Mechanism Design with Costly Information Acquisition

Part IV of Group Decision Making

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Theory Reading Group

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Outline

1. Introduction
2. Gerardi & Yariv, *GEB* 2008, "Information Acquisition in Committees"
3. Gershkov & Szentes, *JET* 2009, "Optimal Voting Schemes with Costly Information Acquisition"
4. Discussion (online environment)

Information is not always free. Sometimes it is costly to acquire information necessary to good decision-making.

True of both individual and group decisions.

Optimal group decision-making procedures need to account for the fact that agents can choose whether or not they are informed.

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- Seeking optimal decision-making procedure (normative exercise)
- Free-rider problem (problem with large groups)
- Tradeoff between informed decision making (aggregation) and providing incentives to invest in information (requires making individual agents' information more important).

- **GY**: SP chooses committee size and decision rule, agents simultaneously choose investment and report.
  - **Main Result**: The ex-ante optimal mechanism may be ex-post inefficient.
- **GS**: Restrict attention to ex-post efficient mechanisms.
  - **Main Result**: Requires sequentiality. After each individual agent reports his type, SP decides whether or not to continue collecting information. Precision threshold for stopping decreases with number of signals already acquired.

Typical jury setup, $N \geq 2$

Two states $\Omega = \{I, G\}$, they occur with probability $P(I)$, $P(G)$

Decisions $\{A, C\}$

$u(d, \omega)$ given by
$u(C, I) = -q$
$u(A, G) = -(1 - q)$
$u(d, \omega) = 0$ otherwise

Each agent can purchase a single signal $s \in \{i, g\}$ of accuracy $p > \frac{1}{2}$ by paying a cost $c > 0$

1. SP chooses *extended mechanism*
   1. committee size $n \leq N$ (assume chooses smallest given level of feasible expected payoffs)
   2. symmetric mapping $\gamma : \{0, 1, ..., n\} \rightarrow [0, 1]$, $\gamma(k) \equiv \Pr(d = C \mid \#_g = k)$
2. Agents observe mechanism, and $j = 1, ..., n$ simultaneously decide whether to purchase signal (restrict attn to pure strategies and collection)
3. Each agent sends a message in $\{i, g\}$ to SP, who uses mechanism to select outcome

$$\sum_{k=0}^{n-1} \binom{n-1}{k} f(k+1;n)(\gamma(k+1)-\gamma(k)) \geq c \qquad (IC_i)$$

$$\sum_{k=0}^{n-1} \binom{n-1}{k} f(k;n)(\gamma(k)-\gamma(k+1)) \geq c \qquad (IC_g)$$

where

$$f(k;n) = -qP(I)(1-p)^k p^{n-k} + (1-q)P(G)p^k(1-p)^{n-k}$$

proxies for difference in payoff between C and A when k of n signals are g

Constraints weight difference in payoff by conviction probabilities given profiles of signals

Define k_n smallest number of signals g such that SP weakly prefers C

SP's problem:

$$\max_{\gamma:\{0,\ldots,n\}\to[0,1]} Eu = -(1-q)P(G) + \sum_{k=0}^{n} \binom{n}{k} f(k;n)\gamma(k) \quad (P_n)$$
$$\text{s.t. } (IC_i),\ (IC_g)$$

$\bar{\gamma}_n$ is sol'n to P_n if it exists, $V(n)$ is exp utility of optimal device

n^* is optimal size, $V(n^*) \geq V(n)\ \forall\ n \leq N$

Consider optimal devices $(n^*, \bar{\gamma}_{n^*})$

Note: SP does not take cost of information into account

γ_n^B is sol'n to maximization problem without constraints

$$\gamma_n^B(k) = 0 \text{ if } k < k_n$$
$$= 1 \text{ if } k \geq k_n$$

Define $\hat{V}(n)$ the expected utility of first best

Proposition 1: *For every* $n \geq 1$, $V(n) = \hat{V}(n)$ *iff* $\hat{V}(n) - \hat{V}(n-1) \geq c$
(Because if $n-1$ have been truthful, last player can guarantee $\hat{V}(n-1)$ by simply saying i or g)

Assume there is at least one committee size for which Bayesian device admissable (n^B denotes greatest: $\hat{V}(n^B) - \hat{V}(n^B - 1) \geq c$, but $\forall n > n^B$, $\hat{V}(n) - \hat{V}(n-1) < c$)

