# The Quanto Theory of Exchange Rates

Lukas Kremens     Ian Martin

April, 2018

# It is notoriously hard to forecast exchange rates

- Much of the literature is organized around the **uncovered interest parity** (UIP) benchmark, which predicts that exchange rate movements should offset interest rate differentials on average, and thereby equalize expected returns across currencies
- Hansen–Hodrick (1980), Fama (1984), and others: UIP fails badly

# Three appealing properties of UIP

1. Based on asset prices alone: observable in real time; no reliance on infrequently updated, imperfectly measured macro statistics
2. No free parameters: nothing to estimate, so no in-sample / out-of-sample issues
3. Straightforward interpretation: represents the expected currency appreciation perceived by a risk-neutral investor

- #1–#3 explain why UIP is such an important benchmark
- #3 also explains why it should never have been expected to work empirically: risk neutral expectation $\mathbb{E}_t^* \neq$ true expectation $\mathbb{E}_t$

# This paper

- We propose an alternative benchmark, the **quanto theory**, that has the three appealing properties, but also allows for risk aversion
- . . . and performs well empirically

# Theory (1)

- Start from a fundamental equation of asset pricing,

$$\mathbb{E}_t \left( M_{t+1} \widetilde{R}_{t+1} \right) = 1$$

  - $\mathbb{E}_t$: expectation conditional on time-$t$ information
  - $M_{t+1}$: SDF that prices dollar payoffs
  - $\widetilde{R}_{t+1}$: any gross dollar return

- Since $\mathbb{E}_t M_{t+1} = 1/R^{\$}_{f,t}$, we can write this as

$$\mathbb{E}_t \widetilde{R}_{t+1} - R^{\$}_{f,t} = -R^{\$}_{f,t} \operatorname{cov}_t \left( M_{t+1}, \widetilde{R}_{t+1} \right)$$

# Theory (2)

- Currency trade: take a dollar, convert to euros, invest at the (gross) euro riskless rate, $R_{f,t}^{€}$, and then convert back to dollars
- $e_t$: price of a euro in dollars, so $€1 = \$e_t$ and $\$1 = €1/e_t$
- Return on currency trade is $R_{f,t}^{€} e_{t+1}/e_t$
- Setting $\widetilde{R}_{t+1} = R_{f,t}^{€} e_{t+1}/e_t$ and rearranging,

$$\mathbb{E}_t \frac{e_{t+1}}{e_t} = \underbrace{\frac{R_{f,t}^{\$}}{R_{f,t}^{€}}}_{\text{UIP forecast}} \underbrace{- R_{f,t}^{\$} \operatorname{cov}_t \left( M_{t+1}, \frac{e_{t+1}}{e_t} \right)}_{\text{risk adjustment}} \tag{1}$$

# Theory (3)

- Sometimes convenient to use risk-neutral notation,

$$\text{time } t \text{ price of a claim to } \$X_{t+1} = \frac{1}{R_{f,t}^{\$}} \mathbb{E}_t^* X_{t+1} = \mathbb{E}_t \left( M_{t+1} X_{t+1} \right)$$

- The identity (1) can be rewritten

$$\mathbb{E}_t^* \frac{e_{t+1}}{e_t} = \frac{R_{f,t}^{\$}}{R_{f,t}^{€}}$$

- Reduces to UIP in a risk-neutral world in which $\mathbb{E}_t^* = \mathbb{E}_t$

# Theory (4)

- The UIP forecast is the expected appreciation perceived by a risk-neutral investor—but this is a very unrealistic perspective
- What about an investor with log utility?
- Answer: depends on the investor's financial wealth, background risk, human capital, etc...
- But if the investor is unconstrained, with wealth fully invested in the market,

$$\mathbb{E}_t \frac{e_{i,t+1}}{e_{i,t}} = \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} + \frac{1}{R^{\$}_{f,t}} \operatorname{cov}^*_t \left( \frac{e_{i,t+1}}{e_{i,t}}, R_{t+1} \right)$$

where $R_{t+1}$ is the return on the market

# Theory (5)

## Result (An identity)

*More generally,*

$$\mathbb{E}_t \frac{e_{i,t+1}}{e_{i,t}} = \underbrace{\frac{R_{f,t}^{\$}}{R_{f,t}^{i}}}_{\text{UIP forecast}} + \underbrace{\frac{1}{R_{f,t}^{\$}} \operatorname{cov}_t^* \left( \frac{e_{i,t+1}}{e_{i,t}}, R_{t+1} \right)}_{\text{quanto-implied risk premium}} - \underbrace{\operatorname{cov}_t \left( M_{t+1} R_{t+1}, \frac{e_{i,t+1}}{e_{i,t}} \right)}_{\text{residual}} \tag{2}$$

