Debt and Deficits:
Fiscal Analysis with Stationary Ratios

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CEPR Paris Symposium
December 2024

Introduction

- What happens when a government's fiscal position deteriorates?
  - Poor returns for bond holders? Fiscal adjustment (rises in tax revenue or cuts in spending)?
- What is the *fiscal position*, anyway?
  - Some seemingly natural definitions are problematic. We suggest an alternative
- We derive an identity that relates the fiscal position to debt returns and fiscal adjustment (a combination of tax and spending growth) — to do variance decompositions
- In US and international data since World War II, a deterioration of the fiscal position forecasts fiscal adjustment in the long run
  - It does not forecast low real returns for bond holders
- In the US, it forecasts a decline in spending over the long run rather than increases in tax revenue
  - International results are similar except in Japan where tax revenue adjusts

Health warning

- This project develops a loglinear intertemporal accounting system to understand the historical dynamics of government debt and deficits
- There is no attempt to identify structural shocks
- There are no causal statements
- There are no counterfactuals
- Any impression I may give to the contrary is unintentional and misleading!

The US debt-GDP ratio appears nonstationary

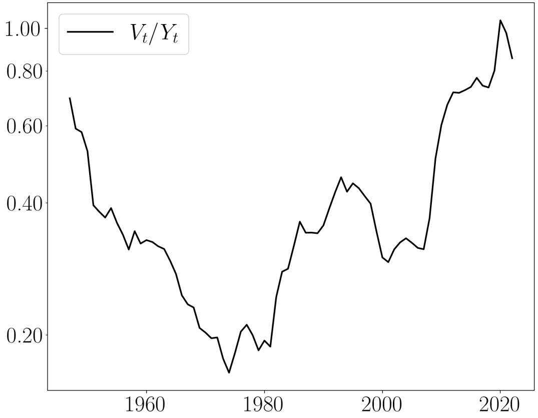

Figure: Post-WW2 US data. Log scale.
Debt at market value from Dallas Fed, GDP from NIPA via FRED.

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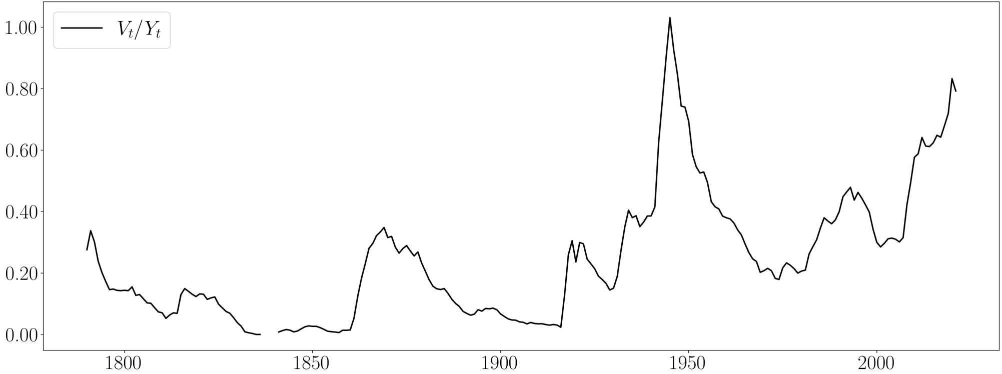

Figure: Long-run data for US. Auto-correlation is 0.98.
From Hall and Sargent (2021) and Johnston and Williamson (2023).

But the surplus-debt ratio appears to be stationary

- Just as a corporation pays dividends to owners of its stock, so the government ‘pays’ the primary surplus to owners of its debt
- This suggests an analogy in which the surplus-debt ratio plays the role of the dividend-price ratio
- **Good news**: In postwar US data, standard unit root tests reject nonstationarity for the surplus-debt ratio
  - Similar findings in G4 (UK, Canada, Japan, Switzerland) and 11 Eurozone countries with available data.

But the surplus-debt ratio appears to be stationary

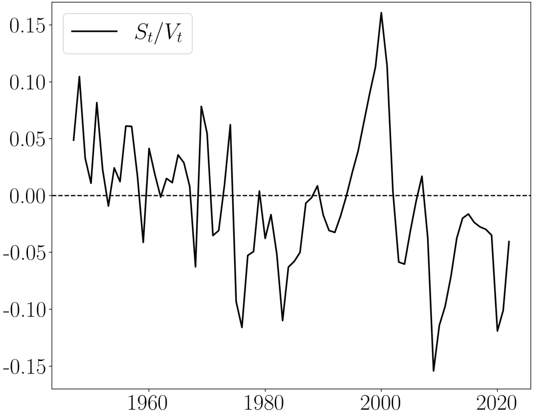

Figure: The surplus-to-debt ratio is stationary in postwar data. US data, 1947–2022. Linear scale

And yet ... the surplus-debt ratio is also a flawed measure

- The surplus-debt ratio has two problems as a measure of the fiscal position. Both are related to the fact that the surplus can be either positive or negative
  1. An exogenous increase in debt, with unchanged surplus, should worsen the fiscal position. But it *increases* the surplus-debt ratio if surplus is negative
  2. The analogy with the dividend-price ratio suggests a Campbell–Shiller-like approximation relating the log surplus-debt ratio to expected future debt returns and surplus growth rates. But the analogy fails: log surplus cannot be defined, as surplus can go negative

