# Linking Decisions and Storable Votes

## Economic Theory Reading Group

Francisco Costa

LSE

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# Outline

1. Jackson & Sonnenschein (Econometrica, 2007)
   Overcoming Incentive Constraints by Linking Decisions
   - Example: Voting
   - Example: Bargaining
   - Model
   - Theorem 1
   - Participation Constraints

2. Alessandra Casella
   Storable Votes and Agenda Control
   - Storable votes
   - Storable votes and agenda power

- Incentive constraints (including participation constraints) usually impose limitations on attaining social efficient outcome.
- The paper shows that by linking (independent) social decisions, the limitations imposed by incentive constraints may disappear.
- By "*budgeting*" agent's decisions we can make truthful revelation incentive compatible and achieve Pareto efficient outcomes.

# Jackson & Sonnenschein (2007)

## Example: Voting

- Two agents making a binary decision $d \in \{a, b\}$.
- Preferences are given by $v_i = v_i(a) - v_i(b)$, where $v_i \in \{-2, -1, 1, 2\}$ with equal probability.
- Suppose a social choice function that maximizes the sum of utilities.
- Social choice function is not implementable.
- The unique social choice function that is anonymous, neutral and maximizes total utility subject to incentives constraints is *voting and flipping a coin in the event of a tie* (May, 1952).

Inefficiency comes from the impossibility to access agents' **intensity of preference** in the event of a tie.

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"However, if two such decisions are linked, we could, for instance, ask the agents to declare that they are of a high type on just one of the two decisions.
Essentially, by linking the decisions together, we can ask: **Which decision do you care more about?**"

- Link $K$ independent decision together and "*budget*" each agent to announce -2 on $K/4$ problems, -1 on $K/4$ problems, and so on.
- Now choose outcome using the social choice function treating announcements as *truthful*.
- It turns out that *agents have incentive to be as truthful as they can*.

With large $K$, law of large numbers comes in, and we converge to truthful revelation and the ex ante efficient decision.

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With large $K$, law of large numbers comes in, and we converge to truthful revelation and the ex ante efficient decision.

- Seller with valuation uniformly distributed from $\{0.1, 0.3, 0.5, 0.7, 0.9\}$
- Buyer with valuation uniformly distributed from $\{0.2, 0.4, 0.6, 0.8, 1\}$.
- Agent's utility is the valuation net of transfers.
- *Social choice function*: trade if and only if buyer's value exceeds seller's value and the price equal to the average valuation.

- There is no incentive compatible social choice function (Myerson and Satterthwaite, 1983).

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- $n$-agent decision problem $(D, U, P)$, where
  - $D$ is finite set of decisions;
  - $U = U_1 \times ... \times U_n$ is a finite set of utility functions $(u_1, ..., u_n)$, where $u_i : D \to \Re$;
  - $P = (P_1, ..., P_n)$ is a profile of probability distributions, where $P_i$ is a distribution over $U_i$.
- Assume $u_i$'s are draw independently.
- A *social choice function* on $(D, U, P)$ is $f : U \to \triangle(D)$.
- $f$ on $(D, U, P)$ *is ex ante Pareto efficient* if $\nexists f'$ on $(D, U, P)$ such that can make at least one voter strictly better off.

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- Given a decision problem $(D, U, P)$ and $K$ linkings, a *linking mechanism* is $(M, g)$ where
  - $M = M_1 \times ... \times M_n$ is a message space;
  - $g : M \to \triangle(D^K)$ is an outcome function.
- Agent's utility of a set of decisions is simply $\sum_k u_i^k(d^k)$.
- Assume that decision problems are independent.

"Given independence and additive separability, there are absolutely **no complementarities across the decision problems**, and so any improvements in efficiency obtained through linking must come from being able to trade decisions off against each other to uncover intensities of preferences."

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- Agent's *strategy* is a mapping $\sigma_i^K : U_i^K \to \triangle(M_i)$.
- Bayesian equilibria.
- A strategy is *approximately truthful* if the agent's announcements always involve as few lies as possible.
- Let $\bar{u}_i = E[u_i(f(u))]$ denote the ex ante expected utility level under the social function $f$.

- Each agent announces utility functions for the $K$ problems (as in a direct revelation mechanism).
- **However**, announcements across the $K$ problems must match the expected frequency distribution.
  - I.e., the number of times that $i$ can announce a given $u_i$ is $K \times P_i(u_i)$.
- The choice is then made according to $f$ based on the announcements.
  - I.e., the decision of $g^K$ for the problem $k$ is $g^K(m) = f(\hat{u}^k)$, where $\hat{u}^k$ is the the announced utility.

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## Theorem 1

Consider a decision problem $(D, U, P)$ and an *ex ante Pareto efficient social choice function* $f$ defined on it. There exists a sequence of linking mechanisms $(M^K, g^K)$ on linked versions of the decision problem such that the following statements hold:

1. There exists a corresponding sequence of Bayesian equilibria that are approximately truthful.
2. The sequence of linking mechanisms together with these corresponding equilibria approximate $f$.
3. Any sequence of approximately truthful strategies for an agent $i$ secures a sequence of utility levels that converge to the ex ante target level $\bar{u}_i$.
4. All sequences of Bayesian equilibria of the linking mechanisms result in expected utilities that converge to the ex ante efficient profile of target utilities of $\bar{u}$ per problem.
5. For any sequence of Bayesian equilibria and any sequence of deviating coalitions, the maximal gain by any agent in the deviating coalitions

The two main aspects of the proof:

1. With a large number of linked problems, there is a high probability that the realized distribution of types will closely match the underlying distribution (*law of large numbers*).
2. Agents have an incentive to be as truthful as possible when faced with this mechanism. This relies on the *ex ante Pareto efficiency* of $f$.

