective rationality induces a strategy-proof social choice function $\mathcal{F}$ on the domain of generalized single-peaked preferences by appropriately reversing this construction (Nehring and Puppe 2002, 2006). A definition of strategy-proofness of F, as opposed to strategy-proofness of $\mathcal{F}$, is given in Dietrich and List (2007c), extending List (2004).

4 Relaxing independence

4.1 Premise-based and related approaches

Perhaps the most discussed alternative to majority voting and propositionwise aggregation more generally is the class of premise-based procedures, applicable to truth-functional agendas in which the subagenda of premises can be chosen so as to consist of mutually independent proposition-negation pairs (see, among others, Kornhauser and Sager 1986, Pettit 2001, List and Pettit 2002, List 2005, Dietrich 2006a, Bovens and Rabinowicz 2006). A *premise-based procedure* is given by applying a suitable propositionwise aggregation rule (such as majority voting) to the premises and deducing the collective judgments on all other propositions (i.e., the conclusions) by logical implication. As an illustration, consider the doctrinal paradox example with individual judgments as shown in Table 1. If the premises are taken to be a, b, $c \leftrightarrow (a \wedge b)$ (and negations) and the conclusion is taken to be c (and its negation), the premise-based procedure (based on majority voting) yields the collective judgment set $\{a, b, c \leftrightarrow (a \wedge b), c\}$, i.e., a 'liable' verdict.

The appeal of a premise-based procedure is that it is collectively rational and that the independence requirement is confined to logically independent propositions. Dietrich (2006a) characterized the premise-based procedure in terms of such a weakened independence condition. A problem, however, is that there does not always exist a unique way to specify premises and conclusions and that different such specifications may lead to different collective judgment sets. For example, on the above agenda containing a, b, $c \leftrightarrow (a \wedge b)$, c (and negations), any three unnegated propositions (and their negations) can form a subagenda of mutually independent premises, setting interpretational issues aside. Using majority voting on the premises, each of these leads to a different collective judgment set in Table 1.

The same example also illustrates another problem of the premise-based procedure: majority voting on the premises may overrule a unanimous judgment on the conclusion, as can be seen by taking a, b, c (and negations) as the premises in Table 1. More generally, Nehring (2005) characterized truth-functional relations between multiple premises and one conclusion in terms of the admitted aggregation rules satisfying independence and monotonicity on the premises and respecting unanimous judgments on the conclusion; for sufficiently complex truth-functional relations, only dictatorial rules have these properties. Relatedly, Mongin (2005) proved that, for sufficiently rich agendas, the only regular aggregation rules satisfying independence restricted to atomic propositions (which one might view as premises) and a propositionwise unanimity condition are dictatorships. A conceptual difference between the two contributions lies in the interpretation of the unanimity requirement on the outcome decision. Nehring (2005) interpreted it as a condition of Paretian welfare rationality, suggesting a potentially deep tension between 'judgment rationality' (reason-basedness) and consequentialist outcome rationality. Mongin (2005) did not adopt the Paretian interpretation, applying the unanimity condition instead to every proposition. His analysis sought to show the robustness of an impossibility under weakening independence.

Bovens and Rabinowicz (2006) and subsequently List (2005) investigated the truth-tracking reliability of the premise-based procedure in cases where the propositions in question have independent truth conditions. Adapting the Condorcet jury theorem to the case of multiple interconnected propositions, they showed that, under a broad range of assumptions, the premise-based procedure leads to more reliable decisions than majority voting on the conclusion. Within this framework, List (2005) also calculated the probability of disagreements between the two procedures under various assumptions and, by implication, the probability of the occurrence of a majority inconsistency.