A way forward

- Instead of surplus growth rates, we work with tax and spending growth rates, and log tax-debt and log spending-debt ratios
- Giannitsarou, Scott and Leeper (2006) and Berndt, Lustig and Yeltekin (2012) use this approach
- They assume that log tax-debt and log spending-debt ratios are stationary, then do a loglinear approximation around their means
  - Empirical problem: neither of these ratios appears to be stationary
  - Conceptual problem: there is no reason to expect either to be stationary. A government's activities can be large or small relative to its debt
  - **Good news:** If surplus-debt is stationary, then tax-debt and spending-debt are cointegrated in levels and approximately cointegrated in logs

The tax-debt and spending-debt ratios appear to be nonstationary

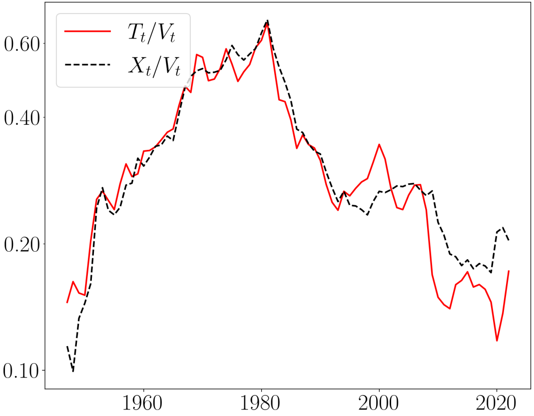

Figure: Tax-debt and spending-debt ratios appear to be nonstationary in postwar data. Log scale

A framework for fiscal analysis

- The gross return on government debt is

$$R_{t+1} = \frac{V_{t+1} + T_{t+1} - X_{t+1}}{V_t}$$

  - V_t market value of debt; T_{t+1} tax revenue; X_{t+1} expenditure; surplus is $S_t = T_t - X_t$

- If expected tax, spending, and debt growth are constant (at G) and expected return on debt is constant (at R), then $R = \mathbb{E}_t \frac{V_{t+1}}{V_t} + \mathbb{E}_t \frac{T_{t+1}}{T_t}\frac{T_t}{V_t} - \mathbb{E}_t \frac{X_{t+1}}{X_t}\frac{X_t}{V_t} = G\left(1 + \frac{S_t}{V_t}\right)$, so

$$\log\left(1 + \frac{S_t}{V_t}\right) = \log R - \log G$$

- We assume $R > G$, so the market value of debt is the present value of future surpluses

A framework for fiscal analysis

- For the general case, rewrite

$$R_{t+1} = \frac{V_{t+1}}{V_t}\left(1 + \frac{S_{t+1}}{V_{t+1}}\right) \quad \text{or, in logs,} \quad r_{t+1} = \Delta v_{t+1} + \log\left(1 + \frac{S_{t+1}}{V_{t+1}}\right)$$

- Linearize in $\tau v_t = \log(T_t/V_t)$ and $xv_t = \log(X_t/V_t)$:

$$\log\left(1 + \frac{S_{t+1}}{V_{t+1}}\right) = \log\left(1 + e^{\tau v_{t+1}} - e^{xv_{t+1}}\right) \approx \underbrace{k + \frac{1-\rho}{1-\beta}\left(\tau v_t - \beta\, xv_t\right)}_{sv_t}$$

where $\mathbb{E}\log(1 + S_t/V_t) = -\log\rho$ and $\tau v_t - \beta\, xv_t$ is stationary

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- sv_t is our proposed measure of the fiscal position
  - It falls when tax falls, spending rises, or debt rises
  - It satisfies the approximate identity $r_{t+1} = \Delta v_{t+1} + sv_{t+1}$

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where $\mathbb{E}\log(1 + S_t/V_t) = -\log\rho$ and $\tau v_t - \beta\, xv_t$ is stationary

- When $R > G$, ρ and β are both less than one (and both equal one when $R = G$)
  - The higher R is relative to G, the larger the primary surplus must be on average
  - Tax must be higher than spending on average so tax growth is more influential

A framework for fiscal analysis

- Rearranging the approximate identity $r_{t+1} = \Delta v_{t+1} + sv_{t+1}$, we have

$$sv_t = (1-\rho)\left[r_{t+1} - \frac{1}{1-\beta}\Delta\tau_{t+1} + \frac{\beta}{1-\beta}\Delta x_{t+1}\right] + \rho\, sv_{t+1}$$

- Solving forward T periods, we have:

$$sv_t = (1-\rho)\sum_{j=0}^{T-1}\rho^j\left[r_{t+1+j} - \frac{1}{1-\beta}\Delta\tau_{t+1+j} + \frac{\beta}{1-\beta}\Delta x_{t+1+j}\right] + \rho^T sv_{t+T}$$

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- Solving forward to an infinite horizon, we have:

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  - A strong fiscal position implies some combination of high log returns on debt, low tax growth, and high spending growth over the infinite future

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- Unconditional mean:

$$\mathbb{E}\, sv_t = \mathbb{E}\, r_t - \underbrace{\left(\frac{1}{1-\beta}\,\mathbb{E}\,\Delta\tau_t - \frac{\beta}{1-\beta}\,\mathbb{E}\,\Delta x_t\right)}_{=g}$$

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- Solving forward to an infinite horizon, we have generalized "$S/V = R - G$":

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  - Tax and spending growth have powerful effects because tax and spending are each large relative to the primary surplus

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$$sv_t = (1-\rho)\sum_{j=0}^{\infty}\rho^j\left[\log \mathbb{E}_t\, R_{t+1+j} - \text{volatility} - \text{skewness..} - \frac{1}{1-\beta}\,\mathbb{E}_t\,\Delta\tau_{t+1+j} + \frac{\beta}{1-\beta}\,\mathbb{E}_t\,\Delta x_{t+1+j}\right]$$

  - A strong fiscal position implies some combination of high log returns on debt, low tax growth, and high spending growth over the infinite future
  - There is a wedge between average log and average simple returns related to higher moments of returns

Empirical preparation: US post WWII

Do we have a plausible imputed debt return?

