

The Logical Space of Democracy

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Abstract: Can we design a perfect democratic decision procedure? Condorcet famously observed that majority rule, our paradigmatic democratic procedure, has some desirable properties, but sometimes produces inconsistent outcomes. Revisiting Condorcet’s insights in light of recent work on the aggregation of judgments, I show that there is a conflict between three initially plausible requirements of democracy: “robustness to pluralism,” “basic majoritarianism,” and “collective rationality.” For all but the simplest collective decision problems, no decision procedure meets these three requirements at once; at most two can be met together. This “democratic trilemma” raises the question of which requirement to give up. Since different answers correspond to different views about what matters most in a democracy, the trilemma suggests a map of the “logical space” in which different conceptions of democracy are located. It also sharpens our thinking about other impossibility problems of social choice and how to avoid them, by capturing a core structure many of these problems have in common. More broadly, it raises the idea of “cartography of logical space” in relation to contested political concepts.

In the run-up to the French Revolution, the polymath and political-science pioneer Nicolas de Condorcet discovered some remarkable properties of majority rule.¹ One of them, described by his celebrated “jury theorem,” is often cited as one of majority rule’s greatest strengths: if each voter is better than random at making a correct judgment on a yes–no question—say, whether a defendant is guilty, or whether a proposed policy will avert a crisis—and different voters are mutually independent, then the probability of a correct majority judgment increases and approaches one, as the size of the electorate increases.² So, under the right conditions, majority decisions track the truth. But the other property of majority rule that Condorcet discovered, now known as “Condorcet’s paradox,” has come to be seen as one of its most notorious weaknesses: even if each voter has impeccably rational preferences, the majority preferences can still be irrational.

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¹ See Nicolas de Condorcet, *Essay sur l’Application de l’Analyse à la Probabilité des Décisions Rendue à la Pluralité des Voix* (Paris, 1785), and for an English translation and commentary, Iain McLean and Arnold B. Urken (eds.), *Classics of Social Choice* (Ann Arbor: University of Michigan Press, 1995).

² See, among many others, Bernard Grofman, Guillermo Owen and Scott L. Feld (1983), “Thirteen theorems in search of the truth,” *Theory and Decision* 15 (1983): 261–78; Philip J. Boland, “Majority Systems and the Condorcet Jury Theorem,” *The Statistician* 38 (1989): 181–89; and Christian List and Robert E. Goodin, “Epistemic Democracy: Generalizing the Condorcet Jury Theorem,” *Journal of Political Philosophy* 9 (2001): 277–306.

Suppose, for example, a third of the electorate prefers option A to option B to option C; a second third prefers B to C to A; and the last third prefers C to A to B. Then majorities prefer A to B, B to C, and yet C to A, a “cyclical” and thus inconsistent outcome.

The significance of Condorcet’s insights cannot be overstated. They have inspired vast bodies of work in philosophy, political science, and economics, with repercussions well beyond these fields. Much of the recent literature on “epistemic democracy” can be traced back to Condorcet’s jury theorem,³ and Condorcet’s paradox is an important precursor of many later discoveries in social choice theory, notably Kenneth Arrow’s and Amartya Sen’s Nobel-Prize-winning impossibility and possibility results on the aggregation of preferences.⁴ Through these influences, Condorcet’s work has left its mark on how we think about collective decision making.

The aim of this paper is to revisit Condorcet’s paradox from a fresh perspective, namely that of recent work on the aggregation of judgments and other propositional attitudes,⁵ and to draw broader lessons for the theory of democracy. I will recast the

³ Contributions include Joshua Cohen, “An Epistemic Conception of Democracy,” *Ethics* 97 (1986): 26–38; Jules L. Coleman and John Ferejohn, “Democracy and Social Choice,” *Ethics* 97 (1986): 6–25; David Estlund, Jeremy Waldron, Bernard Grofman and Scott L. Feld, “Controversy: democratic theory and the public interest: Condorcet and Rousseau revisited,” *American Political Science Review* 83 (1989): 1317–40; Estlund, “Making Truth Safe for Democracy,” in *The Idea of Democracy*, ed. David Copp, Jean Hampton and John E. Roemer (Cambridge: Cambridge University Press, 1993), pp. 71–100; Estlund, “Beyond Fairness and Deliberation: The Epistemic Dimension of Democratic Authority,” in *Deliberative Democracy: Essays on Reason and Politics*, ed. James Bohman and William Rehg (Cambridge/MA: MIT Press, 1997), pp. 173–204; and List and Goodin, “Epistemic Democracy.” In his book, *Democratic Authority: A Philosophical Framework* (Princeton: Princeton University Press, 2009), Estlund distances himself from Condorcet’s jury theorem, but still acknowledges the need to frame his discussion in relation to it.

⁴ E.g., Kenneth J. Arrow, *Social Choice and Individual Values*, (New York: Wiley, 1951/1963), and Amartya K. Sen, *Collective Choice and Social Welfare* (San Francisco: Holden-Day, 1970). A review of the literature is beyond the scope of this paper. William Riker sparked a debate by arguing that these problems challenge the meaningfulness of democracy; see *Liberalism against Populism* (San Francisco: W.H. Freeman and Co., 1982). On the frequency of Condorcet cycles under some simple assumptions, see William V. Gehrlein, “Condorcet’s Paradox,” *Theory and Decision* 15 (1983): 161–97.

⁵ See, among many other works, Christian List and Philip Pettit, “Aggregating Sets of Judgments: An Impossibility Result,” *Economics and Philosophy* 18 (2002): 89–110; List and Pettit, “Aggregating Sets of Judgments: Two Impossibility Results Compared,” *Synthese* 140 (2004): 207–35; Marc Pauly and Martin van Hees, “Logical constraints on judgement aggregation,” *Journal of Philosophical Logic* 35 (2006): 569–85; Franz Dietrich, “Judgment aggregation: (im)possibility theorems,” *Journal of Economic Theory* 126 (2006): 286–98; Dietrich, “A generalised model of judgment aggregation,” *Social Choice and Welfare* 28 (2007): 529–65; Dietrich and List, “Arrow’s theorem in judgment aggregation,” *Social Choice and Welfare* 29 (2007): 19–33; Klaus Nehring and Clemens Puppe, “Abstract Arrovian Aggregation,” *Journal of Economic Theory* 145 (2010): 467–94; and Elad Dokow and Ron Holzman, “Aggregation of binary evaluations,” *Journal of Economic Theory* 145 (2010): 495–511. This literature draws on related work on the “doctrinal paradox” in law and economics, especially Lewis A. Kornhauser and Lawrence G. Sager,

paradox as a conflict between three initially plausible requirements of democracy, which I will call “robustness to pluralism,” “basic majoritarianism,” and “collective rationality.” For all but the simplest collective decision problems, no decision procedure can meet these three requirements at once; at most two of them can be met together. I will call this problem the “democratic trilemma.” Condorcet’s original paradox provides only one illustration of this conflict, and its relationship to the more general trilemma will become clear in the course of the argument.

The democratic trilemma raises the question of which requirement to give up. Since different answers to this question correspond to different views about what matters most in a democracy, the trilemma can be used to draw a map of the logical space in which different conceptions of democracy are located. A conception that relaxes basic majoritarianism, for example, is very different from one that relaxes robustness to pluralism or collective rationality. The resulting map illuminates some of the trade-offs we face when we try to find a compelling conception of democracy. The trilemma also sharpens our thinking about other impossibility results of social choice theory, including some of Arrow’s and Sen’s famous theorems, and how to avoid them. It captures a core structure many of these problems have in common.

While the technical ideas underlying this paper are well-established in existing social-choice-theoretic work (especially on judgment aggregation), the paper’s contribution lies in the way these ideas are used to obtain a novel map of the logical space of democracy. Despite abstracting away from many details of real-world collective decision problems, this alerts us to some surprising trade-offs in the design of democratic

“Unpacking the Court,” *Yale Law Journal* 96: 82–117; and Kornhauser and Sager, “The One and the Many: Adjudication in Collegial Courts,” *California Law Review* 81 (1993): 1–59. The literature also draws on related work in abstract aggregation theory, especially Robert Wilson, “On the Theory of Aggregation,” *Journal of Economic Theory* 10 (1975): 89–99; Ariel Rubinstein and Peter C. Fishburn, “Algebraic Aggregation Theory,” *Journal of Economic Theory* 38 (1986): 63–77; and Nehring and Puppe, “Strategyproof Social Choice on Single-Peaked Domains: Possibility, Impossibility and the Space Between,” working paper (University of California, Davis). For contributions addressed to political philosophers, see, e.g., Pettit, “Deliberative Democracy and the Discursive Dilemma,” *Philosophical Issues* (supplement to *Nous*) 11 (2001): 268–99; Bruce Chapman, “Rational Aggregation,” *Politics, Philosophy and Economics* 1 (2002): 337–54; Kornhauser and Sager, “The Many as One: Integrity and Group Choice in Paradoxical Cases,” *Philosophy and Public Affairs* 32 (2004): 249–76; List and Pettit, “On the Many as One: A Reply to Kornhauser and Sager,” *Philosophy and Public Affairs* 33 (2005): 377–90; and List, “The Discursive Dilemma and Public Reason,” *Ethics* 116 (2006): 362–402. For a survey, see List and Puppe, “Judgment aggregation: a survey,” *The Handbook of Rational and Social Choice*, ed. Paul Anand, Prasanta Pattanaik and Clemens Puppe (Oxford: Oxford University Press, 2009), pp. 457–82.

decision procedures. Over and above this specific contribution, my broader aim is to illustrate the relevance of the axiomatic method for democratic theory more generally (beyond formal social choice theory), and to support the idea of “cartography of logical space” in relation to contested political concepts. The present approach to mapping out the logical space of democracy provides just one example of what such a cartographic exercise might look like. If readers find this approach natural and elegant, and perhaps worth replicating in the case of other contested political concepts, the paper will have achieved its purpose.