SP can induce more than n^B to acquire info only if mechanism aggregates available signals suboptimally
But more information is available in larger committees.....key tradeoff

Proposition 2: *Fix* $P(I), q, p$ *and assume regular environment* (tech assm). *Let* $n^*(c) \leq N$ *denote the optimal committee size when cost is* c. $\exists\ \bar{c} > 0$ *s.t.* $\forall\ c < \bar{c}$, *whenever* $\hat{V}(N) - \hat{V}(N-1) < c$, *then* $V(n^*(c)) < \hat{V}(n^*(c))$
(i.e. optimal device is not Bayesian)

Gerardi & Yariv 2008

Intuition of Main Result

- Consider Bayesian device with $n^B + 1$ agents
  - Difference in payoffs when exactly k_{n^B} others report g but agent sees signal i
  - By definition, $\hat{V}(n^B + 1) - \hat{V}(n^B) < c$
  - Agent has incentive to save cost c by just reporting g (or i)
- Consider device with k_{n^B}
  - Difference in payoffs comes when exactly k_{n^B-1} others report g and agent sees i
  - $\hat{V}(n^B) - \hat{V}(n^B - 1) \geq c$
- Consider combination
  - A when $k < k_{n^B}$, C when $k \geq k_{n^B+1}$, C with prob α when $k = k_{n^B}$
- Distorted device can have strictly higher payoff than Bayesian device, and induces $n^B + 1$ agents to acquire signals (Preview of GS results for ex ante optimal sequential mechanisms)

1. Optimal device ex ante may be ex post inefficient
2. Optimal distortions to ex post efficient rule depend on accuracy of signals (can be described for extreme p)
3. Comparative statics on c, p
    - expected utility monotonic in c, p for given n
    - if restrict to ex-post efficient mechanisms, optimal committee size non-monotonic in p (possible to prefer many uninformative signals to few accurate signals)

Gershkov & Szentes 2009

Key differences:

1. Committtment to ex-post inefficient decisions problematic; look for optimal *ex post efficient* mechanism
2. SP doesn't have to commit to a committee size at outset
3. SP cares about information costs incurred by agents

Main Result: sequentially ask agents to collect info and vote; use decreasing decision threshold

Gershkov & Szentes 2009

The Model – some notation changes (sorry!)

$K \geq 1$ individuals

Two states: A and B, equiprobable (robust to some asymmetry)

Decisions: α and β, where

$$u(\alpha \mid A) = u(\beta \mid B) = 1$$
$$u(\alpha \mid B) = u(\beta \mid A) = 0$$

(where again, robust to asymmetry)

Agents can purchase a signal in $\{a, b\}$ of accuracy p at cost c. This takes no time (i.e. no discounting).

SP maximizes $K_0 Eu - c\bar{L}$ ($\bar{L}$ = expected number of agents who collect info)

Notation: s is a finite sequence of signals; d is the difference in number of a and b signals in s

Note: Ex post efficient decision is to follow the majority of signal reports

Proposition 1: *There exists a weakly decreasing (in single steps) function $g : \mathbb{N} \rightarrow \mathbb{N}$ s.t. if, after asking I agents, the reported signal sequence is such that $|d| > g(I)$, the SP makes the majority decision. Otherwise, SP asks an additional agent.*

1. *If $K = \infty$ then there exists $k \in \mathbb{N}$ such that $g \equiv k$. In addition, $k \rightarrow \infty$ as $K_0 \rightarrow \infty$.*
2. *If $K < \infty$ then for all $I \in \mathbb{N}$, $g(I+1) = g(I)$ or $g(I+1) = g(I) - 1$, and $g(K-1) = 1$.*

1. Restrict attn to sequential mechs in which agents act only once, which are ex post efficient and symmetric wrt A and B, which uniformly order agents for all $K < \infty$
2. Can explicitly characterize IC constraint (both for acquiring signal and truthfully reporting it)
3. Restrict attention to Markovian mechanisms
    1. A state $V(I, d) \equiv \{s \mid I(s) = I, d(s) = d\}$; Markovian mechanisms specify a (possibly random) decision $D \in \{M, (m), C\}$ for each state $V(I, d)$
    2. The optimal mechanism generically involves randomization

