

ABSTRACT

This paper extends work by Burdett and Mortensen (1989) and Mortensen and Vishwanath (1991) and examines equilibria in a matching model with identical firms and workers in which employed as well as unemployed workers receive wage offers. It shows that there are generally a continuum of equilibria often of a form that resembles labour market segmentation in that there are some firms paying high wages and facing an excess supply of labour and some firms paying low wages with an excess demand for labour. The implications of the model are explored.

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ENDOGENOUS LABOUR MARKET SEGMENTATION
IN A MATCHING MODEL

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INTRODUCTION

The idea that labour markets are usefully thought of as being segmented has, for some, been regarded as a helpful way of thinking about the operation of labour markets and a contrast to the competitive model (see, for example, Kerr, 1954, and Doeringer and Piore, 1971 *inter alia*). In its simplest form, the labour market is regarded as being divided into two sectors (although more complicated analyses sometimes differentiate further). The primary sector consists of large firms offering good jobs paying high wages and perhaps offering training and career prospects. Such jobs are in short supply. In contrast secondary sector firms are typically small, jobs are freely available but pay low wages and offer few if any prospects. It is sometimes emphasized, particularly by the more radical believers in segmentation, how the allocation of workers to sectors is not based on objective differences between them (e.g. in quality) and often non-traditional economic factors are emphasized as being important in determining equilibrium.

Although the idea of labour market segmentation has proved attractive to many radical labour economists (e.g. see Gordon, Edwards and Reich, 1982) who believe that it is inconsistent with competitive labour markets, there are a small but

increasing number of papers that attempt to explain segmentation using neoclassical models with relatively minor deviations from the assumptions of a competitive labour market. For example, Bulow and Summers (1986) present an efficiency wage model in which primary sector firms have difficulty monitoring their workers so need to pay a wage premium to prevent their workers from shirking. In contrast secondary sector firms have no problem in monitoring their workers and pay a market-clearing wage. In this model segmentation occurs because of heterogeneity in the monitoring technology across firms. In contrast Albrecht and Vroman (1992) assume that firms are homogeneous but that there is a non-convexity in the monitoring technology and heterogeneity in the reservation wage of workers which causes firms in equilibrium to separate in the way described by Bulow and Summers. Although these models can explain many features of segmentation they do suggest that some form of heterogeneity either in firms or workers is necessary to generate segmentation.

The purpose of this paper is to present a matching model with ex ante identical workers and firms which has something resembling labour market segmentation as an equilibrium outcome. This suggests that heterogeneity is not necessary for segmentation. Also, the model is, in many ways, very close to a competitive labour market model so it is perhaps surprising that an equilibrium that seems so different from the competitive labour market can emerge. One can imagine a competitive labour market as being one in which the following two conditions are satisfied. First, workers are always aware of the wages offered by all firms in the market and, secondly, that there are no job mobility costs so that workers will move from one firm to another on the basis of the smallest wage differentials. This paper maintains this second condition but relaxes the first; we look at a matching model in

which workers do not always have information about where the good jobs are. The model is essentially an extension of the models of Burdett and Mortensen (1989) and Mortensen and Vishwanath (1991) which are matching models with homogeneous workers and firms and on-the-job search. These models have been used to explain equilibrium wage dispersion but it is perhaps more surprising that they can explain labour market segmentation which can be thought of as a very special type of wage dispersion.

Mortensen and Vishwanath do present an equilibrium that resembles a segmented labour market (although they do not emphasize this feature of their model) in which there is a group of large firms paying a high wage (equal to the marginal product of labour) and a group of smaller firms paying lower wages which are below the marginal product of labour (this sector is essentially monopsonistic). However, in this equilibrium the good jobs in the primary sector are not in short supply: any worker matched with a primary sector firm gets a job. But, this is an assumption in Mortensen and Vishwanath rather than a result. What we do in this paper is to relax this assumption and allow for the possibility that primary sector jobs are rationed. What we show is that there are generally equilibria with job rationing in the primary sector. We also show that in this model there is generally a continuum of equilibria. This may be thought to present a problem as we have no theory of equilibrium selection. But for segmented labour market theorists it perhaps represents an opportunity to argue that non-traditional economic factors are important in determining equilibrium. We discuss this in more detail below.

It should be emphasized that the purpose of this paper is not to claim that it is the explanation of labour market segmentation, just that, even in a world of ex ante

homogeneous workers and firms, one does not have to move very far away from the assumptions of a competitive labour market for something resembling labour market segmentation to be an equilibrium outcome. The plan of the paper is as follows. In the next section we present the basic model and analyze equilibrium. In the third section we look at unemployment and welfare issues while in the fourth section we look at possible extensions of the model.

1. THE MODEL

The model presented here essentially follows that presented by Burdett and Mortensen (1989) and Mortensen and Vishwanath (1991). There is a continuum of ex ante identical employers and workers. We normalise the measure of firms to unity and let the measure of workers be M . Both employers and workers are assumed to be infinitely lived.

Each employer is assumed to have a strictly concave revenue function $R(L)$. Each worker receives benefit b when unemployed. Job offers for workers arrive at a rate λ which is the same whether they are employed or unemployed (this is a convenient simplification - nothing of importance depends on it). These job offers are drawn at random from the set of firms. Employed workers receive a wage w (which, as we shall see, generally differs across firms). Employed workers quit to unemployment at an exogenously given rate q , but will also move to another firm if they receive a higher wage offer, the probability of which is endogenous to the model.

We follow Burdett-Mortensen in assuming the process of wage determination is the following. Employers play a one-shot wage-posting game in which the wage they are prepared to pay workers is fixed at the beginning of the game. Workers then move between firms, employment and unemployment with the distribution of employment eventually settling down into a steady state. We assume that employers do not discount the future so they are only concerned with the level of profits in this steady state. Clearly the steady-state level of profits must be the same for all wages that are offered in equilibrium and less for those wages that are not offered. This means that all firms must, in equilibrium, be on the same iso-profit curve as drawn in Fig. 1. As no employer will ever have a wage-employment combination which implies that the marginal product of labour is less than the wage (as they could then increase profits by reducing employment) no firms can be on the downward-sloping part of this curve. This means that, in equilibrium, there will be a strictly monotonic relationship between employer size and wages. Let us denote the equilibrium relationship between employment and wages by $L(w)$. Employers that pay high wages will generally be able to sustain a larger size as they will lose fewer of their workers to firms offering higher wages and will recruit more among workers in firms paying lower wages. Although, the choice variable for the employer is the wage, it is analytically convenient for us to solve the model by thinking of size as the choice variable. Size can be transformed into wages through the (endogenous) relationship $w(L)$ which will be the inverse of the function $L(w)$.

