Self-confirming equilibrium (SCE)
by Fudenberg and Levine

An extensive form solution concept

Motivation 1

- SCE was born from dissatisfaction with Nash Equilibrium as a solution concept (in one shot games)
- How can we motivate NE in this framework?

(in particular, focus on equilibrium in extensive games)

- Learning how to play equilibrium strategies --Fudenberg and Levine (1993b)

- playing the same game repeatedly without dynamic strategic considerations
- impatient or myopic players → little experimentation

- If equilibrium is reached after game is played many times is this equilibrium always going to be a NE?

Motivation 2

- The answer is “No”

(Fudenberg and Levine, 1993b)

- In extensive form games off-equilibrium play is not observed

- All outcomes that result from incomplete learning will satisfy a weaker version of equilibrium than Nash equilibrium: the self-confirming equilibrium (SCE)

- What they do in this paper:

- Discuss properties of SCE
- See when is SCE going to coincide with NE

SCE (informal)

- An end result of a learning model

- In a self-confirming equilibrium:

- Players maximize their payoffs with respect to their beliefs
- Beliefs cannot conflict with the “empirical evidence”

Framework 1

- Dynamic game played once
- Each player i moves at information sets $h_i \in H_i$
- $s = (s_i, s_{-i})$ are pure strategies
- $\hat{\sigma} = (\hat{\sigma}_i, \hat{\sigma}_{-i})$ is equilibrium mixed strategy profile
- $\hat{\pi}_i\ (h_i) = \hat{\pi}(h_i|\hat{\sigma}_i)$ the set of equilibrium behavioural strategies implied by $\hat{\sigma}_i$
- $H(s_i)$ information sets that can be reached if s_i is played
- $\overline{H}(s_i, \sigma_{-i})$ info sets actually reached when (s_i, σ_{-i}) is played

- Let μ_i denote the belief player i has about the behavioural strategy of her opponents

Framework 2

- Payoff of player i with belief μ_i is when playing s_i is:

$$u_i(s_i, \mu_i)$$

- If beliefs are correct:

$\mu_i(\pi_{-i}: \pi_j(h_j) = \widehat{\pi}_j) = 1$ **all $\boldsymbol{h_j}$**

$$u_i(s_i, \mu_i) = u_i(s_i, \hat{\sigma}_{-i})$$

- Players <u>know</u>: the structure of the game, and their own payoffs

Definition of SCE and C-SCE

$(\hat{\sigma}_i, \hat{\sigma}_{-i})$ such that for all $s_i \in supp(\hat{\sigma}_i)$ exists μ_i:

(a) $s_i = argmax_{\{s\}}(u_i(s, \mu_i))$

(b) $\mu_i\{(\boldsymbol{\pi}_{-i}\colon\ \pi_j(h_j) = \hat{\pi}_j(h_j|\sigma_j))\} = 1$

