Mechanism Design in the Presence of Externalities

Sebastian Kodritsch

London School of Economics and Political Science

January 14, 2011

Outline

Externalities in Context of Our Topic

- (My understanding of) Our topic:
  - *mechanism design* approach to group decision making
  - motivation from the opportunities of modern communication technology, most prominently of the internet, relaxing various physical constraints, for instance allowing participation and interaction in "real time" of very *large groups*
  - particular interest in the *procedural details* of possible mechanisms, e. g. of "debates", to deal with the challenges imposed in particular by human limitations such as *limited attention*
- (Negative) Externalities and decision making of (possibly large) groups:
  - externalities may be of two kinds:
    - ⋆ strategic in the sense that different allocations lead to different subsequent games, e. g. foreign relations and nuclear bomb
    - ⋆ non-strategic (immediate/taste-intrinsic), e. g. host of football world cup
  - in the context of group decision making:
    - ⋆ externalities "natural" in social choice
    - ⋆ although today mainly about revenue maximisation/"auctions" still relevant for insights into incentive compatibility and (endogenous) participation constraints

Review (1) — Debates

- Glazer and Rubinstein (GR):
  - uninformed decision maker (DM) with state-dependent preferences over two actions listening to two informed debaters with known opposing preferences
  - due to time/attention constraints only two arguments may be heard
  - optimal debate (procedural + persuasion rule) as minimising the no. of mistakes must be sequential and violates (a form of) "consistency"
- Levy and Razin (LR):
  - again one DM and two agents competing to persuade the former by catching her limited time/attention slots
  - asymmetry of debaters in how much of the scarce time/attention of the DM they need to persuade (less means a simpler argument)
  - agents compete for time/attention slots in an all-pay-auction
  - the simpler argument has an advantage which is enhanced under interim feedback on the first slot's winner (sequentiality)
- In relation to GR, LR...
  - fix the persuasion rule and consider two different procedural rules
  - engage in a positive exercise showing advantage of simplicity

Review (2) — Linking Decisions

- Casella (C):
  - committee faces binary decision each period where preferences private iid across time and members
  - making votes storable allows to indicate intensity of preferences and typically improves welfare ex ante
  - simplified version is a single extra vote to allocate — by signalling his preferences a member controlling the order of decisions (agenda) may benefit
- Jackson and Sonnenschein (JS), inspired by (C):
  - considers Bayesian group decision problem with objective to implement ex-ante Pareto efficient social choice function
  - overcomes incentive constraints by linking decisions across independent copies of the same problem
  - forcing agents' reported preferences (one per problem) to match the frequency distribution exploits intensity of preferences (trading off preferences across decision problems)

1. Introduction and Review

2. Jehiel, Moldovanu and Stacchetti: "How (Not) to Sell Nuclear Weapons", American Economic Review 1996
   - Motivation and Model
   - Complete Information
   - Incomplete Information

3. Two Related Follow-Up Papers

4. Conclusion

Motivation and Novelty

- Motivation
  - for the title: events surrounding Ukraine's inherited nuclear arsenal
    - technologically outdated so uninteresting to buy for superpowers
    - but danger of proliferation changing the power balance
    - US and Russia made arguably enormous transfers of various sorts to Ukraine to encourage its dismantling of nuclear weapons
  - more generally: mechanism design with externalities
    - strategic externalities, e. g. vertical contracting, patents, ...
    - non-strategic externalities, e. g. consumption item that causes pollution
- Novelty in allowing for identity-dependent externalities, i. e. asymmetry; not getting the object may mean different things depending on who else gets it

Environment

- A seller, agent 0, of an indivisible good and set $B \equiv \{1, \ldots, n\}$ of buyers
- Quasi-linear preferences: "allocation utility plus net received payment" $u_i = v_i - p_i$
  - no-trade utilities normalised to zero
  - if buyer i obtains good and each buyer k pays p_k
    - ⋆ seller's payoff is $u_0 = \sum_{j=1}^{n} p_j$
    - ⋆ buyer j's payoff is $u_j = -\alpha_{ij} - p_j$, where $-\alpha_{ii} \equiv \pi_i$ (so i's payoff is $u_i = \pi_i - p_i$) and externalities captured by a matrix

Optimal Mechanism Design

- Seller maximises payoff over mechanisms where usual stages
  1. seller commits to a mechanism
  2. buyers simultaneously decide whether to participate
  3. participants play the mechanism
- Call a mechanism optimal if it has a (Bayes-) Nash equilibrium (NE) with the seller's payoff maximal over all such equilibria of any feasible mechanism
- Two important (implicit) assumptions:
  - externalities fixed (cannot be contracted away)
  - externalities suffered irrespective of participation, hence participation constraints are endogenous
  - seller can allocate object only to participants