Before proceeding to a characterisation of equilibrium we will make one further assumption:

$$R' \left(\frac{\lambda q M}{(\lambda + q)^2} \right) > b \quad (1)$$

This condition ensures that it is profitable for at least some employers to offer a wage strictly above b as $\lambda q M / (\lambda + q)^2$ represents the lower bound to the supply of labour to the firm paying the lowest wage (for details of this see (8a) below). If this condition is not satisfied, the only equilibrium is the uninteresting one where there is chronic excess supply of labour and all employers offer the reservation wage b and only employ labour to the point where $R'(L) = b$.

In characterising the equilibrium, the following results are useful. Many of these results are in Burdett and Mortensen and Mortensen and Vishwanath but they are presented briefly here for convenience.

Result 1: If there is a mass of firms of size L , it must be the case that $R'(L) = w(L)$.

Proof: Obviously employers would never have $R'(L) < w$ as they would then increase profits by reducing employment. Suppose there is a mass of firms of size L and that $R'(L) > w$. We will show that it is possible for a firm to increase its profits by slightly raising its wage so that this cannot be an equilibrium. As the employer could increase profits by raising employment it must be the case that they are using all their existing supply of labour. If, furthermore, they increase their wage offer to $(w + \epsilon)$, their wage costs per worker are only infinitesimally increased but their steady-state labour supply is now increased by a non-infinitesimal amount as they now recruit from the mass of firms who continue to pay w . Hence, profits must be increased.

Comment. The implication of Result 1 is that if there is a mass of firms of any particular size in equilibrium, they must be on the top of the equilibrium iso-profit curve i.e. at point (\hat{w}, \hat{L}) as drawn on Fig. 1. Results 2 and 3 then immediately follow.

Result 2: If, in equilibrium, there is a mass of firms of size \hat{L} , these are the largest firms.

Proof: By inspection of Fig.1 we can see that as any mass of firms are at the top of the equilibrium iso-profit curve, any larger firm on the same iso-profit curve must have $R'(L) < w$ in which case they could increase profits by reducing employment.

Comment: For future use, let us denote the size of the largest firms, if there is a mass point, by \hat{L} , and the wage they pay by \hat{w} . In what follows, these are going to be our primary sector firms.

Result 3: For all firms of size $L < \hat{L}$, we must have $R'(L) > w(L)$.

Proof: Again, by inspection of Fig.1 we can see that any firms not at the mass point must be on the upward-sloping part of the equilibrium iso-profit curve where $R'(L) > w$.

Comment: This result implies that all firms that are not in the primary sector will have the marginal product of labour above the wage so that this sector is essentially monopsonistic. Consequently, employment in these firms will be supply-determined: we will refer to these firms as the secondary sector.

Result 4: If there is any wage dispersion in equilibrium, the lowest wage offer in the market is b .

Proof: From Result 2, if there are at least two wages offered in equilibrium, there cannot be a mass of firms offering the lowest wage. An employer offering the lowest wage will recruit workers only from unemployment. Hence they will maximise profits by offering a wage equal to the reservation wage of the unemployed. As employed and unemployed workers receive job offers with the same probability this will be equal to b .

Result 5: Suppose there are, in equilibrium, some firms of size \underline{L} and some of size \bar{L} where $\underline{L} < \bar{L} < \hat{L}$. Then, there must also be some firms of every size in the interval $[\underline{L}, \bar{L}]$.

Proof: Suppose not. Let us choose (\underline{L}, \bar{L}) so that there are no firms of a size $L \in (\underline{L}, \bar{L})$. We can do this without loss of generality. As, from Result 3, we must have $R'(L) > w$ for these firms, they must be offering different wages if they are to be making the same level of profits. Denote the wages offered by $w(\underline{L})$ and $w(\bar{L})$. Then, as there are no firms of an intermediate size, it must be the case that there are no firms offering a wage w that satisfies $w(\underline{L}) < w < w(\bar{L})$. By assumption and from Result 2, there is not a mass of firms of size \bar{L} . Then, if an employer paying $w(\bar{L})$ cuts its wage to $w = w(\underline{L}) + \epsilon$ for some small positive ϵ , its labour supply will only be infinitesimally reduced but it will make a non-infinitesimal reduction in wage costs. Hence this change will increase profits, so the original situation can not have been an equilibrium.

Putting together Results 1 through 5, we can characterise an equilibrium as the following. A fraction, θ , of firms pay a wage \hat{w} and are of size \hat{L} where $R'(\hat{L}) = \hat{w}$.

These are the largest, highest-paying firms in the economy and we will refer to these as primary sector firms. Their profits are given by:

$$\pi_p(\hat{L}) = R(\hat{L}) - \hat{w}\hat{L} = R(\hat{L}) - \hat{L}.R'(\hat{L}) \quad (2)$$

Among the remaining fraction $(1-\theta)$ of firms, which we will refer to as the secondary sector, there is a continuous, strictly increasing distribution function of employment $G(L)$ on some interval $[\underline{L}, \bar{L}]$ paying a wage $w(L)$ where $w(\underline{L}) = b$. Diagrammatically, we must have something like Fig.1 which is essentially the same diagram used by Mortensen and Vishwanath. In general, there will be a non-zero gap in size and wages between the primary sector firms and the largest secondary sector firms. It seems sensible to say the labour market is segmented if this gap exists.

We want to know what conditions must be satisfied for a combination $\{\theta, \hat{L}, G(L), w(L)\}$ to be an equilibrium.

