

# The Booms and Busts of Beta Arbitrage\*

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This Draft: April 2021

\* We would like to thank Nicholas Barberis, Sylvain Champonnais, Andrea Frazzini, Pengjie Gao, Emmanuel Jurczenko, Ralph Koijen, Toby Moskowitz, Lasse Pedersen, Steven Riddiough, Emil Siriwardane, Dimitri Vayanos, Michela Verardo, Tuomo Vuolteenaho, Yu Yuan, and seminar participants at Arrowstreet Capital, Citigroup Quant Research Conference, Imperial College, London Quant Group, London Business School, London School of Economics, Norwegian School of Economics, Renmin University, UBS Quant Conference, University of Cambridge, University of California in Los Angeles, University of Hong Kong, University of Rotterdam, University of Warwick, 2014 China International Conference in Finance, 2014 Imperial College Hedge Fund Conference, 2014 Quantitative Management Initiative Conference, the 2015 American Finance Association Conference, the 2015 Financial Intermediation Research Society Conference, the 2015 Northern Finance Conference, the 2015 European Finance Association Conference, and the 2016 Finance Down Under Conference for helpful comments and discussions. We are grateful for funding from the Europlace Institute of Finance, the Paul Woolley Centre at the London School of Economics, and the QUANTVALLEY/FdR: Quantitative Management Initiative.

# The Booms and Busts of Beta Arbitrage

## Abstract

Low-beta stocks deliver high average returns and low risk relative to high-beta stocks, an opportunity for professional investors to “arbitrage” away. We argue that beta-arbitrage activity instead generates booms and busts in the strategy’s abnormal trading profits. In times of low arbitrage activity, the beta-arbitrage strategy exhibits delayed correction, taking up to three years for abnormal returns to be realized. In stark contrast, when activity is high, prices overshoot as short-run abnormal returns are much larger and then revert in the long run. We document a novel positive-feedback channel operating through firm-level leverage that facilitates these boom-and-bust cycles.

## I. Introduction

The trade-off of risk and return is a key concept in modern finance. The simplest and most intuitive measure of risk is market beta – the slope in the regression of a security’s return on the market return. In the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965), market beta is the only risk needed to explain expected returns. More specifically, the CAPM predicts that the relation between expected return and beta – the security market line – has an intercept equal to the risk-free rate and a slope equal to the equity premium.

However, empirical evidence indicates that the security market line is too flat on average (Black 1972, Frazzini and Pedersen, 2014) and especially so during times of high expected inflation (Cohen, Polk, and Vuolteenaho 2005), investor disagreement (Hong and Sraer 2016), and market sentiment (Antonioni, Doukas, and Subrahmanyam 2015). These patterns are not explained by other well-known asset pricing anomalies such as size, value, and price momentum.

We study the response of arbitrageurs to this failure of the Sharpe-Lintner CAPM in order to identify booms and busts of beta arbitrage.<sup>1</sup> In particular, we exploit the novel measure of arbitrage activity introduced by Lou and Polk (2021). They argue that traditional measures of such activity are flawed, poorly measuring a portion of the inputs to the arbitrage process, for a subset of arbitrageurs. Lou and Polk’s innovation is to measure the outcome of the arbitrage process, namely, the correlated price impacts that can result in excess return comovement in the spirit of Barberis and Shleifer (2003).<sup>2</sup>

We first confirm that our measure of the excess return comovement, relative to a benchmark asset pricing model, of beta-arbitrage stocks (labelled *CoBAR*) is correlated with existing measures of arbitrage activity. In particular, we find that time variation in the level of institutional holdings in low-beta stocks (i.e., stocks in the long leg of the beta

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<sup>1</sup> Schwert (2003) summarizes the performance of many trading strategies and motivated by his results, argues that practitioners who trade on anomalies after their publication can cause the effects to disappear. McLean and Pontiff (2016) systematically study the out-of-sample and post-publication performance for 97 variables shown to predict stock returns.

<sup>2</sup> See, for example, Barberis, Shleifer and Wurgler (2005), Greenwood and Thesmar (2011), Lou (2012) and Anton and Polk (2014).

strategy), the assets under management of long-short equity hedge funds, aggregate liquidity, and the past performance of a typical beta-arbitrage strategy together forecast roughly 41% of the time-series variation in *CoBAR*. These findings suggest that not only is our measure consistent with existing proxies for arbitrage activity but also that no one single existing proxy is sufficient for capturing time-series variation in arbitrage activity. Indeed, one could argue that perhaps much of the unexplained variation in *CoBAR* represents variation in arbitrage activity missed by existing measures.

After validating our measure in this way, we then forecast the cumulative abnormal returns to beta arbitrage. We first find that when arbitrage activity is relatively high (as identified by the 20% of the sample period with the highest values of *CoBAR*), abnormal returns to beta-arbitrage strategies occur relatively quickly, within the first six months of the trade. In contrast, when arbitrage activity is relatively low (as identified by the 20% of the sample period with the lowest values of *CoBAR*), four-factor-adjusted abnormal returns to beta-arbitrage strategies take much longer to materialize, appearing three years after putting on the trade.

These effects are both economically and statistically significant. When beta-arbitrage activity is low, the four-factor-adjusted abnormal returns on beta arbitrage are insignificantly different from zero in the two years after portfolio formation. For the patient arbitrageur, in year 3, the strategy earns four-factor-adjusted abnormal returns of 0.71% per month with a  $t$ -statistic of 2.90. In stark contrast, for periods when arbitrage activity is high, the four-factor-adjusted abnormal returns to beta arbitrage average 0.89% per month with a  $t$ -statistic of 2.25 in the six months after the trade.

We then show that the stronger performance of beta-arbitrage activities during periods of high beta-arbitrage activity can be linked to subsequent reversal of those profits. In particular, the year 3 four-factor-adjusted abnormal returns are -0.92% per month with an associated  $t$ -statistic of -2.63. As a consequence, the long-run reversal of beta-arbitrage returns varies predictably through time in a striking fashion. The post-formation, year-3 spread in four-factor-adjusted abnormal returns across periods of low arbitrage activity, when abnormal returns are predictably positive, and periods of high arbitrage activity,

when abnormal returns are predictably negative, is -1.63% per month ( $t$ -statistic = -3.83) or about -21% cumulative in that year.<sup>3</sup>

In sum, our results reveal interesting patterns in the relation between arbitrage strategy returns and the arbitrage crowd. When beta-arbitrage activity is low, the returns to beta-arbitrage strategies exhibit significant *delayed* correction. In contrast, when beta-arbitrage activity is high, the returns to beta-arbitrage activities reflect strong *over-correction* due to crowded arbitrage trading. These results are consistent with time-varying arbitrage activity generating booms and busts in beta arbitrage.

We argue that these results are intuitive, as it is difficult to know how much arbitrage activity is pursuing beta arbitrage, and, in particular, the strategy is susceptible to positive-feedback trading. Specifically, successful bets on (against) low-beta (high-beta) stocks result in prices for those securities rising (falling). If the underlying firms are leveraged, this change in price will, all else equal, result in the security's beta falling (increasing) further. Thus, not only do arbitrageurs not know when to stop trading the low-beta strategy, their (collective) trades strengthen the signal. Consequently, beta arbitrageurs may increase their bets precisely when trading is more crowded.<sup>4</sup>

Consistent with our novel positive-feedback mechanism, we show that the cross-sectional spread in betas increases when beta-arbitrage activity is high and particularly so when beta-arbitrage stocks are relatively more levered. As a consequence, stocks remain in the extreme beta portfolios for a longer period of time. Our novel positive feedback channel also has implications for cross-sectional heterogeneity in abnormal returns: we find that our boom and bust beta-arbitrage cycles are particularly strong among high-leverage stocks.

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<sup>3</sup> We use various methods to adjust the standard errors in our return analysis to adjust for the fact that returns are measured over overlapping horizons. The  $t$ -statistics reported in the paper are based on Newey-West adjustments with appropriate lags. The results are robust to other methods of assessing statistical significance. For example, if we bootstrap the standard errors in the aforementioned analysis, the resulting  $t$ -statistic is -3.55. If instead, we measure the joint significance of the *non-overlapping monthly* return spreads in year 3 (of -1.63%/month), we are unable to reject the null at the 1% level.

<sup>4</sup> Note that crowded trading may or may not be profitable, depending on how long the arbitrageur holds the position and how long it takes for any subsequent correction to occur.

A variety of robustness tests confirm our main findings. In particular, we show that controlling for other factors when either measuring *CoBAR* or when predicting beta-arbitrage returns does not alter our primary conclusions a) that the excess comovement of beta-arbitrage stocks forecasts time-varying reversal to beta-arbitrage bets and b) that the beta spread varies with *CoBAR*.

Our findings can also be seen by estimating time variation in the short-run (months 1-6) and long-run (year 3) security market lines, conditioning on *CoBAR*. Thus, the patterns we find are not just due to extreme-beta stocks, but reflect dynamic movements throughout the entire cross section. In particular, we find that during periods of high beta-arbitrage activity, the short-term security market line strongly slopes downward, indicating strong profits to the low-beta strategy, consistent with arbitrageurs expediting the correction of market misvaluation. However, this correction is excessive, as the long-run security market line dramatically slopes upwards. In contrast, during periods of low beta-arbitrage activity, the short-term security market line is weakly upward sloping. During these low-arbitrage periods, we do not find any downward slope to the security market line until the long-run.

A particularly compelling robustness test involves separating *CoBAR* into excess comovement among low-beta stocks occurring when these stocks have relatively high returns (i.e., capital flowing into low beta stocks and pushing up the prices) vs. excess comovement occurring when low-beta stocks have relatively low returns—i.e., upside versus downside comovement. Under our interpretation of the key findings, it is the former that should track time-series variation in expected beta-arbitrage returns, as that particular direction of comovement is consistent with trading aiming to correct the beta anomaly. Though estimates of upside *CoBAR* are naturally much noisier, our evidence confirms the intuition above: our main results are stronger with upside *CoBAR*.

Finally, Shleifer and Vishny (1997) link the extent of arbitrage activity to limits to arbitrage. Based on their logic, trading strategies that bet on firms that are cheaper to arbitrage (e.g., larger stocks, more liquid stocks, or stocks with lower idiosyncratic risk) should have more arbitrage activity. This idea of limits to arbitrage motivates tests examining cross-sectional heterogeneity in our findings. We show that our results

primarily occur in those stocks with the *least* limits to arbitrage: large stocks, liquid stocks, and stocks with low idiosyncratic volatility. This cross-sectional heterogeneity in the effect is again consistent with the interpretation that arbitrage activity causes much of the time-varying patterns we document.

With these patterns in hand, we take a closer look at how our measure reflects investment activity in this strategy in the cross-section of mutual funds and hedge funds. Specifically, we show that the typical long-short equity hedge fund increases its beta-arbitrage exposure when *CoBAR* is relatively high (that is, when short-term beta arbitrage returns are also higher). However, the ability of hedge funds to time the strong overreaction that occurs when *CoBAR* is high declines with assets under management. During booms in beta-arbitrage, small hedge funds have positive exposures to a low-beta factor that are nearly twice as big as their large fund counterparts. In contrast, mutual funds have an insignificant exposure to low-beta strategies which does not vary with *CoBAR*.

The organization of our paper is as follows. Section II summarizes the related literature. Section III describes the data and empirical methodology. We detail our empirical findings regarding beta-arbitrage activity and predictable patterns in returns in section IV, and present key tests of our economic mechanism in Section V. Section VI concludes.

## II. Related Literature

Our results shed new light on the risk-return trade-off, a cornerstone of modern asset pricing research. This trade-off was first established in the Sharpe-Lintner CAPM, which argues that the market portfolio is mean-variance efficient. Consequently, a stock's expected return is a linear function of its market beta, with a slope equal to the equity premium and an intercept equal to the risk-free rate.

However, there is mounting empirical evidence that is inconsistent with the CAPM. Black, Jensen, and Scholes (1972) were the first to show carefully that the security market line is too flat on average. Put differently, the risk-adjusted returns of high beta stocks

are too low relative to those of low-beta stocks. This finding was subsequently confirmed in an influential study by Fama and French (1992). Blitz and van Vliet (2007), Blitz, Pang, and van Vliet (2013), Baker, Bradley, and Taliaferro (2014), and Frazzini and Pedersen (2014) document that the low-beta anomaly is also present in both non-US developed markets as well as emerging markets.