4.2 The sequential priority approach

A premise-based procedure is a special case of a *sequential priority procedure* (List 2004, Dietrich and List 2007b), which can be defined for any agenda. Let an order of priority over the propositions in the agenda be given. Earlier propositions in that order may be interpreted as 'prior to' later ones. For any profile, the collective judgment set is determined as follows. Consider the propositions in the agenda in the given order. For any proposition p, if the collective judgment on p is logically constrained by the collective judgments on propositions considered earlier, then it is deduced from those prior judgments by logical implication. If it is not constrained in this way, then it is made by majority voting or another suitable propositionwise aggregation rule.

By construction, any sequential priority procedure guarantees consistent collective judgment sets. Moreover, for truth-functional agendas, a sequential priority procedure can mimic a premise-based procedure if the premises precede the conclusions in the specified order of priority. But clearly sequential priority procedures can also be defined for non-truth-functional agendas. A key feature of sequential priority procedures is their *path-dependence*: the collective judgment set may vary with changes in the order of priority over the propositions. Necessary and sufficient conditions for such path-dependence were given by List (2004). Dietrich and List (2007b) further showed that the absence of path-dependence is equivalent to strategy-proofness in a sequential priority procedure.

4.3 The distance-based approach

In analogy to the corresponding approach in social choice theory (see e.g., Kemeny 1959), an alternative to propositionwise aggregation is a distance-based approach. Suppose that for a given agenda there is a metric which specifies the distance $d(J, J')$ between any two judgment sets. A *distance-based* aggregation rule determines the collective judgment set so as to minimize the sum of the individual distances. Formally, the collective judgment set for the profile $(J_1, ..., J_n)$ is a solution to

$$\min_J \sum_{i=1}^{n} d(J, J_i),$$

where the minimum is taken over all consistent and complete judgment sets.[18] A natural special case arises by taking d to be the *Hamming distance*, where $d(J, J')$ is the number of propositions in the agenda on which J and J' do not agree. This was proposed and analyzed by Pigozzi (2006) under the name 'fusion operator' (see also Eckert and Klamler 2007), drawing on the theory of belief merging in computer science (Konieczny and Pino-Perez 2002). When applied to the preference agenda in Section 3.2.1 above, this aggregation rule is known as the 'Kemeny rule' (Kemeny 1959; see Merlin and Saari 2000 for a modern treatment).

4.4 The relevance approach

Generalizing each of these specific approaches to relaxing independence, Dietrich (2007b) introduced a *relevance relation* between the propositions in the agenda, reflecting the idea that some propositions are relevant to others. For example, premises or prior propositions may be relevant to conclusions or posterior ones. Aggregation rules are now required to satisfy *independence of irrelevant information*: the collective judgment on any proposition p should depend only on the individuals' judgments on propositions relevant to p. The strength of this constraint depends on how many or few propositions are deemed relevant to each proposition: the fewer such relevant propositions, the

[18] A more general approach could allow also for other functions than the *sum* of individual distances.

stronger the constraint. In the limiting case where each proposition is relevant only to itself, the constraint is maximally strong and reduces to the standard independence condition.

The premise-based, sequential priority and distance-based approaches can all be seen as drawing on particular relevance relations, namely premisehood, linear, and total (i.e., maximally permissive) relevance relations, respectively. Dietrich (2007b) proved several results on aggregation rules induced by general relevance relations, such as arbitrary premisehood or priority relations, which often induce a directed acyclic network over the propositions in the agenda. Whether there exist non-degenerate aggregation rules satisfying independence of irrelevant information depends on the interplay between logical connections and relevance connections.[19]

5 Other themes and contributions

At the time of the writing of this survey, judgment aggregation is still a very active research field in its developing stage. While the results for independent (i.e., propositionwise) aggregation in the case of two-valued logic seem to be near definitive, many important aspects of judgment aggregation are not yet fully explored. In this concluding section, we briefly sketch several other themes and contributions that point towards directions for future research.