First, it must be the case that employment in the largest firms does not exceed the supply of labour to these firms. Each of the largest firms requires $q\hat{L}$ replacement workers. All workers who are not already employed in one of the largest firms will be prepared to move if they receive an offer: the number of such workers who do is $\lambda(M-\theta\hat{L})$. So, we must have:

$$q\hat{L} \leq \lambda(M - \theta\hat{L}) \quad (3)$$

which can be written as:

$$\hat{L} \leq \frac{\lambda M}{q + \lambda\theta} \quad (4)$$

This implied relationship between \hat{L} and θ is represented in Fig.3. Not surprisingly, as the fraction of firms in the primary sector increases, the supply to each individual firm declines as the pool of secondary sector and unemployed workers also declines. Mortensen and Vishwanath (1991) assume that (4) will always be satisfied with equality so that there is never an excess supply of labour to the primary sector firms. But, as we will see, there is no reason why we cannot have an equilibrium in which there is excess supply of labour to the primary sector. One might expect that if this was the case that a firm would undercut the wage \hat{w} . But, if it does so, its labour supply would be reduced by a substantial amount as it would start to lose workers to the mass of firms who continue to pay \hat{w} . As we shall see, it is quite possible that profits are not increased by pursuing this strategy.

If there is an excess supply of labour in the primary sector then a worker who meets such a firm is not guaranteed of being accepted for employment. If, for the moment, we assume that recruits are randomly selected from applicants, the probability of being accepted for a primary sector job, conditional on being matched will, from (3), be:

$$\phi = \frac{q\hat{L}}{\lambda(M - \theta\hat{L})} \quad (5)$$

Now consider employment determination in the secondary sector. As there is a strictly monotonic relationship between wages and employment, workers will always want to move from small to large firms. As, from Result 3, employment in the secondary sector is supply constrained matches with a larger secondary sector

employer will always result in a move. In steady-state equilibrium flows into and out of employment must be equal giving us:

$$[q + \lambda\theta\phi + \lambda(1-\theta)(1-G(L))]L = \lambda \left(M - \theta\hat{L} - (1-\theta) \int_L^{\bar{L}} L \cdot g(L) dL \right). \quad (6)$$

where the term in brackets on the right-hand side of (6) is the number of workers unemployed or working in secondary sector firms of size less than L which is equal to the labour force, M , minus those workers with jobs in the primary sector or with larger secondary sector firms. Solving this is straightforward and leads to:

Proposition 1: If a fraction θ of firms are in the primary sector, employing \hat{L} workers then the density of employment in the secondary sector is given by:

$$g(L) = \frac{1}{2\lambda(1-\theta)} (\lambda q M)^{\frac{1}{2}} L^{-\frac{3}{2}} \text{ where } L \in [L, \bar{L}] \quad (7)$$

and

$$L = \frac{\lambda q M}{\left[\frac{q M}{M - \theta \hat{L}} + \lambda(1-\theta) \right]^2} \quad (8a)$$

$$\bar{L} = \frac{\lambda(M - \theta \hat{L})^2}{q M}. \quad (8b)$$

Proof: See Appendix.

This proposition tells us that given (θ, \hat{L}) we can always deduce the equilibrium distribution of firms in the secondary sector.

An additional equilibrium requirement is that employment in the largest secondary sector firm be no larger than employment in the primary sector firms. If this was not the case, the largest secondary sector firms could not be making the same level of profits as the primary sector firms. Using (8b), this condition can be written as:

$$\bar{L} = \frac{\lambda(M - \theta\hat{L})^2}{qM} \leq \hat{L}. \quad (9)$$

It can readily be checked that equality in (9) gives us a downward-sloping relationship between θ and \hat{L} which is as drawn on Fig.2. For all values of $\theta > 0$ it can be checked that there are feasible values of \hat{L} for which primary sector employment lies between labour supply to these firms and the employment level of the largest secondary sector firm.

We can also use Proposition 1 to derive the profit level of the smallest secondary sector firm which we will denote by $\pi_s(\theta, \hat{L})$. From Result 4 the wage paid is b so that:

$$\pi_s(\theta, \hat{L}) = R(L) - b \cdot L \quad (10)$$

where L is given by (8a).

If (θ, \hat{L}) is an equilibrium that it must be the case that:

$$\pi_s(\theta, \hat{L}) = \pi_p(\hat{L}) \quad (11)$$

If (θ, \hat{L}) satisfy this equal profit condition and the conditions (4) and (9) are satisfied then we can always, by simply reading off from the equilibrium iso-profit locus, find a function $w(L)$ which satisfies:

$$R(L) - w(L).L = \pi_p(\hat{L}) \text{ for all } L \in [L, \bar{L}]. \quad (12)$$

Given the distribution of employment $g(L)$ one could then use this to work out the distribution of wage offers in the secondary sector. Note that in constructing the equilibrium, we are essentially deriving the wages paid last. Although it is analytically convenient to do this, one should remember that it is the wage which is the actual strategic variable for the firm.

To summarize, an equilibrium can be characterized by a pair (θ, \hat{L}) which must satisfy (4), (9) and (11). Given (θ, \hat{L}) one can work out the equilibrium \hat{w} , $g(L)$, $w(L)$. It can be readily checked that if all these conditions are satisfied, all wage offers that are made in equilibrium yield the same profits and no other wage offers yield higher profits so that we do have an equilibrium.

In general equilibrium is not unique. Consider the shape of the equal profit condition (which we will denote by EPC). From (2), $\partial \pi_p(\hat{L}) / \partial \hat{L} > 0$, and from (8a) and (10) $\partial \pi_s(\theta, \hat{L}) / \partial \hat{L} < 0$. Also from (8a) and (10) we have:

$$\partial \pi_s(\theta, \hat{L}) / \partial \theta = [R'(L) - b] \cdot \frac{\partial L}{\partial \theta}$$

and:

$$\frac{\partial L}{\partial \theta} = \frac{-L}{\frac{qM}{M - \theta \hat{L}} + \lambda(1 - \theta)} \left[\frac{qM \hat{L}}{(M - \theta \hat{L})^2} - \lambda \right]. \quad (13)$$

As long as (9) is satisfied $(\partial L / \partial \theta) < 0$ which implies $\partial \pi_s / \partial \theta < 0$. If (9) is not satisfied then $\partial \pi_s / \partial \theta > 0$. This means that once the EPC has entered the region defined by (9) it can never leave it again.

We can also show that the EPC must cut the line defined by (4) from below if it cuts it at all. To see this note that if $\hat{L} = \lambda M / (q + \lambda \theta)$, then, from (8a) $\underline{L} = \lambda q M / (q + \lambda)^2$ i.e. \underline{L} is constant along the line defined by (4). This means that π_s is also constant along this line. Above this line π_s is lower (as, for a given θ), \underline{L} is decreasing in \hat{L} . But as π_p is increasing in \hat{L} this means that the EPC must cut (4) from below (if it cuts it at all).