*where $R_{t+1}$ is an arbitrary dollar return, and the first covariance term is computed using the risk-neutral probability distribution*

# Theory (6)

- Relies only on absence of arbitrage: in particular, must hold in any equilibrium model
- We do not assume complete markets
- We do not assume existence of a representative agent
- We do not assume everyone is rational
- We do not assume everyone is unconstrained
- We do not assume lognormality
- Must hold even for pegged or tightly managed exchange rates

# Theory (7)

- Tension between two goals: want to choose $R_{t+1}$
  (i) to make the second term measurable; and
  (ii) to make the third term small (ideally, negligible)
- We will set $R_{t+1}$ equal to the return on the S&P 500 index
- Then the second term is measurable given quanto forward prices on S&P 500 index
- The third term is zero from the log investor's point of view because $M_{t+1} = 1/R_{t+1}$

# Measuring risk-neutral covariance

## Conventional forward

- A commitment to pay $\$F_t$ in exchange for value of S&P 500 index in dollars, $\$P_{t+1}$. Payoff is $\$(P_{t+1} - F_t)$ at time $t+1$
- To make value equal to zero at initiation, $F_t = \mathbb{E}_t^* \, P_{t+1}$

## Quanto forward

- A commitment to pay €$Q_t$ in exchange for value of S&P 500 index in euros, €$P_{t+1}$. Payoff is € $(P_{t+1} - Q_t)$, or equivalently $\$e_{t+1}(P_{t+1} - Q_t)$, at time $t+1$
- To make value equal to zero at initiation, $Q_t = \frac{\mathbb{E}_t^* \, e_{t+1} P_{t+1}}{\mathbb{E}_t^* \, e_{t+1}}$

- It follows that

$$\frac{Q_t - F_t}{R_{f,t}^{€} P_t} = \frac{1}{R_{f,t}^{\$}} \operatorname{cov}_t^* \left( \frac{e_{t+1}}{e_t}, R_{t+1} \right)$$

# The log investor

## Result

*The exchange-rate appreciation anticipated by a log investor who holds the S&P 500 index can be computed from asset prices via the equation*

$$\mathbb{E}_t \frac{e_{i,t+1}}{e_{i,t}} - 1 = \underbrace{\underbrace{\frac{R_{f,t}^{\$}}{R_{f,t}^{i}} - 1}_{IRD_{i,t}} + \underbrace{\frac{Q_{i,t} - F_t}{R_{f,t}^{i} P_t}}_{QRP_{i,t}}}_{ECA_{i,t}}$$

*Equivalently, the currency risk premium anticipated by such an investor is revealed by QRP:*

$$\mathbb{E}_t \frac{e_{i,t+1}}{e_{i,t}} - \frac{R_{f,t}^{\$}}{R_{f,t}^{i}} = QRP_{i,t}$$

# Beyond the log investor

- We view the log investor as a benchmark
- Well suited for out-of-sample forecasting: no free parameters
- But also allow for nonzero second covariance term in various ways
  - Intercept (captures potential dollar effect)
  - Fixed effects (captures currency-specific but time-invariant effects)
  - Other proxies (both currency-specific and time-varying)
    - $\text{IRD}_{i,t}$
    - $\text{QRP}_{i,t}$
    - Average forward discount, $\overline{\text{IRD}}_t$ (Lustig, Roussanov and Verdelhan, 2014)
    - Log real exchange rate, $\text{RER}_{i,t}$ (Dahlquist and Penasse, 2017)

# Theory: summary

- Intuition: currencies that perform poorly when marginal value of a dollar is high ('bad times') are risky and must earn a risk premium
- Thinking from the perspective of the log investor, the notion of 'bad times' is revealed by the return on the market
- Currencies with positive (risk-neutral) covariance with the market are risky
- Quantos reveal this risk-neutral covariance

