Internet Appendix to
"The Booms and Busts of Beta Arbitrage"

Shiyang Huang
University of Hong Kong
Email: huangsy@hku.hk

Xin Liu
Renmin University of China
Email: xinl@ruc.edu.cn

Dong Lou
London School of Economics and CEPR
Email: d.lou@lse.ac.uk

Christopher Polk
London School of Economics and CEPR
Email: c.polk@lse.ac.uk

This Draft: December 2022

Table A1: Event Time $CoBAR$

This table reports event-time statistics for $CoBAR$, the excess comovement among low beta stocks over the period 1970 to 2016. At the end of each month, all stocks are sorted into deciles based on their lagged-12-month market beta computed using daily returns. Pairwise partial return correlations (after controlling for the Fama-French three factors) for all stocks in the low beta decile are computed based on weekly stock returns in the previous 12 months. $CoBAR$ is the average pair-wise correlation between any two stocks in the low-beta decile in year t. Panel A reports the autocorrelation in $CoBAR$ in event time; that is, we form the beta portfolios in year 0, and compute $CoBAR$ for the same set of low beta stocks in the following 1, 2, and 3 years. Panel B shows the average $CoBAR$ in event time.

Panel A: Autocorrelation in event time

	Year 0	Year 1	Year 2	Year 3
$CoBAR0$	1			
$CoBAR1$	0.142	1		
$CoBAR2$	0.241	0.557	1	
$CoBAR3$	0.245	0.397	0.532	1

Panel B: Average $CoBAR$ in event time

	Mean	Median	Std. Dev.	Min	Max
Year 0	0.104	0.101	0.026	0.037	0.203
Year 1	0.072	0.071	0.027	0.021	0.189
Year 2	0.071	0.068	0.031	0.009	0.187
Year 3	0.073	0.068	0.030	0.022	0.207

Table A2 Forecasting Security Market Line with $CoBAR$

This table shows the estimated function that maps $CoBAR$ into the slope and intercept of the security market line in different time window. At the end of each month, all stocks are sorted into vigintiles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and non-synchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return, in addition to the contemporaneous excess market return. The pre-ranking beta is the sum of the six coefficients from the OLS regression. We then estimate four security market lines based on these 20 portfolios formed in each period: one SML using portfolio returns in months 1-6, months 7-12, year2, and year 3 after portfolio formation; the betas used in these SML regressions are the corresponding post-ranking betas. We regress the intercepts ($intercept_t$) and the slopes ($slope_t$) on a constant, contemporaneous excess market return ($r^e_{M,t}$), and lagged $CoBAR$ ($CoBAR_{t-1}$):

$$intercept_t = a_1 + b_1 r^e_{M,t} + c_1 CoBAR_{t-1} + u_{1,t}$$
$$slope_t = a_2 + b_2 r^e_{M,t} + c_2 CoBAR_{t-1} + u_{2,t}$$

Time	a1	a2	b1	b2	c1	c2	$R^2_{intercept}$	R^2_{slope}
Months 1-6	-0.012	0.012	0.033	**0.956**	**0.180**	**-0.177**	4.63%	51.64%
	(-1.48)	(1.51)	(0.33)	(9.94)	(2.14)	(-2.22)		
Months 7-12	0.000	0.001	0.031	**0.973**	0.078	-0.077	0.33%	42.68%
	(-0.03)	(0.07)	(0.28)	(9.07)	(0.80)	(-0.83)		
Year 2	**0.016**	**-0.015**	0.040	**0.905**	-0.079	0.079	1.17%	32.44%
	(2.36)	(-2.24)	(0.24)	(6.62)	(-1.17)	(1.18)		
Year 3	**0.035**	**-0.038**	0.109	**0.841**	**-0.275**	**0.319**	13.99%	35.11%
	(4.38)	(-4.26)	(0.66)	(5.85)	(-3.68)	(3.75)		

Table A3. Forecasting Security Market Lines with $CoBAR$: Conditional on Future $CoBAR$