Putting this information together the EPC must be in one of the four qualitative forms drawn in Figure 2. If it is of form I, then the only equilibrium is one in which there is no primary sector in the sense that there is no mass of firms paying a higher wage than the others¹. Equilibrium is unique and characterised by a continuous density function of wages and firm size. A prominent special case in which this is the outcome is constant returns to scale (as analyzed in Burdett and Mortensen, 1989).

If the EPC is of form II then there is a range of values of θ strictly between zero and one for each of which there is an equilibrium. The larger is θ i.e. the larger is the proportion of firms in the primary sector, the smaller are the primary sector firms and hence the higher the primary sector wage must be. Let us consider the intuition for why the equilibrium must be of this form. Suppose we start from a point on (4) in which there is no rationing of primary sector jobs. Now let us reduce θ but, for the moment, imagine that \hat{L} is constant, so that there is rationing of primary sector jobs. This increases the supply of labour to the smallest secondary sector firm as more workers are unemployed (from whom they recruit) and fewer workers get offers from primary sector firms and leave. As these firms pay the same wage (b) their profits must be increased. To maintain equilibrium π_p must rise which requires

a fall in the wage paid by primary sector firms and a rise in \hat{L} . For this is to be consistent with equilibrium this rise must be less than the rise in labour supply so that there is still rationing of primary sector jobs in equilibrium.

If the EPC is of the form III then there is a range of values of θ for which there is an equilibrium as in the previous case. But points above the EPC for which $\theta = 1$ are also equilibria. At these points $\pi_p > \pi_s$ but this is not inconsistent with equilibrium as there are no secondary sector firms. In this type of equilibrium, all firms pay the same wage and employment is demand-determined. Finally, if the EPC is of the form IV then only single-wage equilibria exist but there is generally a continuum of them.

The most striking feature of this model is that there is generally a continuum of equilibria. The natural response of most economists to this is to seek ways of eliminating some of them, using some criteria external to the model, ideally ending up with a single equilibrium. Implicitly that is what Mortensen and Vishwanath (1991) do when they restrict attention to the unique equilibrium in which there is no excess demand for labour. A common technique for eliminating equilibria is some form of stability analysis. The model presented here, with its assumption of a once-for-all wage-posting game, is not amenable to an analysis of what happens out of equilibrium, but there are strong grounds for believing that, even in a modified version of the model, stability analysis will not be very helpful. This is because when we have a continuum of equilibria a deviation in a particular direction from any equilibrium simply moves us to another one, implying that none of the equilibria can be locally stable in the conventional sense. However, one could tell stories about a plausible adjustment process. Consider Fig. 3. Suppose the economy is initially at

a point like A where $\pi_p > \pi_s$. As primary sector profits are above secondary sector profits we might expect θ to rise. Eventually, we will run into a point like B on (4) where primary sector employment is equal to labour supply. There are still incentives for firms to enter the primary sector but now they are also likely to compete on wages for the available labour supply. The economy will move down (4) until we reach point C, the Mortensen-Vishwanath equilibrium. However, if the economy started at a point like D, $\pi_p < \pi_s$ so we might reasonably expect θ to fall. This will continue until we reach point E on (9). At this point the primary sector firms are no larger than the largest secondary sector firms but are making lower profits as they are paying higher wages. We might expect θ to fall further, and for primary sector firms to begin to cut their wages. The economy would move up (9) eventually settling at point F. This type of story would suggest that we might expect to see cycling between two equilibria (depending on the shocks hitting the economy) with the economy occasionally getting stuck in other equilibria on the EPC between points C and F. But, there may well be other equally plausible stories of adjustment processes which would produce different results.

Another line of criticism is to wonder whether the conclusions of the model are sensitive to the assumptions made. The assumptions about preferences, technology and the matching process are entirely conventional so we will not consider them. But, the assumptions about the wage-setting process might be important. In the model presented above there are two main assumptions:

- (i) wage-setting is a one-shot game
- (ii) firms must pay all their workers the same wage: individual contracts are not allowed.

We will argue that (i) is not crucial for our results but (ii) is. However we will argue that if we relax assumption (ii) the consequence is that all that is regarded as interesting about the class of models considered here disappears, not just the irritating multiplicity of equilibria in the version presented here.

The assumption of a one-shot wage-setting game is not crucial to deriving the multiplicity of equilibria. One can get a similar result in a model where wage contracts are not infinite in length but are set for only finite periods (although it is not possible to provide such a neat analytical description of the equilibrium in such a case). What sustains the clustering of firms in the primary sector with an excess supply of labour is that if they cut wages their labour supply falls by a lot as they then lose workers to other primary sector firms. Labour supply is not a continuous function of wages offered and employers are faced with a choice of an excess supply or an excess demand for labour. They cannot "fine-tune" their labour supply by adjusting their wage to ensure it is equal to their labour demand. This mechanism is still at work in a model where wages are set for a finite period of time. The inability to fine-tune the labour supply to the firm comes about because of the assumption that there are no job mobility costs. While this is a conventional assumption in much of economics, it does play a crucial role in sustaining the equilibrium here. But, introducing job mobility costs may not eliminate the problem of multiplicity of equilibria as they will make workers' behaviour insensitive to variations in wages over a certain range.

It is worth noting that the problem of being unable to fine-tune the labour supply cannot happen in a competitive (i.e. frictionless) labour market which is one extreme of the parameter space in the model presented here. To see this, suppose

that a single firm initially has an excess supply of labour and, for simplicity, suppose all other firms do as well. Then if this firm cuts wages the supply of labour available to it is all the workers who do not have work in other firms, i.e. exactly the same as before the wage cut. So it must be able to raise profits by reducing wages until its demand for labour equals its supply. We can see this in our model if the frictions disappear. As (q/λ) goes to zero it can be checked that (4) and (9) coincide, that \underline{L} goes to zero (see (8a)) so that the EPC becomes the horizontal axis. The unique equilibrium has $\theta = 1$ and the wage equal to the marginal product of the labour supply to each firm i.e. we have the competitive equilibrium.