# Data

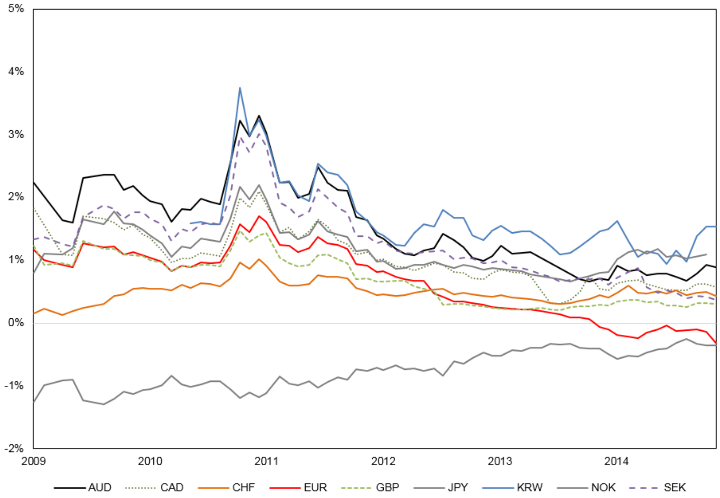
- Monthly data on quanto forwards ($Q_{i,t}$) and conventional forwards ($F_t$) on the S&P 500, obtained from Markit
    - Australian dollar (AUD)
    - Canadian dollar (CAD)
    - Swiss franc (CHF)
    - Danish krone (DKK)
    - Euro (EUR)
    - British pound (GBP)
    - Japanese yen (JPY)
    - Korean won (KRW)
    - Norwegian krone (NOK)
    - Polish zloty (PLN)
    - Swedish krona (SEK)
- Maturities of 6, 12, and **24** months, Dec 2009 to Oct 2015

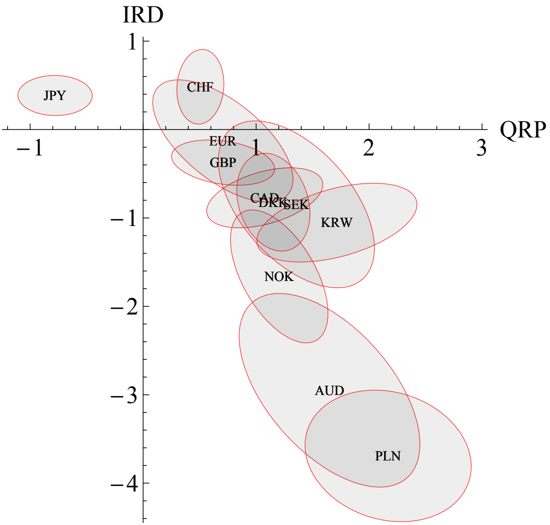
# Currency forecasts, 2yr horizon

## Expected currency appreciation (ECA)

# Currency forecasts, 2yr horizon

## Expected excess returns (QRP)






- IRD and QRP negatively correlated in time series and cross section
- High interest rates $\longleftrightarrow$ high risk premia: carry trade is profitable

# Testing the model

Log investor: $\mathbb{E}_t \frac{e_{i,t+1}}{e_{i,t}} - 1 = \underbrace{\frac{Q_{i,t} - F_t}{R^i_{f,t} P_t}}_{\text{QRP}_{i,t}} + \underbrace{\frac{R^\$_{f,t}}{R^i_{f,t}} - 1}_{\text{IRD}_{i,t}}$

- We test the model by forecasting
  - currency excess return: $\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^\$_{f,t}}{R^i_{f,t}}$
  - currency appreciation: $\frac{e_{i,t+1}}{e_{i,t}} - 1$
- Stylized facts from the literature
  - High-interest-rate currencies have high excess returns (eg, Hansen–Hodrick, 1980; Fama, 1984)
  - Hard to forecast currency appreciation (eg, Meese–Rogoff, 1983)
- Bootstrapped covariance matrices

# Forecasting excess returns (1)

Log investor: $\mathbb{E}_t \frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \text{QRP}_{i,t}$