Of course, the flat security market line is not the only failing of the CAPM (see Fama and French 1992, 1993, and 1996). Nevertheless, since this particular issue is so striking, a variety of explanations have been offered to explain the low-beta phenomenon. Black (1972) and more recently Frazzini and Pedersen (2014) argue that leverage-constrained investors, such as mutual funds, tend to deviate from the capital market line and invest in high beta stocks to pursue higher expected returns, thus causing these stocks to be overpriced relative to the CAPM benchmark.<sup>5,6,7</sup>

Cohen, Polk, and Vuolteenaho (2005) derive the cross-sectional implications of the CAPM in conjunction with the money illusion story of Modigliani and Cohn (1979). They show that money illusion implies that, when inflation is low or negative, the compensation for one unit of beta among stocks is larger (and the security market line steeper) than the rationally expected equity premium. Conversely, when inflation is high, the compensation for one unit of beta among stocks is lower (and the security market line shallower) than what the overall pricing of stocks relative to bills would suggest. Cohen, Polk, and Vuolteenaho provide empirical evidence in support of their theory.

Hong and Sraer (2016) provide an alternative explanation based on Miller's (1977) insights. In particular, they argue that investors disagree about the value of the market portfolio. This disagreement, coupled with short sales constraints, can lead to overvaluation, and particularly so for high-beta stocks, as these stocks allow optimistic investors to tilt towards the market. Along those lines, Kumar (2009) and Bali, Cakici, and Whitelaw (2011) show that high risk stocks can indeed underperform low risk stocks,

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<sup>5</sup> Jylhä (2017) provides evidence in support of this interpretation using Federal Reserve changes in initial margin requirements.

<sup>6</sup> See also Karceski (2002), Baker, Bradley, and Wurgler (2011) and Buffa, Vayanos, and Woolley (2014) for related explanations based on benchmarking of institutional investors.

<sup>7</sup> Kojien and Yogo (2017) provide a general framework for modeling the role of institutions in asset markets.



if some investors prefer volatile, skewed returns, in the spirit of the cumulative prospect theory as modeled by Barberis and Huang (2008). Building on arguments in Stambaugh, Yu, and Yuan (2015), Liu, Stambaugh, and Yuan (2017) attribute the beta anomaly to the positive correlation between market beta and idiosyncratic volatility.<sup>8</sup>

A natural question is why sophisticated investors, who can lever up and sell short securities at relatively low costs, do not fully take advantage of this anomaly and thus restore the theoretical relation between risk and returns. Our paper is aimed at addressing this exact question. Our premise is that professional investors indeed take advantage of this low-beta return pattern, often in dedicated strategies that buy low-beta stocks and/or sell high-beta stocks. However, the total amount of capital that is dedicated to this low-beta strategy is both time varying and unpredictable from a single arbitrageur’s perspective, thus resulting in periods where the security market line remains too flat—i.e., too little arbitrage capital, as well as periods where the security market line becomes overly steep—i.e., too much arbitrage capital.

Not all arbitrage strategies have these issues. Indeed, some strategies have a natural fundamental anchor that is relatively easily observed (Stein 2009). For example, it is straightforward to observe the extent to which an ADR is trading at a price premium (discount) relative to its local share. This ADR premium/discount is a clear signal to an arbitrageur of an opportunity and, in fact, arbitrage activity keeps any price differential small with deviations disappearing within minutes.<sup>9</sup> Importantly, if an unexpectedly large number of ADR arbitrageurs pursue a particular trade, the price differential narrows. An individual ADR arbitrageur can then adjust his or her demand accordingly.

There is, however, no easy anchor for beta arbitrage.<sup>10</sup> Further, we argue that the difficulty in identifying the amount of beta-arbitrage capital is exacerbated by a novel,

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<sup>8</sup> In addition, Campbell, Giglio, Polk, and Turley (2018) document that high-beta stocks hedge time-variation in the aggregate market’s return volatility, offering a potential neoclassical explanation for the low-beta anomaly.

<sup>9</sup> Rösch (2014) studies various properties of ADR arbitrage. For his sample of 72 ADR home stock pairs, the average time it takes until a ADR/home stock price deviation disappears is 252 seconds. For an institutional overview of this strategy, see J.P. Morgan (2014).

<sup>10</sup> Polk, Thompson, and Vuolteenaho (2006) use the Sharpe-Lintner CAPM to relate the cross-sectional beta premium to the equity premium. They show how the divergence of the two types of equity-premium measures implies a time-varying trading opportunity for beta arbitrage. Their methods are quite

indirect positive-feedback channel.<sup>11</sup> Namely, beta-arbitrage trading can lead to the cross-sectional beta spread increasing when firms are levered. As a consequence, stocks in the extreme beta deciles are more likely to remain in these extreme groups, with more extreme beta values, when arbitrage trading becomes excessive. Given that beta arbitrageurs rely on realized beta as their trading signal, this beta expansion resulting from leverage effectively causes a positive feedback loop in the beta-arbitrage strategy.

### III. Data and Methodology

The main dataset used in this study is the stock return data from the Center for Research in Security Prices (CRSP). Following prior studies on the beta-arbitrage strategy, we include in our study all common stocks on NYSE, Amex, and NASDAQ. We then augment the stock return data with institutional ownership in individual stocks provided by Thompson Financial. We further obtain information on assets under management (AUM) of long-short equity hedge funds from Lipper’s Trading Advisor Selection System (TASS). Since the assets managed by hedge funds grow substantially in our sample period, we detrend this variable. In addition, we use fund-level data on hedge fund returns and AUM.

We also construct, as controls, a list of variables that have been shown to predict future beta-arbitrage strategy returns. Specifically, a) following Cohen, Polk, and Vuolteenaho (2005), we construct a proxy for expected inflation using an exponentially weighted moving average (with a half-life of 36 months) of past log growth rates of the producer-price index; b) we also include in our study the sentiment index proposed by Baker and Wurgler (2006, 2007); c) following Hong and Sraer (2016), we construct an aggregate disagreement proxy as the beta-weighted standard deviation of analysts’ long-

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sophisticated and produce signals about the time-varying attractiveness of beta-arbitrage that, though useful in predicting beta-arbitrage returns, are still, of course, quite noisy.

<sup>11</sup> The idea that positive-feedback strategies are prone to destabilizing behaviour goes back to at least DeLong, Shleifer, Summers, and Waldmann (1990). In contrast, negative-feedback strategies like ADR arbitrage or value investing are less susceptible to destabilizing behaviour by arbitrageurs, as the price mechanism mediates any potential congestion. See Stein (2009) for a discussion of these issues.

term growth rate forecasts; d) finally, following Frazzini and Pedersen (2014), we use the Ted spread—the difference between the LIBOR rate and the US Treasury bill rate—as a measure of financial intermediaries’ funding constraints.

We begin our analysis in January 1970 (i.e., our first measure of beta arbitrage crowdedness is computed as of December 1969), as that was when the low-beta anomaly was first recognized by academics.<sup>12</sup> At the end of each month, we sort all stocks into deciles (in some cases vigintiles) based on their pre-ranking market betas. Following prior literature, we calculate pre-ranking betas using daily returns in the past twelve months (with at least 200 daily observations). Our results are similar if we use monthly returns, or different pre-ranking periods. To account for illiquidity and non-synchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return, in addition to the contemporaneous excess market return. The pre-ranking beta is simply the sum of the six coefficients from the OLS regression.

We then compute pairwise partial correlations using 52 (non-missing) weekly returns for all stocks in each decile in the portfolio ranking period. We control for the Fama-French three factors when computing these partial correlations to purge out any comovement in stocks induced by known risk factors. We measure the excess comovement of stocks involved in beta arbitrage (*CoBAR*) as the average pairwise partial correlation in the *lowest* market beta decile. We focus on the low-beta decile as these stocks tend to be larger, more liquid, and have lower idiosyncratic volatility compared to the highest-beta decile; thus, our measurement of excess comovement will be less susceptible to issues related to asynchronous trading and measurement noise.<sup>13</sup> We operationalize this calculation by computing the average correlation of the three-factor residual of every stock in the lowest beta decile with the rest of the stocks in the same decile:

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<sup>12</sup> Though eventually published in 1972, Black, Jensen, and Scholes (1972) had been presented as early as August of 1969. Mehrling’s (2005) biography of Fischer Black details the early history of the low-beta anomaly.

<sup>13</sup> Our results are robust to measuring *CoBAR* as the (minus) cross-correlation between high- and low-beta deciles.

$$CoBAR = \frac{1}{N} \sum_{i=1}^N \text{partialCorr}(\text{retrf}_i^L, \text{retrf}_{-i}^L | \text{mktrf}, \text{smb}, \text{hml}),$$

where  $\text{retrf}_i^L$  is the weekly return of stock  $i$  in the (L)owest beta decile,  $\text{retrf}_{-i}^L$  is the weekly return of the equal-weight lowest beta decile excluding stock  $i$ , and  $N$  is the number of stocks in the lowest beta decile. We have also measured  $CoBAR$  using returns that are orthogonalized not only to the Fama-French factors but also to each stock's industry return or to other risk factors, and our conclusions continue to hold. We present these and many other robustness tests in Table IV.

In the following period, we then form a zero-cost portfolio that goes long the value-weight portfolio of stocks in the lowest market beta decile and short the value-weight portfolio of stocks in the highest market beta decile. We track the cumulative abnormal returns of this zero-cost long-short portfolio in months 1 through 36 after portfolio formation. To summarize the timing of our empirical exercise, year 0 is our portfolio formation year (during which we also measure  $CoBAR$ ), year 1 is the holding year, and years 2 and 3 are our post-holding period, to detect any (conditional) long-run reversal to the beta-arbitrage strategy.

#### IV. Main Results

We first document simple characteristics of our arbitrage activity measure. Table I Panel A indicates that there is significant excess correlation among low-beta stocks on average and that this pairwise correlation varies substantially through time; specifically, the mean of  $CoBAR$  is 0.10 varying from a low of 0.04 to a high of 0.20.

Panel B of Table I examines  $CoBAR$ 's correlation with existing measures linked to time variation in the expected abnormal returns to beta-arbitrage strategies. We find that  $CoBAR$  is high when disagreement is high, with a correlation of 0.27.  $CoBAR$  is also positively correlated with the Ted spread, consistent with a time-varying version of Black (1972), though the Ted spread does not forecast (or in some cases forecasts in the wrong direction) time variation in expected abnormal returns to beta-arbitrage strategies (Frazzini and Pederson 2014).  $CoBAR$  is negatively correlated with the expected inflation

measure of Cohen, Polk, and Vuolteenaho (2005). However, in results not shown, the correlation between expected inflation and *CoBAR* becomes positive for the subsample from 1990-2016, consistent with arbitrage activity eventually taking advantage of this particular source of time-variation in beta-arbitrage profits. There is little to no correlation between *CoBAR* and sentiment.

Figure 1 plots *CoBAR* as of the end of each December. Note that we do not necessarily expect a trend in this measure. Though there is clearly more capital invested in beta-arbitrage strategies, in general, markets are also deeper and more liquid. Nevertheless, after an initial spike in December 1971, *CoBAR* trends slightly upward for the rest of the sample. However, there are clear cycles around this trend. These cycles tend to peak before broad market declines. Also, note that *CoBAR* is essentially uncorrelated with market volatility. A regression of *CoBAR* on contemporaneous realized market volatility produces a loading of 0.01 with a  $t$ -statistic of -0.36.

Consistent with our measure tracking arbitrage activity, Appendix Table A1 shows that *CoBAR* is persistent in event time. Specifically, the correlation between *CoBAR* measured in year 0 and year 1 for the same set of stocks is 0.14. In fact, year 0 *CoBAR* remains highly correlated with subsequent values of *CoBAR* for the same stocks all the way out to year 3. The average value of *CoBAR* remains high as well. Recall that in year 0, the average excess correlation is 0.10. We find that in years 1, 2, and 3, the average excess correlation of these same stocks remains around 0.07.<sup>14</sup>

#### IV.A. Determinants of *CoBAR*

To confirm that our measure of beta-arbitrage is sensible, we estimate regressions forecasting *CoBAR* with four variables that are often used to proxy for arbitrage activity. The first variable we use is the aggregate institutional ownership (*Inst Own*) of the low-beta decile—i.e., stocks in the long leg of the beta strategy—based on 13F filings. We include institutional ownership as these investors are typically considered smart money,

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<sup>14</sup> *CoBAR* is essentially uncorrelated with a similar measure of excess comovement based on the fifth and sixth beta deciles.

at least relative to individuals, and we focus on their holdings in the low-beta decile as we do not observe their short positions in the high-beta decile. We also include the assets under management (*AUM*) of long-short equity hedge funds, the prototypical arbitrageur. Finally, we include a measure of the past profitability of beta-arbitrage strategies, the realized four-factor alpha of Frazzini and Pedersen’s BAB factor. Intuitively, more arbitrageurs should be trading the low-beta strategy after the strategy has performed well in recent past.