This table shows the estimated function that maps $CoBAR$ into the excess slope and intercept of the security market line for different subsamples divided by future $CoBAR$. We rank the entire $CoBAR$ time-series into terciles. In Panel A, we include months with $CoBAR$ computed from the future 12 months within the bottom tercile. In Panel B, we include months with $CoBAR$ computed from the future 12 months within the top tercile. At the end of each month, all stocks are sorted into vigintiles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and non-synchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return, in addition to the contemporaneous excess market return. The pre-ranking beta is the sum of the six coefficients from the OLS regression. We then estimate two security market lines based on these 20 portfolios formed in months 1-6 and year 3 after portfolio formation; the betas used in these SML regressions are the corresponding post-ranking betas. We regress the intercepts ($intercept_t$) and the slopes ($slope_t$) on a constant, contemporaneous excess market return ($r^e_{M,t}$), and lagged $CoBAR$ ($CoBAR_{t-1}$):

$$intercept_t = a_1 + b_1 r^e_{M,t} + c_1 CoBAR_{t-1} + u_{1,t}$$
$$slope_t = a_2 + b_2 r^e_{M,t} + c_2 CoBAR_{t-1} + u_{2,t}$$

The excess slope is defined as $g_0 + g_1 CoBAR_{t-1}$, where $g_0 \equiv a_2/b_2$ and $g_1 \equiv c_2/b_2$. The excess intercept is computed as $h_0 + h_1 CoBAR_{t-1}$, where $h_0 \equiv a_1 - a_2 b_1/b_2$ and $h_1 \equiv c_1 - c_2 b_1/b_2$. *t*-statistics computed using the delta method are in parentheses. 5% statistical significance is indicated in bold.

Panel A: Future Low $CoBAR$				
Time	g_0	g_1	h_0	h_1
Months 1-6	0.006	-0.113	-0.002	0.082
	(0.50)	(-0.90)	(-0.22)	(0.77)
Year 3	-0.034	0.261	**0.030**	-0.215
	(-1.76)	(1.50)	(2.26)	(-1.80)

Panel B: Future High $CoBAR$				
Time	g_0	g_1	h_0	h_1
Months 1-6	0.013	**-0.190**	-0.018	**0.238**
	(1.40)	(-2.16)	(-1.54)	(1.99)
Year 3	**-0.051**	**0.460**	**0.046**	**-0.393**
	(-4.53)	(4.27)	(3.73)	(-3.25)

Table A4: Smarter Beta-Arbitrage Strategies

This table reports monthly returns to a smarter beta-arbitrage strategy that exploits the time-varying overreaction and subsequent reversal present in standard beta arbitrage strategies. Specifically, we first time the standard beta-arbitrage strategy using current $CoBAR$. If $CoBAR$ is above the 80th percentile, we go long the long-short beta-arbitrage strategy for the next six months. Otherwise, we short that portfolio over that time period. In addition, if $CoBAR$ from two years ago is below the 20th percentile, we go long for the next twelve months the long-short beta-arbitrage strategy based on beta estimates from two years ago. Otherwise, we short that portfolio, again for the next twelve months. In Panel A, the percentile break points of the $CoBAR$ distribution are identified using the entire $CoBAR$ time series (thus an in-sample analysis). In Panel B, the percentile break points are identify based solely on its prior distribution (an out-of-sample analysis); we skip the first three years of our sample to compute the initial distribution. In both panels, we use a seven-factor model that includes: the Fama and French (2015) five-factors (market, size, value, investment and profitability), the Carhart (1997) momentum factor, the lottery factor (FMAX) from Bali, Brown, Murray, and Tang (2017), and Frazzini and Pedersen's (2014) betting-against beta (BAB) factor. 5% statistical significance is indicated in bold.