$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \alpha + \beta\,\text{QRP}_{i,t} + \gamma\,\text{IRD}_{i,t} + \varepsilon_{i,t+1} \tag{22}$$

$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \alpha + \beta\,\text{QRP}_{i,t} + \varepsilon_{i,t+1} \tag{23}$$

$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \alpha + \gamma\,\text{IRD}_{i,t} + \varepsilon_{i,t+1} \tag{24}$$

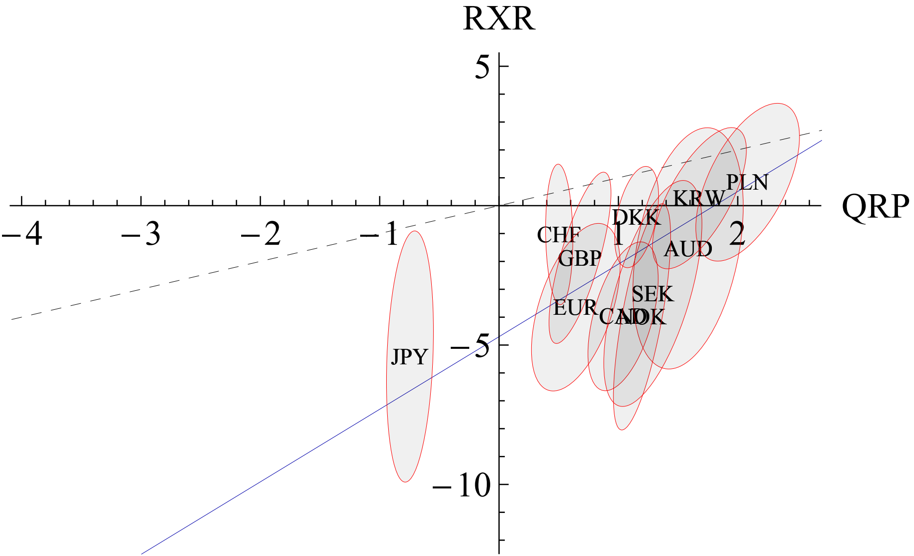
- UIP: $\alpha = \beta = \gamma = 0$
- We hope to find positive and significant $\beta$
- Log investor: $\alpha = 0$, $\beta = 1$, $\gamma = 0$ in (22) and (23)

# Forecasting excess returns (2)

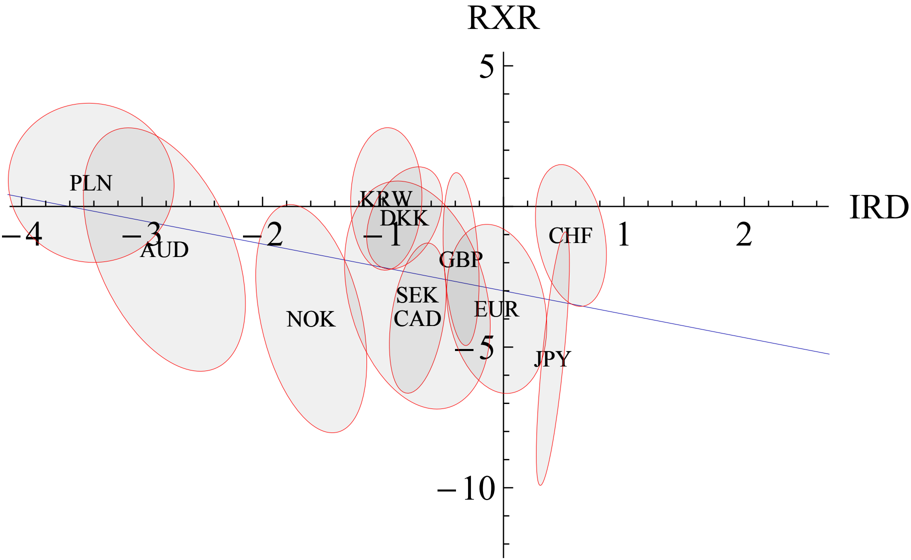
| | pooled | | | | | | currency fixed effects | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Regression | (22) | | (23) | | (24) | | (22) | | (23) | | (24) | |
| $\alpha$ | -0.048 | (0.020) | -0.047 | (0.019) | -0.030 | (0.014) | | | | | | |
| QRP, $\beta$ | 3.394 | (1.734) | 2.604 | (1.127) | | | 5.456 | (2.046) | 4.995 | (1.565) | | |
| IRD, $\gamma$ | 0.769 | (1.040) | | | -0.832 | (0.651) | 0.717 | (1.411) | | | -1.363 | (1.001) |
| $R^2$ | 19.13 | | 17.43 | | 3.88 | | 22.60 | | 22.03 | | 2.77 | |