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## Participation Constraints

- Participation constraints are relevant in settings where agents can choose or not to participate in the mechanism. ▸ E.g., bargaining
- Consider a decision problem $(D, U, P)$ where some decision $e \in D$ has a special designation (e.g., outside option).
- First stage, agents submit announcements from $M_i^K$ and decisions are given by $g^K(m^K)$.
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## Participation Constraints: Modified mechanism

- If any agent chooses not to participate, then $e$ is selected on all problems, otherwise the outcomes are $g^K(m^K)$.
- Agents choose to participate on the ex post stage, hence the strongest participation constraint will be satisfied.
- Note, however, that an agent chooses to participate in the whole linking mechanism or not to participate at all.

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- Note, however, that an agent chooses to participate in the whole linking mechanism or not to participate at all.

Consider any ex ante efficient $f$ that satisfies a strict participation constraint of any sort: ex ante, interim, or ex post. Consider the two-stage linking mechanisms with a participation decision as described previously. For every $K$, there exists an approximately truthful perfect Bayesian equilibrium of the modified two-stage linking mechanism such that the resulting social choice function satisfies an ex post (and thus interim and ex ante) participation constraint and the sequence of these equilibria approximate $f$.

# Alessandra Casella

## Storable Votes and Agenda Control

- "Storable votes". Games and Economic Behavior 2005, 51, 391–419
- "Storable Votes and Agenda Order Control: Theory and Experiments". Working paper, October 2008.

- Consider a committee, with heterogeneous members, that meets regularly over time to vote on binary proposals that affect all of its members.
- When decisions are taken by majority vote, we face the same problem as before: you cannot exploit the intensity of preferences.
- Simple alternative: although each member continues to accrue one new vote at each meeting, suppose **votes are storable**.
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- In the case of 2 voters, ex ante welfare is higher with storable votes than with non-storable votes.
- When the number of voters is larger, the conclusion continue to hold if one of the following conditions is satisfied:

  (i) the number of voters is above a minimum threshold;
  (ii) preferences are not too polarized;
  (iii) the horizon is long enough.
- Other mechanisms may lead to similar results too, but storable votes have the advantage of being extremely simple.
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# Storable Votes and Agenda Control (Casella, 2008)

## Overview

- Members of a committee are each granted a single extra **bonus vote** to cast as desired.
- I.e., in addition to a regular vote for each decision (*non storable*), each voter is endowed with one "bonus vote".
- The same idea: a voting scheme that elicit and reward voters intensity of preferences.

- Potential concerns on agenda manipulation.

Is the efficiency comparison to simple majority voting robust to the endogenous determination of the agenda's order?

- Two approaches:
  - (i) No agenda power;
  - (ii) A committee member sets agenda's order.

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- Two approaches:
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- Agenda order is given and in addition to their regular votes, agents are endowed with a single indivisible bonus vote to be cast freely over any of the proposals.
- This mechanism can achieve welfare gains over majority voting if
  1. the value of the bonus vote is not too large;
  2. either the number of voters is even or large enough;
  3. the differences in intensity of preferences across proposals are important enough.

- At the start of the game, the chair decides and announces the order of the agenda.

Agenda's order in this framework acquires the character of a **cheap talk** message: the chair is in a position to transmit information about his priorities and his planned use of the bonus vote.

- The game has multiple equilibria that differ in the precision of the information conveyed.
  - A babbling equilibrium exists;
  - But, informative equilibria also exist, with varying degrees of information.

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The effect of agenda power on the expected aggregate welfare effect is ambiguous.

- In equilibrium, when information is transmitted the chair effectively commits to casting his bonus vote on a subset of decisions only.
- The commitment is valuable because it reduces competition on the decisions he cares most about.
  - Chair's expected utility is higher, and the power to set the order of the agenda is valuable.
- As for the other voters, the end result is ambiguous: by avoiding competition with the chair, in equilibrium they face higher competition from other non-chair voters on the remaining decisions.

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- A sequence of linking mechanisms defined on increasing numbers of linked problems, $\{(M^1, g^1), \ldots, (M^K, g^K), \ldots\}$, and a corresponding sequence of Bayesian equilibria, $\{\sigma^K\}$, *approximate* $f$ if

$$\lim_K \left[ \max_{k \leq K} \Pr \left\{ g_k^K \left( \sigma^K(u) \right) \neq f(u^k) \right\} \right] = 0.$$

where $g_k(m)$ is the marginal distribution under $g$ on $k$th decision where the agents communicated $m$.

Back Back Theorem

- We say that $f$ satisfies
  - an *ex ante participation constraint* if $E[u_i(f(u))] \geq E[u_i(e)]$, $\forall i$.
  - an *interim participation constraint* if $E[u_i(f(u))|u_i] \geq u_i(e)$, $\forall i, u_i$.
  - an *ex post participation constraint* if $u_i(f(u)) \geq u_i(e)$, $\forall i, u$.

Back