ALPHA	RM-RF	SMB	HML	UMD	RMW	CMA	FMAX	BAB
				Panel A: In-Sample				
0.43%	**0.52**	0.10	0.02	**-0.16**				
(2.31)	(11.20)	(1.57)	(0.18)	(-2.40)				
0.42%	**0.52**	0.14	0.07	**-0.16**	0.11	-0.13		
(2.11)	(10.99)	(1.85)	(0.72)	(-2.35)	(0.79)	(-0.71)		
0.53%	**0.38**	0.00	0.21	**-0.17**	**0.43**	-0.00	**0.37**	
(2.78)	(7.83)	(0.03)	(1.91)	(-2.82)	(3.26)	(-0.00)	(5.00)	
0.59%	**0.47**	0.11	**0.25**	-0.11	**0.46**	0.03	**0.21**	**-0.34**
(3.10)	(9.19)	(1.33)	(2.14)	(-1.60)	(3.54)	(0.18)	(2.82)	(-4.24)
				Panel B: Out-of-Sample				
0.43%	**0.53**	0.09	-0.04	**-0.21**				
(2.35)	(11.52)	(1.51)	(-0.40)	(-3.09)				
0.45%	**0.51**	0.12	0.06	**-0.20**	0.05	-0.22		
(2.32)	(11.01)	(1.58)	(0.61)	(-2.92)	(0.35)	(-1.16)		
0.57%	**0.38**	-0.01	0.19	**-0.21**	**0.36**	-0.09	**0.36**	
(2.97)	(7.71)	(-0.16)	(1.79)	(-3.38)	(2.65)	(-0.50)	(4.88)	
0.63%	**0.47**	0.10	**0.24**	**-0.14**	**0.39**	-0.06	**0.19**	**-0.35**
(3.31)	(9.11)	(1.14)	(2.05)	(-2.09)	(2.94)	(-0.34)	(2.60)	(-4.42)

Table A5: Beta Expansion, Time-Series Analysis, Robustness

This table examines time-series beta expansion associated with arbitrage trading under alternative specifications as of Panel A, Table VI. In each specification, the dependent variable is the beta spread between the high-beta and low-beta deciles (ranked in year t) in year t+1. $CoBAR$ is the average pairwise partial weekly three-factor residual correlation within the low-beta decile over the past 12 months. $Leverage$ is a quintile dummy based on the average value-weight book leverage of the bottom and top beta deciles. We also include in the regression an interaction term between $CoBAR$ and $Leverage$. Reported below is the coefficient on the interaction of $CoBAR$ and $Leverage$. In Panel A, we consider different subsample results. Row 1 shows the baseline results which are also reported in Table III. In Rows 2 and 3, we exclude the tech bubble crash and the recent financial crisis from our sample. In Panel B, we explore alternative definitions of $CoBAR$. In row 1, we control for the UMD factor in computing $CoBAR$. In row 2, we control for both large- and small-cap HML in computing $CoBAR$. In row 3, we control for the Fama-French five factor model that adds profitability and investment to their three-factor model. In row 4, we control for the Fama-French five factors and the UMD factor. In row 5, we control for the Fama-French five factors, the UMD factor, and the lottery factor from Bali, Brown, Murray, and Tang (2017). In row 6, we perform the entire analysis on an industry-adjusted basis by sorting stocks into beta deciles within industries. In row 7, we instead measure the correlation between the high and low-beta portfolios, with a low correlation indicating high arbitrage activity. In Panel C, we replace $CoBAR$ with residual $CoBAR$ from a time-series regression where we purge from $CoBAR$ variation linked to, respectively, $CoMOM$ and $CoValue$ (Lou and Polk, 2021; rows 1-2), the average pair-wise correlation in the market (row 3), the BAB factor (Frazzini and Pedersen, 2014; row 4), the lagged 36-month volatility of the BAB factor (row 5), market volatility over the past 24 months (row 6), a trend (row 7), lagged $CoBAR$ (where we hold the stocks in the low-beta decile constant but calculate $CoBAR$ using returns from the previous year; row 8), smoothed past inflation (Cohen, Polk, and Vuolteenaho, 2005; row 9), a sentiment index (Baker and Wurgler, 2006; row 10), aggregate analyst forecast dispersion (Hong and Sraer, 2014; row 11), the TED Spread (Frazzini and Pedersen, 2014; row 12), the TED volatility (row 13), and the AR(2) residual of financial leverage (Chen and Lu, 2019; row 14). Standard errors are shown in brackets. *, **, *** denote significance at the 10%, 5%, and 1% level, respectively.