SCE: for all $h_j \in \overline{H}\ (s_i, \sigma_{-i})$

Consistent SCE: for all $h_j \in H(s_i)$

NE: for all h_j

Preview of Results

1. SCE = NE are the same in simultaneous games

2. SCE = NE

if you have observed deviators and unitary and independent beliefs

Theorem 1

In simultaneous games, SCE = NE

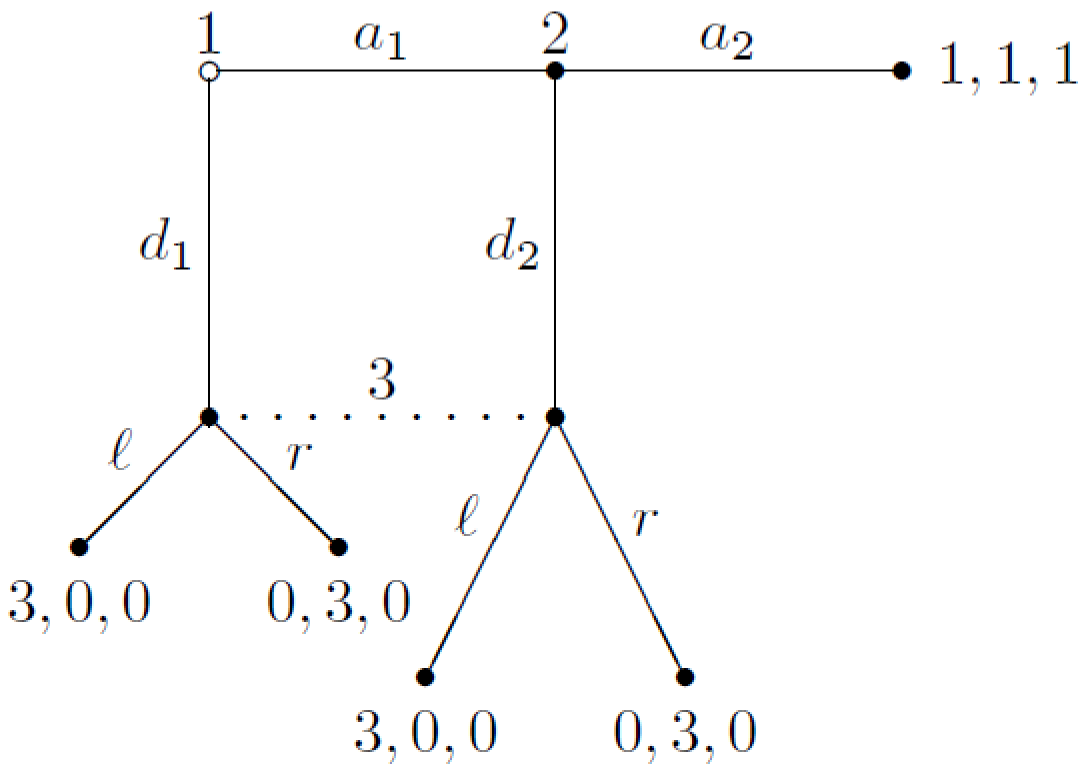
Proof:

1. Immediate from the fact that there are no out-of-equilibrium information sets

Remark: every NE is a SCE

Example 1

(Non-consistent SCE)



Theorem 2

In games with observed deviators, SCE = C-SCE

Steps of the proof:

1. Define the set of beliefs that (in equilibrium) matter for an agent's decision - Relevant beliefs
2. Show that (with observed deviators) restrictions on beliefs imposed by consistency will be on non-relevant beliefs
3. Therefore, C-SCE = SCE with observed deviators

Observed deviators

For any new information set reached after player i deviates $h_j \in H(s_i', s_{-i})$ there will be no deviation of some other player that reaches the same h_j. That is $h_j \notin H(s_i, s_{-i}')$ for any s_{-i}'

Relevant information sets

- Define **relevant information sets** for player i as all h_j such that:

$$\exists s_i: \qquad P\left(h_j \middle| s_i, \mu_i\right) > 0$$

- We can show that if μ_i and $\mu_i{}'$ only differ on non-relevant information sets:

$$u_i(s_i, \mu_i) = u_i(s_i, \mu_i{}') \qquad \text{any } s_i$$

Proof:

1. Consistency will impose additional restrictions on player i beliefs on each

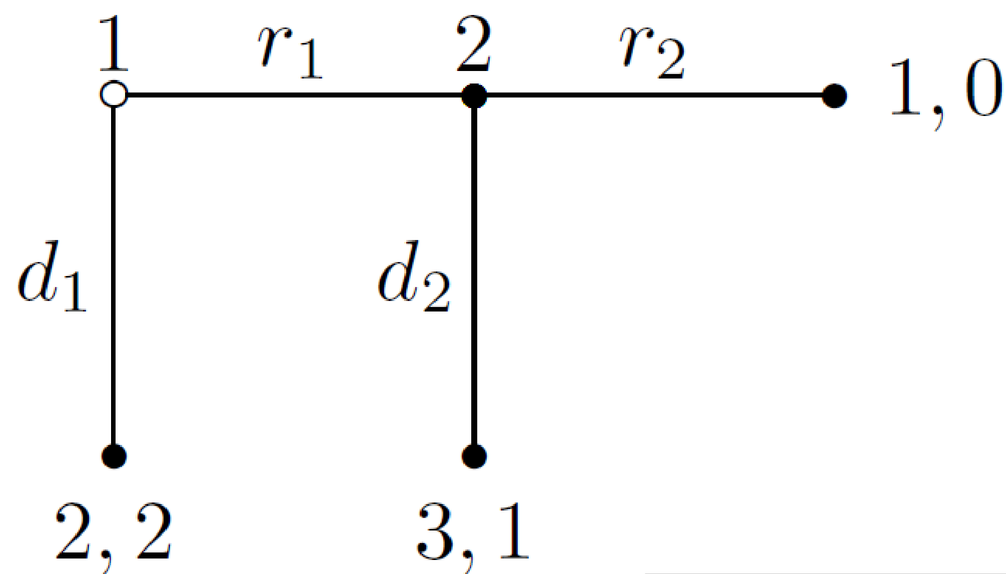
$h_j \in H(s_i)\backslash\overline{H}\ (s_i, \sigma_{-i})$