5.1 Rationality relaxations

The possibility of judgment aggregation under weaker rationality constraints is the subject of several contributions. List and Pettit (2002) observed that, for a sufficiently large supermajority threshold, *(symmetrical) supermajority rules* – where any proposition is accepted if and only if it is accepted by a specified supermajority of individuals – guarantee consistency of collective judgments,[20] and unanimity rule in addition guarantees *deductive closure* (i.e., implications of accepted propositions are also accepted). More generally, Dietrich and List (2007c) provided necessary and sufficient conditions under which quota rules satisfy each of consistency, deductive closure and completeness.

Gärdenfors (2006) proved an impossibility theorem showing that, under a particular agenda richness assumption, any independent aggregation rule satisfying universal

[19] Arrow's theorem for weak preferences turns out to be a corollary of one of these results (see Footnote 15).

[20] As a sufficient condition on the supermajority threshold q, they gave $q > \frac{k-1}{k}$, where $k = \frac{|X|}{2}$, a result from List's 2001 doctoral dissertation; it also follows from the intersection property, generalized to the case of collective consistency. Dietrich and List (2007b) showed that this can be improved to a necessary and sufficient condition by defining k to be the size of the largest minimal inconsistent subset of X.

domain and unanimity, and generating consistent and deductively closed (but not necessarily complete) collective judgments is *weakly oligarchic*, in the sense that there exists a smallest subset $M \subseteq N$ such that, for all profiles $(J_1, ..., J_n)$, $F(J_1, ..., J_n) \supseteq \bigcap_{i \in M} J_i$.

Generalizations of this result were given by Dietrich and List (2006a) and Dokow and Holzman (2006). The common finding is that, if collective rationality is weakened to the conjunction of consistency and deductive closure (and also if the completeness requirement is dropped at the individual level), the agenda conditions leading to dictatorships in the full-rationality case lead to *oligarchies (with empty default)*, whereby there exists a subset $M \subseteq N$ such that, for all profiles $(J_1, ..., J_n)$, $F(J_1, ..., J_n) = \bigcap_{i \in M} J_i$. More precisely, Theorems 3,4,6 and 7 continue to hold if in their respective statements 'non-dictatorial' is strengthened to 'non-oligarchic' and 'full rationality' is weakened to 'consistent and deductively closed' (optionally, full rationality at the individual level can also be replaced by consistency and deductive closure).[21] Dietrich and List (2006a) provided applications to the aggregation of partial orderings (including a variant of Gibbard's oligarchy theorem for strict preferences) and to the group identification problem (see above); Dokow and Holzman (2006) derived Gibbard's original oligarchy theorem and Arrow's theorem for weak preferences as corollaries.

More recently, Dietrich and List (2007d) provided a characterization of agendas leading to dictatorships when the rationality requirement at both individual and collective levels is weakened to consistency alone, dropping both completeness and deductive closure.

5.2 Multi-valued logic and general logics

Pauly and van Hees (2006) and van Hees (2007) extended the model of judgment aggregation by allowing more than two degrees of acceptance, at both individual and collective levels. Thus they considered the aggregation of multi-valued truth functions. Building on, and generalizing, their impossibility results on systematicity and independence for two-valued logic, they showed that strong impossibility results arise even in this multi-valued context.

As mentioned above, Dietrich (2007a) developed a model of judgment aggregation in general logics, which allows the agenda to contain more expressive propositions than those of standard propositional logic. He argued that most realistic judgment aggregation problems and most standard examples of the discursive dilemma involve propositions that contain not only classical operators ('not', 'and', 'or', ...) but also non-classical ones, such as subjunctive conditionals (see above), modal operators ('it is necessary/possible that') or deontic operators ('it is obligatory/permissible that'). The

[21] The Dietrich and List (2006a) results in addition drop the consistency requirement at the collective level.

general-logics model uses an arbitrary language $\mathcal{L}$ with a notion of consistency satisfying the minimal conditions stated in Footnote 1. This includes many familiar logics: classical and non-classical ones, propositional and predicate ones, and logics whose logical connections are defined relative to a given set of constraints $C \subseteq \mathcal{L}$ such as the rationality constraints in the preference aggregation problem. Most theorems of the literature hold in general logics.