I. THE UNIVERSE OF COLLECTIVE DECISION PROBLEMS

Collective decision problems occur at many levels of social organization. While national and local elections, referenda and legislative decisions are the most familiar examples, collective decision problems occur in a great variety of social units, ranging from families, local communities and private organizations at one end of the spectrum to international and global bodies such as the European Union and the United Nations Security Council at the other. Outside conventional political contexts, collective decisions are also made in epistemic contexts, such as in expert panels, collegial courts, groups of scientific collaborators, and fact-finding commissions.

To model decision problems in general, it is useful to think of them as involving the formation of intentional attitudes towards certain propositions, which subsequently determine the decision makers’ actions. Depending on the context, these attitudes can be “representational,” such as beliefs, or “motivational,” such as preferences. Representational attitudes encode the way an agent represents the world as being, while motivational attitudes encode the way the agent wants the world to be. A rational agent then acts, roughly speaking, so as to satisfy his or her motivational attitudes in accordance with his or her representational attitudes.⁶

For example, if I prefer to drink coffee and believe there is coffee available next door, I may rationally act by going there, assuming no complicating factors. This picture

⁶ For a discussion of the roles of intentional attitudes in individual and collective agency, see List and Pettit, *Group Agency: The Possibility, Design and Status of Corporate Agents* (Oxford: Oxford University Press, 2011). The underlying model of agency goes back to David Hume, *A Treatise of Human Nature*, reprinted from the original and edited by L.A. Selby-Bigge (Oxford: Clarendon Press, 1739/1896). See also Daniel Dennett, *The Intentional Stance* (Cambridge/MA: MIT Press, 1987).

applies not just to individuals but also to groups making decisions. A state troubled by poverty may form the preference to lift people out of poverty, and the belief that a particular policy will achieve this goal, and then act by adopting the policy. Political decisions usually involve the formation of motivational attitudes, such as the preference to reduce poverty or to increase economic growth, and epistemic decisions involve the formation of representational attitudes, such as the belief that a defendant is guilty, or that current greenhouse gas emissions will lead to a global temperature increase. On an epistemic interpretation of politics, political decisions resemble, or centrally involve, epistemic decisions, but I do not need to take a stand on this issue here.

Regardless of whether representational or motivational attitudes are formed, most decision problems are subject to certain constraints of consistency. If one forms the belief that p and the belief that if p then q , for instance, one cannot consistently form the belief that $\text{not-}q$ as well. Here the consistency constraints come from logic. Similarly, if one prefers A to B and B to C , one cannot consistently prefer C to A , as in Condorcet's paradox. In this case, the consistency constraints are given by the standard rationality requirements on preferences, especially transitivity.

While Condorcet—and later Arrow, Sen and most of contemporary social choice theory—focused on decision problems involving the *ranking of options*, we can lift this restriction by considering a larger class of decision problems, defined in terms of the *formation of propositional attitudes*. A decision problem in this sense is given by:

- (i) a set of propositions, together with their negations, which are to be accepted or rejected—call this set the “agenda”—and
- (ii) some constraints specifying which combinations of propositions can be consistently accepted and which not.⁷

Solving the decision problem then requires arriving at a set of accepted propositions that meets the specified consistency constraints and, ideally, also the constraint of “completeness.” The latter requires that, for every proposition–negation pair on the

⁷ For different versions of this logic-based model of a decision problem, at increasing levels of generality, see List and Pettit, “Aggregating Sets of Judgments,” Dietrich, “A generalised model of judgment aggregation,” and Dietrich and List, “The aggregation of propositional attitudes: towards a general theory,” *Oxford Studies in Epistemology* 3 (2010): 215–34. For a related abstract model not based on logic, see Nehring and Puppe, “Abstract Arrovian Aggregation,” and Dokow and Holzman, “Aggregation of binary evaluations.”

agenda, either the proposition or its negation be accepted. The interpretation of “acceptance” and “rejection” depends on the kind of attitudes in question. In the representational case, accepting a proposition means believing it to be true; in the motivational case, it can mean something like desiring or preferring it to be true.

Expressed in this model, the agenda in a preference aggregation problem as considered by Condorcet and Arrow contains pairwise ranking propositions of the form:

- “A is preferred (or preferable) to B,”
- “B is preferred (or preferable) to C,”
- “A is preferred (or preferable) to C,”

and the reverse of each.⁸ Accepting all three propositions, for example, which corresponds to preferring A to B to C, is deemed to be consistent, while accepting the first two propositions and the negation of the last is not, since it involves a violation of transitivity: a preference for A over B, for B over C, and yet for C over A. Any ranking of a set of options in an order of preference can be formally translated into a set of accepted propositions of the kind just introduced, where the rationality constraints on preferences, such as transitivity, turn into corresponding consistency constraints on the admissible acceptance–rejection patterns across propositions.

In a paradigmatic epistemic decision problem, to give an example not involving rankings, the agenda might contain:

- an empirical premise, such as “CO₂ emissions are above a certain threshold,”
- a causal claim, such as “if emissions are above that threshold, then the global temperature will increase by 2 degrees Celsius,” and
- a prediction, such as “the global temperature will increase by 2 degrees Celsius,”

and their negations. Here again, accepting all three propositions is consistent, while accepting the first two and the negation of the last is not, this time interpreting consistency as in standard propositional logic. Since the need to arrive at collective

⁸ This construction is formally given in List and Pettit, “Two Impossibility Results Compared,” and Dietrich and List, “Arrow’s theorem in judgment aggregation,” but the idea of a propositional representation of preferences goes back to Georges-Théodule Guilbaud, “Theories of the General Interest, and the Logical Problem of Aggregation,” in *Readings in Mathematical Social Science*, ed. Paul F. Lazarsfeld and Neil W. Henry (Cambridge/MA: MIT Press, 1966), pp. 262–307; and even to Condorcet’s *Essay*. The present representation can be interpreted in various ways, as implicit in the contrast between “preferred” and “preferable” in the formulation of ranking propositions; the details do not matter for present purposes.

judgments on logically connected issues arises in many decision-making bodies, from courts and expert panels to commercial corporations and other purposive organizations, the kinds of decision problems not based on rankings are common in the real world.

It should be evident that a large class of collective decision problems can be expressed in the present model: not just the preference ranking problems of standard social choice theory but also all decision problems in which “true–false” or “yes–no” judgments have to be made on a set of propositions, including very complex ones. Thus we have at our disposal a very general model of collective decision problems.⁹ The challenge now is to find a procedure that a group can use to solve such problems.

II. THE UNIVERSE OF COLLECTIVE DECISION PROCEDURES

Just as there are many different collective decision problems, so there are many different collective decision procedures, ranging from democratic and participatory ones to hierarchical and even dictatorial ones. How can we think about such procedures systematically?

Decision procedures can be studied as objects in their own right. This is the subject-matter of social choice theory. Historically, social choice theorists studied the properties of *specific* such procedures. While Condorcet focused on the properties of majority rule, his contemporary Jean-Charles de Borda advocated an alternative, which ranks options by summing up the “scores” implicit in the voters’ rankings of them, such as a score of k for each voter’s top-ranked among k options, a score of $k-1$ for the second-ranked option, and a score of 1 for the bottom option.¹⁰ Others similarly studied decision procedures they considered interesting or politically salient. In the 19th century, for example, Charles Dodgson, better known as Lewis Carroll, devoted some of his mathematical work—his

⁹ The model can be further generalized so as to capture decision problems that involve the formation of non-binary propositional attitudes, as discussed in Dietrich and List, “The aggregation of propositional attitudes.” But since real-world democratic decisions typically take a discrete—especially binary—form (requiring the acceptance or rejection of certain propositions, or the ranking of certain options in an order of preference), I set the non-binary case aside here.

¹⁰ For an English translation of, and commentary on, Borda’s proposal, see McLean and Urken, *Classics of Social Choice*.

day job while he was not writing *Alice in Wonderland*—to investigating electoral systems based on proportional representation.¹¹

But we can also study decision procedures in another way. Instead of focusing on *specific* procedures, we can consider the logical space of *all possible* decision procedures, and ask which of them satisfy certain requirements. This is the “axiomatic” approach pioneered by Arrow.¹² Here, however, I will apply the approach not just to preference ranking problems, which Arrow considered, but to the larger class of decision problems introduced in the last section.¹³ Suppose, then, a group of individuals seeks to arrive at collective attitudes, each in the form of acceptance or rejection, on a given agenda of propositions. We can model a decision procedure as an input–output scheme, formally a function, which takes the individuals’ attitudes towards the relevant propositions as input and produces collective attitudes as output, as shown in Figure 1.