Preliminaries

- Simple starting point as buyers only decide whether to participate or not; a buyer's strategy space is $[0, 1]$ which is a probability of participation
- Separates effects due to externalities from those due to incentive constraints ensuring truthfulness
- Some preliminaries
  - assume explicitly that externalities are negative, i. e. $\forall i \; \forall j \; \alpha_{ij} \geq 0$
  - also assume there exists a smallest money unit $\epsilon > 0$ with $\forall i \; \forall j \; \pi_i - \epsilon \geq 0$ and $\alpha_{ij} - \epsilon \geq 0$
  - define $\alpha^i \equiv \max_{k \neq i} \alpha_{ki}$ as worst utility for i and let $v(i)$ a selection from $\{j \in B | \alpha_{ji} = \alpha^i\}$ as worst allocation for i
  - use $B^* \subseteq B$ to denote the set of participating buyers
  - for subsets of $A \subseteq B$ define $w(A) \equiv \min\{A\}$ whenever $A \neq \emptyset$ and $w(\emptyset) = 0$
  - define $W \equiv \max_{i \in B} \{\pi_i - \sum_{j \neq i} \alpha_{ij}\}$ as maximal total welfare *from trade*

A Mechanism

Consider the following mechanism and call it Γ

1. $B^* = \emptyset$ implies no trade, no payments
2. $|B^*| \leq n-2$ implies
   - allocation to $w(B^*)$ (as arbitrary rule), let $i = w(B^*)$ and $j = w(B^* \setminus \{i\})$
   - payments of any buyer $k \in B^*$
     - ⋆ winner $k = i$ pays $p_k = \pi_i + \alpha_{ji} - \epsilon$
     - ⋆ losers $k \neq i$ pay $p_k = \alpha_{jk} - \alpha_{ik} - \epsilon$ (this may be a subsidy)
3. $B^* = B \setminus \{h\}$ for some $h \in B$ implies allocation to $v(h)$ and payments as above where let $i = v(h)$
4. $B^* = B$ has two subcases
   1. if $W < 0$ then no trade but payments $p_k = \alpha^k - \epsilon \geq 0$
   2. if $W \geq 0$ then allocation to some i with $\pi_i - \sum_{j \neq i} \alpha_{ij} = W$, and payments
      - ⋆ if $k = i$ then $p_k = \pi_i + \alpha^i - \epsilon$
      - ⋆ if $k \neq i$ then $p_k = \alpha^k - \alpha_{ik} - \epsilon \geq 0$

(Approximate) Optimality

- Γ satisfies the following Proposition 1:
  1. for every buyer, participation is a strictly dominant strategy, hence full participation is the unique NE with revenue to the seller of

$$\overline{R} = \left(\sum_{i \in B} \alpha^i - \epsilon\right) + \max\{0, W\} = \left(\sum_{i \in B} \alpha^i\right) - n\epsilon + \max\{0, W\}$$

  2. for any feasible mechanism G and any NE σ of G with associated revenue $R(\sigma) = R$, have $R \leq \overline{R} + n\epsilon = \left(\sum_{i \in B} \alpha^i\right) + \max\{0, W\}$; if ties must be broken in favour of non-participation then $R \leq \overline{R}$
- Proof: (sufficient to prove for σ pure and deterministic allocation)
  1. all immediate from strict dominance; for latter go over various cases to find that, irrespective of σ_{-i}, participation worth exactly $\epsilon > 0$ to buyer i
  2. if no trade then must be that $R \leq \sum_{i \in B^*} \alpha^i \leq \overline{R} + n\epsilon$; if allocation to i then $R \leq \pi_i + \alpha^i + \left(\sum_{j \in B^* \setminus \{i\}} \alpha^j - \alpha_{ij}\right)$, which, by $\alpha^j - \alpha_{ij} \geq 0$ and $B^* \subseteq B$, no greater than $\pi_i + \alpha^i + \left(\sum_{j \in B \setminus \{i\}} \alpha^j - \alpha_{ij}\right) = \left(\sum_{j \in B} \alpha^j\right) + \pi_i - \left(\sum_{j \in B \setminus \{i\}} \alpha_{ij}\right)$, itself no greater than $\left(\sum_{i \in B} \alpha^i\right) + W$; finally note only use of ϵ in Γ to break ties and thus generate the strictness of dominance