Pauly (2007) explored the role of language in judgment aggregation from a different perspective. He investigated the richness of the language required to express the conditions (such as unanimity, independence, systematicity etc.) needed for characterizing various aggregation rules. This approach allowed him to derive some non-axiomatizability results, showing that certain aggregation rules cannot be axiomatically characterized unless a sufficiently rich language is used to express the axioms.

5.3 Domain restrictions

If the condition of universal domain is dropped and the domain of admissible profiles of individual judgment sets is suitably restricted, it becomes possible to satisfy all the other conditions on aggregation rules introduced above. Several domains are known on which majority voting is consistent. One such domain is the set of all profiles of consistent and complete individual judgment sets satisfying a condition called unidimensional alignment (List 2003). A profile is *unidimensionally aligned* if the individuals can be aligned from left to right such that, for each proposition in the agenda, the individuals accepting the proposition are either all to the left, or all to the right, of those rejecting it. Dietrich and List (2006b) provided several more general domain restriction conditions guaranteeing consistent majority judgments, including a local variant of unidimensional alignment, under which the relevant left-right alignment of the individuals can be different for each minimal inconsistent subset of the agenda, and some conditions that do not require complete individual judgment sets.

5.4 Judgment aggregation with disagreements on connections between premises and conclusion

M. Miller (2007) offered a generalization of the truth-functional case of judgment aggregation by considering agendas consisting of several premises and a conclusion, where individuals may disagree about the logical connection between the former and the latter. The rationale behind this extension is that different individuals may reason in different ways and thus use different decision principles for the same decision. M. Miller (2007) proved an impossibility result showing that, again, certain types of oligarchic rules are the only collectively rational aggregation rules satisfying some reasonable conditions.

5.5 Liberal paradox

In some judgment aggregation problems, some individuals or subgroups may have expert knowledge on certain propositions or be particularly affected by them. One may then wish to assign to these individuals or subgroups the right to determine the collective judgment on those propositions. Dietrich and List (2004) investigated how such rights constrain the available aggregation rules. Among other results, they showed that, for a large class of agendas, the assignment of rights to two or more individuals or subgroups is inconsistent with the unanimity condition. This result generalizes Sen's famous 'liberal paradox' (1970), as it also applies to the preference agenda where its conditions reduce to Sen's original conditions. Dietrich and List (2004) further identified domain restriction conditions under which the conflict between rights and the unanimity condition can be avoided.

In a related vein, Nehring (2005) shows that if an aggregation rule treats a proposition and its negation symmetrically, any differential treatment of voters as experts across propositions leads to potential violations of unanimity.

5.6 Bayesian approaches

A natural step is to abandon the discrete, and mostly binary, nature of the evaluation of propositions. Continuous evaluations of propositions arise, for example, from a probabilistic interpretation of propositions or from their interpretation as economic variables. Claussen and Røisland (2005) analyzed the discursive dilemma in economic environments in which judgements involve quantitative assessments of variables. They showed that the original discursive dilemma (with majority voting on 'premise variables') is robust with respect to the generalization to a continuous setting.

Nehring (2007) proposed a 'Bayesian' model of group choice and showed that there does not generally exist any anonymous aggregation rule that is independent on the premises and always respects individuals' unanimous preferences over the outcome.

Bradley, Dietrich and List (2006) applied insights from the theory of judgment aggregation to the aggregation of Bayesian networks, which consist of a causal relevance relation over some variables and a probability distribution over them. While some standard impossibility and possibility results also apply to the aggregation of causal relevance relations, a possibility result holds for the aggregation of the associated probability distributions.

Although these contributions underline the robustness of some of the impossibility results derived in the binary case, the Bayesian approach seems to offer new possibilities not yet explored.

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