- We impute the return on government debt from the time series of market value and primary surpluses
- We confirm the plausibility of the implied return series by regressing it on contemporaneous variables that explain the returns on short-term and long-term government debt
  - the short-term realized real interest rate (nominal rate minus realized inflation)
  - the change in the long-term bond yield.
- These regressions have high explanatory power and coefficients with the right sign and strong statistical significance

Do we have a plausible imputed debt return?

$$r_t = \alpha + \beta \left(\text{short yield}_{t-1\to t} - \text{realised inflation}_t\right) + \gamma\, \Delta\text{long yield}_t + \varepsilon_t$$

country	α	NW_{se}	β	NW_{se}	γ	NW_{se}	R^2	obv.
	0.00	[0.01]	1.56	[0.17]	—	—	52.8%	76
USA	0.02	[0.00]	—	—	−4.17	[0.50]	49.8%	76
	0.01	[0.00]	1.56	[0.19]	−2.98	[0.39]	74.6%	76

Table: US postwar sample

Do we have a plausible imputed debt return?

country	α	nominal short yield	inflation	Δ long yield	slope	R^2	obv.
USA	0.030	1.040	-1.650	-2.742	-0.258	78.5%	76
	[0.010]	[0.167]	[0.206]	[0.354]	[0.458]		

Table: US postwar sample

Choosing the linearization parameters

- In our 1947–2022 data, the sample mean surplus-debt ratio is negative for US
  - Mixed findings in 15 other countries with shorter samples: Japan appears negative, Eurozone, UK, Canada, Switzerland are positive.
- Negative number contradicts the theory we are using which requires a positive population mean
  - We set $\rho = 0.999$ to come close to the data while remaining consistent with the theory
- We then determine β by choosing the best fit for linear approximation

$$\min_{\beta} \sum_{t} \left(\log\left(1 + S_t/V_t\right) - k - \frac{1-\rho}{1-\beta}\left(\tau v_t - \beta\, xv_t\right) \right)^2$$

- When estimating dynamics of the data, we impose theoretically motivated means of $\mathbb{E}\, r_t$, $\mathbb{E}\, \Delta\tau_t$, and $\mathbb{E}\, sv_t$ rather than using sample means (using average GDP growth for tax and spending).

Our measure of the fiscal position, sv_t, and the surplus-debt ratio

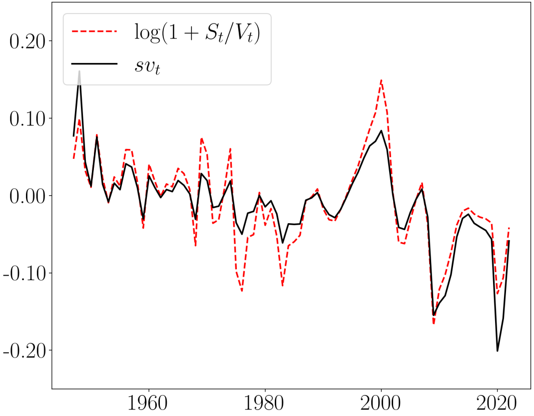

Figure: sv_t and $\log(1 + S_t/V_t)$, US data 1947-2022.

Some unit root tests

	r_t	$\Delta\tau_t$	Δx_t	Δv_t	Δy_t
t-stat	-6.976	-5.873	-10.954	-5.067	-8.072
p-value	0.000	0.000	0.000	0.000	0.000
auto-corr	0.223	0.226	0.191	0.493	0.079

Table: USA sample 1947-2022, ADF tests with AIC lags.

Some unit root tests

	vy_t	τy_t	xy_t	τv_t	xv_t	sv_t
t-stat	-1.086	-4.642	-2.152	-1.469	-2.565	-4.271
p-value	0.720	0.000	0.224	0.549	0.100	0.000
auto-corr	0.976	0.662	0.814	0.960	0.972	0.747

Table: USA sample 1947-2022, ADF tests with AIC lags.

Empirical results: US post WWII

A variance decomposition for sv_t

- Recall that

$$sv_t = (1-\rho)\sum_{j=0}^{\infty}\rho^j\left[r_{t+1+j} - \frac{1}{1-\beta}\Delta\tau_{t+1+j} + \frac{\beta}{1-\beta}\Delta x_{t+1+j}\right]$$

- Hence, over an infinite horizon

$$1 = \frac{\mathrm{cov}\left(sv_t, (1-\rho)\sum_{j=0}^{\infty}\rho^j\,\mathbb{E}_t\, r_{t+1+j}\right)}{\mathrm{var}\; sv_t} + \frac{\mathrm{cov}\left(sv_t, -(1-\rho)\sum_{j=0}^{\infty}\rho^j\,\mathbb{E}_t\,\frac{1}{1-\beta}\Delta\tau_{t+1+j}\right)}{\mathrm{var}\; sv_t} + \\ + \frac{\mathrm{cov}\left(sv_t, (1-\rho)\sum_{j=0}^{\infty}\rho^j\,\mathbb{E}_t\,\frac{\beta}{1-\beta}\Delta x_{t+1+j}\right)}{\mathrm{var}\; sv_t}$$