Remarks

- Further properties to note about Γ:
  - as $\epsilon \longrightarrow 0$ it becomes optimal
  - its unique NE involves efficient allocation
  - endogenous outside options allow seller to exploit buyers' "worst fears" and extract the latter on top of full social surplus (approximately)
  - indeed, equilibrium payoff of any i is $\epsilon - \alpha^i \leq 0$
  - extends to general case where externalities may be of either sign; there also use lowest possible utility as threat to non-participants
  - subsidies occur only off equilibrium to guarantee strict dominance of participation when some other buys stay out
  - collusion-proof if rule out side-payments within coalition and require no deviating subcoalitions, e. g. consider coalition B
- Seller's ability to commit allows powerful threats
- On motivating example of Ukraine: US and Russia pay Ukraine their worst fears not to sell to anybody, and although they might collude to not pay it is hard to imagine them paying e. g. countries in the Middle East not to buy

1. Introduction and Review

2. Jehiel, Moldovanu and Stacchetti: "How (Not) to Sell Nuclear Weapons", American Economic Review 1996
   - Motivation and Model
   - Complete Information
   - Incomplete Information

3. Two Related Follow-Up Papers

4. Conclusion

Private Types

- Add incentive/elicitation problem with basic assumptions:
  - $\forall j \neq i \ \alpha_{ij} = \alpha_i$ so externality caused is non-discriminatory
  - each buyer i's private and independent type is $t_i = (t_i^1, t_i^2) = (\pi_i, \alpha_i)$, i. e. buyers have private information on their own payoff and the externality they cause to others; let $t_0 = (0, 0)$
- Further assumptions and notation
  - $t_i \in T_i = [\underline{\pi}_i, \overline{\pi}_i] \times [\underline{\alpha}_i, \overline{\alpha}_i] \subset \mathbb{R}^2_+$ independently distributed with density f_i (thus focus on negative externalities again); denote $\underline{t}_i \equiv (\underline{\pi}_i, \underline{\alpha}_i)$
  - $t \in T \equiv \times_{i \in B} T_i$ and $\phi \equiv \times_{i \in B} f_i$ where $T_{-i} \equiv \times_{j \in B \setminus \{i\}} T_j$ and similarly ϕ_{-i}
  - for any i let $(t_i, t_{-i}) \equiv t$
  - recall $\Delta^n = \{z = (z_0, z_1, \ldots, z_n) \in \mathbb{R}^{n+1}_+ | \sum_{i=0} z_i = 1\}$

Some Mechanisms

- Apply revelation principle to consider only direct mechanisms with the property of having a Bayes-NE (BNE) in which all buyers participate and report their types truthfully
- Consider therefore direct mechanisms where buyers simultaneously submit a report, each buyer i from $T_i \cup \{\emptyset\}$ where $\emptyset$ means non-participation, and which are of class (x, p, ρ) such that[1]
  1. for case of full participation
     - $p : T \longrightarrow \Delta^n$ with $p_i(t)$ the probability that i receives the object under report t
     - $x_i : T \longrightarrow \mathbb{R}$ with $x_i(t)$ i's payment to the seller when the report is t
  2. for cases of exactly one non-participant
     - $\rho = (\rho^1, \rho^2, \ldots, \rho^n)$ with $\rho^i : T_{-i} \longrightarrow \Delta^n$ so $\rho^i_j(t_{-i})$ the probability that $j \in T_{-i}$ receives the object under report t_{-i}
     - no payments of any sort for any reports of participants
  3. for case of more than one non-participant no trade and no payments

[1] Just need to prevent unilateral deviations from full participation so may consider (x, p, ρ) as an equivalence class; any two direct mechanisms with the same (x, p, ρ) are equivalent for the purposes here.

Incentive Compatibility (IC)

- Recall that focus on the case of full participation
- Define the following functions $q_i : T_i \to [0,1]$ and $y_i : T_i \to \mathbb{R}$
  - $q_i(t_i) = \int_{T_{-i}} p_i(t_i, t_{-i})\phi_{-i}(t_{-i})dt_{-i}$ as i's conditional expected probability assignment
  - $y_i(t_i) = \int_{T_{-i}} x_i(t_i, t_{-i})\phi_{-i}(t_{-i})dt_{-i}$ as i's conditional expected payment
- If others report truthfully then a buyer i of type t_i reporting s_i has expected utility (EU)

$$U_i(s_i, t_i) \equiv q_i(s_i)\pi_i - \left(\sum_{j \in B \setminus \{i\}} \int_{T_{-i}} p_j(s_i, t_{-i})\alpha_j \phi_{-i}(t_{-i})dt_{-i} \right) - y_i(s_i)$$