- QRP positive and economically large in every specification and substantially increases $R^2$
- Coefficient on QRP is even larger than the log investor predicts
- Fixed effects are a departure from the log investor benchmark: they capture currency-specific, time-invariant component of residual covariance term (and they matter)

# Forecasting excess returns (3)



# Forecasting excess returns (3)



# Forecasting currency appreciation (1)

Log investor: $\mathbb{E}_t \frac{e_{i,t+1}}{e_{i,t}} - 1 = \text{QRP}_{i,t} + \text{IRD}_{i,t}$

$$\frac{e_{i,t+1}}{e_{i,t}} - 1 = \alpha + \beta\,\text{QRP}_{i,t} + \gamma\,\text{IRD}_{i,t} + \varepsilon_{i,t+1} \tag{25}$$

$$\frac{e_{i,t+1}}{e_{i,t}} - 1 = \alpha + \beta\,\text{QRP}_{i,t} + \varepsilon_{i,t+1} \tag{26}$$

$$\frac{e_{i,t+1}}{e_{i,t}} - 1 = \alpha + \gamma\,\text{IRD}_{i,t} + \varepsilon_{i,t+1} \tag{27}$$

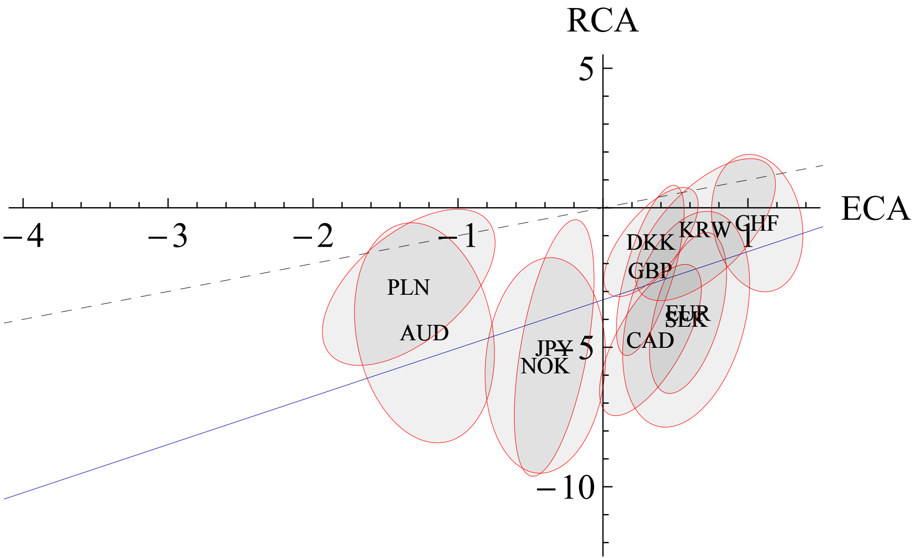
- UIP: $\alpha = \beta = 0$, $\gamma = 1$
- Log investor: $\alpha = 0$, $\beta = \gamma = 1$ in (25)