Now consider the assumption that the firm must pay all its workers the same wage and that it cannot negotiate individual contracts with workers. This is a crucial assumption as the following argument shows. In the equilibria where there is excess supply of labour in the primary sector some workers who are matched with a primary sector firm are not getting employment because the wage is too high. But there are gains from trade to be had. The worker could offer to work for a slightly lower wage and acceptance of such a deal would make both worker and employer better-off. These individual contracts will undermine the excess supply of labour. But, it is important to realize that allowing individual contracts in this type of model has serious consequences far beyond simply eliminating equilibria with an excess supply of labour. This is because even in Burdett and Mortensen and Mortensen and Vishwanath there are unconsummated matches when a high wage worker meets a low wage firm. It is in the interest of such a firm to offer a higher wage to that worker alone. Similarly it is in the interest of a low-wage employer about to lose a worker to match their higher wage offer. It is not clear that equilibrium wage

dispersion (which is the main attraction of those models) can survive the introduction of individual contracts which essentially allow employers to act as discriminating monopsonists. Employers would like to offer every worker a contract "I will pay you b until you receive a better offer when I will match that offer". If all employers do this workers will never get paid above b .

So, for models of the Burdett and Mortensen type to be interesting at all we need to rule out individual contracts. While one can argue that this is a reasonable assumption on the grounds of descriptive accuracy for the sorts of labour markets to which the models plausibly apply, it is unsatisfactory from a theoretical point of view. But, if individual contracts are allowed, asymmetric information about the marginal product of labour, the reservation wage and the non-pecuniary aspects of work, and problems of verifying external offers would combine to make individual bargaining costly and inefficient and would require substantial personnel resources to be employed by firms. In such circumstances simple fixed wage contracts have the virtue of simplicity and are not subject to informational problems.

This section has shown how a very simple matching model can generate equilibria that are rather complex and which often resemble those suggested by labour market segmentation theorists. In the following sections, we explore some further aspects of the model.

2. UNEMPLOYMENT AND WELFARE

There are two sources of unemployment in this model. First, there is the unemployment that arises because of the frictions in the labour market which mean

that it takes time for workers who lose their jobs to find a new match. Secondly, in equilibria where there is an excess supply of labour in the primary sector, not all matches between unemployed workers and primary sector firms are consummated. The flows into and out of unemployment are given by:

$$[\lambda\theta\phi + \lambda(1-\theta)]U = q(M-U) \quad (14)$$

where U is the number of workers unemployed in steady-state equilibrium. Rearranging (14) and using (5) gives us:

$$U = \frac{qM}{\left[\frac{qM}{M-\theta\hat{L}} + \lambda(1-\theta) \right]} \quad (15)$$

Let us consider how unemployment varies along the equilibrium locus like that of type III in Fig.3. As θ falls, the equilibrium level of \hat{L} rises causing the primary sector profit level to rise which requires rising secondary sector profits which requires a rise in \underline{L} the employment level of the smallest firm. By inspection of (8a), \underline{L} can only rise if the denominator of the right-hand side rises. By comparison of (8a) and (15) we can see that this implies that unemployment must also be rising. So as we move up the equilibrium locus we are raising unemployment. Unemployment is minimized when we reach a point in (4) where there is no excess supply of labour in the primary sector. Then, there is only frictional unemployment with $U = qM/(q+\lambda)$.

As the unemployment associated with excess supply of labour in the primary sector appears to be unnecessary, it is tempting to conclude that government policies should be directed towards trying to eliminate it. But it is not necessarily the case

that welfare and unemployment are negatively related. If we measure welfare by the net surplus generated by the economy², it is given by:

$$V = \theta(R(\hat{L}) - b.\hat{L}) + (1 - \theta) \int_L^{\bar{L}} (R(L) - b.L)g(L)dL. \quad (16)$$

If we compare equilibria with $\theta = 1$ then it is the case that equilibria with lower unemployment have higher welfare as more workers are in work and the marginal product of labour (which is the same in all firms) is above the value of leisure.

But comparisons among equilibria with $\theta < 1$ are not so simple. As θ varies, the total number of workers in employment changes but so does the distribution of employment. As θ rises, employment is shifted from the secondary sector where, under our assumption that all employers are homogeneous, the marginal product of labour is high to the primary sector where it is low. This tends to reduce welfare. The easiest way to show that this effect may dominate is by simulation, as shown in Table 1. Note, that, depending on the reservation wage, welfare may or may not (ceteris paribus) be a decreasing function of θ . This type of result occurs for many parameter values.

One of the implications of this is that policies to encourage primary sector employment relative to secondary sector employment as has been suggested in much of the segmented labour markets literature (for example, Katz and Summers, 1989) may not necessarily be desirable. The usual argument is that as the secondary sector is assumed to be competitive, the wage is equal to the marginal product of labour in both segments implying that the marginal product of labour is higher in the high-wage primary sector. This is not the case in the model here as the secondary sector is essentially monopsonistic with the marginal product of labour above the wage, and

the marginal product of labour higher in the small secondary sector firms. Of course, the model presented here is as simplistic as the others in the literature; what this suggests is that it may be dangerous to base strong policy conclusions on such models.

An interesting question is whether there are any government policies that would definitely improve welfare in this economy. It is worth noting that the maximum level of welfare occurs on (4) where $\theta=1$. Then employment is at its maximum level and is evenly distributed across all firms. One policy that could be used to ensure that the EPC does go through this point is a minimum wage policy. Suppose the government sets a (perfectly enforceable) minimum wage of \bar{w} . Then all that alters from the previous analysis is that the lowest wage firm pays \bar{w} instead of b . π_s will be reduced so the EPC must shift downwards. Obviously, one can find a minimum wage such that the EPC cuts the $\theta = 1$ line at or below E. If the minimum wage is set at:

$$\bar{w} = R' \left(\frac{\lambda M}{\lambda + q} \right) \quad (17)$$

then the equilibrium configuration must be of Form IV. The only equilibria are then single wage equilibria, with wages at or above the statutory minimum.

Notice from this discussion, there is no way that the government can ensure that there is a unique equilibrium at the welfare maximum because of the shape of the EPC. It should also be noted that raising unemployment benefits will have a similar effect.