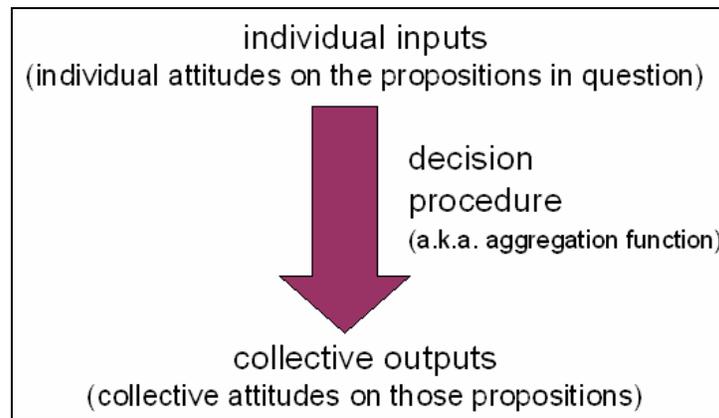


Figure 1: A decision procedure

Majority rule, under which the collective attitude towards any proposition is the attitude held by the majority of individuals, is only one example of such a function, and there are many other possibilities. But once we have also thought of various supermajority and unanimity rules, dictatorial rules (under which one individual’s attitudes always prevail), and perhaps weighted majority rules (under which different people have different weights in determining the collective attitudes), our intuitions tend

¹¹ See Iain S. McLean, Alistair McMillan, and Burt L. Monroe (eds.), *A mathematical approach to proportional representation: Duncan Black on Lewis Carroll* (Dordrecht: Kluwer, 1996).

¹² For an excellent discussion of the axiomatic method in social choice theory, see William Thomson, “On the axiomatic method and its recent applications to game theory and resource allocation,” *Social Choice and Welfare* 18 (2000): 327–86.

¹³ I thereby follow the technical literature on the aggregation of judgments and other propositional attitudes, as briefly reviewed in earlier notes.

to run out, and we find it harder to come up with further examples. How large, then, is the logical space of possible decision procedures for a given collective decision problem?

A simple example shows that, notwithstanding the limits of our imagination, it is truly enormous. Suppose a three-member group wishes to make a decision between just two options: whether A is preferable to B or the other way round. Table 1 lists all 8 ($=2^3$) possible combinations of individual attitudes on this issue, which we can here simply interpret as the individuals' votes. To specify a decision procedure, we must assign to each combination of votes (each row of the table) a collective choice; so we must complete the right-most column by replacing each question mark with a choice of A or B.

| | Individual 1 | Individual 2 | Individual 3 | Collective |
|--------------------------------------|--------------|--------------|--------------|------------|
| 1 st possible combination | A | A | A | ? |
| 2 nd possible combination | A | A | B | ? |
| 3 rd possible combination | A | B | A | ? |
| 4 th possible combination | A | B | B | ? |
| 5 th possible combination | B | A | A | ? |
| 6 th possible combination | B | A | B | ? |
| 7 th possible combination | B | B | A | ? |
| 8 th possible combination | B | B | B | ? |

Table 1: Possible combinations of individual attitudes

Each way of completing the column thus corresponds to one particular decision procedure. If we replace every question mark with an A, for instance, we obtain the “constant-A procedure,” under which the group always prefers A to B, regardless of its members' attitudes—a logically possible but obviously unattractive procedure. An equally unattractive possibility is the “constant-B procedure,” where every question mark is replaced by a B, again ignoring all individuals' attitudes. Majority rule is defined by replacing any question mark with an A when there are more As than Bs in the relevant row, and with a B otherwise. A dictatorship of one individual is defined by pasting a fixed individual's attitudes into the right-most column. If the right-most column replicates individual 1's column, for instance, then individual 1 always determines the group's choice between A and B. An even more perverse possibility is to put an A in the right-most column whenever there is a B in individual 1's column and *vice versa*. This

corresponds to an “inverse dictatorship,” under which the group’s attitudes are always the opposite of a fixed individual’s attitudes.

Since there are two possible collective choices, A or B, for each of the eight rows, there are 2^8 ways of completing the table, and hence 256 possible decision procedures. This is already a sizeable number, given that we are only dealing with a three-member group making a single binary decision. What happens if we increase the group size or consider more complex decision problems?

Suppose the group size grows from three to ten, but we keep the restriction to two options. By a straightforward extension of the reasoning just outlined, the number of logically possible decision procedures increases from 2^8 to 2^{1024} , since there are now 1024 ($=2^{10}$) possible combinations of individual attitudes, corresponding to a table of 1024, rather than 8, rows. Thus the number of possible decision procedures in this case is:

17976931348623159077293051907890247336179769789423065727343008
 11577326758055009631327084773224075360211201138798713933576587
 89768814416622492847430639474124377767893424865485276302219601
 24609411945308295208500576883815068234246288147391311054082723
 7163350510684586298239947245938479716304835356329624224137216.

This exceeds the number of elementary particles in the universe according to standard estimates, and we have not even considered more complex decision problems or larger group sizes. So we are faced with a dramatic combinatorial explosion: the logical space of possible decision procedures grows exponentially with increasing group size and increasing complexity of the decision problem.¹⁴

How can we make sense of this vast logical space? How can we ensure, in particular, that our choice of procedure is not merely *ad hoc*, or driven by a lack of imagination? This is where the axiomatic method can come into play. By specifying some requirements that any “good” decision procedure is expected to satisfy, we can narrow down the space of possibilities. Let me introduce three such requirements.

¹⁴ Generally, if there are x admissible combinations of individual inputs and y admissible collective outputs, there are y^x possible decision procedures. If each individual has z different choices, then x can be further expressed as z^n , where n is the group size. In the examples just discussed, $y = 2$, $z = 2$, and $n = 3$ or $n = 10$.

III. THREE INITIALLY PLAUSIBLE REQUIREMENTS OF DEMOCRACY

Although there is considerable disagreement on what exactly a decision procedure must look like in order to count as democratic, the following seem to be widely accepted necessary (though arguably not sufficient) requirements.

Robustness to pluralism: The decision procedure is able to function under conditions of pluralism, that is, it accepts as admissible input any possible combination of individual attitudes on the propositions on the given agenda, subject only to the constraints of individual consistency and completeness.¹⁵

In particular, no combinations of individual attitudes must be ruled out in advance as admissible inputs to the decision procedure, so long as they satisfy some minimal constraints of formal rationality. One might ideally want the procedure to cope even with inputs violating those constraints, since good democratic decisions are needed regardless of how rational people's attitudes are. But robustness to pluralism turns out to be very demanding already, and thus strengthening it further may not be feasible.

Basic majoritarianism: A necessary condition for the collective acceptance of any proposition on the agenda is its majority acceptance.

This majoritarian requirement is a "basic" one as it takes majority acceptance to be only a *necessary* condition for the collective acceptance of any proposition, not automatically a *sufficient* condition. Basic majoritarianism is thus less demanding than the standard majority principle and compatible with a wide range of decision procedures apart from majority rule itself.¹⁶ Supermajority or unanimity rules, for instance, which require more than majority support for the acceptance of any proposition, also meet the requirement.¹⁷

¹⁵ This is the universal-domain requirement in the literature on judgment or attitude aggregation. In the special case of preferences, it reduces to Arrow's universal domain condition.

¹⁶ Notice, further, that basic majoritarianism does *not* imply some of the other standard conditions commonly used to prove impossibility results in formal work on aggregation, such as "independence" or "systematicity." Independence requires that the collective attitude on each proposition depend only on individual attitudes on that proposition, not on individual attitudes on other conditions. Systematicity adds to this the requirement that the pattern of dependence be the same for all propositions.

¹⁷ For discussions of such rules, see, e.g., Goodin and List, "Special Majorities Rationalized," *British Journal of Political Science* 36 (2006): 213–41; and Dietrich and List, "Judgment aggregation by quota rules: majority voting generalized," *Journal of Theoretical Politics* 19 (2007), 391–424. By contrast, weighted majority rules, under which different individuals have different voting weights, do not generally satisfy the present basic majoritarian requirement. Harry Brighouse and Marc Fleurbaey advocate such

Collective rationality: The decision procedure produces as collective output consistent and complete attitudes on the propositions on the agenda.¹⁸

Collective rationality requires the procedure to deliver what may count as a full solution to any decision problem brought to it: a set of accepted propositions that respects the relevant consistency constraints (the “consistency” part) and leaves no proposition–negation pairs on the agenda undecided (the “completeness” part).