All else equal, we expect *CoBAR* to be lower if markets are more liquid. However, as arbitrage activity is endogenous, times when markets are more liquid may also be times when arbitrageurs are more active. Indeed, Cao, Chen, Liang, and Lo (2013) show that hedge funds increase their activity in response to increases in aggregate liquidity. Following Cao, Chen, Liang, and Lo, we further include past market liquidity as proxied by the Pastor and Stambaugh (2003) liquidity factor (*PS liquidity*) in our regressions to measure which channel dominates.

All regressions in Table II include a trend to ensure that our results are not spurious. We also report a second specification that includes variables that arguably should forecast beta-arbitrage returns: the inflation, sentiment, and disagreement indices as well as the Ted spread. We measure these variables contemporaneously with *CoBAR* as we will be running horse races against these variables in our subsequent analysis.

Regression (1) in Table II documents that *Inst Own*, *AUM*, and *PS liquidity* forecast *CoBAR*, with an  $R^2$  of approximately 41%.<sup>15</sup> Regression (2) shows that three of the extant predictors of beta-arbitrage returns help explain *CoBAR*. The Ted spread, adds some incremental explanatory power, with the sign of the coefficient consistent with arbitrageurs taking advantage of potential time-variation in beta-arbitrage returns linked to this channel. Indeed, as we show later, the Ted spread does a poor job forecasting beta-arbitrage returns in practice, perhaps because arbitrageurs have compensated appropriately for this potential departure from Sharpe-Lintner pricing. The disagreement

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<sup>15</sup> We choose to forecast *CoBAR* in a predictive regression rather than explain *CoBAR* in a contemporaneous regression simply to reduce the chance of a spurious fit. However, our results are robust to estimating contemporaneous versions of these regressions.

measure and inflation rate also help explain variation in *CoBAR*. In both specifications, past profitability of a prototypical beta-arbitrage strategy strongly forecasts relatively high arbitrage activity going forward. It seems reasonable that strong past performance of an investment strategy may result in the strategy becoming more popular.

Overall, these findings make us comfortable in our interpretation that *CoBAR* is related to arbitrage activity and distinct from existing measures of opportunities in beta arbitrage. As a consequence, we turn to the main analysis of the paper, the short- and long-run performance of beta-arbitrage returns, conditional on *CoBAR*.

#### IV.B. Forecasting Beta-Arbitrage Returns

Table III forecasts the abnormal returns on the standard beta-arbitrage strategy as a function of investment horizon, conditional on *CoBAR*. Panel A examines Fama and French (1993) three-factor-adjusted returns while Panel B studies abnormal returns relative to the four-factor model of Carhart (1997).<sup>16</sup> In each panel, we measure the average abnormal returns in the first six months subsequent to the beta-arbitrage trade, months 7 through 12, and then those occurring in years two and three. We also report the average abnormal returns across years two and three combined. These returns are measured as a function of *CoBAR* as of the end of the beta formation period. In particular, we split the sample into five *CoBAR* quintiles.

We focus on the four-factor-adjusted estimates. Pursuing beta arbitrage when arbitrage activity is low takes patience. Four-factor-adjusted abnormal returns are statistically insignificant in the first year for the bottom four *CoBAR* groups. Abnormal returns only become statistically significant for the lowest *CoBAR* group in the third year. In the lowest *CoBAR* period, the four-factor alpha is 0.71% per month with an associated *t*-statistic of 2.90.<sup>17</sup>

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<sup>16</sup> We compute the four-factor alpha for each event month after portfolio formation by taking the average across all the calendar months in our sample.

<sup>17</sup> We have also separately examined the long and short legs of beta arbitrage (i.e., low-beta vs. high-beta stocks). Around 40% of our return effect comes from the long leg, and the remaining 60% from the short leg.

However, as beta-arbitrage activity increases, the abnormal returns arrive sooner and stronger. For the highest *CoBAR* group, the four-factor-adjusted abnormal returns average 0.89% per month in the six months immediately subsequent to the beta-arbitrage trade. This finding is statistically significant with a  $t$ -statistic of 2.25, though the difference between abnormal returns in high and low *CoBAR* periods, despite being economically large at 0.62%/month, is statistically insignificant.

The key finding of our paper is that these quicker and stronger beta-arbitrage returns can be linked to subsequent reversal in the long run. Specifically, in year three, the four-factor-adjusted abnormal return to beta arbitrage when *CoBAR* is high is -0.92% per month, with a  $t$ -statistic of -2.63. These abnormal returns are dramatically different from their corresponding values when *CoBAR* is low; the difference in year 3 four-factor-adjusted abnormal returns is -1.63% per month ( $t$ -statistic = -3.83).<sup>18</sup>

Since splitting the long run at year 2 is arbitrary, Table III also reports the results combining years 2 and 3 together. The patterns remain, and these estimates are monotonically decreasing in *CoBAR*.

Figure 2 summarizes these patterns by plotting the cumulative four-factor-adjusted abnormal returns to beta arbitrage during periods of high and low *CoBAR*, accumulating abnormal returns up to 60 months post portfolio formation. We include in the plot the cumulative four-factor-adjusted returns during median *CoBAR* periods as well. This figure clearly shows that there is a significant delay in abnormal trading profits to beta arbitrage when beta-arbitrage activity is low. However, when beta-arbitrage activity is high, beta arbitrage results in price overshooting, as evidenced by the initial price run-up and subsequent reversal that we document. We argue that trading of the low-beta anomaly is initially stabilizing, then, as the trade becomes crowded, turns destabilizing, causing prices to overshoot. The bottom panel of Figure 2 further shows that the run-up to the beta arbitrage strategy during high *CoBAR* periods starts in the formation period, consistent with the view that arbitrageurs may use shorter windows to calculate beta. If this

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<sup>18</sup> We postpone the discussion of conditional abnormal returns to beta arbitrage (as shown in the last row of Table III) to Section V.A.1.



formation-period run-up is included in the accumulation of abnormal return, we can account for effectively all of the reversal.

#### IV.C. Robustness of Key Results

Table IV examines variations to our methodology to ensure that our finding of time-varying reversal of beta-arbitrage profits is robust. For simplicity, we report the difference in returns to the beta strategy between the high and low *CoBAR* groups in two versions of the long run (year 3 and years 2-3). For reference, the first row of Panel A Table IV reports the baseline results from Table III Panel B.

In row two of Panel A, we conduct the same analysis for the sub-period before our sample (1927-1969). Of course, this sample not only predates the discovery of the low-beta anomaly but also is a period where there is much less arbitrage activity in general, at least explicitly organized as such. Thus, this period could be thought of as a placebo test of our story. Consistent with our paper’s explanation, we find no statistically significant link between *CoBAR* and the reversal of beta-arbitrage returns. Our remaining subsample analysis excludes potential outlier years. We find that our results remain robust if we exclude the tech bubble crash (2001) or the recent financial crisis (2007-2009) from our sample.

In Panel B, we consider alternative definitions of *CoBAR*. In row 1, we control for UMD when computing *CoBAR*. In row 2, we separate HML into its large cap and small cap components. In row 3, we report results based on Fama and French (2015a, 2015b) five-factor returns. In row 4, we perform the entire analysis on an industry-adjusted basis by sorting stocks into beta deciles within industries. Row 5 uses the correlation between the high-beta and low-beta deciles as a measure of arbitrage activity, with lower values indicating more activity.

Rows 6 and 7 split *CoBAR* into upside and downside components. Specifically, we measure the following

$$CoBAR^U = \frac{1}{N} \sum_{i=1}^N \text{partialCorr}(\text{retrf}_i^L, \text{retrf}_{-i}^L | \text{mktrf}, \text{smb}, \text{hml}, \text{retrf}^L \geq \text{median}(\text{retrf}^L))$$

$$CoBAR^D = \frac{1}{N} \sum_{i=1}^N \text{partialCorr}(retrf_i^L, retrf_{-i}^L | mktrf, smb, hml, retrf^L < \text{median}(retrf^L))$$

Separating *CoBAR* in this way allows us to distinguish between excess comovement tied to strategies buying low-beta stocks (such as those followed by beta arbitrageurs) and strategies selling low-beta stocks (such as leveraged-constrained investors modeled by Black (1972)). Consistent with our interpretation, we find that only *CoBAR*<sup>U</sup> forecasts time variation in the short- and long-run expected returns to beta arbitrage (whereas *CoBAR*<sup>D</sup> does not).

Panel C documents that our results are robust to replacing *CoBAR* with residual *CoBAR*. In particular, we orthogonalize *CoBAR* to measures of arbitrage activity in momentum and value (Lou and Polk, 2021), the average correlation in the market (Pollet and Wilson 2010), the past volatility of beta-arbitrage returns, the volatility of market returns over the twelve-month period corresponding to the measurement of *CoBAR*, a trend, lagged *CoBAR* (the year -1 average pairwise excess correlation of low-beta stocks identified in year 0), smoothed past inflation (Cohen, Polk, and Vuolteenaho, 2005), the sentiment index of Baker and Wurgler (2006), aggregate analyst forecast dispersion (Hong and Sraer, 2014), and the TED Spread (Frazzini and Pedersen, 2014). Finally, in Panel D, we measure abnormal returns using a six-factor model that augments the Fama-French five-factor model with momentum.

In all cases, *CoBAR* continues to predict time-variation in year 3 returns. The estimates are always economically significant, with most point estimates larger than 1% per month. Statistical significance is always strong as well, with most *t*-statistics larger than 3. Taken together, these results confirm that our measure of crowded beta arbitrage *robustly* forecasts times of strong reversal to beta-arbitrage strategies.

#### IV.D. Predicting the Security Market Line

Our results can also be seen from time variation in the shape of the security market line (SML) as a function of lagged *CoBAR*. Such an approach can help ensure that the time-variation we document is not restricted to a small subset of extreme-beta stocks, but

instead is a robust feature of the cross-section. (We argue that beta arbitrage activity can affect the entire cross-section of stocks rather than just the extreme deciles, because arbitrageurs may bet against the low-beta anomalies by selecting portfolio weights that are inversely proportional to the market beta.) At the end of each month, we sort all stocks into 20 value-weighted portfolios by their pre-ranking betas. We track these 20 portfolios' returns both in months 1-6 and months 25-36 after portfolio formation to compute short-term and long-term post-ranking betas, and, in turn, to construct our short-term and long-term security market lines.

For the months 1-6 portfolio returns, we then compute the post-ranking betas by regressing each of the 20 portfolios' value-weighted monthly returns on market excess returns. Following Fama and French (1992), we use the entire sample to compute post-ranking betas. That is, we pool together these six monthly returns across all calendar months to estimate portfolio beta. We estimate post-ranking betas for months 25-36 in a similar fashion. The two sets of post-ranking betas are then labelled  $\beta_1^1, \dots, \beta_{20}^1$  and  $\beta_1^{25}, \dots, \beta_{20}^{25}$ .

To calculate the intercept and slope of the short-term and long-term security market lines, we estimate the following cross-sectional regressions:

$$\text{short-term SML: } XRet_{i,t}^1 = \textit{intercept}_t^1 + \textit{slope}_t^1 \beta_i^1,$$

$$\text{long-term SML: } XRet_{i,t}^{25} = \textit{intercept}_t^{25} + \textit{slope}_t^{25} \beta_i^{25},$$

where  $XRet_{i,t}^1$  is portfolio  $i$ 's monthly excess returns in months 1 through 6, and  $XRet_{i,t}^{25}$  is portfolio  $i$ 's monthly returns in months 25 through 36. These two regressions then give us two time-series of coefficient estimates of the intercept and slope of the short-term and long-term security market lines:  $(\textit{intercept}_t^1, \textit{slope}_t^1)$  and  $(\textit{intercept}_t^{25}, \textit{slope}_t^{25})$ , respectively. As the average excess returns and post-ranking betas are always measured at the same point in time, the pair  $(\textit{intercept}_t^1, \textit{slope}_t^1)$  fully describes the security market line in the short run, while  $(\textit{intercept}_t^{25}, \textit{slope}_t^{25})$  captures the security market line two years later.