3. APPLICATIONS AND EXTENSIONS

a. Discrimination

In the equilibria where there is excess supply of labour to the primary sector there must be some mechanism for rationing the availability of these jobs. In this paper we assumed that such rationing was random. Yellen (1984) argued that firms faced with an excess supply of labour could costlessly discriminate against certain groups of workers if they chose to do so. This view was criticised by Bulow and Summers (1986) who, in the context of their shirking model, argued that if, say, women were discriminated against by primary sector firms, the wage they would have to be paid to prevent them from shirking would fall and a primary sector firm could increase profits by employing them rather than men. However, they emphasized that if the wages paid by any firm to men and women were constrained to be equal, discrimination as suggested by Yellen could exist and the equilibrium would resemble segregation.³

In the model presented here, this sort of discriminatory equilibrium exists but it is also possible to have discriminatory equilibria where two otherwise identical groups of workers receive different wages within the same firm. For example it is simple, if extremely tedious, to show that for some parameter values, there is an equilibrium in which men work only in primary sector jobs receiving a high wage but with some job rationing, while women are all in low-paying secondary sector jobs. This perhaps should not be too surprising given the multiplicity of equilibria in the model without discrimination.

One can also have discrimination based not on the exogenous characteristics of workers but on the basis of their labour market status. The simplest way to

illustrate this is in the context of the model presented in this paper is to suppose that primary sector employers would rather hire workers who are currently employed than those who are unemployed, perhaps because they perceive (rightly or wrongly) the unemployed as less productive (Blanchard and Diamond, 1990, have looked at the implications of this sort of view in a different type of matching model). This will tend to raise equilibrium unemployment as the exit rate from unemployment now falls. But it will also affect the wages offered by secondary sector firms. Unemployed workers will now be prepared to work for a wage lower than b as secondary sector employment will improve the chances of getting a primary sector job at some point in the future. Although this selection rule is costless for primary sector employers, it will generally have non-zero social costs or benefits. Indeed, in the example given above it may well be socially desirable for primary sector employers to discriminate in favour of the unemployed as hiring a worker from a secondary sector firm is moving a worker from a firm where their marginal product of labour is high to one where it is low.

b. Firm and Worker Heterogeneity

In the model presented above, both workers and firms were assumed to be identical. This is because one of the aims of this paper is to show how segmentation can arise even in a market peopled by homogeneous agents. But in the real world both workers and firms are obviously heterogeneous and much of the literature on labour market segmentation has emphasized, for example, how the nature of firms is different in the two sectors and how workers in the primary sector are generally equipped with more skills.

If some firms are inherently more productive than others (their marginal product of labour is higher for given employment) then it is straightforward to show that there is a positive relationship in equilibrium between firm productivity, wages and employment (Burdett and Mortensen show this formally for their model; the generalisation is simple). The productive firms will tend to end up in the primary sector as the segmentation literature suggests. To consider another possible example, suppose that some firms need to give their workers firm-specific skills for them to be productive. There is an incentive for these firms to be in the primary sector as labour turnover is lower in this sector which means that training costs will be lower. So one could then explain why workers in primary sector firms tend to have more skills than those in secondary sector firms.

We could also introduce worker heterogeneity e.g. by allowing workers to differ in their productivity. We would then find that the high-wage firms would need to be more strict in their hiring policies as they will only want to hire workers whose estimated productivity is above their wage. Consequently, workers in high wage firms will tend, on average, to be of higher quality and lower quality workers will tend to end up in the secondary sector.

What this section suggests is that an enriched version of the model considered here is likely to be able to explain many of the correlations between segment and employer and worker characteristics that are often suggested.

c. **Macroeconomic Implications**

In the discussion above, the continuum of equilibria was presented as something of a "problem" of the model. But, one view that has become quite

fashionable in recent years might actually see the multiplicity of equilibria and lack of determinacy as an advantage. For example, Blanchard and Summers (1988) argue that we need to move away from natural rate models with a unique equilibrium if we are to be able to explain the macroeconomic behaviour of OECD countries. They argue for models with what they call "fragile equilibria" in which unemployment depends sensitively on current and past events. One can obviously argue that the model presented here has the desired properties. For example, it is conceivable that money could be non-neutral if, although a money supply increase leaves the equilibrium locus unchanged, it causes the economy to move along the locus (e.g. because an unchanged primary sector money wage is still an equilibrium).

Although many possibilities and opportunities are opened up by such models, there is an urgent need for a theory of equilibrium selection to complete the model. At the moment, there are really no ground rules among economists for what is regarded as an acceptable theory of equilibrium selection and until such work is done, discussion about the ability of the types of models analyzed here to explain macroeconomic behaviour will remain very speculative.

4. CONCLUSIONS

This paper has looked at a very simple matching model with identical firms and workers in which employers post wages and both employed and unemployed workers receive job offers. Although, this model is not very far from the competitive model in terms of its assumptions, it may have equilibria which more resemble the descriptions of segmented labour market theorists than competitive markets. There is a primary sector with large firms offering high wages, with an

excess supply of labour and low labour turnover. There is a secondary sector with small firms offering low wages with an excess demand for labour (this segment is essentially monopsonistic) and with high labour turnover. In general, there is a continuum of equilibria opening up opportunities for non-traditional economic factors to be important in determining equilibrium. It was suggested that simple extensions of the model could explain why the better firms tend to be in the primary sector, why primary sector workers tend to receive more training and why discrimination might persist. It is perhaps surprising that such a simple model can have such a rich set of possibilities and that these emerge as soon as one moves even the smallest distance away from the assumption of a frictionless competitive labour market.

What is missing from this paper is evidence that labour market segmentation exists and is at least in part explained by the sort of mechanisms presented here. Dickens and Lang (1985) have provided some empirical evidence for the existence of segmented labour markets although their approach was not based on the model presented here. The model presented here is in some sense testable; it suggests the hypothesis that employment is demand determined in the primary sector and supply determined in the secondary sector. In future work, we hope to try to implement such a test. But, even in the absence of such evidence, the fact that something resembling segmentation can emerge in such a traditional economic model can, perhaps, provide an impetus to the empirical study of segmentation.

ENDNOTES

- * London School of Economics and Centre for Economic Performance. The Centre for Economic Performance is funded by the Economic and Social Research Council.
1. Note that, if there is an equilibrium with $\theta=0$, then the equilibrium condition (11) can be written as $\pi^P < \pi_S$ as there are no primary sector firms in the equilibrium. Similarly if $\theta=1$, (11) should be altered to $\pi^P > \pi_S$.
 2. Note that here we are assuming $R(L)$ represents the social as well as private return to employment and that b represents the disutility of labour. If we have imperfect competition in product markets and unemployment benefits then these are additional reasons why low unemployment may be associated with higher welfare.
 3. However, even this equilibrium could not survive if the firm could choose different effort levels for men and women as, for a given wage, firms would be able to set a higher effort level for women without inducing them to shirk.