- Mechanism (x, p, ρ) is incentive compatible for buyer i if

$$\forall s_i \in T_i \; \forall t_i \in T_i \quad U_i(t_i, t_i) \geq U(s_i, t_i)$$

and it satisfies IC if incentive compatible for all buyers

Participation Constraints (PCs)

- As in the case of complete information outside options endogenous
- Since they constrain revenue an optimal mechanism (x, p, ρ) must use the harshest possible threats in ρ
  - let $v(i, t_{-i})$ a selection from $\arg\max_{j \in B \setminus \{i\}} \{\alpha_j\}$
  - then specify $\rho^i_{v(i,t_{-i})}(t_{-i}) = 1$ and thus for $j \neq v(i, t_{-i})$ $\rho^i_j(t_{-i}) = 0$
- Then the EU of i when all but i participate and report truthfully is

$$\mathcal{A}_i \equiv - \int_{T_{-i}} \alpha_{v(i,t_{-i})} \phi_{-i}(t_{-i}) dt_{-i}$$

- Mechanism (x, p, ρ) satisfies the PCs for buyer i if

$$\forall t_i \in T_i \quad U_i(t_i, t_i) \geq \mathcal{A}_i$$

and it satisfies the PCs if so for all buyers

Reduced Problem

- We have reduced the set of mechanisms to consider to that of direct mechanisms of class (x, p, ρ) which satisfy IC and the PCs
- Given ρ is pinned down, let us write such mechanisms as (x, p)
- Now if (x, p) is feasible then also $(\overline{x}, p)$ with $\overline{x}_i(t) \equiv y_i(t_i)$ is feasible
  - thus no loss in having i's payment depend only on her own report
  - specify mechanisms then as (y, p)
- The seller's mechanism design problem is then

$$\max \sum_{i \in B} \int_{T_i} y_i(t_i) f_i(t_i) dt_i$$

over such mechanisms (y, p) which satisfy IC and the PCs

Characterisation of IC

- Proposition 2: a mechanism (y, p) with conditional probability assignment functions $(q_i)_{i \in B}$ satisfies IC if and only if
  1. $\forall i \ \forall \alpha_i$ have $q_i(\cdot, \alpha_i)$ non-decreasing
  2. $\forall i \ \forall \pi_i$ have $q_i(\pi_i, \cdot)$ constant
  3. $\forall i \ \forall t_i$ have for $\underline{U}_i \equiv U_i(\underline{t}_i, \underline{t}_i)$

$$y_i(t_i) = -\underline{U}_i + q_i(t_i)\pi_i - \int_{\underline{\pi}_i}^{\pi_i} q_i(v, \alpha_i)dv - \sum_{j \in B \setminus \{i\}} \int_{T_{-i}} p_j(t_i, t_{-i})\alpha_j \phi_{-i}(t_{-i})dt_{-i}$$

- Proof skipped but seems intuitive that
  1. must not punish truthful revelation of higher utility
  2. nor make payoff dependent on the reported externality as otherwise may substitute among the two components of reports
  3. note that with this payment schedule have, where $s_i = (\hat{\pi}_i, \hat{\alpha}_i)$,
     - $U_i(s_i, t_i) = \underline{U}_i + \int_{\underline{\pi}_i}^{\hat{\pi}_i} q_i(v, \hat{\alpha}_i)dv + q_i(\hat{\pi}_i, \hat{\alpha}_i)(\pi_i - \hat{\pi}_i)$
     - $U_i(t_i, t_i) = \underline{U}_i + \int_{\underline{\pi}_i}^{\pi_i} q_i(v, \alpha_i)dv \geq \underline{U}_i$ so participation for $\underline{t}_i$ must bind at optimum, i. e. $\forall i \ \underline{U}_i = \mathcal{A}_i$

Optimal Mechanism Design

- Define first a kind of (inverse) hazard ratio

$$h_i(t_i) \equiv \int_{\pi_i}^{\overline{\pi}_i} \frac{f_i(v, \alpha_i)}{f_i(\pi_i, \alpha_i)} dv$$