We then examine how these intercepts and slopes vary as a function of our measure of beta-arbitrage capital. As can be seen from the top panel of Figure 3, the intercept of

the short-term security market line significantly increases in *CoBAR*, and its slope significantly decreases in *CoBAR*. During high *CoBAR*—i.e., high beta-arbitrage capital—periods, the short-term security market line strongly slopes downward, indicating strong profits to the low-beta strategy, consistent with arbitrageurs expediting the correction of market misevaluation. In contrast, during low *CoBAR*—i.e., low beta-arbitrage capital—periods, the short-term security market line is weakly upward sloping and the beta-arbitrage strategy, as a consequence, unprofitable, consistent with delayed correction of the beta anomaly.

The pattern is completely reversed for the long-term security market line. The intercept of the long-term security market line is significantly negatively related to *CoBAR*, whereas its slope is significantly positively related to *CoBAR*. As can be seen from the bottom panel of Figure 3, two years after high *CoBAR* periods, the long-term security market line turns upward sloping; indeed, the slope is so steep (resulting in a negative intercept) that the beta strategy loses money, consistent with over-correction of the low beta anomaly by crowded arbitrage trading. In contrast, after low *CoBAR* periods, the long-term security market line turns downward sloping, reflecting eventual profitability of the low-beta strategy in the long run.

#### IV.E. Smarter Beta-Arbitrage Strategies

One way to measure the economic importance of these boom and bust cycles is through an *out-of-sample* calendar-time trading strategy.<sup>19</sup> We combine these time-varying overreaction and subsequent reversal patterns as follows. We first time the standard beta-arbitrage strategy using current *CoBAR*. If *CoBAR* is above the 80<sup>th</sup> percentile (of its distribution up to that point), we invest in the long-short beta-arbitrage strategy studied in Table III for the next six months. Otherwise, we short that portfolio over that time period. We skip the first three years of our sample to compute the initial distribution as well as show in-sample results in Panel A of Table V for the sake of comparison.

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<sup>19</sup> This calendar-time approach also ensures that our Newey-West standard errors (for example, the standard errors of Table III) are not misleading about the statistical significance of our findings.

In addition, if *CoBAR* from three years ago is below the 20<sup>th</sup> percentile (of its prior distribution), we long for the next twelve months the long-short beta-arbitrage strategy based on beta estimates from three years ago. Otherwise, we short that portfolio, again for the next twelve months.

This “smarter” beta-arbitrage strategy harvests beta-arbitrage profits much more wisely than unconditional bets against beta. As can be seen from Panel B of Table V, the four-factor alpha is 45 basis points per month with a  $t$ -statistic of 2.46. The six-factor alpha (where we add the investment and profitability factors of Fama and French, 2015a) remains high at 47 basis points per month ( $t$ -statistic of 2.42). Finally, if we also include the BAB factor of Frazzini and Pedersen (2014) as a seventh factor, the abnormal return increases to 62 basis points per month with a  $t$ -statistic of 3.25. By comparison, the standard value-weight beta-arbitrage strategy yields a four-factor alpha of 0.02% per month ( $t$ -statistic = 0.08) in our sample period.

We have also estimated conditional regressions where we interact each factor with *CoBAR* to control for conditional risk exposures. The alpha from this regression is significantly larger at 0.83% per month ( $t$ -statistic of 2.65).<sup>20</sup>

## V. Testing the Economic Mechanism

The previous section documents rich cross-sectional and time-series variation in expected returns linked to our proxy for arbitrage activity and the low-beta anomaly. In this section, we delve deeper to test specific aspects of the economic mechanism behind these patterns. Our interpretation of these patterns makes specific novel predictions in terms of the role of firm leverage, the limits to arbitrage, and the reaction of sophisticated investors to these patterns.

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<sup>20</sup> We construct an “even-smarter” beta arbitrage strategy by further exploiting differences between high-leverage and low-leverage firms. In particular, we divide all stocks into four quartiles based on their lagged leverage ratios. We then go long the smart-beta-strategy formed solely with high-leverage stocks and short the smart-beta strategy solely with low-leverage stocks. This “even-smarter” beta strategy yields a monthly alpha of 46bp ( $t$ -statistic = 2.68) after controlling for the Fama and French (2016) five factors, momentum factor, the Frazzini and Pedersen (2014) BAB factor, as well as our “smarter” beta-arbitrage portfolio studied in Table V.

## V.A. Beta Expansion

Beta arbitrage can be susceptible to positive-feedback trading. Successful bets on (against) low-beta (high-beta) stocks result in prices of those securities rising (falling). If the underlying firms are leveraged, this change in price will, all else equal, result in the security's beta falling (increasing) further.<sup>21</sup> Thus, not only do arbitrageurs not know when to stop trading the low-beta strategy, their (collective) trades also affect the strength of the signal. Consequently, beta arbitrageurs may increase their bets precisely when trading becomes crowded and the expected profitability of the strategy has decreased.

We test this prediction in Table VI. The dependent variable is the spread in betas across the high and low value-weight beta decile portfolios, denoted *BetaSpread*, as of the end of year 1. The independent variables include lagged *CoBAR*, the beta-formation-period value of *BetaSpread* (computed from the same set of low- and high- beta stocks as the dependent variable), the average book leverage quintile (*Leverage*) across the high and low beta decile portfolios, and an interaction between *CoBAR* and *Leverage*. Note that since we estimate beta using 52 weeks of stock returns, the two periods of beta estimation that determine the change in *BetaSpread* do not overlap. (Our results are robust to including a time trend in the regression.)

Regression (1) in the Panel A of Table VI shows that when *CoBAR* is relatively high, future *BetaSpread* is also high, controlling for lagged *BetaSpread*. A one-standard-deviation increase in *CoBAR* forecasts an increase in *BetaSpread* of roughly 6% (of the average beta spread). Regression (2) shows that this is particular true when *Leverage* is also high. If beta-arbitrage bets were to contain the highest book-leverage quintile stocks, a one-standard deviation increase in *CoBAR* would increase *BetaSpread* by nearly 10%.

These results are consistent with a positive feedback channel for the beta-arbitrage strategy that works through firm-level leverage. In terms of the economic magnitude of this positive feedback loop, we draw a comparison with the price momentum strategy.

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<sup>21</sup> The idea that, all else equal, changes in leverage drive changes in equity beta is, of course, the key insight behind Proposition II of Modigliani and Miller (1958).

The formation-period spread for a standard price momentum bet in the post-1963 period is around 115%, while the momentum profit in the subsequent year is close to 12% (e.g., Lou and Polk, 2021). Put differently, if we attribute price momentum entirely to positive feedback trading, such trading increases the initial return spread by about 10% (12% divided by 115%) in the subsequent year, which is similar in magnitude to the positive feedback channel we document for beta arbitrage. Table VI Panel B confirms that these results are robust to the same methodological variations as in Table IV.

Table VII turns to firm-level regressions to document the beta expansion our story predicts. In particular, we estimate panel regressions forecasting beta with lagged *CoBAR*. At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. The dependent variable is *Post Ranking Beta*, the change in stock beta from year  $t$  to  $t+1$  (again, we use non-overlapping periods). In addition to *CoBAR*, we also include *Distance*, the difference between a stock’s pre-formation beta and the average pre-formation beta in year  $t$ . *Leverage* is the book leverage of the firm, measured in year  $t$ . We also include all double and triple interaction terms of *CoBAR*, *Distance*, and *Leverage*. Other control variables include the lagged firm size, book-to-market ratio, lagged one-month and one-year stock return, and the prior-year idiosyncratic volatility. Time-fixed effects are included in Columns 3 and 4. Note that since *CoBAR* is a time-series variable, it is subsumed by the time dummies in those regressions.

In all four regressions, stocks with higher *Distance* have a higher *Post Ranking Beta*, consistent with betas being persistent. This persistence is higher when *CoBAR* is relatively high. Our main focus is on the triple interaction among *CoBAR*, *Distance*, and *Leverage*. The persistence in a firm’s beta is significantly stronger when *CoBAR* and *Leverage* are high. Taken together, these results are consistent with beta-arbitrage activity causing the cross-sectional spread in betas to expand.

As a natural extension, our positive feedback channel suggests that booms and busts of beta arbitrage should be especially strong among more highly levered stocks. Appendix Figure A1 reports results where the sample is split based on leverage. Specifically, at the beginning of the holding period, we sort stocks into four equal groups

using book leverage. For each leverage quartile, we compute the *CoBAR* return spread – i.e., the difference in four-factor alpha to the beta arbitrage strategy between high and low *CoBAR* periods. Reported in the figure is the cumulative difference in the *CoBAR* return spread between the highest and lowest leverage quartiles over the five years after portfolio formation.

As can be seen from the figure, the difference in the *CoBAR* return spread rises substantially in the first twelve months, by about 20% ( $1.67\% \times 12$ ). It then mostly reverses in the subsequent years. For example, the cumulative *CoBAR* return spread in year 4 is roughly -7% ( $-0.60\% \times 12$ ). Both are significant at 5%. This finding is consistent with our novel positive feedback channel facilitating excessive arbitrage trading activity that can potentially destabilize prices.

In results not reported, we have also confirmed that leverage splits enhance the profitability of the calendar-time strategies studied in Table V. Specifically, we go long a version of the trading strategy restricted to the top quartile of firms based on leverage and go short the corresponding low leverage (bottom quartile) version. The resulting in-sample alpha is 46 basis points per month with a  $t$ -statistic of 2.68. The out-of-sample estimate still generates a statistically-significant 31 basis points per month.

### V.A.1. Conditional Attribution

Given that beta is moving with *CoBAR*, we also estimate conditional performance attribution regressions (that is, we allow for the possibility that portfolio betas and expected market and factor returns comove in the time series). The last rows of Table III Panel A and Panel B report the results of those regressions. We find that the long-run reversal of beta-arbitrage profits remains; there is again an economically large reversal of beta-arbitrage profits when *CoBAR* is high. Figure 4 plots the conditional security market line in the short and long-run as a function of lagged *CoBAR*. It is easy to see from the figure our result that beta expansion and destabilization go hand-in-hand: the range of average beta across the 20 beta-portfolios is much larger during high *CoBAR* periods than in low *CoBAR* periods.



## V.B. *Low Limits to Arbitrage*

We interpret our findings as consistent with arbitrage activity facilitating the correction of the slope of the security market line in the short run. However, in periods of crowded trading, arbitrageurs can cause price overshooting. In Table VIII, we exploit cross-sectional heterogeneity to provide additional support for our interpretation. All else equal, arbitrageurs prefer to trade stocks with low idiosyncratic volatility (to reduce tracking error), high liquidity (to facilitate opening/closing of the position), and large capitalization (to increase strategy capacity). As a consequence, we split our sample along each of these dimensions. In particular, we rank stocks into quartiles based on the variable in question (as of the beginning of the holding period); we label the quartile with the weakest limits to arbitrage as “Low LTA” and the quartile with the strongest limits to arbitrage as “High LTA.” Our focus is on the long-run reversal associated with periods of high *CoBAR*.

The first two columns report results based on market capitalization, the third and fourth based on idiosyncratic volatility, and the final two based on illiquidity. The first column of each pair shows the difference in four-factor alpha to the beta arbitrage strategy between high *CoBAR* periods and low *CoBAR* periods in Year 3 while the second column shows the difference occurring in Years 2-3.

For each of the three proxies for low limits to arbitrage, we find economically and statistically significant differences in the predictability of year 3 (and years 2-3) returns. In summary, Table VIII confirms that our effect is stronger among stocks with weaker limits of arbitrage, exactly where one expects arbitrageurs to play a more important role.