# Forecasting currency appreciation (2)

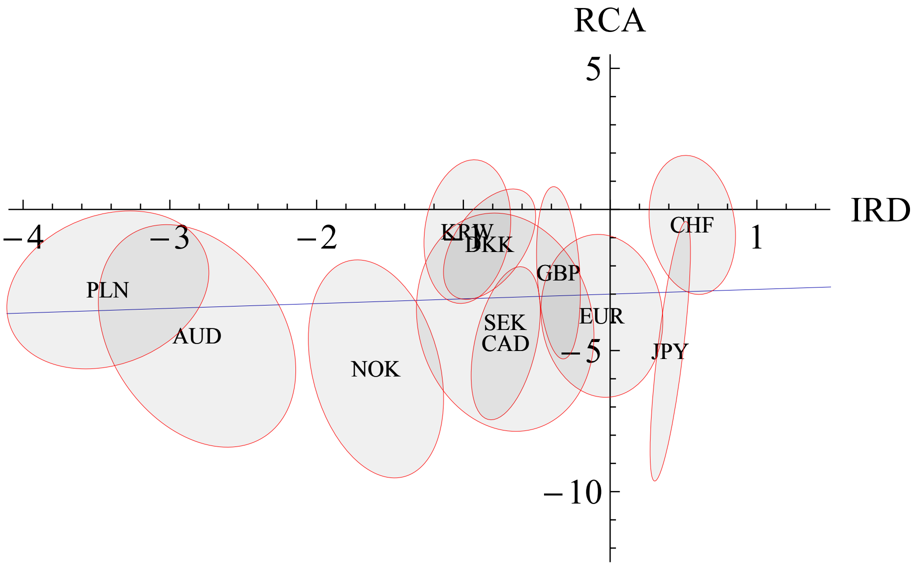
| | pooled | | | | | | currency fixed effects | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Regression | (25) | | (26) | | (27) | | (25) | | (26) | | (27) | |
| $\alpha$ | -0.048 | (0.020) | -0.045 | (0.019) | -0.030 | (0.014) | | | | | | |
| QRP, $\beta$ | 3.394 | (1.726) | 1.576 | (1.172) | | | 5.456 | (2.046) | 4.352 | (1.682) | | |
| IRD, $\gamma$ | 1.769 | (1.045) | | | 0.168 | (0.651) | 1.717 | (1.414) | | | -0.363 | (1.007) |
| $R^2$ | 16.01 | | 6.63 | | 0.16 | | 20.56 | | 17.16 | | 0.20 | |

- Mechanical link to previous coefficients, so our interest is in $R^2$
- Using interest-rate differentials alone, no evidence of forecastability
- Adding QRP dramatically increases $R^2$, with and without FEs
- Again, coefficient on QRP is even larger than the theory predicts

# Forecasting currency appreciation (3)



# Forecasting currency appreciation (3)



# Forecasting excess returns: beyond the log investor

$$\mathbb{E}_t \frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \mathrm{QRP}_{i,t} - \mathrm{cov}_t\left(M_{t+1}R_{t+1}, \frac{e_{i,t+1}}{e_{i,t}}\right)$$

| | currency fixed effects | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Regressor | univariate | | bivariate | | 3-variate | | 4-variate | |
| QRP, $\beta$ | 4.995 | (1.565) | 5.654 | (1.402) | 3.799 | (1.657) | 3.541 | (1.836) |
| IRD, $\gamma$ | | | | | | | -1.059 | (1.573) |
| $\overline{\mathrm{IRD}}$, $\delta$ | | | | | -5.060 | (1.605) | -4.266 | (1.538) |
| RER, $\zeta$ | | | -0.413 | (0.136) | -0.780 | (0.159) | -0.804 | (0.188) |
| $R^2$ | 22.03 | | 35.40 | | 43.56 | | 44.09 | |

- Consider other specifications in search of the 'residual' covariance term: QRP; IRD; average forward discount, $\overline{\mathrm{IRD}}$ (Lustig, Roussanov and Verdelhan 2014); real exchange rate, RER (Dahlquist and Penasse 2017)
- Table reports $R^2$-maximizing univariate, ..., 4-variate specifications

# Joint tests of statistical significance

Asymptotic $p$-value / bootstrapped small-sample $p$-value

| | pooled | | | currency fixed effects | | |
|---|---|---|---|---|---|---|
| Regression | (22) | (23) | (25) | (22) | (23) | (25) |
| $\alpha = \gamma = 0, \beta = 1$ | 0.029 / 0.357 | | | | | |
| $\alpha = 0, \beta = 1$ | | 0.039 / 0.342 | | | | |
| $\alpha = 0, \beta = \gamma = 1$ | | | 0.030 / 0.340 | | | |
| $\beta = 1, \gamma = 0$ | 0.342 / 0.546 | | | 0.029 / 0.256 | | |
| $\beta = 1$ | | 0.155 / 0.299 | | | 0.011 / 0.163 | |
| $\beta = 1, \gamma = 1$ | | | 0.339 / 0.493 | | | 0.029 / 0.238 |

