Dynamic Wars of Attrition
Developments and Directions

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Dynamic Wars of Attrition

Plan for today:

- Introduction & Motivation

- Early Literature

- Shubik, JCR 1971

- Seb presents Abreu and Gul, ECMA 2000

Motivation to the Topic

Many models of bargaining assume that no direct cost incurred while bargaining except for the delay cost faced when no agreement is reached.

This section of the reading group considers papers that depart from such assumption by assuming that players incur a positive cost while bargaining.

Such costs have been modeled:

- as direct costs
- as incomplete information on payoffs
- as incomplete information on behavior type.

The main objectives are the characterization of:

- equilibrium behavior and payoffs;
- resolution of conflict and delay.

Classical Literature

- Rubinstein ECMA 1982 & ECMA 1985, Kreps Wilson 1982.

- The Dollar Auction: A Paradox in Non-Cooperative Behavior, Journal of Conflict Resolution 1971.

- The Theory of Games and the Evolution of Animal Conflicts, Maynard Smith, Journal of Theoretical Biology 1974.

- Bargaining with Two-Sided Incomplete Information, Chatterjee & Samuelson, RES 1987 & OR 1989.

- The War of Attrition in Continuous Time with Complete Information, Hendricks, Weiss & Wilson, IER 1988.

Shubik 1971 – The Dollar Auction

Consider the following dynamic game:

- a dollar is at auction;
- bidders must bid multiples of 5 cents;
- bidders can increase their bid over time;
- the auction ends when nobody bids for a fixed amount of time;
- the dollar is won by the player with the highest bid;
- all players pay their bid.

In informal experiments bidders appear to escalate in bids, since marginally increasing the winning bid discontinuously increases payoff by 1 dollar.

The auctioneer often collects more than a dollar in bids.

Bidders occasionally end up bidding more than the full dollar.

Shubik 1971 – The Dollar Auction

This game has an unsatisfactory PNE in which a player bids 1 dollar and everyone else zero. In this equilibrium all players get a payoff of 0.

Escalation could appears if players were concerned of their relative gains and losses, since the cost of inflicting a damage of 1 dollar to the rival would be only 5 cents. This would put the auctioneer at an advantage.

The cooperative analysis instead, puts the auctioneer at a disadvantage, since the characteristic function satisfies:

$$V(A) = -0.95, V(1, 2) = 0.95, V(1) = V(2) = V(A, 1) = V(A, 2) = 0$$

Shubik claims that the lack of communication and threats appears to play a central role in the process of communication.

The Dynamic War of Attrition – Hendricks 1988

The classical description of a dynamic war of attrition appears in Hendricks, Weiss, Wilson 1988.

It consist of a two-player game in which the two competitors choose the time in $[0, 1]$ in which to concede to their opponent.

The payoff of each competitor is described by a map:

$$U_1(t_1, t_2) = \begin{cases} L_1(t_1) & \text{if} & t_1 < t_2 \\ T_1(t_1) & \text{if} & t_1 = t_2 \\ W_1(t_2) & \text{if} & t_1 > t_2 \end{cases}$$

For the game to be a war of attrition payoffs must satisfy:

- $L_1(t) < W_1(t)$ for $t \in (0, 1)$
- $T_1(t) < W_1(t)$ for $t \in [0, 1)$
- $L_1'(t) < 0$ for $t \in [0, 1)$

The Dynamic War of Attrition – Hendricks 1988

In such an environment there always exists at least one degenerate equilibrium in which either:

- one of the two players concedes at 0 or,
- both concede only at 1.

However, non-degenerate equilibria in which players mix over an interval $(0, t)$ may also exist.

The analysis characterizes all non-degenerate equilibria and presents necessary and sufficient conditions on payoffs for their existence.

The authors also show that when players cannot commit to the exact date of concession, only equilibria in which a player concedes for sure at the initial stage may fail to be SPE.