- Then the optimal mechanism solves

$$\max_{p: T \to \Delta^n} -\left(\sum_{i \in B} \mathcal{A}_i\right) + \int_T \left(\sum_{i \in B} [\pi_i - (n-1)\alpha_i - h_i(t_i)] p_i(t)\right) \phi(t) dt$$

  s. t. parts 1 and 2 of proposition 2

- At optimum must be that $p_i((\pi_i, \cdot), t_{-i})$ constant (as q_i) so can write p as $p(\pi)$ where $\pi = (\pi_1, \ldots, \pi_n)$
- By first integrating over externalities in the above expression for revenue the problem becomes one of one-dimensional types

Illustration

- Now consider the case where $f_i(t_i) = f(\pi_i)g(\alpha_i)$ so utility and externality independent and both identically distributed across buyers
  - let $E \equiv \int_{\underline{\alpha}}^{\overline{\alpha}} vg(v)dv$, $\Pi \equiv [\underline{\pi}, \overline{\pi}]^n$ and $f(\pi_1) \cdots f(\pi_n) \equiv \phi(\pi)$
  - then $h_i(t_i) = \frac{1-F(\pi_i)}{f(\pi_i)}$ and $\mathcal{A}_i = -(n-1)\int_{\underline{\alpha}}^{\overline{\alpha}} v(G(v))^{n-2}g(v)dv \equiv \mathcal{A}$
  - expected revenue therefore is equal to

$$-n\mathcal{A} + \int_{\Pi} \left(\sum_{i \in B} \left[\pi_i - \frac{1-F(\pi_i)}{f(\pi_i)} - (n-1)E \right] p_i(\pi) \right) \phi(\pi) d\pi$$

- Remarks (in comparison with complete information):
  - sell to $i^* \equiv \arg\max_{i \in B} \pi_i$ provided above $[...] \geq 0$
  - now "virtual valuations" $\pi - \frac{1-F(\pi)}{f(\pi)} < \pi$ so inefficiency
  - may even have inefficient sale, e. g. i with very high π_i and α_i
  - recognise qualitative features of earlier Γ

Optimal Mechanisms with Externalities

Complementary paper by same authors "Multidimensional Mechanism Design for Auctions with Externalities", JET 1999

- in AER 1996 deal with incomplete information in simplified setting
  - buyers' private information concerned their own utility and externality caused
  - externalities were non-discriminatory
  - although two-dimensional types eventually reduces to one-dimensional problem
- in relation to this, in JET 1999
  - consider again mechanisms of class (x, p, ρ) but for the case where a buyer's type is the $n+1$-dimensional payoff vector consisting of her payoff for every possible allocation
  - characterise IC which allows to eliminate payment functions from variables
  - the PCs are not only endogenous but also depend on a buyer's *own* type; show that although they need not bind for the "lowest" type it is sufficient to check a "critical" type (the one closest to a type not suffering any externalities and with lowest possible own utility)
  - study anonymous auctions in symmetric settings where can deal with IC

Efficient Mechanisms with Externalities

Jehiel and Moldovanu, "Efficient Design with Interdependent Valuations", Econometrica 2001

- study efficient mechanisms for social choice
- while have Vickrey-Clarke-Groves mechanisms to deal with allocative externalities here allow for informational externalities; illustration in multi-object setting
  - an alternative is a partition (whole allocation over buyers) of M objects
  - each agent receives a signal about each possible bundle (a subset of M) and has her valuation for each alternative (utility)
  - "pure" private values means utility depends only on own bundle and signal about own bundle
  - allocative externalities: valuation depends also on whole partition
  - informational externalities: valuation depends also on others' signals
- characterise IC and show impossibility results about efficiency and IC (without requiring budget-balancedness)

1. Introduction and Review
2. Jehiel, Moldovanu and Stacchetti: "How (Not) to Sell Nuclear Weapons", American Economic Review 1996
   - Motivation and Model
   - Complete Information
   - Incomplete Information
3. Two Related Follow-Up Papers
4. Conclusion

A Few Points to Conclude

- Looked at optimal mechanisms in presence of (negative) externalities to in particular capture transactions that change future interaction
- From complete information:
  - outcome always efficient when no incentive constraints
  - endogeneity of participation constraints interacts with revenue maximisation to enable seller to extract revenue from buyers who do not receive anything
  - in particular seller extracts revenue for not selling, and indeed will not sell if externalities are large
- From incomplete information:
  - incentive constraints cause outcome to not always be efficient
  - in particular may also have inefficient selling whereas without externalities inefficiency only due to supply restriction
  - apart from efficiency other qualitative features of the optimal mechanism under complete information carry over
- Note also: commitment to mechanism by seller critical for optimality, optimality of simultaneous mechanisms here