$DepVar = BetaSpread_{t+1}$		
	Estimate	Std. Dev.
Panel A: Subsamples		
(1) Full Sample:1970-2016	0.433***	[0.117]
(2) Excluding 2001	0.443***	[0.112]
(3) Excluding 2007-2009	0.341**	[0.142]
Panel B: Alternative definitions of $CoBAR$		
(1) Controlling for UMD	0.441***	[0.116]
(2) Controlling for Large/Small-Cap HML	0.374***	[0.127]
(3) Controlling for FF Five Factors	0.453***	[0.129]
(4) Controlling for FF Five Factors + UMD	0.434***	[0.126]
(5) Controlling for FF Five Factors + UMD + FMAX	0.459***	[0.125]
(6) Controlling for Industry Factors	0.504***	[0.128]
(7) Correl btw High and Low Beta Stocks	0.097**	[0.041]
Panel C: Residual $CoBAR$		
(1) Controlling for $CoMOM$	0.240**	[0.116]
(2) Controlling for $CoValue$	0.233**	[0.116]
(3) Controlling for MKT CORR	0.423***	[0.120]
(4) Controlling for BAB	0.415***	[0.120]
(5) Controlling for Vol(BAB)	0.431***	[0.110]
(6) Controlling for Vol(MKT)	0.303**	[0.118]
(7) Controlling for Trend	0.347***	[0.120]
(8) Controlling for Pre-formation $CoBAR$	0.484***	[0.119]
(9) Controlling for Inflation	0.436***	[0.120]
(10) Controlling for Sentiment	0.443***	[0.119]
(11) Controlling for Disagreement	0.300**	[0.127]
(12) Controlling for TED Spread	0.464***	[0.126]
(13) Controlling for TED Volatility	0.479***	[0.136]
(14) Controlling for Financial Leverage	0.292**	[0.148]

Table A6: Even Smarter Beta-Arbitrage Strategies

This table reports monthly returns to an even smarter beta-arbitrage strategy that exploits the time-varying overreaction and subsequent reversal present in standard beta arbitrage strategies. That is, we first sort stocks into high and low leverage subgroups and then take the difference in smart beta arbitrage returns between the two leverage-sorted groups. In Panel A, the percentile break points of the $CoBAR$ distribution are identified using the entire $CoBAR$ time series (thus an in-sample analysis). In Panel B, the percentile break points are identify based solely on its prior distribution (an out-of-sample analysis); we skip the first three years of our sample to compute the initial distribution. In both panels, we use a nine-factor model that includes: the Fama and French (2015) five-factors (market, size, value, investment and profitability), the Carhart (1997) momentum factor, the lottery factor (FMAX) from Bali, Brown, Murray, and Tang (2017), Frazzini and Pedersen's (2014) betting-against beta (BAB) factor, and the smart-beta-strategy factor. 5% statistical significance is indicated in bold.