## V.C. *Time-series and Cross-sectional Variation in Fund Exposures*

We next use our novel measure of beta-arbitrage activity to understand time-series and cross-sectional variation in the performance of long-short/market-neutral hedge funds, typically considered to be the classic example of smart money; as well as active mutual funds, who are subject to more stringent leverage and short-sale constraints. Appendix Table A2 reports estimates of panel regressions of monthly fund returns on the Fama-

French-Carhart four-factor model augmented with the beta-arbitrage factor of Frazzini and Pedersen (2014). In particular, we allow the coefficient on the Frazzini-Pedersen betting-against-beta (BAB) factor to vary as a function of *CoBAR*, a fund's AUM, and the interaction between these two variables. To capture variation in a fund's AUM, we create a dummy-variable, *SizeRank*, that takes the value of zero if the fund is in the smallest-AUM tercile (within the active mutual fund or long-short equity hedge fund industry, depending on the returns being analyzed) in the previous month, one if it is in the middle tercile, and two otherwise. The first two columns analyze hedge fund returns while the last two columns analyze active mutual fund returns.

We find that the typical long-short equity hedge fund increases their exposure to the BAB factor when *CoBAR* is relatively high. For the 20% of the sample period that is associated with the lowest values of *CoBAR*, the typical hedge fund's BAB loading is -0.063. This loading increases by 0.017 for each increment in *CoBAR* rank. (It is noteworthy that the average long-short hedge fund is loading negatively on the BAB factor – i.e., on average, funds are tilting towards high beta stocks.)

Adding the interaction with AUM reveals that the ability of hedge funds to time beta-arbitrage strategies is decreasing in the size of the fund's assets under management. These findings seem reasonable as we would expect large funds to be unable to time a beta-arbitrage strategy as easily as smaller (and presumably nimbler) funds.

The typical small fund's exposure increases by 0.030 for each increase in *CoBAR* rank. Thus, when *CoBAR* is in the top quintile, the typical small hedge fund's BAB loading is 0.047. In contrast, large hedge funds' BAB loading moves by 0.016 from the bottom to the top *CoBAR* quintiles, a much smaller increase in exposure to beta arbitrage. Indeed, when *CoBAR* is high, small hedge funds have loadings on BAB that are nearly twice as large.

As can be seen from columns 3 and 4, there is a vastly different pattern in the market exposures of mutual funds. To start, mutual funds have an average market beta that is larger than one. Second, none of the interactions are statistically significant. In particular, mutual funds' loadings on market risk do not vary with *CoBAR*, our proxy for the strategy's crowdedness.

## V.D. Fresh versus Stale Beta

Though beta-arbitrage activity may cause the beta spread to vary through time, for a feedback loop to occur, beta arbitrageurs must base their strategies on fresh estimates of beta rather than on stale estimates. (Note that the autocorrelation in a stock's market beta is far less than one.) Consistent with this claim, we show that our predictability results decay as a function of beta staleness.

We repeat the previous analysis of section IV.B, but replacing our fresh beta estimates (measured over the most recent year) with progressively staler ones. In particular, we estimate betas in each of the five years prior to the formation year. As a consequence, both the resulting beta strategy and the associated *CoBAR* are different for each degree of beta staleness. For each of these six beta strategies, we regress the four-factor alpha of the strategy in months 1-6 and year 3 on its corresponding *CoBAR*.

Appendix Figure A2 plots the resulting regression coefficients (results for months 1-6 plotted with a blue square and results for year 3 plotted with a red circle) as a function of the degree of staleness of beta. We find that both the short-run and long-run predictability documented in section IV.B decay as the beta signal becomes more and more stale. Indeed, strategies using beta estimates that are five years old display no predictability. These results are consistent with the feedback loop we propose.<sup>22</sup>

## VI. Conclusion

We study the response of arbitrageurs to a flat security market line. Using an approach to measuring arbitrage activity first introduced by Lou and Polk (2021), we document booms and busts in beta arbitrage. Specifically, we find that when arbitrage activity is relatively low, abnormal returns on beta-arbitrage strategies take much longer to materialize, appearing only two to three years after putting on the trade. In sharp contrast, when arbitrage activity is relatively high, abnormal returns on beta-arbitrage

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<sup>22</sup> Appendix Figure A3 documents that the unconditional CAPM alpha for a beta-arbitrage strategy also significantly declines with beta staleness.

strategies occur relatively quickly, within the first six months of the trade. These strong abnormal returns then revert over the next three years. Thus, our findings are consistent with arbitrageurs exacerbating the time-variation in the expected return to beta arbitrage we document.

We provide evidence on a novel positive feedback channel for beta-arbitrage activity. Since the typical firm is levered and given the mechanical effects of leverage on equity beta (Modigliani and Miller 1958), buying low-beta stocks and selling high-beta stocks may cause the cross-sectional spread in betas to increase. We show that this beta expansion occurs when beta-arbitrage activity is high and particularly so when stocks typically traded by beta arbitrageurs are highly levered. Thus, beta arbitrageurs may actually increase their bets when the profitability of the strategy has decreased. Indeed, we find that the short-run abnormal returns to high-leverage beta-arbitrage stocks more than triples before reverting in the long run.

Interestingly, the *unconditional* four-factor alpha of a value-weight beta-arbitrage strategy over our 1970-2019 sample is close to zero, much lower than the positive value one finds for earlier samples (also see Novy-Marx and Velikov, 2020). Thus, it seems that arbitrageurs' response to Black, Jensen, and Scholes's (1972) famous finding has been right *on average*. However, our conditional analysis reveals rich time-series variation that is consistent with the general message of Stein (2009): Arbitrage activity faces a significant coordination problem for unanchored strategies that have positive feedback characteristics.

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Table I: Summary Statistics

This table provides characteristics of *CoBAR*, the *excess* comovement among low beta stocks over the period 1970-2016 (we then examine beta arbitrage returns in the following three years, so the return sample ends in 2019). At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and non-synchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return, in addition to the contemporaneous excess market return. The pre-ranking beta is the sum of the six coefficients from the OLS regression. Pairwise partial return correlations (controlling for the Fama-French three factors) for all stocks in the bottom beta decile are computed based on weekly stock returns in the previous 12 months. *CoBAR* is the average pair-wise correlation between any two stocks in the low-beta decile in the previous 12 months. *Inflation* is the smoothed inflation rate used by Cohen, Polk, and Vuolteenaho (2005), who apply an exponentially weighted moving average (with a half-life of 36 months) to past log growth rates of the Producer Price Index. *Sentiment* is the sentiment index proposed by Wurgler and Baker (2006, 2007). *Disagreement* is the beta-weighted standard deviation of analysts' long-term growth rate forecasts, as used in Hong and Sraer (2016). *TED Spread* is the difference between the LIBOR rate and the US Treasury bill rate. Panel A reports the summary statistics of these variables. Panel B shows the time-series correlations among these variables for the entire sample period.

Panel A: Summary Statistics					
Variable	N	Mean	Std. Dev.	Min	Max
<i>CoBAR</i>	564	0.104	0.026	0.037	0.203
Inflation	564	0.003	0.002	-0.001	0.008
Sentiment	564	0.015	0.939	-2.420	3.200
Disagreement	420	0.054	1.012	-1.277	3.593
TED Spread	372	0.588	0.428	0.118	3.353

Panel B: Correlation					
	<i>CoBAR</i>	Inflation	Sentiment	Disagreement	TED Spread
<i>CoBAR</i>	1				
Inflation	-0.272	1			
Sentiment	0.024	-0.361	1		
Disagreement	0.271	-0.242	0.132	1	
TED Spread	0.290	0.277	0.007	-0.211	1

Table II: Determinants of *CoBAR*

This table reports regressions of *CoBAR*, described in Table I, on lagged variables plausibly linked to arbitrage activity in the post-1993 period (constrained by the availability of the hedge fund AUM data). At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and non-synchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return, in addition to the contemporaneous excess market return. The pre-ranking beta is the sum of the six coefficients from the OLS regression. The dependent variable in the regressions, *CoBAR*, is the average pairwise partial weekly return correlation in the low-beta decile over 12 months. *Inst Own* is the aggregate institutional ownership of the low-beta decile, *AUM* is the logarithm of the total assets under management of long-short equity hedge funds (detrended). *BAB Alpha* is the realized four-factor alpha of Frazzini and Pedersen's BAB factor. *Inflation* is the smoothed inflation rate used by Cohen, Polk, and Vuolteenaho (2005), who apply an exponentially weighted moving average (with a half-life of 36 months) to past log growth rates of the producer price index. *Sentiment* is the sentiment index proposed by Wurgler and Baker (2006, 2007). *Disagreement* is the beta-weighted standard deviation of analysts' long-term growth rate forecasts, as used in Hong and Sraer (2012). *TED Spread* is the difference between the LIBOR rate and the US Treasury bill rate. We also include in the regression the Pastor-Stambaugh liquidity factor (*PS Liquidity*). A trend dummy is included in all regression specifications. All independent variables are standardized to have a mean of zero and standard deviation of one, so that the coefficient represents the effect of a one-standard-deviation change in the independent variable on *CoBAR*. Standard errors are shown in brackets. \*, \*\*, \*\*\* denote significance at the 10%, 5%, and 1% level, respectively.

<i>DepVar</i>	<i>CoBAR<sub>t</sub></i>	
	[1]	[2]
<i>Inst Own<sub>t-1</sub></i>	0.023*** [0.002]	0.011*** [0.002]
<i>AUM<sub>t-1</sub></i>	0.004** [0.001]	0.003** [0.001]
<i>BAB Alpha<sub>t-1</sub></i>	0.003* [0.001]	0.006*** [0.001]
<i>Inflation<sub>t</sub></i>		-0.003** [0.001]
<i>Sentiment<sub>t</sub></i>		0.001 [0.002]
<i>Disagreement<sub>t</sub></i>		0.012*** [0.002]
<i>TED Spread<sub>t</sub></i>		0.011*** [0.002]
<i>PS Liquidity<sub>t</sub></i>	0.007*** [0.001]	0.011*** [0.001]
TREND	Yes	Yes
Adj-R <sup>2</sup>	0.413	0.567
No. Obs.	288	288

Table III: Forecasting Beta-arbitrage Returns with *CoBAR*

This table reports returns to the beta arbitrage strategy as a function of lagged *CoBAR*. At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and non-synchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return, in addition to the contemporaneous excess market return. The pre-ranking beta is the sum of the six coefficients from the OLS regression. All months are then classified into five groups based on *CoBAR*, the average pairwise partial weekly return correlation in the low-beta decile over the past 12 months. Reported below are the returns to the beta arbitrage strategy (i.e., to go long the value-weight low-beta decile and short the value-weight high-beta decile) in each of the three years after portfolio formation during 1970 to 2016, following low to high *CoBAR*. Panels A and B report, respectively, the average monthly three-factor alpha and Carhart four-factor alpha of the beta arbitrage strategy. “5-1” is the difference in monthly returns to the long-short strategy following high vs. low *CoBAR*; “5-1 Conditional” is the difference in conditional abnormal returns (i.e., allowing for risk loadings to vary as a function of *CoBAR*) following high vs. low *CoBAR*. We compute *t*-statistics, shown in parentheses, based on standard errors corrected for serial-dependence up to 24 lags. 5% statistical significance is indicated in bold.