- Asymptotic tests reject the quanto theory, largely due to negative intercept (strong dollar over the sample period)
- Small-sample tests do not reject the quanto theory predictions

# Out-of-sample forecasting (1)

- For out-of-sample forecasts, we return to the log investor case, since this gives us a formula with no free parameters and no fixed effects
- We focus on forecasting differential returns on currencies: eg, the relative performance of the yen and the euro vis-à-vis the dollar
- By doing so, we avoid making our results sensitive to the performance of the base currency over our short sample period
- Dollar-neutral $R^2_{OS}$ for quanto theory (Q) versus benchmark (B)

$$R^2_{OS} = 1 - \frac{\sum_i \sum_j \sum_t (\varepsilon^Q_{i,t} - \varepsilon^Q_{j,t})^2}{\sum_i \sum_j \sum_t (\varepsilon^B_{i,t} - \varepsilon^B_{j,t})^2} \quad \text{and} \quad R^2_{OS,i} = 1 - \frac{\sum_j \sum_t (\varepsilon^Q_{i,t} - \varepsilon^Q_{j,t})^2}{\sum_j \sum_t (\varepsilon^B_{i,t} - \varepsilon^B_{j,t})^2}$$

where $\varepsilon^Q_{i,t}$ and $\varepsilon^B_{i,t}$ are forecast errors for quanto theory and benchmark

- Benchmarks: UIP, random walk, and PPP

# Out-of-sample forecasting (2)

$$\text{Quanto theory:} \quad \mathbb{E}_t^Q \frac{e_{i,T}}{e_{i,t}} - 1 = \text{QRP}_{i,t} + \text{IRD}_{i,t}$$

$$\text{UIP:} \quad \mathbb{E}_t^U \frac{e_{i,T}}{e_{i,t}} - 1 = \text{IRD}_{i,t}$$

$$\text{Constant:} \quad \mathbb{E}_t^C \frac{e_{i,T}}{e_{i,t}} - 1 = 0$$

$$\text{PPP:} \quad \mathbb{E}_t^P \frac{e_{i,T}}{e_{i,t}} - 1 = \left( \frac{\pi^{\$}_{t-12\to t}}{\pi^{i}_{t-12\to t}} \right)^2 - 1$$

- Natural competitor models: no free parameters

# Out-of-sample forecasting (3)

| Benchmark | IRD | Constant | PPP |
|---|---|---|---|
| $R^2_{OS}$ | 10.91 | 9.57 | 26.05 |
| $R^2_{OS,AUD}$ | 9.71 | 0.93 | 11.42 |
| $R^2_{OS,CAD}$ | 6.24 | 6.55 | 21.31 |
| $R^2_{OS,CHF}$ | 1.40 | 16.37 | 11.43 |
| $R^2_{OS,DKK}$ | 10.22 | 7.71 | 23.36 |
| $R^2_{OS,EUR}$ | 7.65 | 5.36 | 24.56 |
| $R^2_{OS,GBP}$ | 2.98 | 9.74 | 32.35 |
| $R^2_{OS,JPY}$ | 19.21 | 9.59 | 33.74 |
| $R^2_{OS,KRW}$ | 21.98 | 17.09 | 34.71 |
| $R^2_{OS,NOK}$ | 3.43 | 12.86 | 18.97 |
| $R^2_{OS,PLN}$ | 13.25 | 8.32 | 19.62 |
| $R^2_{OS,SEK}$ | 7.68 | 5.88 | 28.22 |
| DM $p$-value | 0.039 | 0.000 | 0.000 |

# A change of perspective (1)

- From the perspective of the US log investor,

$$\mathbb{E}_t \frac{e_{i,t+1}}{e_{i,t}} - 1 = \text{IRD}_{i,t} + \text{QRP}_{i,t}$$

- For a log investor who is fully invested in the currency-*i* stock market,

$$\mathbb{E}_t^i \frac{1/e_{i,t+1}}{1/e_{i,t}} - 1 = \text{IRD}_{1/i,t} + \text{QRP}_{1/i,t}$$