ALPHA	RM-RF	SMB	HML	UMD	RMW	CMA	FMAX	BAB	Smarter Beta
Panel A: In-Sample									
0.47%	0.01	-0.01	0.06	**0.09**					
(2.92)	(0.16)	(-0.16)	(0.91)	(2.31)					
0.47%	0.01	0.00	0.07	**0.09**	0.03	-0.02			
(2.81)	(0.17)	(0.03)	(0.82)	(2.25)	(0.40)	(-0.19)			
0.48%	-0.01	-0.02	0.09	**0.08**	0.08	-0.00	0.06		
(2.90)	(-0.23)	(-0.32)	(1.07)	(2.15)	(0.87)	(-0.01)	(0.98)		
0.49%	0.00	-0.00	0.10	**0.10**	0.09	0.00	0.03	-0.06	
(2.89)	(0.02)	(-0.02)	(1.16)	(2.15)	(0.93)	(0.03)	(0.45)	(-0.66)	
0.49%	0.00	-0.00	0.10	**0.10**	0.09	0.00	0.03	-0.06	-0.00
(2.85)	(0.03)	(-0.02)	(1.16)	(2.13)	(0.95)	(0.03)	(0.45)	(-0.65)	(-0.01)
Panel B: Out-of-Sample									
0.34%	0.03	0.00	0.07	**0.09**					
(2.13)	(0.70)	(0.01)	(1.02)	(2.31)					
0.32%	0.04	0.00	0.04	**0.09**	0.02	0.06			
(1.98)	(0.83)	(0.06)	(0.46)	(2.15)	(0.23)	(0.51)			
0.35%	0.01	-0.03	0.07	**0.08**	0.09	0.09	0.08		
(2.11)	(0.19)	(-0.39)	(0.80)	(2.09)	(0.93)	(0.73)	(1.34)		
0.35%	0.01	-0.02	0.07	0.09	0.09	0.09	0.07	-0.01	
(2.09)	(0.23)	(-0.29)	(0.80)	(1.93)	(0.95)	(0.74)	(1.05)	(-0.15)	
0.37%	0.02	-0.02	0.08	0.08	0.10	0.09	0.08	-0.02	-0.02
(2.16)	(0.39)	(-0.25)	(0.88)	(1.85)	(1.07)	(0.73)	(1.11)	(-0.26)	(-0.49)

Table A7: Beta Arbitrage Timing Ability

This table reports regressions of monthly mutual funds' and hedge funds' returns on a conditional five-factor model (the Fama-French-Carhart four-factor model augmented with the beta-arbitrage factor of Frazzini and Pedersen (2014)). At the end of each month, we track mutual funds' and hedge funds' performance in the next six months. The dependent variable in columns (1) and (2) is the average monthly excess return of long-short equity hedge funds over that six-month window. Columns (3) and (4) use the average monthly excess returns of actively-managed mutual funds instead. We attribute those returns to $mktrf$, smb, hml, umd, and BAB, the four-factor adjusted portfolio return from buying bottom-beta-decile stocks and shorting top-beta-decile stocks. We allow the loading on BAB to be a function of lagged $CoBAR$ (quintile ranks from 1 to 5) and $SizeRank$, the lagged cross-sectional ranking of the fund's assets under management. To compute $SizeRank$, we first rank all funds (within the respective hedge fund or mutual fund subset) into three groups as of the previous month, and then assign the value 2 if the fund is in the highest group, 1 if the fund is in the middle group, and 0 if the fund is in the lowest group. Standard errors, shown in bracket, are clustered at both fund and month levels. *, **, *** denote significance at the 10%, 5%, and 1% level, respectively.

	Equity Hedge Funds		Equity Mutual Funds	
	[1]	[2]	[3]	[4]
$mktrf$	0.354***	0.354***	1.036***	1.036***
	[0.056]	[0.056]	[0.022]	[0.022]
smb	0.254***	0.254***	0.208***	0.208***
	[0.040]	[0.040]	[0.024]	[0.024]
hml	-0.036	-0.036	0.106***	0.106***
	[0.033]	[0.033]	[0.033]	[0.033]
umd	0.014	0.014	0.031**	0.031**
	[0.020]	[0.020]	[0.015]	[0.015]
BAB	-0.080***	-0.103***	0.023	0.036
	[0.022]	[0.029]	[0.023]	[0.026]
$CoBAR$	0.001***	0.001**	0.000	0.000
	[0.000]	[0.000]	[0.000]	[0.000]
$BAB \times CoBAR$	0.017**	0.030***	-0.002	0.002
	[0.008]	[0.009]	[0.008]	[0.009]
$SizeRank$		-0.001**		-0.000*
		[0.000]		[0.000]
$BAB \times SizeRank$		0.023		-0.013
		[0.014]		[0.013]
$CoBAR \times SizeRank$		-0.000		-0.000
		[0.000]		[0.000]
$BAB \times CoBAR \times SizeRank$		-0.013***		-0.004
		[0.004]		[0.004]
Adj-R^2	0.018	0.019	0.461	0.462
No. Obs.	222,842	222,842	430,237	430,237