A lot could be said about each of the three requirements, and in the end we may wish to revise or redefine them. For the moment, I will treat them as “democratic platitudes” capturing commonly held intuitions about what a democratic procedure should minimally look like.¹⁹ As Robert Dahl distinguished between narrower and broader notions of democracy in his classic account of procedural democracy, so the present requirements might be best thought of as *prima facie* necessary conditions on a democratic procedure narrowly construed, that is, on a procedure that is democratic relative to a given agenda of issues and a given demos of decision-makers.²⁰ Most people will expect a full-blown democracy to meet stronger requirements, including requirements on the specification of the demos and the composition of the agenda.

rules on the grounds of a particular proportionality principle; see “Democracy and Proportionality,” *Journal of Political Philosophy* 18 (2008): 137–55. They suggest that, in certain collective decisions, each decision-maker’s voting weight should be proportional to his or her “stake” in the decision. Depending on the precise pattern of stakes, and on whether a simple or qualified version of weighted majority voting is used, the resulting decision procedure may well overrule the majority attitude—not, however, the attitude supported by more than half of the total voting weight. Although Brighthouse and Fleurbaey’s proposal may require a revision in the formulation of “basic majoritarianism,” it is not generally immune to the kinds of technical difficulties illustrated by what I call the “democratic trilemma.”

¹⁸ This is the collective-rationality requirement in the literature on judgment aggregation. Again, in the special case of preferences, it reduces to Arrow’s ordering condition.

¹⁹ For present purposes, I set aside the distinct case of procedures for ranking distributions of certain goods or units of welfare in an order of social preference. The application of social choice theory to such problems of distributive justice is very important, but raises somewhat different challenges as compared to the paradigmatic cases of democratic decision making (in elections, referenda, legislatures, committees etc.). On the justice-theoretic side of social choice theory, see, e.g., John E. Roemer, *Theories of Distributive Justice* (Cambridge/MA: Harvard University Press, 1996); and Marc Fleurbaey, *Fairness, Responsibility, and Welfare* (New York: Oxford University Press, 2008).

²⁰ See, e.g., Robert A. Dahl, “Procedural Democracy,” in *Philosophy, Politics and Society*, 5th series, ed. Peter Laslett and James S. Fishkin (Oxford: Blackwell, 1979).

IV. THE DEMOCRATIC TRILEMMA

Can we find a decision procedure that meets the three requirements? In light of the sheer size of the logical space of possibilities, one would expect to find at least one such procedure, and possibly many. Surprisingly, however, the following result holds:

The democratic trilemma: For all but the simplest collective decision problems, there exists no decision procedure satisfying robustness to pluralism, basic majoritarianism, and collective rationality. At most two of these requirements can be met at once.²¹

I sketch a general proof in the Appendix. To illustrate the trilemma more informally, let me give some simple examples of how the conflict between the three requirements arises.

Example 1: weapons of mass destruction

Suppose a multi-member government is making judgments on the following three propositions and their negations:²²

- “Country X has weapons of mass destruction.”
- “We should invade country X if and only if it has weapons of mass destruction.”
- “We should invade country X.”

Since different cabinet members may disagree on these issues, the government needs a procedure to arrive at its collective judgments. Suppose, for the sake of argument, that there exists a procedure satisfying robustness to pluralism, basic majoritarianism, and collective rationality. I will show that this supposition leads to a contradiction.

By robustness to pluralism, the procedure must be able to cope with conditions of pluralism. In particular, it must accept as admissible input the combination of individual judgments shown in Table 2. Here one third of the group holds that there are weapons of mass destruction and that the presence of such weapons is necessary and sufficient for an

²¹ As formally stated here, this result is more general than the familiar observation that majority voting does not generally secure consistent collective attitudes, which has been shown at different levels of generality in the literature on judgment aggregation. For the versions of this standard result most closely related to the present point, see List and Pettit, “Aggregating Sets of Judgments,” Dietrich and List, “Judgment aggregation by quota rules,” Nehring and Puppe, “The structure of strategy-proof social choice – Part I: General characterization and possibility results on median spaces,” *Journal of Economic Theory* 135 (2007): 269–305; and Nehring and Puppe, “Abstract Arrovian Aggregation.” Note that the present contribution is not so much the formal result, but rather its substantive interpretation.

²² This example was introduced in Dietrich and List, “Judgment aggregation by quota rules.”

invasion, and therefore supports an invasion. A second third agrees that the presence of weapons is necessary and sufficient for an invasion but thinks that there are no such weapons, and thus opposes an invasion. The final third does not think that there are any weapons but does not consider their presence necessary for an invasion, and can therefore consistently support an invasion on different grounds, for example for economic reasons.

| | “There are WMD.” | “We should invade if and only if there are WMD.” | “We should invade.” |
|------------------------|------------------|--|---------------------|
| 1/3 of the individuals | Yes | Yes | Yes |
| 1/3 of the individuals | No | Yes | No |
| 1/3 of the individuals | No | No | Yes |

Table 2: The cabinet members’ judgments

What should the collective judgments be? By basic majoritarianism, the collective judgment on each proposition, including any negated proposition, must either be the majority judgment on the proposition or be silent on it. So the government as a whole can either form the view that there are no weapons of mass destruction, following the majority, or take no opinion on this issue; adopting the collective view that there *are* weapons would breach basic majoritarianism. Similarly, the government can either form the view that an invasion should take place if and only if there are such weapons, again following the majority, or take no opinion on this issue; it cannot accept the negation of the proposition consistently with basic majoritarianism. Finally, the government can either form the view that an invasion should take place, following the majority, or take no opinion on this issue; accepting the negation would go against basic majoritarianism.

Collective rationality, however, will not be satisfied unless the collective judgments are both consistent and complete. The completeness requirement rules out the possibility of taking no opinion on some proposition–negation pairs on the agenda, and thus the collective view must be that there are no weapons of mass destruction, that the presence of such weapons is necessary and sufficient for an invasion, but that there should be an invasion nonetheless. This violates consistency, and so the decision procedure does not satisfy collective rationality, contrary to our supposition.

Example 2: global warming

Suppose an expert panel, such as the Intergovernmental Panel on Climate Change, is making judgments on the following three propositions and their negations:

- “The atmospheric CO₂ concentration will increase above 500 ppm by 2050.”
- “If atmospheric CO₂ increases above this level by 2050, the Greenland ice shield will melt by 2150.”
- “The Greenland ice shield will melt by 2150.”

Different experts may, of course, disagree on these propositions, and the panel may wish to resolve these disagreements democratically. Suppose, for the sake of argument, that there exists a decision procedure satisfying robustness to pluralism, basic majoritarianism and collective rationality. As before, robustness to pluralism requires the procedure to cope with any individual inputs fed into it, assuming they are consistent and complete, and so it must admit the combination of judgments shown in Table 3, where each expert holds a perfectly consistent and complete set of views on the issues in question.

| | “CO ₂ will increase above 500 ppm.” | “If CO ₂ increases above this level, the Greenland ice will melt.” | “The Greenland ice will melt.” |
|----------|--|---|--------------------------------|
| Expert 1 | Yes | Yes | Yes |
| Expert 2 | No | Yes | No |
| Expert 3 | Yes | No | No |

Table 3: The experts’ judgments

By basic majoritarianism, the expert panel must either adopt the majority judgment on each proposition on the agenda or be silent on it. In light of the majority views, the expert panel must therefore accept the judgment that atmospheric CO₂ will increase above 500 ppm by 2050, or be silent on this issue. Likewise, it must either hold that *if* CO₂ increases above this level, *then* the Greenland ice shield will melt by 2150, or be silent on this issue. And finally, it must either hold that the Greenland ice shield will *not* melt by 2150, or be silent on this issue. Basic majoritarianism precludes the collective acceptance of any proposition rejected by a majority.

Collective rationality, as before, requires the collective judgments to be both consistent and complete. Again, completeness rules out the possibility of forming no judgment on some proposition–negation pairs on the agenda, and hence the expert panel

is constrained to judge that CO₂ will increase above 500 ppm, that *if* CO₂ increases above this level, *then* the Greenland ice will melt, and yet that the ice will *not* melt, an inconsistent set of judgments. As in the earlier example, the decision procedure fails to satisfy collective rationality, contrary to our supposition.

Condorcet's paradox as a special case

Before I turn to the three “horns” of the trilemma, I should clarify its relationship with Condorcet’s paradox. As noted, Condorcet focused on decision problems involving the ranking of options, rather than the more general class of decision problems considered here. But as also noted, the decision problems that give rise to Condorcet’s paradox can be re-expressed in the present model, by taking the agenda to contain the propositions “A is preferred/preferable to B,” “B is preferred/preferable to C,” “A is preferred/preferable to C,” and their opposites. What are the properties of this agenda?