Looking for optimal Markovian mechanism

$$\max_G K_0 \sum_{V \in V(K)} [\rho(V, M : G) P(|d(V)|) + \rho(V, m : G) Q\,|d(V)|] - c\bar{L}(G)$$

$$\text{s.t. } \frac{p-q}{4pq} \sum_{V \in V(K)} \rho(V, M : G) P(|d(V)|)[|d(V)| - (p-q) l(V)] + \frac{p-q}{4pq} \sum_{V \in V(K)} \rho(V, m : G) Q(|d(V)|)[-|d(V)| - (p-q) l(V)] \geq c\bar{L}$$

Continuation Mechanisms (c.m.)
– $G(I,d)$ a Markovian mechanism that follows reaching $V(I,d)$ – note interaction through IC constraint
– Efficiency of mechanisms: $e(G(I,d)) = \frac{\Delta W(G(I,d))}{|\Delta IC(G(I,d))|}$ a measure of the tradeoff between effects on objective function and IC constraint

4. Use of a c.m. must increase the posterior in expectation
5. Monotonicity: for a given posterior, a longer sequence implies less efficient to continue asking more
6. If a c.m. used with positive probability in some state, can't stop in another state if a more efficient c.m. is available
7. Don't acquire more info if it can't possibly change the decision
8. Either no randomization or randomization only in $V(\hat{I}, |\hat{d}|)$

Theorem 1: *Suppose that the first-best mechanism does not satisfy the incentive compatibility constraint. Let $G^\star$ be an ex ante optimal mechanism among the ex post efficient ones. Then, there exists a decreasing step function $f : \mathbb{N} \to \mathbb{N}$, and $N \in \mathbb{N}$, such that*

$$\text{for all } I \in \mathbb{N},\ f(I+1) = f(I) \text{ or } f(I+1) = f(I) - 1, \text{ and } f(N) = 1$$

Let

$$T = \{V : f(I(V)) = |d(V)|, f(I(V) - 1) = f(I(V)) + 1\}.$$

$G^\star$ is defined by the following three conditions:

1. *If $V \notin T$ and $f(I(V)) \leq |d(V)|$ then $p(M : V, G^\star) = 1$*
2. *If $V \notin T$ and $f(I(V)) > |d(V)|$ then $p(C : V, G^\star) = 1$*
3. *If $V \in T$ then $p(M : V, G^\star) \geq 0$, $p(C : V, G^\star) > 0$*

Furthermore, generically, there exists an optimal ex post efficient mechanism for which there are only two states, V_1, $V_2 \in T$, such that $p(M : V_i, G^\star) > 0$ for $i = 1, 2$. In addition, if $K < \infty$, then $N = K$.

Optimal mechanism (if first best not incentive compatible) is to:

- Ask agents sequentially to acquire and report signals, without letting them know their position or any previous reports
- Stop acquiring and make efficient decision when posterior exceeds a cutoff, which is decreasing (weakly) in the number of reports
  - The cutoff jumps down by at most 1 for an additional report
- There is a bound $N \leq K$ s.t. SP never asks more than $N + 1$ agents
- Generically, the mech will involve randomization
  - But only in relatively few states that satisfy certain criteria

Note: theorem holds even when $K = \infty$ (solely through the IC constraint)

Sequential mechanism beats out simultaneous mechanism through:

1. Cost efficiency
   - Example: $K = K_0 = 9$, $p = 2/3$, $c = 0.04$
   - Simultaneous asks 5. If $s = (a, a, a)$, SP incurs cost of two signals which won't change the decision
   - Sequential stops asking when cost of information exceeds expected benefit ($s = (a, a, a, b, a)$)
   - What if introduce discounting?
2. Efficient incentive provision
   - Agents acquire information only if sufficiently likely that they are pivotal
   - Provide opportunities to be pivotal by sometimes acting on imprecise posteriors
   - Decreasing threshold means that acting on imprecise posteriors occurs more often after long sequences than short
   - Exploit difference in actual and conditional probability of long sequences to provide incentive at low cost

Ex ante optimal mechanisms

1. The ex ante optimal mechanism sometimes involves ex post inefficiency (as in GY)
2. Ex ante optimal mechanism is also ex post efficient provided p is small enough (for a given K)

Wrapping up – online decisions

Issues from Ronny's introduction

- Information aggregation – here not just concerned with aggregating, but with generating information for better decision-making
- What's bad about large groups? – free rider problem
- Congestion – here time constraints not an issue (info collected instantaneously); if this were not true, risk of decisions taking a long time (issue of discounting), but also ignorance of position in sequence becomes questionable (back to simultaneous?)
- Attention constraints – not addressed, but worth noting computation intensity
- Sequentiality – crucial in GS, but not for usual reasons