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- Hence, over a finite horizon T

$$1 = \frac{\text{cov}\left(sv_t, (1-\rho)\sum_{j=0}^{T-1}\rho^j\,\mathbb{E}_t\, r_{t+1+j}\right)}{\text{var}\, sv_t} + \frac{\text{cov}\left(sv_t, -(1-\rho)\sum_{j=0}^{T-1}\rho^j\,\mathbb{E}_t\,\frac{1}{1-\beta}\Delta\tau_{t+1+j}\right)}{\text{var}\, sv_t} + \\ + \frac{\text{cov}\left(sv_t, (1-\rho)\sum_{j=0}^{T-1}\rho^j\,\mathbb{E}_t\,\frac{\beta}{1-\beta}\Delta x_{t+1+j}\right)}{\text{var}\, sv_t} + \frac{\text{cov}\left(sv_t, \rho^T\,\mathbb{E}_t\, sv_{t+T}\right)}{\text{var}\, sv_t}$$

A model free approach: local projection

horizon	return	fiscal adjustment	future sv	spending ratio
1	0.0	26.2	73.7	67.9
	[0.0]	[7.3]	[7.3]	[20.8]
3	0.0	56.1	43.8	70.7
	[0.1]	[10.8]	[10.8]	[21.4]
10	0.0	77.1	22.9	79.3
	[0.1]	[24.5]	[24.5]	[29.1]

Table: Till 10 years' horizon, with NW standard error.

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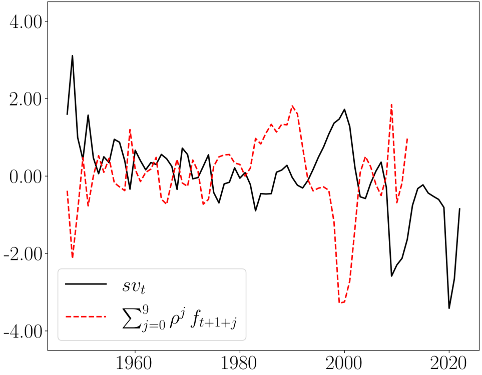
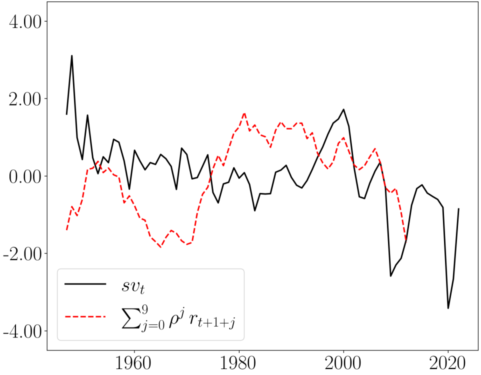


(a) Fiscal Adjustment



(b) Return

Choice of VAR(1) system

We discipline the choice of variables using our log-linear identity

$$sv_t = (1-\rho)\left[r_{t+1} - \frac{1}{1-\beta}\Delta\tau_{t+1} + \frac{\beta}{1-\beta}\Delta x_{t+1}\right] + \rho\, sv_{t+1}$$

- Unconstrained VAR(1) with all four variables would not work: the identity imposes a constraint on the coefficient matrix
- We omit Δx_{t+1} and impute its forecast using the identity — this works if we keep sv_t in the system so the information set is consistent whichever variable is omitted
- Baseline VAR(1) system is $(r_{t+1}, \Delta\tau_{t+1}, \Delta y_{t+1}, sv_{t+1}, sv_t)$
- Impute Δx_{t+1} and fiscal adjustment $f_{t+1} = \Delta\tau_{t+1} - \beta\,\Delta x_{t+1}$

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VAR(1) estimation using 'demeaned series', US data, 1947-2022

	r_{t+1}	$\Delta\tau_{t+1}$	Δy_{t+1}	sv_{t+1}	Δx_{t+1}	f_{t+1}
r_t	0.292	−0.293	0.009	−0.138	0.121	−0.414
	[0.122]	[0.103]	[0.050]	[0.072]	[0.190]	[0.224]
$\Delta\tau_t$	−0.041	0.034	−0.038	−0.122	0.400	−0.365
	[0.111]	[0.094]	[0.045]	[0.066]	[0.172]	[0.203]
Δy_t	0.005	1.578	0.178	0.539	−0.037	1.616
	[0.310]	[0.263]	[0.126]	[0.184]	[0.483]	[0.570]
sv_t	0.362	−0.536	−0.105	0.838	−0.047	−0.488
	[0.184]	[0.155]	[0.075]	[0.109]	[0.286]	[0.338]
sv_{t-1}	−0.092	0.146	0.141	−0.170	0.657	−0.509
	[0.191]	[0.162]	[0.078]	[0.114]	[0.298]	[0.352]
R^2	14.50%	51.51%	9.15%	69.44%	20.39%	33.52%