APPENDIX

Proof of Proposition 1: The Distribution of Employment in the Secondary Sector

Differentiating (6) with respect to L yields:

$$q + \lambda\theta\phi + \lambda(1-\theta)(1-G(L)) = 2\lambda(1-\theta)Lg(L) \quad (\text{A1})$$

which can be written as:

$$\frac{\partial \log [q + \lambda\theta\phi + \lambda(1-\theta)(1-G(L))]}{\partial \log L} = -\frac{1}{2} \quad (\text{A2})$$

Integrating (A2) and taking exponents yields:

$$q + \lambda\theta\phi + \lambda(1-\theta)(1-G(L)) = kL^{-\frac{1}{2}} \quad (\text{A3})$$

for some constant of integration k. Now, from (6), \bar{L} will be given by:

$$\bar{L} = \frac{\lambda(M-\theta\hat{L})}{q + \lambda\theta\phi} \quad (\text{A4})$$

Using (A4) in (A3) and the fact that $G(\bar{L}) = 1$, we derive:

$$k = [\lambda(M-\theta\hat{L})(q + \lambda\theta\phi)]^{\frac{1}{2}} \quad (\text{A5})$$

Now, from (5) we can derive that:

$$q + \lambda\theta\phi = qM/(M-\theta\hat{L}) \quad (\text{A6})$$

Using (A3) and (A6) we can derive that:

$$\frac{qM}{M-\theta\hat{L}} + \lambda(1-\theta)[1-G(L)] = (\lambda qM/L)^{\frac{1}{2}} \quad (\text{A7})$$

We can also use (A7) to solve for \underline{L} , the employment level of the smallest firm as

$G(\underline{L}) = 0$. We obtain:

$$L = \frac{\lambda qM}{\left[\frac{qM}{M - \theta \hat{L}} + \lambda(1 - \theta) \right]^2} \quad (\text{A8})$$

Differentiating (A7) we also obtain $g(L)$ as given in (7).

TABLE 1

Welfare and Unemployment

θ	b=0.02		b=0.04	
	welfare	unemployment rate	welfare	unemployment rate
0.65	*	*	*	*
0.70	0.771	0.091	*	*
0.75	0.769	0.090	0.535	0.104
0.80	0.768	0.089	0.523	0.103
0.85	0.770	0.087	0.515	0.101
0.90	0.773	0.085	0.511	0.100
0.95	0.776	0.083	0.510	0.097

Note:

These simulations were carried out assuming the production function was of the form $R(L)=L^{0.8}$, $q=0.2$, $\lambda=4$, $M=1$. A grid of size 0.05 was carried out for θ . Non-existence of equilibrium is denoted by an asterisk.

