## Derivation of Wage Elasticity of Labour

## Demand and Profits

The production function is assumed to be given by:

$$Y_{it} = \left[a_1(A_{it}N_{it})^{\rho} + a_2K_{it}^{\rho}\right]^{\frac{\xi}{\rho}} \quad \textbf{(a1)}$$

where $Y_{it}$ is output, $N_{it}$ employment, $K_{it}$ capital and $A_{it}$ labour-augmenting technical progress. Assume the demand curve for the firm's product is given by:

$$Y_{it} = \left(P_{it} | P_t\right)^{-\frac{1}{\theta}} G_t \quad \textbf{(a2)}$$

where $P_{it}$ is the firm's own price, $P_t$ the aggregate price level and $G_t$ some measure of aggregate demand.

From (a2) we can write real profits as:

$$\Pi_{it} = G_t^{\theta} Y_t^{1-\theta} - \frac{W_{it}}{P_t}(1 + \tau_t)N_{it} \quad \textbf{(a3)}$$

By differentiating (a1) we obtain:

$$\frac{\partial Y_{it}}{\partial N_{it}} = \xi . a_1 . A_{it}\left(\frac{Y_{it}}{A_{it}N_{it}}\right)^{1-\rho} Y_{it}^{\frac{\rho(\xi-1)}{\xi}} \quad \textbf{(a4)}$$

By differentiating (a3), the first-order condition for profit maximisation is:

$$(1-\theta)G_t^{\theta}Y_{it}^{-\theta}\frac{\partial Y_{it}}{\partial N_{it}} = \frac{W_{it}}{P_t}(1 + \tau_t) \quad \textbf{(a5)}$$

Substitute (a4) in (a5), take logs and differentiating with respect to the log of $w_{it}$ yields:

$$\theta \frac{\partial \log Y_{it}}{\partial \log N_{it}} \cdot \frac{\partial \log N_{it}}{\partial \log W_{it}} + (1-\rho)\left[\frac{\partial \log Y_{it}}{\partial \log N_{it}} - 1\right]\frac{\partial \log N_i}{\partial \log W_i}$$

$$+ \frac{\rho(\xi - 1)}{\xi} \frac{\partial \log Y_{it}}{\partial \log N_{it}} \cdot \frac{\partial \log N_{it}}{\partial \log W_{it}} = 1 \qquad \textbf{(a6)}$$

Re-arranging, we obtain:

$$\epsilon_{Nt} = -\frac{\partial \log N_{it}}{\partial \log W_{it}} = \frac{1}{(1-\rho) - \left(1 - \theta - \frac{\rho}{\xi}\right)\frac{\partial \log Y_{it}}{\partial \log N_{it}}} \qquad \textbf{(a7)}$$

Now, using (a2) in (a5) we can derive:

$$\frac{\partial \log Y_{it}}{\partial \log N_{it}} = \frac{1}{1-\theta} \cdot \frac{W_{it}(1+\tau_t)N_{it}}{P_{it}Y_{it}} = \frac{\phi_{it}}{1-\theta} \qquad \textbf{(a8)}$$

Substituting (a8) in (a7) and imposing symmetry yields (18).

From the envelope condition,

$$\frac{\partial \Pi_{it}}{\partial W_{it}} = -\frac{(1+\tau_t)N_{it}}{P_t} \qquad \textbf{(a9)}$$

So,

$$\epsilon_{\Pi t} = -\frac{\partial \log \Pi_{it}}{\partial \log W_{it}} = \frac{W_{it}(1+\tau_t)N_{it}}{P_{it}Y_{it} - W_{it}(1+\tau_t)N_{it}} = \frac{\phi_{it}}{1-\phi_{it}} \qquad \textbf{(a10)}$$

which, imposing symmetry, is (19).