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	[0.122]	[0.103]	[0.05]	[0.072]	[0.19]	[0.224]
$\Delta\tau_t$	−0.041	0.034	−0.038	−0.122	0.400	−0.365
	[0.111]	[0.094]	[0.045]	[0.066]	[0.172]	[0.203]
Δy_t	0.005	1.578	0.178	0.539	−0.037	1.616
	[0.310]	[0.263]	[0.126]	[0.184]	[0.483]	[0.570]
sv_t	0.362	−0.536	−0.105	0.838	−0.047	−0.488
	[0.184]	[0.155]	[0.075]	[0.109]	[0.286]	[0.338]
sv_{t-1}	−0.092	0.146	0.141	−0.170	0.657	−0.509
	[0.191]	[0.162]	[0.078]	[0.114]	[0.298]	[0.352]
R^2	14.50%	51.51%	9.15%	69.44%	20.39%	33.52%

$$f_{t+1} = \Delta\tau_{t+1} - \beta\Delta x_{t+1}.$$

Variance decomposition for sv_t in US based on VAR(1) models

- We merge the contribution from tax and spending and call it fiscal adjustment
  - A three-way decomposition of return, fiscal adjustment and future sv
- We report the contribution from spending to the fiscal adjustment component in the 4th column, which we call the spending ratio
- We show bootstrapped 95% confidence intervals of those four quantities
- The variance decomposition depends on the variables included in the $VAR(1)$ system
  - Our baseline is $(r_t, \Delta\tau_t, \Delta y_t, sv_t, sv_{t-1})$
  - We check robustness by: 1) adding additional state variables that explain return: 1-year real yield, 10- minus 1-year yield spread; 2) comparing with 'local projection' results (already shown).

A variance decomposition for sv_t in US

horizon	return	fiscal adjustment	future sv	spending ratio
1	0.0	24.4	76.9	66.4
	[0.0, 0.0]	[11.1, 37.8]	[63.5, 90.2]	[34.7, 88.2]
3	0.1	70.7	30.5	66.9
	[0.0, 0.1]	[35.6, 99.1]	[2.1, 65.7]	[32.8, 99.1]
10	0.1	99.7	1.5	74.1
	[0.0, 0.3]	[73.1, 102.1]	[−0.9, 28.1]	[29.9, 118.3]
∞	0.1	101.2	0.0	74.4
	[0.0, 0.3]	[101.0, 101.3]	[−0.0, 0.0]	[28.5, 121.8]

Table: According to VAR(1) system $(r_t, \Delta\tau_t, \Delta y_t, sv_t, sv_{t-1})$.

A variance decomposition for sv_t in US

horizon	return	fiscal adjustment	future sv	spending ratio
1	0.0	24.4	76.9	66.4
	[0.0, 0.0]	[11.1, 37.8]	[63.5, 90.2]	[34.7, 88.2]
3	0.1	70.7	30.5	66.9
	[0.0, 0.1]	[35.6, 99.1]	[2.1, 65.7]	[32.8, 99.1]
10	0.1	99.7	1.5	74.1
	[0.0, 0.3]	[73.1, 102.1]	[−0.9, 28.1]	[29.9, 118.3]
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Table: According to VAR(1) system $(r_t, \Delta\tau_t, \Delta y_t, sv_t, sv_{t-1})$.

A variance decomposition for sv_t in US

horizon	return	fiscal adjustment	future sv	spending ratio
1	0.0	24.4	76.9	66.4
	$[0.0, 0.0]$	$[11.1, 37.8]$	$[63.5, 90.2]$	$[34.7, 88.2]$
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∞	0.1	101.2	0.0	74.4
	$[0.0, 0.3]$	$[101.0, 101.3]$	$[-0.0, 0.0]$	$[28.5, 121.8]$

Table: According to VAR(1) system $(r_t, \Delta\tau_t, \Delta y_t, sv_t, sv_{t-1})$.

A variance decomposition for sv_t in US

horizon	return	fiscal adjustment	future sv	spending ratio
1	0.0	24.2	77.1	65.4
	$[0.0, 0.0]$	$[11.3, 36.8]$	$[64.5, 90.0]$	$[36.9, 87.5]$
3	0.1	71.6	29.7	68.3
	$[0.0, 0.1]$	$[35.8, 103.0]$	$[-1.8, 65.5]$	$[38.9, 98.7]$
10	0.1	100.1	1.1	81.6
	$[-0.1, 0.2]$	$[70.0, 106.6]$	$[-5.2, 31.1]$	$[40.5, 127.2]$
∞	0.1	101.3	0.0	82.4
	$[-0.2, 0.4]$	$[100.9, 101.5]$	$[-0.0, 0.0]$	$[31.9, 142.4]$

Table: Robustness: based on VAR(1) system $(r_t, \Delta\tau_t, \Delta y_t, sv_t, sv_{t-1}, yr_{1,t}, spr_{1\to 10,t})$.

One more stationary ratio...?