Panel A: Fama-French-Adjusted Beta-arbitrage Returns											
		Months 1-6		Months 7-12		Year 2		Year 3		Years 2&3	
Rank	Obs.	Est.	t-stat	Est.	t-stat	Est.	t-stat	Est.	t-stat	Est.	t-stat
1	112	<b>0.50%</b>	(2.09)	<b>0.74%</b>	(2.98)	<b>0.78%</b>	(2.82)	<b>0.94%</b>	(4.06)	<b>0.82%</b>	(3.49)
2	113	0.05%	(0.18)	<b>0.59%</b>	(2.25)	<b>0.78%</b>	(3.03)	0.41%	(1.94)	<b>0.63%</b>	(3.09)
3	113	-0.37%	(-1.32)	0.45%	(1.58)	0.46%	(1.81)	0.50%	(1.95)	<b>0.50%</b>	(-2.20)
4	113	-0.11%	(-0.41)	0.41%	(1.48)	0.00%	(-0.01)	0.19%	(0.60)	0.11%	(0.41)
5	113	<b>1.28%</b>	(3.24)	0.38%	(0.89)	0.01%	(0.02)	-0.59%	(-1.67)	-0.29%	(-1.70)
5-1		0.78%	(1.72)	-0.37%	(-0.75)	-0.77%	(-1.42)	<b>-1.53%</b>	(-3.55)	<b>-1.11%</b>	(-3.21)
5-1(Cond)		0.24%	(0.59)	-0.77%	(-1.75)	-0.99%	(-1.92)	<b>-1.52%</b>	(-3.75)	<b>-1.22%</b>	(-3.94)

Panel B: Four-Factor Adjusted Beta-arbitrage Returns											
		Months 1-6		Months 7-12		Year 2		Year 3		Years 2&3	
Rank	Obs.	Est.	t-stat	Est.	t-stat	Est.	t-stat	Est.	t-stat	Est.	t-stat
1	112	0.27%	(1.17)	0.49%	(1.89)	<b>0.55%</b>	(1.97)	<b>0.71%</b>	(2.90)	<b>0.59%</b>	(2.30)
2	113	-0.09%	(-0.33)	0.27%	(1.01)	<b>0.53%</b>	(2.09)	0.22%	(0.93)	0.40%	(1.80)
3	113	<b>-0.62%</b>	(-2.24)	0.18%	(0.62)	0.13%	(0.47)	0.23%	(0.86)	0.21%	(0.81)
4	113	-0.39%	(-1.46)	0.14%	(0.54)	-0.28%	(-0.80)	-0.09%	(-0.26)	-0.17%	(-0.51)
5	113	<b>0.89%</b>	(2.25)	0.13%	(0.32)	-0.15%	(-0.36)	<b>-0.92%</b>	(-2.63)	<b>-0.54%</b>	(-2.50)
5-1		0.62%	(1.37)	-0.36%	(-0.75)	-0.70%	(-1.35)	<b>-1.63%</b>	(-3.83)	<b>-1.13%</b>	(-3.05)
5-1 (Cond)		0.08%	(0.21)	<b>-1.04%</b>	(-2.44)	<b>-1.18%</b>	(-2.51)	<b>-1.87%</b>	(-4.77)	<b>-1.50%</b>	(-4.55)

## Table IV: Robustness Checks

This table reports returns to the beta arbitrage strategy as a function of lagged *CoBAR*. At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and non-synchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return, in addition to the contemporaneous excess market return. The pre-ranking beta is the sum of the six coefficients from the OLS regression. All months are then classified into five groups based on *CoBAR*, the average pairwise partial weekly return correlation in the low-beta decile over the past 12 months. Reported below is the difference in four-factor alpha to the beta arbitrage strategy between high *CoBAR* periods and low *CoBAR* periods. Year zero is the beta portfolio ranking period. In Panel A, we consider different subsample results. Row 1 shows the baseline results which are also reported in Table III. Row 2 shows the same analysis for the earlier sample (1927-1969) as a placebo test. In rows 3 and 4, we exclude the tech bubble crash and the recent financial crisis from our sample. In Panel B, we explore alternative definitions of *CoBAR*. In row 1, we control for the UMD factor in computing *CoBAR*. In row 2, we control for both large- and small-cap HML in computing *CoBAR*. In row 3, we control for the Fama-French five factor model that adds profitability and investment to their three-factor model. In row 4, we perform the entire analysis on an industry-adjusted basis by sorting stocks into beta deciles within industries. In row 5, we instead measure the correlation between the high and low-beta portfolios, with a low correlation indicating high arbitrage activity. In Rows 6 and 7, we examine upside and downside *CoBAR*, as distinguished by the median low-beta portfolio return. In Panel C, we replace *CoBAR* with residual *CoBAR* from a time-series regression where we purge from *CoBAR* variation linked to, respectively, *CoMOM* and *CoValue* (Lou and Polk, 2021; rows 1-2), the average pair-wise correlation in the market (row 3), the lagged 36-month volatility of the BAB factor (Frazzini and Pedersen, 2013; row 4), market volatility over the past 24 months (row 5), a trend (row 6), lagged *CoBAR* (where we hold the stocks in the low-beta decile constant but calculate *CoBAR* using returns from the previous year; row 7), smoothed past inflation (Cohen, Polk, and Vuolteenaho, 2005; row 8), a sentiment index (Baker and Wurgler, 2006; row 9), aggregate analyst forecast dispersion (Hong and Sraer, 2014; row 10), and the TED Spread (Frazzini and Pedersen, 2014; row 11). In Panel D, abnormal returns to the beta arbitrage strategy are calculated relative to the Fama-French 5 factor model plus the momentum factor. We compute *t*-statistics, shown in parentheses, based on standard errors corrected for serial-dependence up to 24 lags. 5% statistical significance is indicated in bold.

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Four-Factor Adjusted Beta-arbitrage Returns

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	Year 3		Years 2-3	
	Estimate	t-stat	Estimate	t-stat
<u>Panel A: Subsamples</u>				
(1) Full Sample: 1970-2010	<b>-1.63%</b>	(-3.83)	<b>-1.13%</b>	(-3.05)
(2) Placebo test: Early sample: 1927-1969	0.15%	(0.32)	0.17%	(0.53)
(3) Excluding 2001	<b>-1.55%</b>	(-3.75)	<b>-1.04%</b>	(-2.54)
(4) Excluding 2007-2009	<b>-1.60%</b>	(-3.53)	<b>-1.13%</b>	(-2.85)
 <u>Panel B: Alternative definitions of <i>CoBAR</i></u>				
(1) Controlling for UMD	<b>-1.65%</b>	(-4.02)	<b>-1.18%</b>	(-3.05)
(2) Controlling for Large/Small-Cap HML	<b>-1.59%</b>	(-3.66)	<b>-1.08%</b>	(-2.76)
(3) Controlling for FF Five Factors	<b>-1.42%</b>	(-3.05)	<b>-1.04%</b>	(-2.66)
(4) Controlling for Industry Factors	<b>-1.21%</b>	(-2.66)	<b>-0.98%</b>	(-2.51)
(5) Correl btw High and Low Beta Stocks	<b>-1.12%</b>	(-3.08)	<b>-1.00%</b>	(-2.87)
(6) Upside <i>CoBAR</i>	-0.80%	(-1.82)	-0.69%	(-1.80)
(7) Downside <i>CoBAR</i>	-0.39%	(-0.92)	0.04%	(0.15)
 <u>Panel C: Residual <i>CoBAR</i></u>				
(1) Controlling for CoMomentum	<b>-1.68%</b>	(-4.02)	<b>-1.20%</b>	(-3.28)
(2) Controlling for CoValue	<b>-1.77%</b>	(-4.18)	<b>-1.23%</b>	(-3.18)
(3) Controlling for MKT CORR	<b>-1.76%</b>	(-4.33)	<b>-1.23%</b>	(-3.38)
(4) Controlling for Vol(BAB)	<b>-1.60%</b>	(-3.89)	<b>-1.13%</b>	(-3.19)
(5) Controlling for Mktvol24	<b>-1.63%</b>	(-3.83)	<b>-1.13%</b>	(-3.05)
(6) Controlling for Trend	<b>-1.62%</b>	(-3.60)	<b>-1.15%</b>	(-2.92)
(7) Controlling for Pre-formation <i>CoBAR</i>	<b>-1.67%</b>	(-3.97)	<b>-1.25%</b>	(-3.37)
(8) Controlling for Inflation	<b>-1.73%</b>	(-4.18)	<b>-1.17%</b>	(-3.39)
(9) Controlling for Sentiment	<b>-1.65%</b>	(-3.89)	<b>-1.18%</b>	(-3.23)
(10) Controlling for Disagreement	<b>-2.24%</b>	(-5.64)	<b>-1.55%</b>	(-4.17)
(11) Controlling for TED spread	<b>-1.87%</b>	(-4.01)	<b>-1.36%</b>	(-3.57)
 <u>Panel D: Beta arbitrage alpha based on</u>				
Controlling for FF5+MOM	<b>-1.57%</b>	(-4.03)	<b>-1.16%</b>	(-3.33)

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Table V: Smarter Beta-Arbitrage Strategies

This table reports monthly returns to a smarter beta-arbitrage strategy that exploits the time-varying overreaction and subsequent reversal present in standard beta arbitrage strategies. Specifically, we first time the standard beta-arbitrage strategy using current *CoBAR*. If *CoBAR* is above the 80th percentile, we go long the long-short beta-arbitrage strategy studied in Table III for the next six months. Otherwise, we short that portfolio over that time period. In addition, if *CoBAR* from three years ago is below the 20th percentile, we go long for the next twelve months the long-short beta-arbitrage strategy based on beta estimates from three years ago. Otherwise, we short that portfolio, again for the next twelve months. In Panel A, the percentile break points of the *CoBAR* distribution are identified using the entire *CoBAR* time series (thus an in-sample analysis). In Panel B, the percentile break points are identify based solely on its prior distribution (an out-of-sample analysis); we skip the first three years of our sample to compute the initial distribution. In both panels, we use a seven-factor model that includes: the Fama and French (2015) five-factors (market, size, value, investment and profitability), the Carhart (1997) momentum factor, and Frazzini and Pedersen’s (2014) betting-against beta (BAB) factor. 5% statistical significance is indicated in bold.

ALPHA	RM-RF	SMB	HML	UMD	RMW	CMA	BAB
Panel A: In-Sample							
<b>0.46%</b> (2.43)	<b>0.53</b> (10.92)	0.11 (1.72)	0.03 (0.34)	<b>-0.15</b> (-2.23)			
<b>0.44%</b> (2.24)	<b>0.52</b> (10.58)	<b>0.18</b> (2.43)	0.07 (0.74)	<b>-0.15</b> (-2.20)	0.12 (0.84)	-0.15 (-0.80)	
<b>0.60%</b> (3.07)	<b>0.55</b> (12.03)	<b>0.24</b> (3.31)	0.20 (1.82)	-0.07 (-1.06)	<b>0.33</b> (2.51)	-0.03 (-0.19)	<b>-0.45</b> (-5.38)
Panel B: Out-of-Sample							
<b>0.45%</b> (2.46)	<b>0.53</b> (11.59)	0.10 (1.57)	-0.02 (-0.26)	<b>-0.20</b> (-2.93)			
<b>0.47%</b> (2.42)	<b>0.51</b> (10.93)	<b>0.16</b> (2.08)	0.06 (0.60)	<b>-0.19</b> (-2.78)	0.06 (0.40)	-0.23 (-1.22)	
<b>0.62%</b> (3.25)	<b>0.55</b> (12.25)	<b>0.22</b> (2.94)	0.19 (1.72)	-0.11 (-1.59)	<b>0.27</b> (1.98)	-0.11 (-0.64)	<b>-0.45</b> (-5.55)

Table VI: Beta Expansion, Time-Series Analysis

This table examines time-series beta expansion associated with arbitrage trading. Panel A reports the baseline regression. The dependent variable is the beta spread between the high-beta and low-beta deciles (ranked in year  $t$ ) in year  $t+1$ . *CoBAR* is the average pairwise partial weekly three-factor residual correlation within the low-beta decile over the past 12 months. *Leverage* is a quintile dummy based on the average value-weight book leverage of the bottom and top beta deciles. We also include in the regression an interaction term between *CoBAR* and *Leverage*. Panel B reports a battery of robustness checks. The dependent variable in all rows is the beta spread between the high-beta and low-beta deciles in year  $t+1$ . Reported below is the coefficient on the interaction of *CoBAR* and *Leverage*. In Subpanel A, we consider different subsample results. Row 1 shows the baseline results which are also reported in Table III. In Rows 2 and 3, we exclude the tech bubble crash and the recent financial crisis from our sample. In Subpanel B, we explore alternative definitions of *CoBAR*. In row 1, we control for the UMD factor in computing *CoBAR*. In row 2, we control for both large- and small-cap HML in computing *CoBAR*. In row 3, we control for the Fama-French five factor model that adds profitability and investment to their three-factor model. In row 4, we perform the entire analysis on an industry-adjusted basis by sorting stocks into beta deciles within industries. In row 5, we instead measure the correlation between the high and low-beta portfolios, with a low correlation indicating high arbitrage activity. In rows 6 and 7, we examine upside and downside *CoBAR*, as distinguished by the median low-beta portfolio return. In Subpanel C, we replace *CoBAR* with residual *CoBAR* from a time-series regression where we purge from *CoBAR* variation linked to, respectively, *CoMOM* and *CoValue* (Lou and Polk, 2021; rows 1-2), the average pair-wise correlation in the market (row 3), the lagged 36-month volatility of the BAB factor (Frazzini and Pedersen, 2014; row 4), market volatility over the past 24 months (row 5), a trend (row 6), lagged *CoBAR* (where we hold the stocks in the low-beta decile constant but calculate *CoBAR* using returns from the previous year; row 7), smoothed past inflation (Cohen, Polk, and Vuolteenaho, 2005; row 8), a sentiment index (Baker and Wurgler, 2006; row 9), aggregate analyst forecast dispersion (Hong and Sraer, 2014; row 10), and the TED Spread (Frazzini and Pedersen, 2014; row 11). Standard errors are shown in brackets. \*, \*\*, \*\*\* denote significance at the 10%, 5%, and 1% level, respectively.