Its central feature for present purposes is that it permits the occurrence of patterns of consistent and complete individual attitudes for which the resulting majority attitudes are inconsistent relative to the standard constraints on preferences, especially transitivity. The combination of preferences that Condorcet used to illustrate this point involves a third of the electorate preferring A to B to C, a second third preferring B to C to A, and the last third preferring C to A to B. When translated into the present framework, as shown in Table 4, these preferences are structurally similar to the experts’ attitudes in Table 3 above, though of course interpretationally different. So, by essentially the same argument as in the government and expert-panel examples, there is no decision procedure satisfying robustness to pluralism, basic majoritarianism and collective rationality here.²³

²³ Expressed in terms of the formal conditions for the democratic trilemma stated in the Appendix, the present agenda has a minimally inconsistent subset of three propositions: the set consisting of “A is preferred/preferable to B,” “B is preferred/preferable to C,” “C is preferred/preferable to A.” This set is *inconsistent* relative to the standard constraints on preferences, by violating transitivity, and *minimally* so, since the removal of any of its elements breaks the preference cycle and thereby restores consistency. Generally, if we wish to rank k options in an order of preference, the largest minimally inconsistent subset of the agenda contains k propositions, corresponding to a preference cycle of length k . This shows that, like Condorcet’s paradox, the democratic trilemma applies to a preference aggregation problem if and only if there are three or more options to be ranked. See also List and Pettit, “Two Impossibility Results Compared,” Dietrich and List, “Arrow’s theorem in judgment aggregation,” and “Judgment aggregation by quota rules.”

| | “A is pref. to B.” | “B is pref. to C.” | “A is pref. to C.” |
|-----------------------|--------------------|--------------------|--------------------|
| 1/3 of the electorate | Yes | Yes | Yes |
| 1/3 of the electorate | No | Yes | No |
| 1/3 of the electorate | Yes | No | No |

Table 4: Condorcet’s paradox revisited

Condorcet’s paradox can thus be interpreted as a special case of the more general democratic trilemma. The trilemma quantifies over *all but the simplest* collective decision problems involving the formation of intentional attitudes (of an acceptance–rejection kind) and *all* decision procedures and shows that, whichever procedure we pick, at least one of our three requirements will be violated. Condorcet’s paradox shows that, in a decision on how to rank three or more options, the *particular* procedure of majority rule fails to deliver a rational collective output for the *particular* combination of inputs constructed by Condorcet.

In short, Condorcet did identify what can go wrong with majority rule in decisions involving the ranking of options, but he did not have the axiomatic tools to conceptualize this as an instance of a more general trade-off between different requirements on a democratic procedure. At the end of this paper, I will relate the democratic trilemma to the best-known modern generalization of Condorcet’s paradox, Arrow’s impossibility theorem, and to one of Sen’s famous results.

V. A MAP OF THE LOGICAL SPACE

I have shown that there is a conflict between three initially plausible requirements of democracy. Any decision procedure will either fail to be fully robust to pluralism in its input, or sometimes overrule majorities, or sometimes deliver an incomplete or even inconsistent collective output. Depending on which “horn” of the trilemma we choose, and how we do so, we arrive at a different conception of democracy. In what follows, I will provide a rough map of the resulting logical space. I will keep the discussion as non-technical as possible and focus on general ideas, rather than details. A more fine-grained map could be given by drawing further on technical work in social choice theory.

Relaxing robustness to pluralism

One way to avoid the trilemma is to restrict the domain of admissible inputs to the democratic procedure. On this approach, the procedure is no longer required to cope with all possible combinations of consistent and complete individual attitudes, but only with those that satisfy some additional constraints. Crucially, those constraints must be strong enough to ensure that a procedure such as majority rule will never generate inconsistent outputs. The weakened robustness requirement will then be compatible with basic majoritarianism and collective rationality.

If the procedure had to cope only with unanimous individual attitudes, to give the simplest example, then majority rule would obviously work without violating basic majoritarianism or collective rationality, assuming individual rationality. However, full unanimity is only sufficient, but not necessary for this happy result. Even certain forms of partial consensus or cohesion among individual attitudes are enough. In the technical literature, several “domain-restriction” conditions on combinations of individual attitudes have been identified that are sufficient to guarantee consistent majority outcomes.²⁴

The most easily interpretable such conditions take the form of requiring a certain kind of “meta-consensus” among the individuals, a consensus not on what the right attitudes are, but on what the disagreement is about.²⁵ Suppose, for example, the individuals can be aligned from left to right on some cognitive or political axis, such as from socio-economic left to socio-economic right, from secular to religious, or from urban to rural. Suppose, further, this axis structures the disagreement among the individuals in the sense that, for every proposition on the agenda, the individuals accepting the proposition lie on the opposite side of those rejecting it. This pattern may be plausible at least in some political contexts. Left-leaning individuals and right-leaning

²⁴ See Duncan Black, “On the Rationale of Group Decision-Making,” *Journal of Political Economy* 56 (1948): 23–34. Other key contributions include K.-I. Inada, “A Note on the Simple Majority Decision Rule,” *Econometrica* 32 (1964): 525–31; and Amartya K. Sen, “A Possibility Theorem on Majority Decisions,” *Econometrica* 34 (1966): 491–99. For a survey of the literature and more recent results, see Wulf Gaertner, *Domain Conditions in Social Choice Theory* (Cambridge: Cambridge University Press, 2001); and Dietrich and List, “Majority voting on restricted domains,” *Journal of Economic Theory* 145 (2010): 512–43.

²⁵ On the notion of “meta-consensus” and the pattern of “unidimensional alignment” as discussed here, see List, “Two Concepts of Agreement,” *The Good Society* 11 (2002): 72–9. For some further, subsequent notions of “meta-consensus,” see John Dryzek and Simon Niemeyer, “Reconciling pluralism and consensus as political ideals,” *American Journal of Political Science* 50 (2006): 634–49.

ones often fall on opposite sides with respect to many political issues, from redistributive policies to military interventions. If this pattern holds, the majority attitudes will coincide with the individual attitudes of whoever occupies the median position on the relevant left-right axis, that is, the individual who has an equal number of others to the left and to the right. So long as the median individual's attitudes are consistent, majority rule then produces a perfectly consistent outcome, and even one that can be seen as a compromise by occupying a centrist political position. Another well-known domain-restriction condition that allows a "meta-consensus" interpretation, and which applies specifically to Condorcetian preference aggregation problems, is that of "single-peakedness."²⁶ Here it is not the individuals, but the decision options, which have to be aligned from left to right on some shared cognitive or political axis. The details do not matter for present purposes. The point is that *if* we require the decision procedure to work only under conditions of suitably restricted—or "structured"—pluralism, *then* the democratic trilemma goes away.

How plausible is this escape route from the trilemma? When we design a democratic decision procedure for a given group or society, how much can we rely on the occurrence of the necessary forms of cohesion or partial consensus among the decision makers? In answer to this question, we can distinguish between "exogenous" and "endogenous" approaches to limiting pluralism. On the exogenous approach, a certain level of cohesion or attitudinal homogeneity is taken to be a *precondition* for democratic decision making. The key idea is that democracy cannot get off the ground unless pluralism in the relevant group or society is sufficiently limited. Communitarians or liberal proponents of the traditional nation state, for example, may find this idea plausible.²⁷ They are likely to interpret the democratic trilemma as reinforcing a point they already make in other contexts, namely that democracy works well only when individual attitudes are sufficiently cohesive—in particular, when the members of the relevant society, while disagreeing on many issues, do not disagree too deeply. Similarly, some versions of the view, held by some political liberals, that democracy can cope only

²⁶ See Black, "On the Rationale of Group Decision-Making." The significance of Black's result was emphasized even by the critic of populist democracy, William Riker, in *Liberalism against Populism* (at p. 128): "If, by reason of discussion, debate, civic education, and political socialization, voters have a common view of the political dimension (as evidenced by single-peakedness), then a transitive outcome is guaranteed."

²⁷ Classic contributions include Michael Walzer, *Spheres of Justice* (New York: Basic Books, 1983); and David Miller, *On Nationality* (Oxford: Oxford University Press, 1995).

with “reasonable” disagreement, under a sufficiently demanding notion of “reasonableness,” may be interpreted as suggesting certain constraints on pluralism as a precondition for democratic decision making.²⁸

On the endogenous approach to limiting pluralism, by contrast, the required level of cohesion among the decision makers is not taken to be a precondition for setting up democratic institutions in the first place, but is expected as the outcome of certain structuration processes *within the democratic system*. In particular, the aim is to set up democratic institutions so as to shape individual attitudes through education or deliberation in a way that enables consistent majoritarian decision making.²⁹ Those democrats who emphasize the importance of opinion formation through education, political participation and deliberation, beyond the democratic decision procedure narrowly construed, are likely to support this response to the democratic trilemma. Even in Robert Dahl’s primarily procedurally oriented account of democracy, the presence, or promotion, of an “enlightened understanding” among the decision makers, which can be seen as a deliberative democratic ideal, is one of the broader requirements of democracy, over and above the thinnest necessary conditions.³⁰

Of course, the present proposal is a demanding one, and its success depends on certain contingent features of the democratic process. The central question is whether the patterns of individual attitudes that give rise to inconsistent majority outcomes as in Condorcet’s paradox or in the government and expert-panel examples would go away after sufficient deliberation. Evidence from deliberative opinion polls suggests that, under certain conditions, deliberation in groups of a few hundred people can promote the forms of cohesion needed to avoid inconsistent majority outcomes,³¹ but the generalizability of those findings to larger settings has not yet been fully explored.