- We look for other cointegrating relationships, and find one in the US sample
- The tax-GDP ratio is stationary
  - This may reflect political economy considerations that limit the extent to which tax revenue can vary as a fraction of GDP
  - Jiang, Sargent, Wang and Yang (2022) cite Keynes (1923) arguing that tax-GDP has an upper bound that is politically supportable
- No other fiscal variables are so closely related to GDP: spending-GDP, surplus-GDP, and debt-GDP ratios are all nonstationary

The tax-GDP ratio appears to be stationary in the US

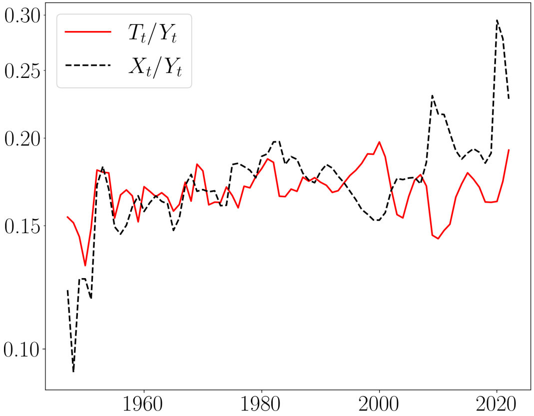


Figure: Spending-to-GDP is also nonstationary, but tax-to-GDP is stationary. US data. Log scale

Including τy_t in VAR(1) estimation

	r_{t+1}	$\Delta\tau_{t+1}$	Δy_{t+1}	sv_{t+1}	τy_{t+1}	Δx_{t+1}	f_{t+1}
r_t	0.221	−0.205	0.033	−0.164	−0.239	0.288	−0.492
	[0.123]	[0.101]	[0.050]	[0.074]	[0.088]	[0.185]	[0.232]
$\Delta\tau_t$	−0.147	0.164	−0.002	−0.161	0.166	0.650	−0.483
	[0.119]	[0.097]	[0.049]	[0.072]	[0.085]	[0.178]	[0.224]
Δy_t	0.221	1.313	0.105	0.619	1.208	−0.544	1.855
	[0.318]	[0.260]	[0.130]	[0.192]	[0.228]	[0.476]	[0.599]
sv_t	0.272	−0.424	−0.074	0.804	−0.350	0.165	−0.589
	[0.183]	[0.150]	[0.075]	[0.111]	[0.132]	[0.275]	[0.346]
τy_t	0.218	−0.268	−0.074	0.080	0.806	−0.501	0.241
	[0.103]	[0.084]	[0.042]	[0.062]	[0.074]	[0.154]	[0.193]
sv_{t-1}	−0.082	0.135	0.138	−0.166	−0.004	0.634	−0.498
	[0.186]	[0.152]	[0.076]	[0.112]	[0.134]	[0.279]	[0.351]
R^2	19.69%	57.10%	12.91%	70.35%	74.41%	31.37%	34.99%

The role of the tax-GDP ratio in the US

- If we include the tax-GDP ratio in the system, the stabilizing force on tax growth narrows the confidence interval of the 'spending ratio'

horizon	return	fiscal adjustment	future sv	spending ratio
1	0.0 $[0.0, 0.0]$	24.5 $[11.7, 36.8]$	76.8 $[64.5, 89.6]$	66.8 $[42.7, 86.5]$
3	0.1 $[0.0, 0.1]$	73.4 $[38.5, 102.7]$	27.9 $[-1.4, 62.8]$	74.7 $[55.4, 97.6]$
10	0.0 $[-0.1, 0.1]$	100.6 $[77.4, 107.5]$	0.7 $[-6.2, 24.0]$	102.1 $[90.5, 131.9]$
∞	0.0 $[-0.1, 0.2]$	101.3 $[101.2, 101.4]$	0.0 $[0.0, 0.0]$	101.1 $[89.4, 137.7]$

Table: VAR(1) system includes $(r_t, \Delta\tau_t, \Delta y_t, sv_t, sv_{t-1}, \tau y_t)$

Interpretation

- A weak fiscal position is typically resolved by declines in the growth rate of spending, rather than by increases in tax revenue or poor returns for bondholders
- In historical US data, government debt returns have modest variability and low correlation with the fiscal position, so returns play little role at any horizon
  - Contrast with the Campbell–Shiller (1988) finding that returns are the dominant driver of fluctuations in the market dividend-price ratio
  - The fiscal theory of the price level postulates changes in real debt valuation in response to exogenous shocks to taxes or spending
  - It remains possible that the FTPL holds, but the US government has chosen not to change taxes or spending in a way that requires volatile real debt returns
- Taxes play little role in the long run because taxes are linked to GDP and fiscal variables do not strongly predict long-run GDP growth (there are offsetting short- and medium-run effects)

Decomposing responses to tax and expenditure shocks

Variance decomposition of short term tax or spending news

Our framework allows us to analyse the behaviour of tax and spending separately.

$$sv_t = (1-\rho)\sum_{j=0}^{T-1}\rho^j\left[r_{t+1+j} - \frac{1}{1-\beta}\underbrace{(\Delta\tau_{t+1+j} - \beta\Delta x_{t+1+j})}_{\text{fiscal adjustment}}\right] + \rho^T sv_{t+T}$$

We ask whether deficits driven by shocks to taxes look different from deficits driven by shocks to spending.

- The "news operator" is: $\Delta\,\mathbb{E}_{t+1} = \mathbb{E}_{t+1} - \mathbb{E}_t$
- Variance of 'one period' tax (or spending) news could be decomposed into
  - news about returns
  - news about fiscal adjustment
  - news about the longer term fiscal position

Variance decomposition of short term tax or spending news

Our framework allows us to analyse the behaviour of tax and spending separately.

$$0 = (1-\rho)\sum_{j=0}^{T-1}\rho^j\left[\Delta\,\mathbb{E}_{t+1}\,r_{t+1+j} - \frac{1}{1-\beta}\Delta\,\mathbb{E}_{t+1}\,f_{t+1+j}\right] + \rho^T\Delta\,\mathbb{E}_{t+1}\,sv_{t+T}$$

We ask whether deficits driven by shocks to taxes look different from deficits driven by shocks to spending.