Panel A: Baseline Regression		
<i>DepVar</i>	<i>BetaSpread</i> <sub><math>t+1</math></sub>	
	[1]	[2]
<i>BetaSpread</i>	0.244***	0.246***
	[0.058]	[0.057]
<i>CoBAR</i>	1.314**	0.320
	[0.545]	[0.645]
<i>Leverage</i>		-0.033**
		[0.015]
<i>CoBAR * Leverage</i>		0.433***
		[0.117]
Adj-R <sup>2</sup>	0.090	0.113
No. Obs.	564	564

Panel B: Robustness Checks		
$DepVar = BetaSpread_{t+1}$		
	Estimate	Std. Dev.
<u>Subpanel A: Subsamples</u>		
(1) Full Sample:1970-2016	0.433***	[0.117]
(2) Excluding 2001	0.443***	[0.112]
(3) Excluding 2007-2009	0.341**	[0.142]
<u>Subpanel B: Alternative definitions of <i>CoBAR</i></u>		
(1) Controlling for UMD	0.441***	[0.116]
(2) Controlling for Large/Small-Cap HML	0.374***	[0.127]
(3) Controlling for FF Five Factors	0.453***	[0.129]
(4) Controlling for Industry Factors	0.504***	[0.128]
(5) Correl btw High and Low Beta Stocks	0.097**	[0.041]
(6) Upside <i>CoBAR</i>	0.528***	[0.169]
(7) Downside <i>CoBAR</i>	0.279*	[0.145]
<u>Subpanel C: Residual <i>CoBAR</i></u>		
(1) Controlling for <i>CoMOM</i>	0.240**	[0.116]
(2) Controlling for <i>CoValue</i>	0.233**	[0.116]
(3) Controlling for MKT CORR	0.423***	[0.120]
(4) Controlling for Vol(BAB)	0.431***	[0.110]
(5) Controlling for Mktvol24	0.303**	[0.118]
(6) Controlling for Trend	0.347***	[0.120]
(7) Controlling for Pre-formation <i>CoBAR</i>	0.484***	[0.119]
(8) Controlling for Inflation	0.436***	[0.120]
(9) Controlling for Sentiment	0.443***	[0.119]
(10) Controlling for Disagreement	0.300**	[0.127]
(11) Controlling for TED Spread	0.464***	[0.126]



Table VII: Beta Expansion, Cross-Sectional Analysis

This table reports panel regressions of post-ranking stock beta on lagged *CoBAR*. At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and non-synchronous trading, on the right-hand side of the regression, we include five lags of the excess market return, in addition to the contemporaneous excess market return. The pre-ranking beta is the sum of the six coefficients from the OLS regression. The dependent variable is the post-ranking stock beta from year  $t$  to  $t+1$  (non-overlapping periods). The main independent variable is lagged *CoBAR*, the average pairwise excess weekly return correlation in the low-beta decile over the past 12 months. *Distance* is the difference between a stock's pre-ranking beta and the average pre-ranking beta in year  $t$ . *Leverage* is the book leverage of the firm, measured in year  $t$ . We also include all double and triple interaction terms of *CoBAR*, *Distance*, and *Leverage*. Other (unreported) control variables include lagged firm size, book-to-market ratio, past one-year return, idiosyncratic volatility (over the prior year), and past one-month return. Time-fixed effects are included in Columns 3 and 4 (since *CoBAR* is a time-series variable, it is subsumed by the time dummies). Standard errors, shown in brackets, are double clustered at both the firm and year-month levels. \*, \*\*, \*\*\* denote significance at the 10%, 5%, and 1% level, respectively.

<i>DepVar</i>	<i>Post Ranking Beta<sub>t+1</sub></i>			
	[1]	[2]	[3]	[4]
<i>CoBAR</i>	-0.943*** [0.144]	-0.924*** [0.139]		
<i>Distance</i>	0.269*** [0.029]	0.253*** [0.032]	0.253*** [0.026]	0.226*** [0.029]
<i>CoBAR * Distance</i>	0.842*** [0.264]	0.640** [0.291]	0.640*** [0.242]	0.629** [0.264]
<i>Leverage</i>		-0.005*** [0.001]		-0.003** [0.001]
<i>CoBAR * Leverage</i>		0.025* [0.014]		0.023* [0.013]
<i>Leverage * Distance</i>		-0.006 [0.005]		0.005 [0.005]
<i>CoBAR * Leverage * Distance</i>		0.234*** [0.048]		0.096** [0.042]
Time Fixed Effects	No	No	Yes	Yes
Adj-R2	1,265,762	1,265,762	1,265,762	1,265,762
No. Obs.	0.258	0.263	0.318	0.320

Table VIII: Limits to Arbitrage

This table reports returns to the beta arbitrage strategy as a function of lagged *CoBAR* in various subsamples ranked by proxies for limits to arbitrage (LTA) (as of the beginning of the holding period). At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and non-synchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return, in addition to the contemporaneous excess market return. The pre-ranking beta is the sum of the six coefficients from the OLS regression. All months are then classified into five groups based on *CoBAR*, the average pairwise partial return correlation in the low-beta decile over the past 12 months. Reported below is the difference in four-factor alpha to the beta arbitrage strategy between high *CoBAR* periods and low *CoBAR* periods. Year zero is the beta portfolio ranking period. “Low LTA” corresponds to the subsample of stocks with low limits to arbitrage, and “high LTA” corresponds to the subsample with high limits to arbitrage. “Low-High” is the difference in monthly portfolio alpha between the two subsamples. We measure limits to arbitrage using three common proxies. In columns 1-2, we rank stocks into quartiles based on market capitalization; we label the top quartile as “Low LTA” and the bottom quartile as “High LTA.” In columns 3-4, we rank stocks into quartiles based on idiosyncratic volatility with regard to the Carhart four-factor model; we label the bottom quartile as “Low LTA” and the top quartile as “High LTA.” In columns 5-6, we rank stocks into quartiles based on the illiquidity measure of Amihud (2002); we label the bottom quartile as “Low LTA” and the top quartile as “High LTA.” We compute *t*-statistics, shown in parentheses, based on standard errors corrected for serial-dependence up to 24 lags. 5% statistical significance is indicated in bold.

	Market Cap		Idiosyncratic Volatility		Illiquidity	
	Year 3	Years 2-3	Year 3	Years 2-3	Year 3	Years 2-3
Low LTA	<b>-1.68%</b> (-3.73)	<b>-1.17%</b> (-2.86)	<b>-1.70%</b> (-4.35)	<b>-1.19%</b> (-3.52)	<b>-1.61%</b> (-3.42)	<b>-1.07%</b> (-2.57)
High LTA	-0.13% (-0.26)	-0.16% (-0.44)	-0.40% (-0.84)	-0.34% (-0.89)	-0.44% (-0.95)	-0.34% (-0.94)
Low-High	<b>-1.55%</b> (-2.82)	<b>-1.01%</b> (-2.39)	<b>-1.30%</b> (-2.76)	<b>-0.85%</b> (-2.37)	<b>-1.17%</b> (-2.52)	<b>-0.73%</b> (-1.97)

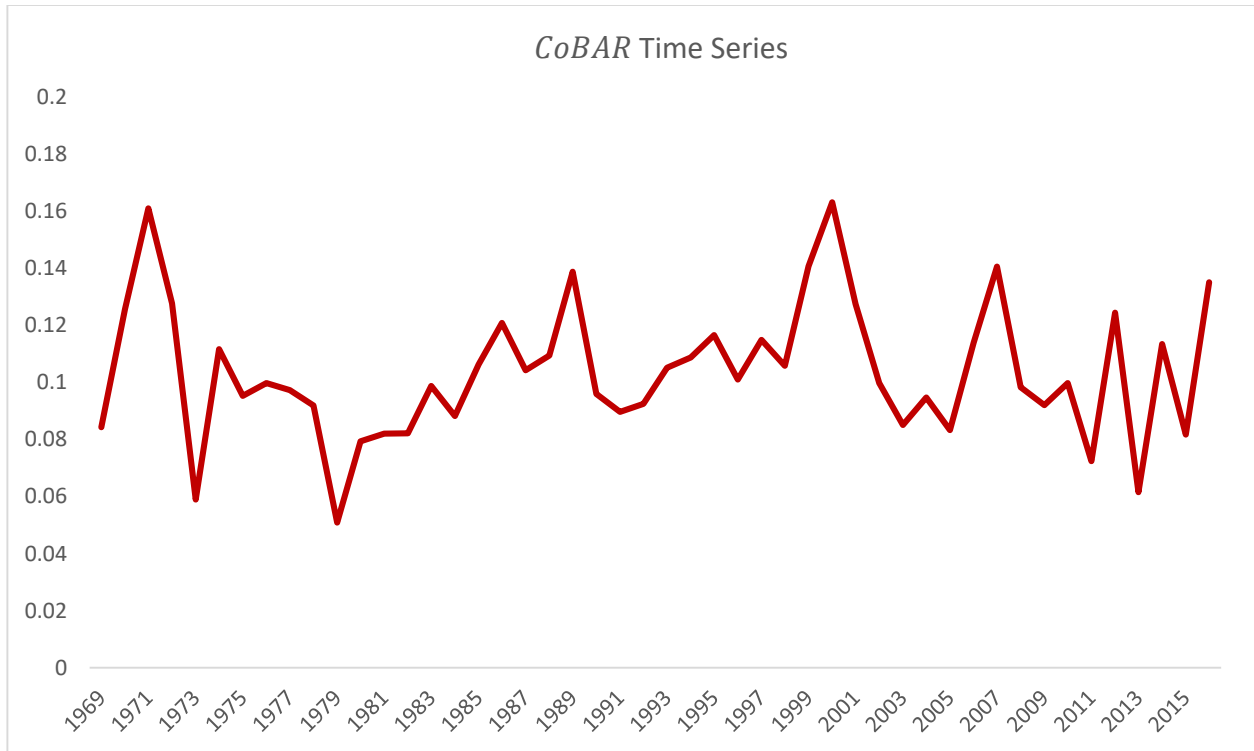


Figure 1: This figure shows the time series of the December observations of the *CoBAR* measure. At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and non-synchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return, in addition to the contemporaneous excess market return. The pre-ranking beta is the sum of the six coefficients from the OLS regression. *CoBAR* is the average pairwise partial return correlation in the low-beta decile measured in the ranking period. We begin our analysis in 1970, as it is the year when the low-beta anomaly was first recognized by academics. The time series average of *CoBAR* is 0.10.