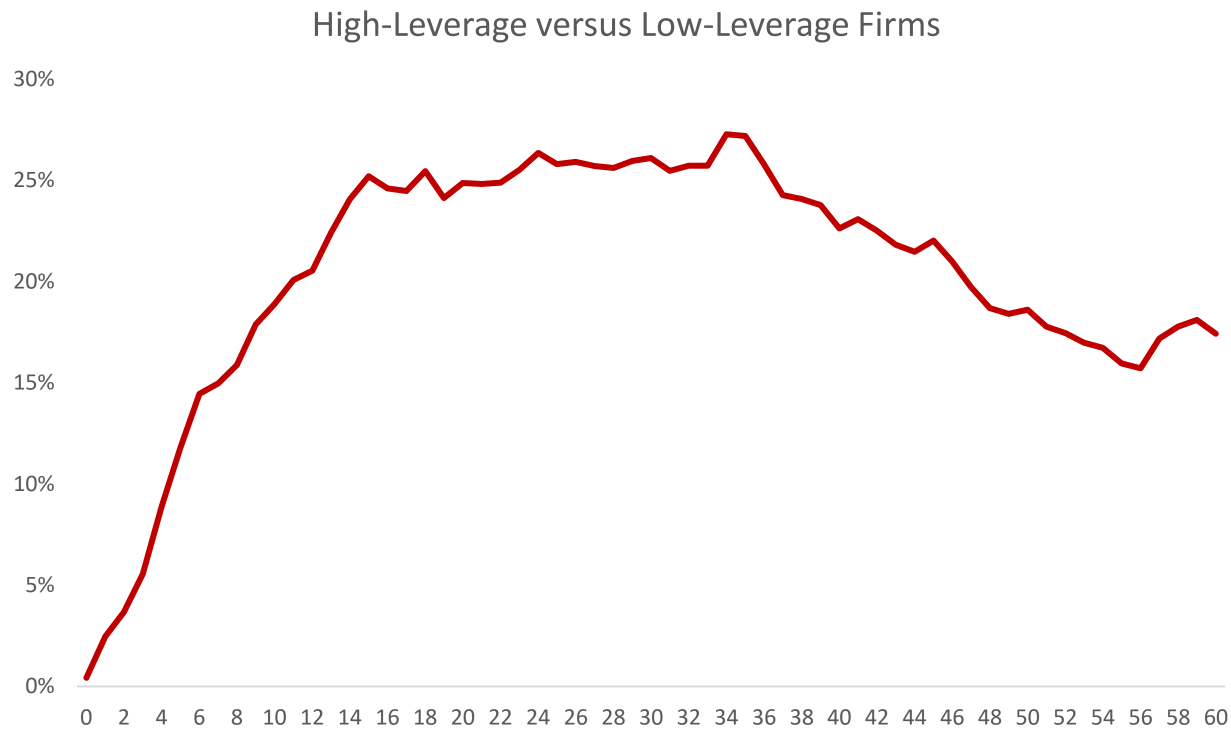

Figure A1: This figure shows how the relation between $CoBAR$ and subsequent beta arbitrage returns varies with firm leverage. At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and non-synchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return, in addition to the contemporaneous excess market return. The pre-ranking beta is the sum of the six coefficients from the OLS regression. All months are then sorted into five groups based on $CoBAR$, the average pairwise weekly three-factor residual correlation within the low-beta decile over the previous 12 months. At the beginning of the holding period, we sort stocks into four equal groups using book leverage. For each leverage quartile, we compute the $CoBAR$ return spread – i.e., the difference in seven-factor alpha (Fama-French five factors, the momentum factor, and the lottery factor from Brown, Murray, and Tang (2017)) to the beta arbitrage strategy (i.e., a portfolio that longs the value-weight low-beta decile and shorts the value-weighted high-beta decile) between high and low $CoBAR$ periods. The solid red curve shows the cumulative difference in the $CoBAR$ return spread between the highest and lowest leverage quartiles over the five years after portfolio formation. This difference in the $CoBAR$ return spread is 1.67%/month in year one and is -0.52%/month in year four, both significant at 10%.