- The "news operator" is: $\Delta\,\mathbb{E}_{t+1} = \mathbb{E}_{t+1} - \mathbb{E}_t$
- Variance of 'one period' tax (or spending) news could be decomposed into
  - news about returns
  - news about fiscal adjustment
  - news about the longer term fiscal position

Variance decomposition of short term fiscal news

Table: Short-run tax news, postwar US data.

horizon	return	fiscal adjustment	future sv	spending ratio
1	-0.1	-38.3	139.8	100.0
	$[-0.2, 0.0]$	$[-49.1, -28.1]$	$[129.6, 150.7]$	$[100.0, 100.0]$
3	0.0	13.1	88.4	-171.9
	$[-0.1, 0.2]$	$[-60.8, 71.7]$	$[29.7, 162.3]$	$[-1249.1, 1519.5]$
10	0.2	100.6	0.7	26.5
	$[0.0, 0.4]$	$[54.5, 127.9]$	$[-26.7, 46.7]$	$[-5.1, 58.5]$
∞	0.2	101.3	0.0	26.6
	$[-0.1, 0.4]$	$[101.1, 101.6]$	$[0.0, 0.0]$	$[4.7, 83.3]$

Variance decomposition of short term fiscal news

Table: Short-run spending news, postwar US data.

horizon	return	fiscal adjustment	future sv	spending ratio
1	-0.1	-9.8	140.8	0.0
	$[-0.2, 0.0]$	$[-16.4, -4.8]$	$[115.4, 171.5]$	$[0.0, 0.0]$
3	-0.0	44.9	86.0	106.2
	$[-0.2, 0.1]$	$[-5.7, 75.3]$	$[41.6, 144.8]$	$[-9.8, 273.8]$
10	0.0	129.7	1.3	117.0
	$[-0.2, 0.2]$	$[88.6, 156.3]$	$[-17.8, 45.8]$	$[108.8, 143.8]$
∞	0.0	131.0	0.0	115.5
	$[-0.2, 0.2]$	$[106.4, 158.0]$	$[0.0, 0.0]$	$[108.2, 139.4]$

Sensitivity analysis of ρ and β

Sensitivity analysis of ρ for variance decomposition of sv_t

Table: Using system $(r_t, \Delta\tau_t, \Delta y_t, sv_t, sv_{t-1})$, Infinite horizon

ρ	r	β	approx. error	λ_{max}	return	fiscal	spending ratio
0.999	0.031	0.997	0.052	0.646	0.1	101.2	74.3
0.995	0.035	0.985	0.059	0.726	0.7	100.6	69.6
0.990	0.040	0.970	0.079	0.810	2.1	99.2	63.6
0.980	0.050	0.945	0.160	0.895	6.6	94.7	55.4
0.970	0.060	0.927	0.284	0.926	11.6	89.7	53.0
0.960	0.071	0.914	0.439	0.941	16.0	85.3	54.2
0.950	0.081	0.904	0.614	0.949	19.6	81.8	56.8
0.900	0.135	0.871	1.630	0.969	27.8	73.5	71.1

Sensitivity analysis of ρ for variance decomposition of sv_t

Table: Using system $(r_t, \Delta\tau_t, \Delta y_t, sv_t, sv_{t-1}, \tau y_t)$, Infinite horizon

ρ	r	β	approx. error	λ_{max}	return	fiscal	spending ratio
0.999	0.031	0.997	0.052	0.614	0.1	101.3	101.1
0.995	0.035	0.985	0.059	0.683	0.4	100.9	101.2
0.990	0.040	0.970	0.079	0.784	1.3	100.0	101.0
0.980	0.050	0.945	0.160	0.891	5.2	96.1	99.6
0.970	0.060	0.927	0.284	0.928	10.3	91.0	96.8
0.960	0.071	0.914	0.439	0.943	14.9	86.4	93.8
0.950	0.081	0.904	0.614	0.952	18.8	82.5	91.2
0.900	0.135	0.871	1.630	0.971	27.7	73.6	84.5

Sensitivity analysis of β for variance decomposition of sv_t

Table: Using system $(r_t, \Delta\tau_t, \Delta y_t, sv_t, sv_{t-1})$, Infinite horizon

ρ	r	β	approx. error	λ_{max}	return	fiscal	spending ratio
0.999	0.031	0.997	0.052	0.646	0.1	101.2	74.3
		0.991	0.154	0.704	0.4	101.0	71.1
		0.999	0.319	0.634	0.1	101.3	75.0
0.995	0.035	0.985	0.059	0.726	0.7	100.6	69.6
		0.954	0.158	0.91	4.1	97.2	50.1
		0.993	0.325	0.664	0.3	101.0	73.2
0.970	0.060	0.927	0.284	0.926	11.6	89.7	53.0
		0.719	0.734	0.999	32.5	68.8	72.1
		0.955	0.45	0.845	5.1	96.2	62.5

Sensitivity analysis of β for variance decomposition of sv_t

Table: Using system $(r_t, \Delta\tau_t, \Delta y_t, sv_t, sv_{t-1}, \tau y_t)$, Infinite horizon