Which are the information sets that result from deviations from other players

2. These information sets are different from the ones player i could get by deviating
3. Therefore they are not relevant that is

$$P\left(h_j \middle| s_i, \mu_i\right) = 0$$

Example 2:
(non-unitary SCE)



SCE: $\left(\frac{1}{2}.[d_1] + \frac{1}{2}[r_1]\ ,\ \ d_2\right)$

Definition of NE, SCE and C-SCE

$(\hat{\sigma}_i, \hat{\sigma}_{-i})$ such that for all $s_i \in supp(\hat{\sigma}_i)$ exists μ_i:

(a) $s_i = argmax_{\{s\}}(u_i(s, \mu_i))$

(b) $\mu_i\{(\boldsymbol{\pi}_{-i}\text{: } \pi_j(h_j) = \pi_j(h_j|\hat{\sigma}_j))\} = 1$

SCE:	for all $h_j \in \bar{H}\ (s_i, \sigma_{-i})$
Consistent SCE:	for all $h_j \in H(s_i)$
NE:	for all h_j

Claim: SCE differs from NE if:

1. Players have inconsistent beliefs about the play of opponents [example 1]

2. Correlated uncertainty: A player's subjective uncertainty about the play of other opponents might be correlated

3. A player can use different beliefs to justify different strategies in support of a SCE [example 2]

Theorem 4
If beliefs are unitary and independent then,
SCE (w/ observed deviators) = NE

Intuition of the Proof:

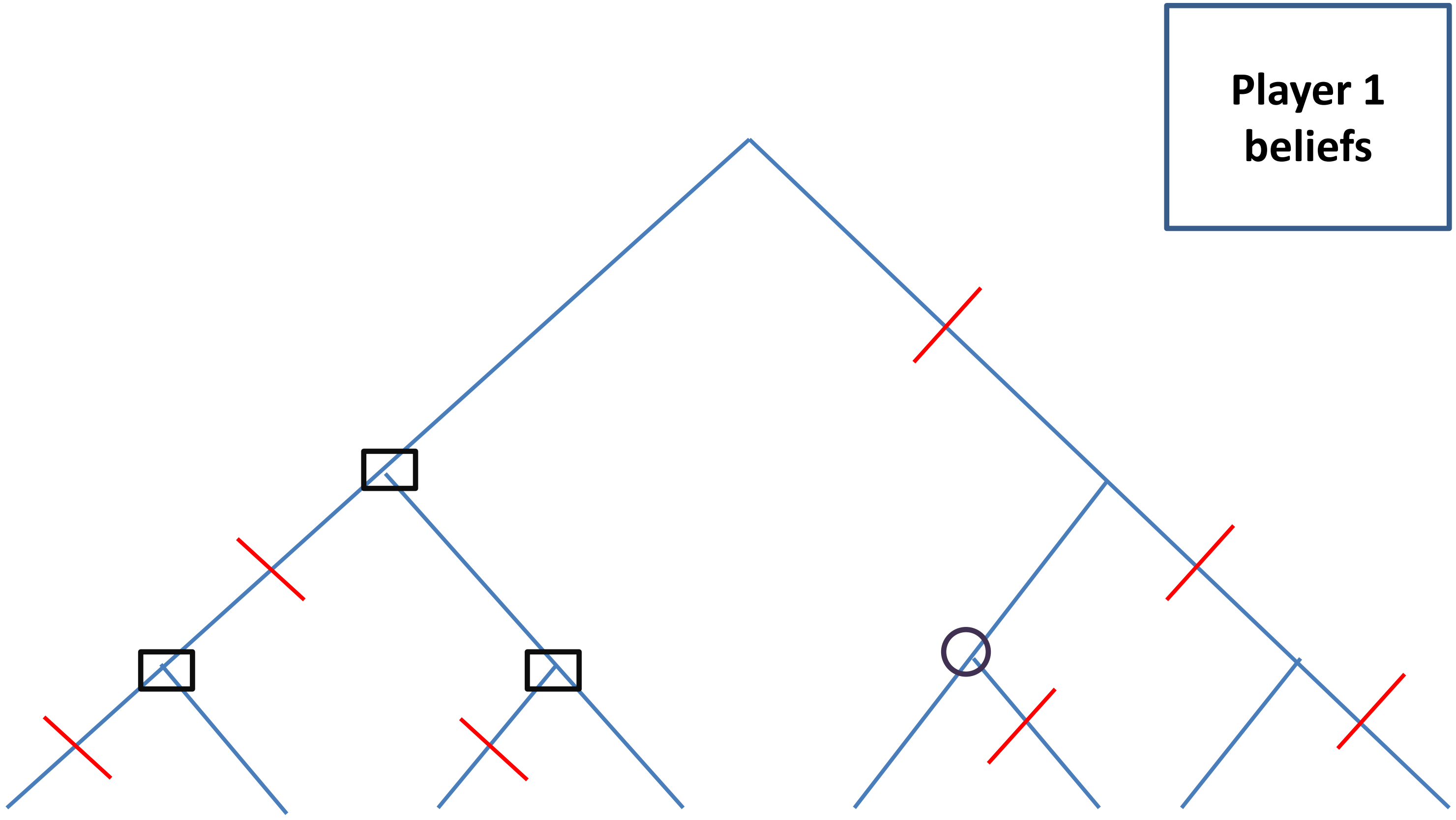
- Beliefs being unitary and independent rule out point 2. and 3.
- Any SCE (with observed deviators) can be replicated as a NE if beliefs are unitary and independent:

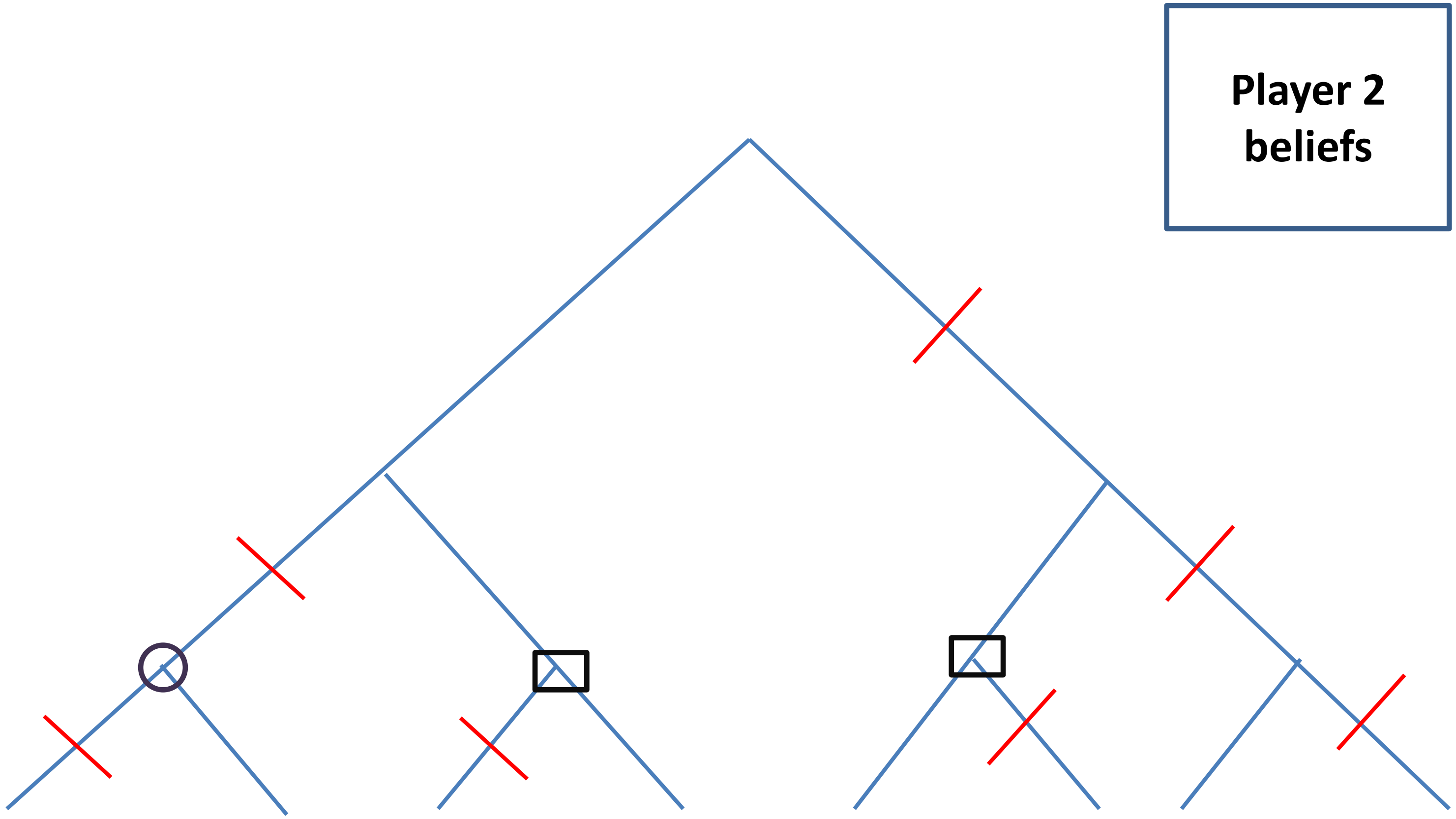
1. Fix a SCE with observed deviators σ
2. Construct σ' new strategy profile such that:

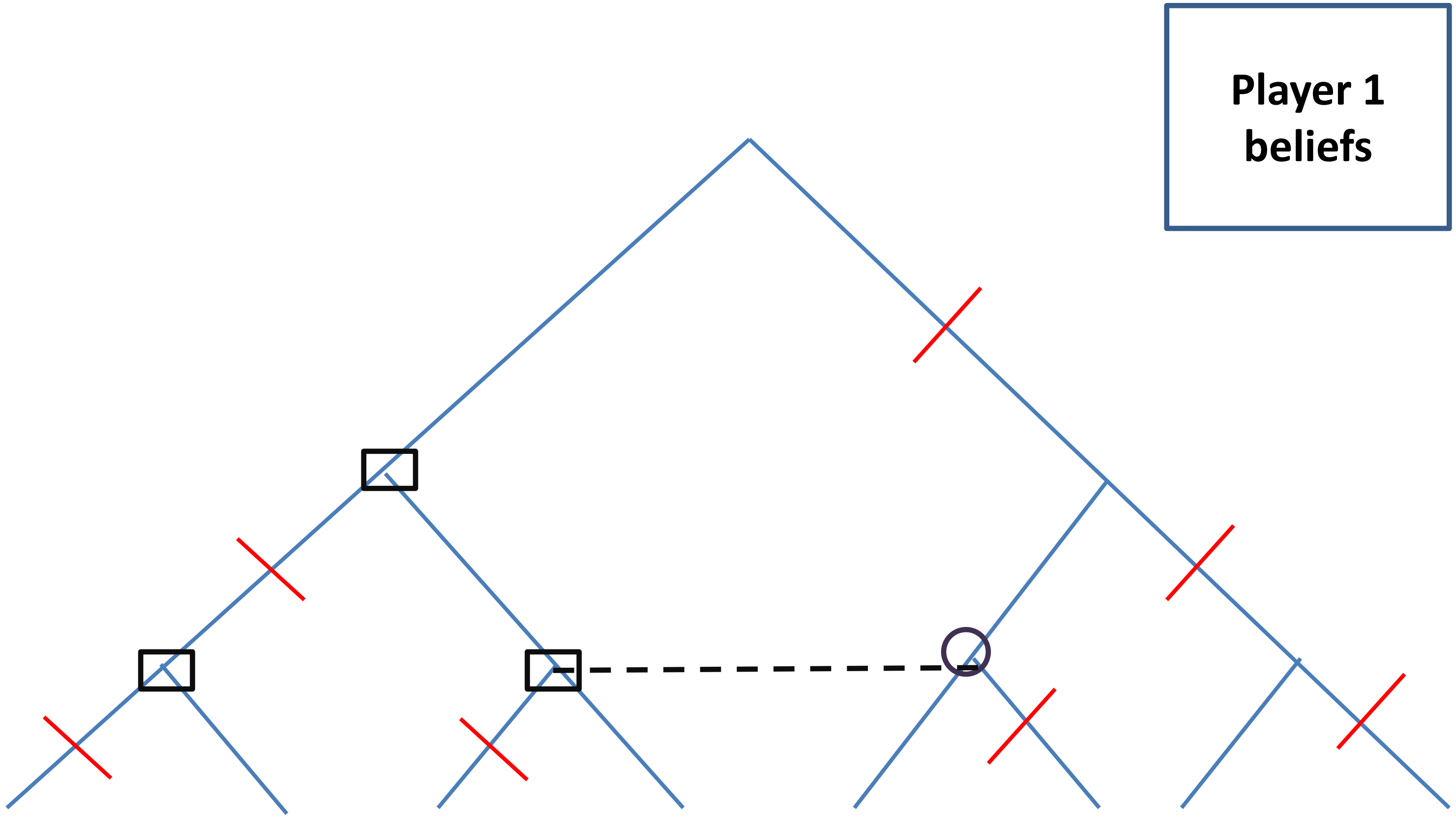
- $\hat{\pi}_k(\sigma_k')(h_k)$ is how player i beliefs player k will play at info sets if h_k is relevant for player i and is off equilibrium

3. Show that this new profile σ' is indeed a NE

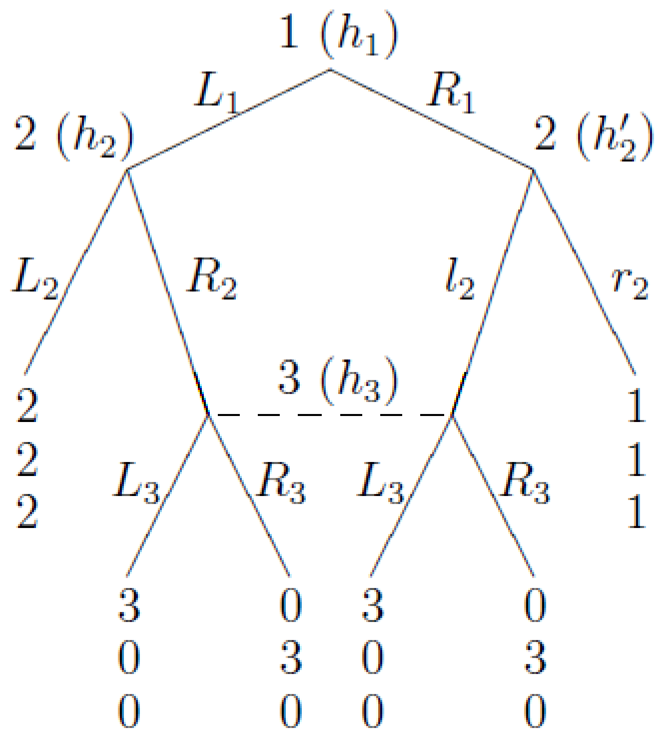
Intuition of proof


Player 1
beliefs


Player 2
beliefs


Player 1
beliefs

Counter-example
(kamada, 2010)



Conclusion

1. SCE can be different from NE in 3 ways:

-Heterogeneous beliefs

-Correlated equilibrium over beliefs (sunspot)

-Subjective correlated beliefs

2. NE = SCE if:

-Unitary + independent beliefs and (strong) consistency