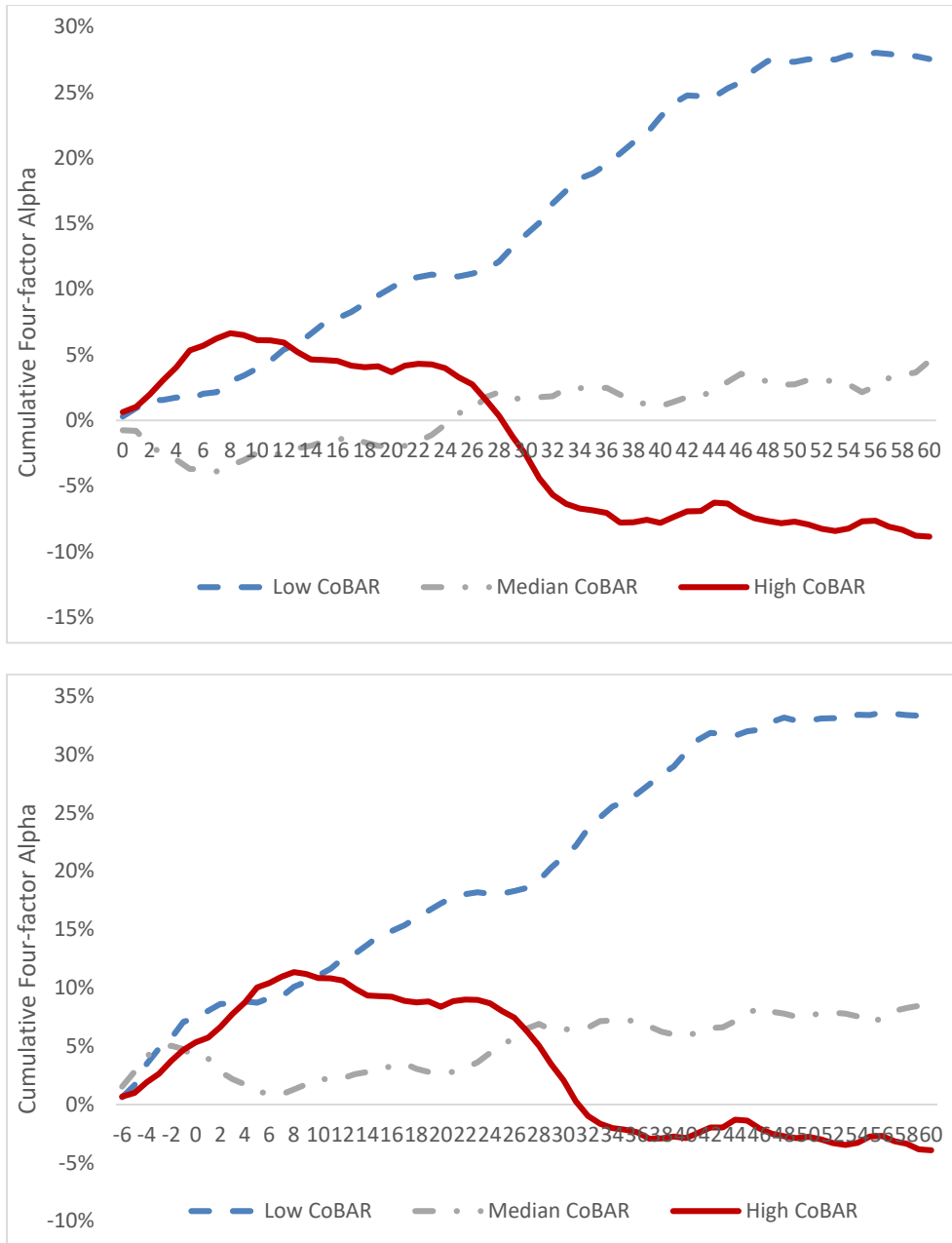


Figure 2: This figure shows returns to the beta arbitrage strategy as a function of lagged *CoBAR*. At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and non-synchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return, in addition to the contemporaneous excess market return. The pre-ranking beta is the sum of the six coefficients from the OLS regression. All months are then sorted into five groups based on *CoBAR*, the average pairwise weekly three-factor residual correlation within the low-beta decile over the previous 12 months. The red curve shows the cumulative Carhart four-factor alpha to the beta arbitrage strategy (i.e., a portfolio that is long the value-weight low-beta decile and short the value-weighted high-beta decile) formed in high *CoBAR* periods, the dotted blue curve shows the cumulative Carhart four-factor alpha to the beta arbitrage strategy formed in periods of low *CoBAR*, and the grey curve shows the cumulative Carhart four-factor alpha to the beta arbitrage strategy formed in periods of median *CoBAR*.

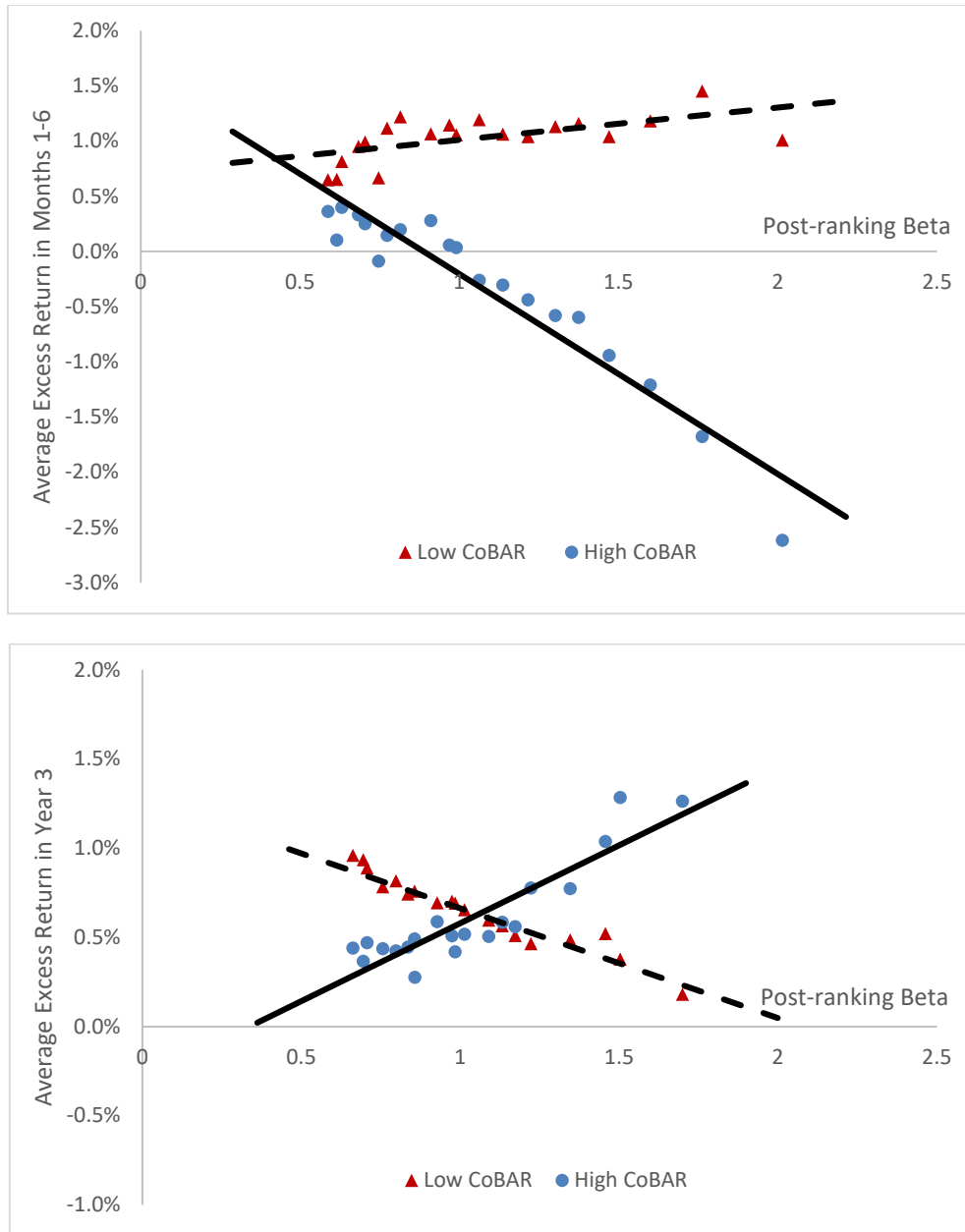


Figure 3: This figure shows the security market line as a function of lagged *CoBAR*. At the end of each month, all stocks are sorted into vigintiles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and non-synchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return, in addition to the contemporaneous excess market return. The pre-ranking beta is the sum of the six coefficients from the OLS regression. We then estimate two security market lines based on these 20 portfolios formed in each period: one SML using portfolio returns in months 1-6 (the top panel), and the other using portfolio returns in year 3 after portfolio formation (the bottom panel); the betas used in these SML regressions are the corresponding post-ranking betas. The Y-axis reports the average monthly excess returns to these 20 portfolios, and the X-axis reports the post-ranking betas of these portfolios. Beta portfolios formed in high *CoBAR* periods are depicted with a blue circle and fitted with a solid line, and those formed in low *CoBAR* periods are depicted with a red triangle and fitted with a dotted line.

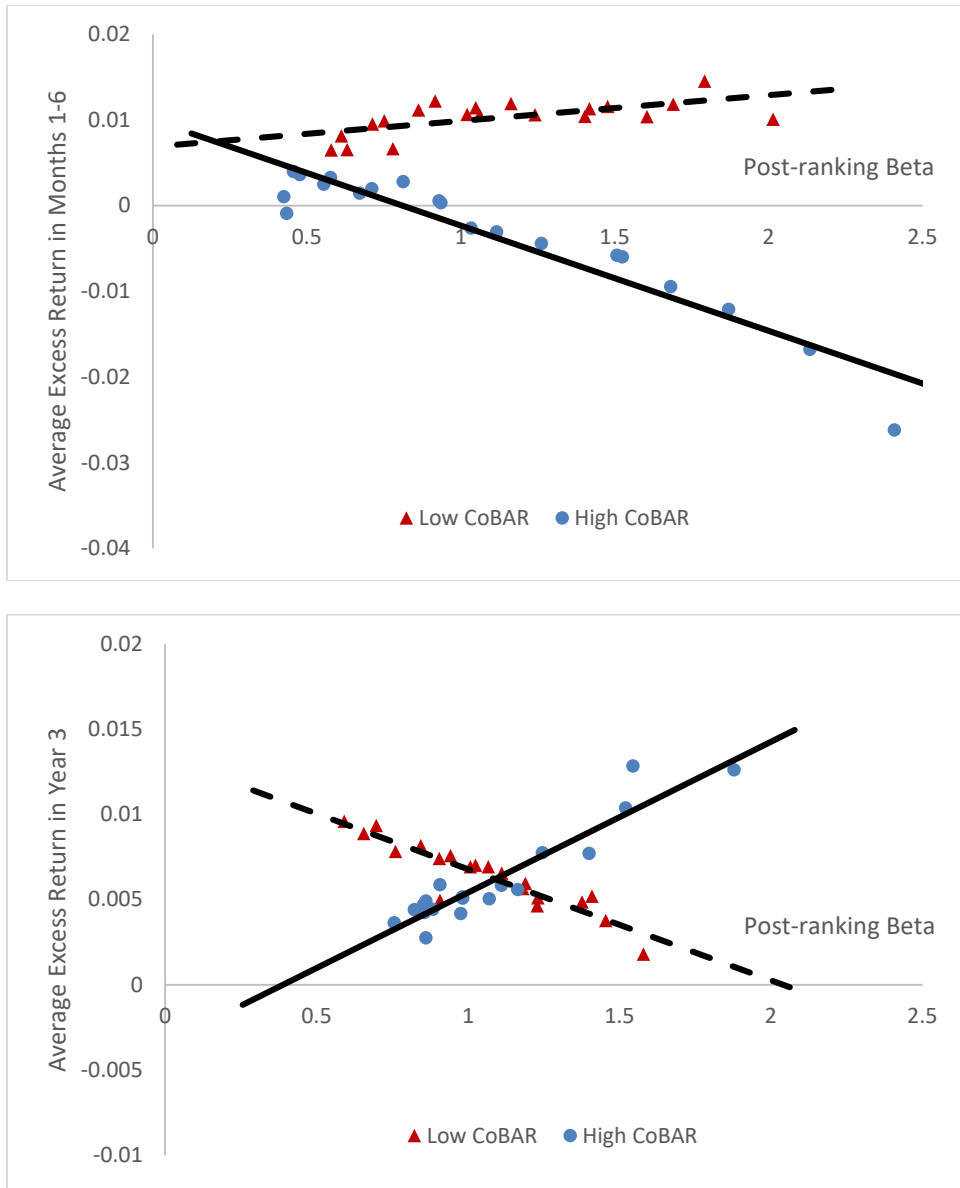


Figure 4: This figure shows the *conditional* security market line as a function of lagged *CoBAR* (i.e., where betas are allowed to vary with *CoBAR*). At the end of each month, all stocks are sorted into vigintiles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and non-synchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return, in addition to the contemporaneous excess market return. The pre-ranking beta is the sum of the six coefficients from the OLS regression. We then estimate two *conditional* security market lines based on these 20 portfolios: one SML using portfolio returns in months 1-6 (the top panel), and the other using portfolio returns in year 3 after portfolio formation (the bottom panel); the betas used in these SML regressions are the corresponding post-ranking betas. The Y-axis reports the average monthly excess returns to these 20 portfolios, and the X-axis reports the post-ranking beta of these portfolios. Beta portfolios formed in high *CoBAR* periods are depicted with a blue circle and fitted with a solid line, and those formed in low *CoBAR* periods are depicted with a red triangle and fitted with a dotted line.