- If the US investor expects the euro to appreciate by 2%, does the European investor expect the dollar to depreciate by roughly 2%?
- Yes (empirically)

# A change of perspective (2)

- But must take into account Siegel's "paradox":

$$\mathbb{E}_t \frac{e_{i,t+1}}{e_{i,t}} \geq \left( \mathbb{E}_t \frac{1/e_{i,t+1}}{1/e_{i,t}} \right)^{-1}$$

- So if both investors have the same expectations,

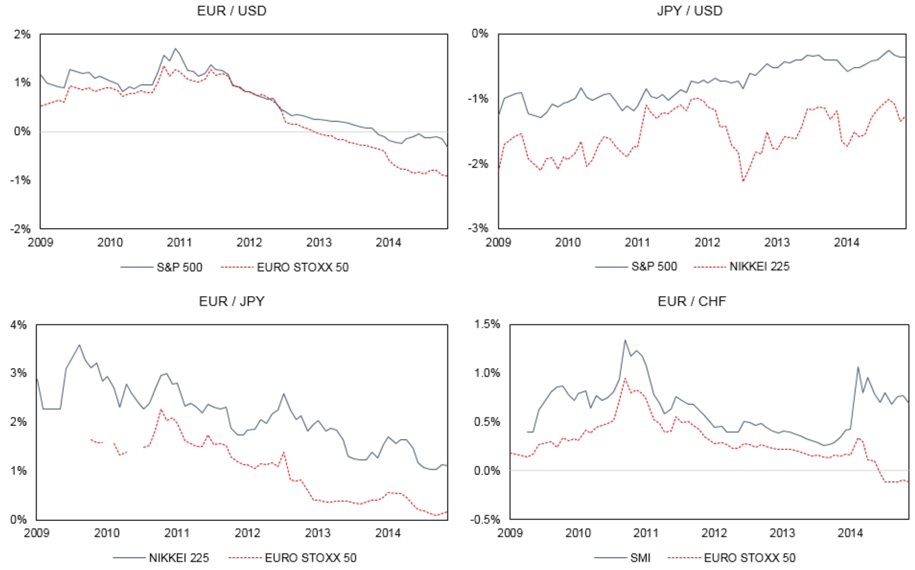
$$\log\left(1 + \mathrm{ECA}_{i,t}\right) \geq -\log\left(1 + \mathrm{ECA}_{1/i,t}\right)$$

- Difference between the two sides depends on variability of $e_{i,t+1}$
- If $e_{i,t+1}$ is lognormal, the difference equals $\mathrm{var}_t \log e_{i,t+1}$
- More generally,

$$\text{difference} = 2 \sum_{n \text{ even}} \frac{\kappa_{n,t}}{n!}$$

where $\kappa_{n,t}$ is the $n$th conditional cumulant of $\log e_{i,t+1}$

# A change of perspective (3)



# Risk-neutral covariance vs. true covariance (1)

- Theory says that risk-neutral covariance is the relevant measure
- The distinction matters: the carry trade is more correlated with the market in bad times (Lettau, Maggiori and Weber, 2014)
- Risk-neutral and realized covariances are strongly positively correlated in the cross-section and in the time-series
- QRP is driven out by lagged realized covariance as a forecaster of realized covariance
- But the resulting covariance forecast is driven out by QRP as a currency forecaster

# Risk-neutral covariance vs. true covariance (2)

- We find that risk-neutral covariance exceeds (proxied) true covariance in magnitude for every currency $i$ in our dataset
- This implies that at least one of the following three options is false
  1. The market has a positive risk premium
  2. Currency $i$ has a positive risk premium
  3. Currency $i$, the market return, and the SDF are lognormal
- Most plausible that #3 is false (and consistent with the existence of a volatility smile in FX and equity markets)
- International finance models that assume lognormality cannot hope to match our empirical findings

# Conclusions

- Our identity provides a new line of attack for currency forecasting
- Expected currency appreciation equals interest-rate differential plus quanto risk premium plus residual $\longleftarrow$ zero for log investor
- QRP is negatively correlated with UIP forecast: ‘predicts’ the existence of the carry trade
- QRP itself is highly economically & statistically significant in forecasting regressions
- Outperforms UIP, random walk, and PPP in forecasting differential currency movements out-of-sample