²⁸ For a related discussion, see Thomas Christiano, *The Constitution of Equality: Democratic Authority and Its Limits* (Oxford: Oxford University Press, 2008).

²⁹ See, e.g., David Miller, “Deliberative Democracy and Social Choice,” *Political Studies* 40 [special issue] (1992): 54–67; Jack Knight and James Johnson, “Aggregation and Deliberation: On the Possibility of Democratic Legitimacy,” *Political Theory* 22 (1994): 277–96; and John Dryzek and Christian List, “Social Choice Theory and Deliberative Democracy: A Reconciliation,” *British Journal of Political Science* 33 (2003): 1–28.

³⁰ See, e.g., Dahl, “Procedural Democracy.”

³¹ See Christian List, Robert C. Luskin, James Fishkin and Iain McLean, “Deliberation, Single-Peakedness, and the Possibility of Meaningful Democracy: Evidence from Deliberative Polls,” working paper (London School of Economics, 2000/2006).

In summary, the first escape route from the democratic trilemma should appeal to communitarians as well as deliberative democrats, although they may be divided on whether the exogenous or the endogenous variant is more attractive.

Relaxing basic majoritarianism

A second way to avoid the trilemma is to relax the requirement that majority acceptance is generally necessary for the collective acceptance of any proposition, and instead to allow the majority to be overruled on some propositions on the agenda. Like the previous escape route from the trilemma, the present route can take several forms.

On one variant, which we may call the “exogenous” one, different propositions are explicitly given a different status, and each proposition’s status is taken to determine whether, and if so when, the majority attitude on it can be overruled. For example, some propositions may be deemed “prior” to others, where the order of priority is given on a temporal, logical or epistemic basis and where respecting the majority attitudes on propositions higher in that order is taken to be more important than respecting the majority attitudes on lower ones.³² So, if the majority view on a fairly peripheral matter clashes with the majority commitment on a more fundamental, perhaps constitutional matter, then the majority on the derivative matter may be overruled for the sake of achieving consistency with the more fundamental commitment, but not the other way round. An extension of this idea would be to make the attitudes on some privileged propositions, such as propositions about “inalienable rights,” completely unreviseable.

In the technical literature, several decision procedures have been formalized that capture the idea of prioritizing some propositions over others and letting the propositions’ status determine whether, and when, majority attitudes can be overruled.³³ Some of these procedures have become known as “premise-based” or “sequential priority procedures” and involve taking majority votes on certain fundamental “premises” first and then using the resulting majority attitudes as a basis for deriving the collective attitudes on other,

³² For non-technical discussions of this idea, see Pettit, “Deliberative Democracy and the Discursive Dilemma,” List and Pettit, “Aggregating Sets of Judgments,” and Chapman, “Rational Aggregation.”

³³ See, e.g., List, “A Model of Path-Dependence in Decisions over Multiple Propositions,” *American Political Science Review* 98 (2004): 495–513; Dietrich, “Aggregation theory and the relevance of some issues to others,” working paper (London School of Economics, 2006); and Franz Dietrich and Philippe Mongin, “The premise-based approach to judgment aggregation,” *Journal of Economic Theory* 145 (2010): 562–82.

less fundamental propositions—the “conclusions.” As before, the technical details do not matter here. In the real world, some forms of precedent-based decision making, as well as some forms of judicial review, can be seen as instances of this exogenous way of relaxing basic majoritarianism. The judicial version is, of course, precisely the kind of restriction on majoritarianism that Jeremy Waldron, in his much-cited critique of judicial review, objects to.³⁴ Interestingly, Waldron’s argument against judicial review rests on the premise that citizens share a certain commitment to rights, broadly defined, which can in fact be interpreted as a constraint on pluralism in the present terms.

A second way to relax basic majoritarianism may be described as the “endogenous” one. Here, there is no proposition on which the majority is always guaranteed to prevail. Instead, whether the majority attitude is allowed to stand as the collective attitude on any proposition is endogenous to the democratic process. For example, the decision-making group may engage in some collective “reflective equilibrium” process, in which the group seeks to arrive at the most coherent collective extrapolation, or rational reconstruction, of its members’ attitudes.³⁵ The group may begin by taking the majority attitudes on all propositions as its provisional attitudes and then identify the most plausible—perhaps least invasive—way to revise them to achieve overall coherence. In the real world, certain forms of legislative deliberation or collective reasoning, when successful, might approximate this ideal. In technical work, so-called “distance-based” decision procedures have been proposed, according to which the collective attitudes are chosen to minimize the total “distance” from individual attitudes, subject to the constraint of collective consistency.³⁶

Another example of a decision procedure that sacrifices basic majoritarianism in what I have described as an endogenous manner, while satisfying robustness to pluralism

³⁴ E.g., Jeremy Waldron, “The Core of the Case Against Judicial Review,” *Yale Law Journal* 115 (2006): 1346–406.

³⁵ The idea of “reflective equilibrium” goes back, of course, to John Rawls, *A Theory of Justice* (Cambridge/MA: Harvard University Press, 1971). In a judgment-aggregation context, a proposal along the present lines is discussed in List and Pettit, *Group Agency*, ch. 3.

³⁶ See, e.g., Sébastien Konieczny and Ramón Pino Pérez, “Merging information under constraints: a logical framework,” *Journal of Logic and Computation* 12 (2002): 773–808; Gabriella Pigozzi, “Belief merging and the discursive dilemma: an argument-based account to paradoxes of judgment aggregation,” *Synthese* 152 (2006): 285–98; and Michael K. Miller and Dan Osherson, “Methods for distance-based judgment aggregation,” *Social Choice and Welfare* 32 (2009): 575–601. In the special case of preference aggregation, the so-called “Kemeny method” is a well-known such distance-based decision procedure.

and collective rationality, is Borda's method mentioned earlier, which is applicable in the case of preference aggregation. Since this method ranks options by summing up the scores implicit in the voters' rankings of them, it always produces a consistent and complete outcome, but it can—depending on the pattern of individual preferences—sometimes yield pairwise rankings of options that reverse the corresponding majority preferences.³⁷ Again, my aim here is not to offer a detailed account of the present escape route from the democratic trilemma, but just to locate it on the map.

In summary, there are some recognizably democratic approaches to collective decision making that give up basic majoritarianism. These may involve precedent-based decision making, judicial review, something like the Borda method, or alternatively collective reasoning so as to implement the idea of a “reflective equilibrium.” It is interesting to observe how these approaches differ from the ones we encountered in the earlier discussion of relaxing robustness to pluralism.

Relaxing collective rationality

The third way to avoid the democratic trilemma is to give up the requirement that collective attitudes must always be consistent and complete. We can relax this requirement in at least two ways: we can drop either the consistency part or the completeness part (or both). Since we are interested in collective decision making, however, the collective attitudes must provide some degree of action-guidance, and violations of consistency would compromise this very significantly, if not rule it out altogether.³⁸ So, although giving up collective consistency is a logical possibility, I will not pursue it further here. Instead, I will focus on relaxing collective completeness. At first, we may also be reluctant to go along that route. The idea that the group should form a decisive view on every proposition–negation pair on the agenda seems well-motivated

³⁷ On arguments for and against the Borda method, see, for example, the debate between Mathias Risse, who advocates a variant of the Kemeny method (which, he argues, is Condorcetian in spirit), and Donald Saari, who advocates Borda's method. See Risse, “Arrow's Theorem, Indeterminacy, and Multiplicity Reconsidered,” *Ethics* 111 (2001): 706–34; and Saari, “Capturing the ‘Will of the People’,” *Ethics* 113 (2003): 333–49; followed by Risse, “Why the count de Borda cannot beat the Marquis de Condorcet,” *Social Choice and Welfare* 25 (2005): 95–113; and Saari, “Which is better: the Condorcet or Borda winner?” *Social Choice and Welfare* 26 (2006): 107–29.

³⁸ It is well-known that inconsistent attitudes make a decision maker—individual as well as collective—vulnerable to various forms of strategic exploitation by others, quite apart from the fact that, at least in classical logical terms, anything can be derived from an inconsistent set of commitments.

by the fact that all these propositions were put on the agenda in the first place—presumably because a decision on them was needed. But since one of the three requirements leading to the trilemma has to go, it makes sense to explore what happens if we relax collective completeness.

Again, it is useful to distinguish between “exogenous” and “endogenous” variants of the present route. On an “exogenous” approach, the agenda is partitioned in advance into those propositions that require adjudication, and those on which no collective attitude is needed. A procedure such as majority rule is then applied to the first set of propositions, while the group refrains from forming attitudes on the second.³⁹ Crucially, the first set of propositions must be specified so as to avoid any non-trivial logical connections between them, because otherwise the majority attitudes on them could still be inconsistent. But, assuming no such connections between propositions in the first set, and collective “abstention” on the second, overall consistency will be achieved, albeit at the cost of producing incomplete collective attitudes. A decision procedure along these lines is sometimes called a “conclusion-based procedure.” In the earlier government example, the cabinet as a whole might vote only on whether to invade country X, while not forming any collective views on the underlying empirical and normative premises.