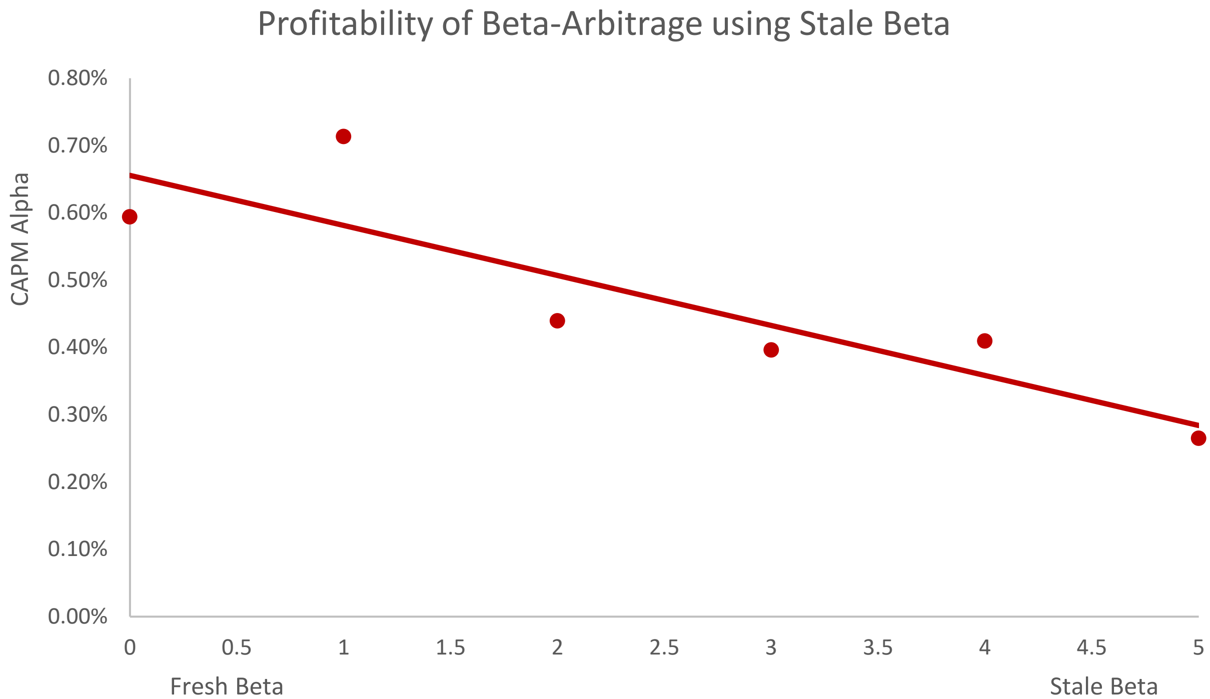

Figure A2: This figure shows how the post-holding return to beta-arbitrage strategies decays as stale estimates of beta are used to form beta-arbitrage strategy. At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and non-synchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return, in addition to the contemporaneous excess market return. The pre-ranking beta is the sum of the six coefficients from the OLS regression. We then compute the strategy return as the value-weight low-beta decile return minus the value-weight high-beta decile return. We then repeat the analysis using stale betas, computed from daily returns in each of the prior 5 years (thus having different beta portfolios as of time zero for each degree of beta staleness). We plot the corresponding beta-arbitrage strategies' CAPM alphas (averaged over the first six months after portfolio formation) for each of the six beta-arbitrage strategies, ranging from fresh beta to five-year stale beta.