ρ	r	β	approx. error	λ_{max}	return	fiscal	spending ratio
0.999	0.031	0.997	0.052	0.614	0.1	101.3	101.1
		0.991	0.154	0.664	0.2	101.2	101.4
		0.999	0.319	0.605	0.0	101.3	101.1
0.995	0.035	0.985	0.059	0.683	0.4	100.9	101.2
		0.954	0.158	0.914	2.6	98.7	102.9
		0.993	0.325	0.624	0.1	101.2	100.9
0.970	0.060	0.927	0.284	0.928	10.3	91.0	96.8
		0.719	0.734	1.002	32.4	68.9	79.7
		0.955	0.450	0.815	4.0	97.3	98.5

International evidence

G4 countries: UK, Canada, Japan, and Switzerland

Table: G4, ADF tests of ratios

country		vy_t	τy_t	xy_t	τv_t	xv_t	sv_t
GBR	t-stat	−1.218	−2.291	−1.018	−1.449	−1.765	−2.399
	p-value	0.666	0.175	0.747	0.559	0.398	0.142
CAN	t-stat	−1.92	−1.546	−2.405	−2.195	−1.766	−2.293
	p-value	0.323	0.511	0.140	0.208	0.397	0.174
JPN	t-stat	−1.786	0.136	0.110	−2.068	−4.892	−2.406
	p-value	0.387	0.968	0.967	0.257	0.000	0.140
CHE	t-stat	−0.669	−1.899	−3.338	−0.786	−0.574	−3.023
	p-value	0.855	0.333	0.013	0.823	0.877	0.033

Plausibility of imputed return from identity

Table: UK, Canada, Japan, and Switzerland

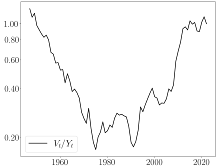
country	α	nominal short yield	inflation	Δ long yield	slope	R^2	obv.
GBR	0.004	2.054	0.853	−16.697	3.676	60.2%	76
	[0.015]	[0.549]	[0.640]	[2.053]	[1.788]		
CAN	0.096	0.394	−1.977	−4.115	−0.904	64.5%	33
	[0.015]	[0.180]	[0.422]	[0.771]	[0.497]		
JPN	0.008	0.930	−0.440	−2.868	1.176	38.3%	25
	[0.004]	[1.163]	[0.360]	[1.825]	[1.070]		
CHE	0.030	0.151	1.128	−7.314	−1.653	71.7%	23
	[0.008]	[0.322]	[0.586]	[0.842]	[0.627]		

Choices of theoretical parameters across countries

Table: Theoretical choices parameters for G4

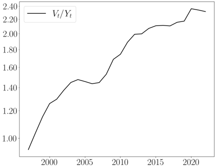
country	start	ρ	β	sv	sv (data)	r	r (data)	g (data)	$r-g$ (data)
GBR	1947	0.967	0.952	0.034	0.034	0.088	0.083	0.054	0.030
CAN	1989	0.960	0.947	0.041	0.041	0.062	0.067	0.021	0.046
JPN	1997	0.999	0.994	0.001	-0.023	0.007	0.019	0.006	0.013
CHE	1999	0.970	0.970	0.031	0.031	0.050	0.025	0.019	0.006

Some plots

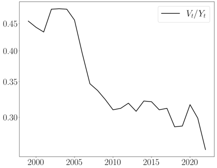


(a) UK

(b) Canada



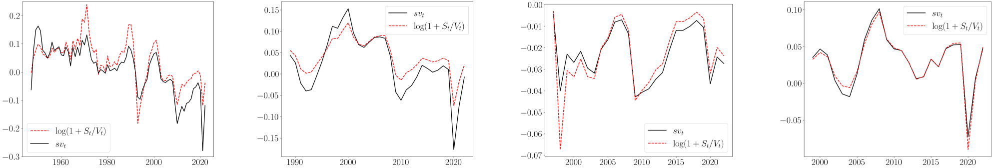
(c) Japan



(d) Switzerland

Figure: The debt-to-GDP ratios in G4

Some plots



(a) UK (b) Canada (c) Japan (d) Switzerland

Figure: The surplus-to-debt ratios in G4

Main results for G4

Table: Variance decomposition of fiscal position sv_t for G4 at $T = 10$

country	return	fiscal adjustment	future sv	spending ratio
GBR	1.5	86.4	13.4	105.2
	$[-3.1, 6.6]$	$[49.3, 104.8]$	$[-1.9, 48.1]$	$[37.6, 213.1]$
CAN	3.7	97.8	1.6	78.1
	$[0.2, 8.4]$	$[50.9, 104.0]$	$[-4.3, 47.0]$	$[51.1, 163.8]$
JPN	-0.2	77.6	26.8	-25.6
	$[-0.8, 0.2]$	$[1.9, 134.9]$	$[-31.0, 102.5]$	$[-461.3, 75.1]$
CHE	2.6	104.6	-2.7	78.5
	$[-0.1, 6.4]$	$[84.9, 131.5]$	$[-29.7, 16.5]$	$[34.4, 140.1]$

Summary

- Our framework uses identities to organize the time-series analysis of historical data
- We have not identified structural shocks and cannot make causal statements or explore counterfactuals
- However, the identities in our paper are in a convenient form to be combined with typical loglinear macro models, whether in the DSGE tradition or the NK tradition
- In the US, shocks to the fiscal position are associated with long-run adjustment in spending more than taxes, with a negligible contribution from returns
  - International evidence is comparable, except for Japan where tax adjustment appears more important
- We think it is important to distinguish the separate influences of taxes and spending
  - Consistent with the distinction drawn by Alesina, Favero and Giavazzi (2020) between tax-driven and spending-driven austerity