FIGURE 1

Summary of Equilibrium

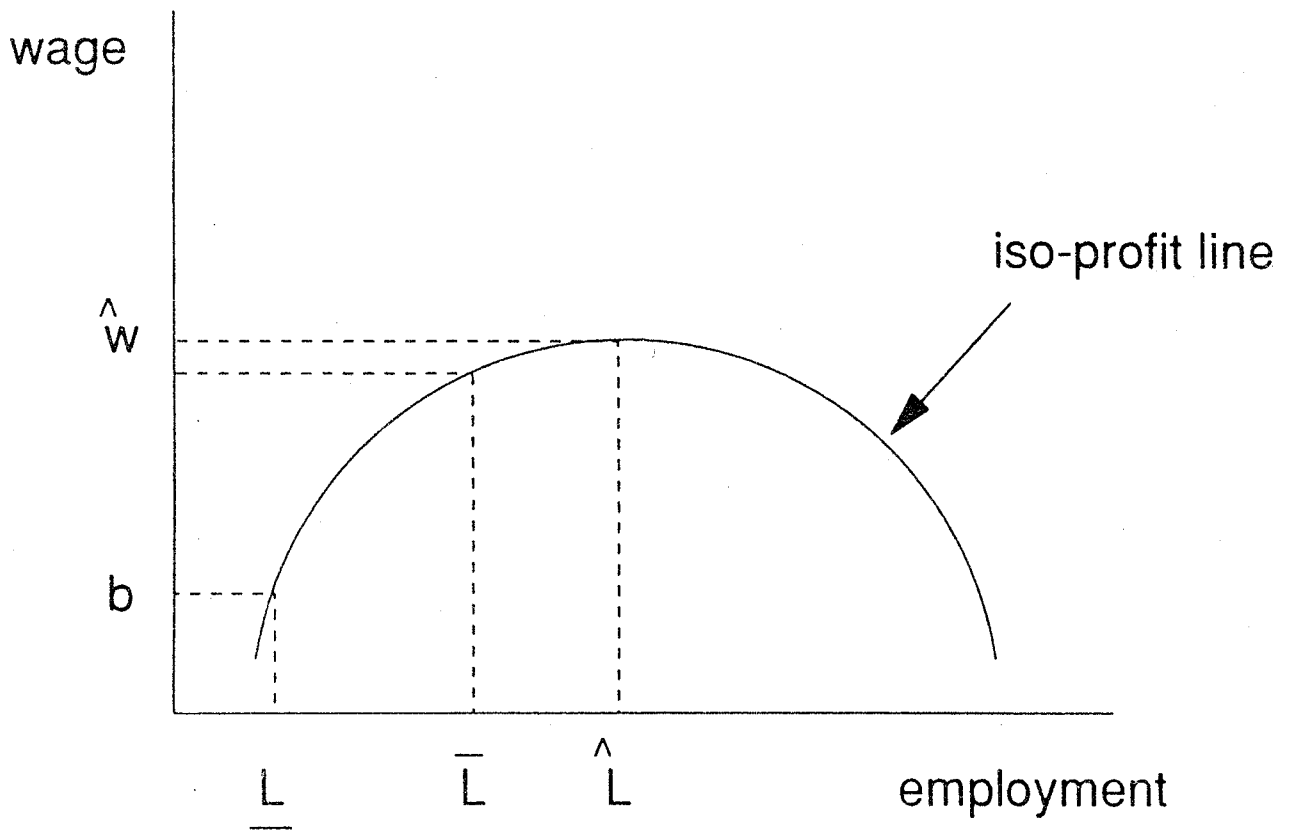


FIGURE 2

The Different Types of Equilibria

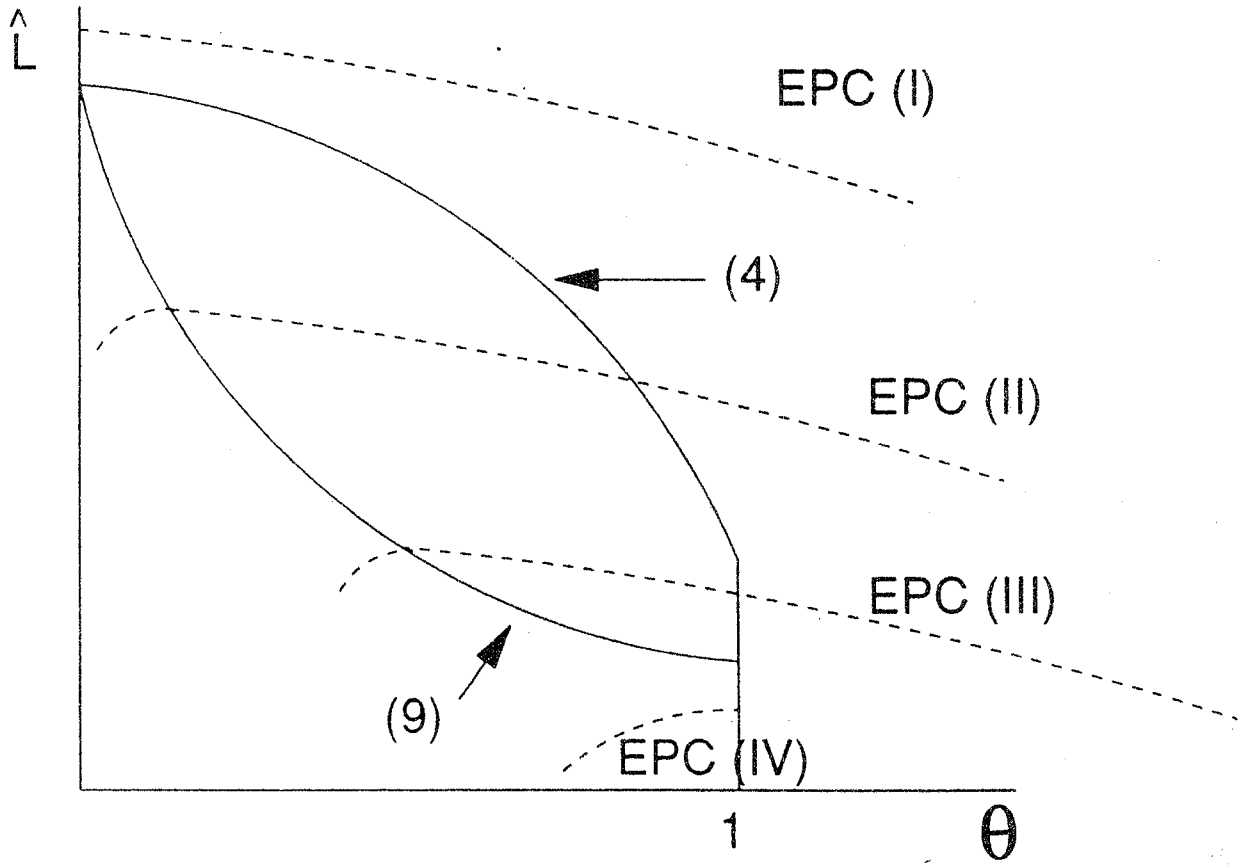
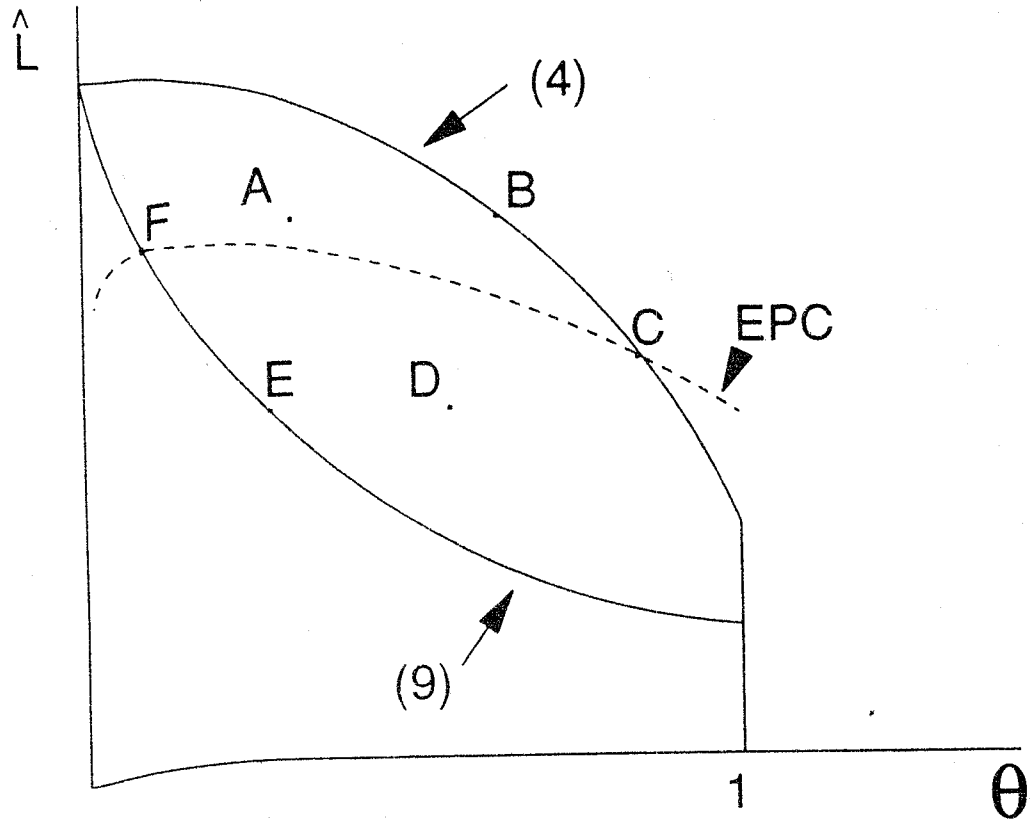


FIGURE 3

A Stability Story



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