We may or may not find this approach plausible. On some conceptions of democracy, making decisions not only on action-propositions but also on underlying reasons is a key requirement for democratic accountability, which would be violated here.⁴⁰ However, Cass Sunstein has advocated an approach along the present lines in certain judicial settings, suggesting that judges should make decisions in a thin and foundationally uncommitted manner. A similar proposal could be made with regard to some international contexts or deeply divided societies, where agreements on anything beyond narrowly defined action-propositions may be too difficult to reach.⁴¹ Sunstein has introduced the term “incompletely theorized agreements” to refer to agreements on thinly specified practical matters without agreeing on the more fundamental supporting reasons. Incompletely theorized decision making is a real-world example of the present approach

³⁹ List and Pettit, “Aggregating Sets of Judgments,” and List, “The Discursive Dilemma and Public Reason.”

⁴⁰ See, e.g., Pettit, “Deliberative Democracy and the Discursive Dilemma.”

⁴¹ See, in particular, Cass Sunstein, *Political Conflict and Legal Agreement*, Tanner Lectures on Human Values (Harvard University, 1994); and *One case at a time: judicial minimalism on the Supreme Court* (Cambridge/MA: Harvard University Press, 1999).

to relaxing collective completeness. Likewise, a Rawlsian “overlapping consensus” can be interpreted as an incompletely theorized agreement on a public conception of justice without an agreement on any underlying comprehensive moral doctrines.

Depending on whether the distinction between “practical conclusions” and “underlying reasons” can be drawn in advance, or whether it is the outcome of the political process, incompletely theorized decision making may instantiate not only an exogenous approach to relaxing collective completeness, but also an endogenous one. On the endogenous approach, the agenda is not partitioned in advance into those propositions on which a collective decision is needed and those on which it is not, but a collective attitude on *any* proposition is formed only if there is a sufficiently strong agreement on it or, in the limit, a consensus. It can be shown that a supermajority rule with a sufficiently high threshold for the acceptance of any proposition (as well as for the acceptance of its negation) always guarantees consistent collective judgments.⁴² A feature of such a procedure is that even relatively small minorities can veto the collective acceptance of any proposition. While this may protect minorities from the tyranny of the majority, a negative consequence is that even groups with relatively mild disagreements will be at risk of stalemate, that is, often unable to reach any decisions. In the real world, many international organizations, including the UN Security Council or the EU Council of Ministers operate in this strongly consensual manner. Despite the difficulties involved, the approach is a possible response to the democratic trilemma.⁴³

Both exogenous and endogenous approaches to relaxing collective completeness give up the idea that the group as a whole should live up to the same stringent demands on intentional attitudes that we typically expect an individual decision maker to meet. These approaches thus reflect a “minimalist” conception of democracy, under which a democratic group is not viewed as a collective agent in its own right, but only as a loose collection of disparate individuals.⁴⁴

⁴² The acceptance threshold for any proposition must be above $(k-1)/k$, where k is the size of the largest “minimal inconsistent” subset of the agenda (a measure of its “complexity”). See Dietrich and List, “Judgment aggregation by quota rules.” In the examples above, $k = 3$ and thus a decision procedure requiring more than two thirds of the individuals for the acceptance of any proposition would guarantee consistent—albeit incomplete—collective attitudes.

⁴³ Relatedly, on “symmetrical supermajority rules,” see Goodin and List, “Special Majorities Rationalized.”

⁴⁴ On the notion of group agency, see List and Pettit, *Group Agency*.

Figure 2 shows the simple map of the logical space of democracy I have sketched. The large oval represents the space of all possible decision procedures for a given non-trivial decision problem, and the three circular regions represent the sets of decision procedures satisfying the requirements introduced. Graphically, the democratic trilemma consists in the fact that the intersection of the three circular regions is empty, while there is a non-empty overlap between any two of them; it is these regions that I have explored.

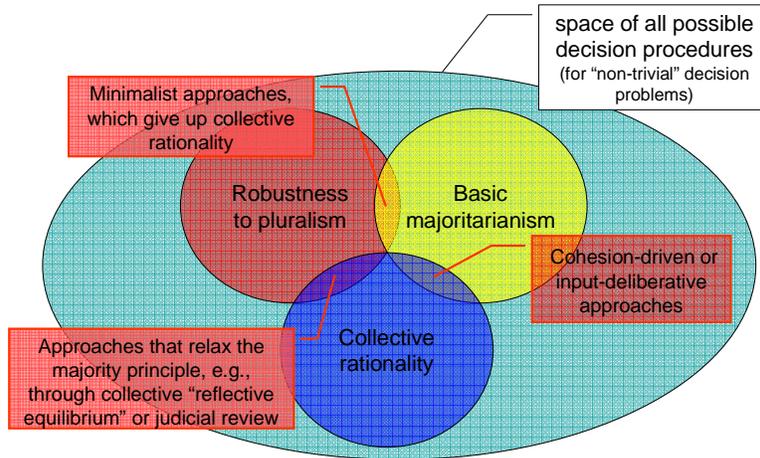


Figure 2: The logical space of democracy

VI. THE DEMOCRATIC TRILEMMA AS A GENERAL TEMPLATE

In conclusion, it is helpful to put the trilemma and the present cartographic exercise into a broader context. I have identified a conflict between the requirements of robustness to pluralism, basic majoritarianism and collective rationality. More abstractly, these can be described as “input,” “responsiveness” and “output” conditions on a democratic procedure. Input conditions specify what inputs the procedure should accept; responsiveness conditions specify how the procedure’s outputs should respond to these inputs; and output conditions specify what formal or other constraints the resulting outputs should obey. Robustness to pluralism, basic majoritarianism and collective rationality are perhaps the most paradigmatic instances of such conditions.

Although this terminology is not normally used, several prominent impossibility results of social choice theory can be interpreted as conflicts between certain input, responsiveness and output conditions, on the model of the democratic trilemma. I will briefly illustrate this by reference to Arrow’s impossibility theorem and Sen’s “liberal

paradox.” Both results concern preference aggregation problems in Condorcet’s tradition, where, as discussed above, a group of individuals has to rank some options in an order of collective preference, based on the group members’ individual preferences.

Arrow’s theorem states that, when there are three or more options, there exists no preference aggregation procedure satisfying the following five conditions:⁴⁵

Universal domain: The procedure accepts as admissible input any possible combination of rational individual preference orderings on the given options.

Ordering: The procedure produces as collective output a rational collective preference ordering on the given options.

Weak Pareto principle: If all individuals prefer an option A to another option B, then so does the group.

Independence of irrelevant alternatives: The collective preference between any two options depends only on the individual preferences between these two options, not on individual preferences involving other options.

Non-dictatorship: There is no fixed individual who always determines the collective preference.

The main point for present purposes is that these five conditions can be neatly subdivided into input, output and responsiveness conditions, following the template of the democratic trilemma. Universal domain and ordering are simply what the conditions of robustness to pluralism and collective rationality reduce to when applied to the case of preference aggregation, and they therefore constitute Arrow’s input and output conditions. Arrow’s other three conditions—the weak Pareto principle, independence of irrelevant alternatives, and non-dictatorship—are instances of responsiveness conditions; they constrain the way the individual inputs and the collective outputs hang together. In fact, the latter three conditions, together with universal domain, are satisfied by majority rule, which falls short only with regard to ordering, as we have already seen. In essence, Arrow’s theorem can be described as a variant of the democratic trilemma in the case of

⁴⁵ See Arrow, *Social Choice and Individual Values*.

preference aggregation, where basic majoritarianism is replaced by the conjunction of the weak Pareto principle, independence of irrelevant alternatives and non-dictatorship.

Next, let us turn to Sen's theorem on the impossibility of a Paretian liberal. Sen's result states that there exists no preference aggregation procedure satisfying the following four conditions:⁴⁶

Universal domain: As in Arrow's theorem.

Ordering: As in Arrow's theorem, although Sen's result also holds for a slightly weaker acyclicity condition.

Weak Pareto principle: As in Arrow's theorem.

Minimal liberalism: At least two individuals—call them i and j —are decisive on at least one pair of options each; that is, there is at least one pair of options on which i always determines the collective preference, and another pair on which j does so. (These pairs of options are usually interpreted as affecting the personal spheres of i and j , respectively.)

Again, we can reinterpret this result in the present terms. As already noted, universal domain and ordering are special cases of the democratic trilemma's input and output conditions. The responsiveness conditions of Sen's liberal paradox—the weak Pareto principle and minimal liberalism—depart further from the one of the democratic trilemma. In Sen's result, the emphasis is placed on respect for individual rights rather than basic majoritarianism. Still, it is instructive to see that Sen's theorem, too, can be presented as a conflict between an input condition, an output condition and a particular set of responsiveness conditions (albeit not of a majoritarian kind).

In this way, both Sen's and Arrow's theorems lend themselves to a cartographic exercise akin to the one sketched in this paper. Since each of these theorems uses multiple responsiveness conditions, not just one, we can naturally arrive at an even more fine-grained analysis of the logical space of relevant possibilities, by relaxing each of the responsiveness conditions individually. The main lesson for present purposes, however,

⁴⁶ See Amartya K. Sen, "The Impossibility of a Paretian Liberal," *Journal of Political Economy* 78 (1970): 152–57.

is that the democratic trilemma offers a general template in terms of which many social-choice-theoretic results can be framed, as illustrated in Table 5.⁴⁷

| | Democratic trilemma | Arrow's theorem | Sen's liberal paradox |
|------------------------------------|--|--|---|
| Relevant decision problems | Binary propositional attitude aggregation in general | Preference aggregation | Preference aggregation |
| Input condition(s) | Robustness to pluralism | Universal domain | Universal domain |
| Responsiveness condition(s) | Basic majoritarianism | Weak Pareto principle Indep. of irr. alternatives Non-dictatorship | Weak Pareto principle Minimal liberalism |
| Output condition(s) | Collective rationality | Ordering | Ordering/acyclicity |

Table 5: Conditions leading to an impossibility

Although I have focused here on drawing a simple map of the logical space of democracy, the idea of “cartography of logical space” can be generalized to many other contested political concepts, well beyond the realm of social choice theory and the theory of democracy. For any such concept, a cartographic exercise of the present kind would involve the following steps.⁴⁸

First identify a number of desiderata that any good instantiation of the concept might be expected to satisfy. It is perfectly acceptable for this initial list of desiderata to be such that no single philosophical view endorses them all. If these desiderata are mutually compatible, we are in the happy position of being able to fulfil them all at once, and we do not face any trade-offs. More often than not, however, the different desiderata will be in conflict, and this explains why the concept in question is a contested one.

The next step is to formulate the logically weakest variants of the desiderata for which their mutual incompatibility persists. In this “thinned-out” formulation, the desiderata should be “maximally independent” from one another: if there are k desiderata in total, any subset of $k-1$ or fewer of them should be mutually consistent and compatible

⁴⁷ This template is relevant not only to impossibility results, but also to possibility results where certain conditions characterize a *non-empty* class of decision procedures. Many such results can be interpreted in the present terms too, by classifying the relevant conditions as input, responsiveness and output conditions.

⁴⁸ For a comprehensive review and helpful methodological discussion of the axiomatic method as it is used in a number of different areas of economic theory, see again Thomson, “On the axiomatic method and its recent applications to game theory and resource allocation.”

with the negation of the others. Arriving at this thinned-out set of desiderata may require a certain amount of “reflective equilibration” on the part of the cartographer.

Once we have reached this point, we are able to draw a map of the relevant logical space, following the template of the democratic trilemma. And while we will not have resolved the conflict between the different desiderata, we will have achieved a better understanding of what the conflict is about and what solutions are and are not possible. With essentially contested concepts such as democracy, this may be the best we can do.

APPENDIX: THE GENERALITY OF THE TRILEMMA

The two examples I have given in the main text show that, for the given decision problems, there do not exist any decision procedures satisfying robustness to pluralism, basic majoritarianism and collective rationality. To see that this problem goes beyond these examples, we must introduce some additional concepts.

As noted, any decision problem can be modelled in terms of the agenda of propositions under consideration, together with the appropriate consistency constraints. By definition, the agenda is not a consistent set, since it contains proposition–negation pairs. But some of its subsets are consistent, while others are not. If the agenda contains propositions ‘p’, ‘if p then q’, ‘q’ and their negations, for example, as in the expert-panel case, then the subset consisting of the three un-negated propositions is consistent, while the subset consisting of ‘p’, ‘if p then q’, and ‘not-q’ is not. One class of subsets that is particularly important for our analysis is the class of “minimally inconsistent” ones. A set of propositions is called “minimally inconsistent” if it is inconsistent but all its proper subsets are consistent (so the removal of any of its members suffices to restore consistency). The set consisting of ‘p’, ‘if p then q’, and ‘not-q’, for example, is minimally inconsistent, since it is inconsistent but the removal of any one of the three propositions suffices to restore consistency. The set consisting of ‘p’, ‘not-p’, and ‘q’, by contrast, is not minimally inconsistent, since its inconsistency does not go away even after removing some of its elements: if we remove ‘q’, the inconsistency between ‘p’ and ‘not-p’ remains in place. It turns out that one can characterize the generality of the democratic trilemma in terms of what the minimally inconsistent subsets of the agenda are like.

The generality of the democratic trilemma: The trilemma applies to a given decision problem *if and only if* the agenda under consideration has at least one minimally inconsistent subset of three or more propositions.⁴⁹

To establish this, it suffices to show that combinations of individual attitudes structurally similar to those in the government and expert-panel examples can occur *if and only if* the agenda has at least one minimally inconsistent subset of three or more propositions. Let me first consider the “only if” part of this claim, and then turn to the “if” part.

It should be evident that just as majority rule can never generate any inconsistent outputs when there is only a single proposition–negation pair to be voted on, majority rule cannot generate any inconsistencies when every minimally inconsistent subset of the agenda is structurally similar to a proposition–negation pair. Since majority rule always produces complete collective attitudes (setting aside ties⁵⁰) and satisfies robustness to pluralism and basic majoritarianism, it then constitutes a decision procedure satisfying our three requirements. Thus the democratic trilemma does not arise when the agenda has no minimally inconsistent subset of more than two propositions.

The other direction of the argument requires showing that *if* the agenda has at least one such subset, *then* the trilemma arises. Suppose there is a minimally inconsistent subset of the agenda containing three or more propositions; call it *S*. Let us pick out three distinct elements of *S* and label them ‘*p*’, ‘*q*’, and ‘*r*’. Suppose, for the sake of argument, there is a decision procedure satisfying robustness to pluralism, basic majoritarianism and collective rationality. Consider the combination of individual attitudes shown in Table 6. For simplicity, Table 6 shows only the individuals’ attitudes towards the propositions in *S*, not towards all propositions on the agenda. Although the individuals can be assumed to have views on those other propositions too, these can be set aside for present purposes. Notice that each individual rejects precisely one of the propositions in *S*, and so his or her

⁴⁹ For proofs that majority voting guarantees consistent collective attitudes if and only if the agenda has no minimally inconsistent subset of more than two propositions, see Dietrich and List, “Judgment aggregation by quota rules,” and Nehring and Puppe, “The structure of strategy-proof social choice.” For earlier related results, see Nehring and Puppe, “Strategyproof Social Choice on Single-Peaked Domains.” Strictly speaking, without technical caveats, the “if and only if” claim requires an odd number of individuals.

⁵⁰ Majority ties are possible only when the number of individuals is even. I set the technicalities raised by ties aside here and implicitly assume, for the sake of simplicity, an odd number of individuals.

attitudes are consistent (since S is minimally inconsistent, the rejection of any one proposition in it suffices to restore consistency among the rest).

| | 'p' | 'q' | 'r' | Other propositions in S (if any) |
|------------------------|-----|-----|-----|------------------------------------|
| 1/3 of the individuals | Yes | Yes | No | All accepted |
| 1/3 of the individuals | Yes | No | Yes | All accepted |
| 1/3 of the individuals | No | Yes | Yes | All accepted |

Table 6: A combination of individual attitudes

By robustness to pluralism, the given combination of individual attitudes is admissible as input to the decision procedure. What are the resulting collective attitudes? By basic majoritarianism, the group must either adopt the majority attitude on any given proposition, or be silent on it. In light of the majority views displayed in Table 6, the group must either accept each of 'p', 'q', and 'r', as well as all other propositions in S (if any), or be silent on one or more of these propositions. The completeness part of collective rationality, however, rules out silence with respect to any of these propositions, so that the group has no choice but to accept all of 'p', 'q', 'r' and the rest of the propositions in S . But this means that all of S , an inconsistent set, is collectively accepted, which violates the consistency part of the collective rationality requirement. This completes the argument.

It should now be clear what is meant by the claim that the democratic trilemma applies to "all but the simplest collective decision problems": it applies to all decision problems except those in which all minimally inconsistent subsets of the agenda have two or fewer elements.⁵¹ The latter are the "simplest" decision problems insofar as they are structurally similar to one or several logically unconnected binary choices. Note that although the construction of individual attitudes in Table 6 implicitly assumed the overall number of individuals to be divisible by three, the argument can be generalized to any number of individuals. For the purposes of this paper, I set these technicalities aside.⁵²

⁵¹ For further discussions of this structure condition on an agenda (the "simplicity" or "median" property), see Nehring and Puppe, "The structure of strategy-proof social choice," Dietrich and List, "Judgment aggregation by quota rules," and Nehring and Puppe, "Abstract Arrovian Aggregation."

⁵² To avoid a trivial conflict between basic majoritarianism and collective completeness in the case of a majority tie, one may demand completeness only in the absence of a majority tie. The trilemma can then be shown to hold for all group sizes except 2 and, in case the largest minimally inconsistent subset of the agenda has exactly three elements, 4. This follows from a result in Dietrich and List, "Judgment aggregation by quota rules."