

Repeated Games with Local Monitoring

Developments and Directions

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Repeated Games with Local Monitoring

Can reputations be established in societies?

Plan for today:

- Introduction & Motivation
- Classical Results:
 - Uniform Random Matching
 - Sequential Interactions
- Recent Developements:
 - Deterministic Matching (Network)
 - Common Knowledge
 - Simultaneous Interactions
- Our (Michele & I) Findings:
 - Deterministic Matching
 - Without Common Knowledge
 - Simultaneous Interactions

Why do should we care about reputations with local information?

- Trade Models with Quality
- Dynamic Pricing Models in Geographical Markets
- Social Norms in Large Societies
- Local Public Goods Games

What one would want?

- Efficient of Equilibrium Payoffs
- Stability to Random Mistakes
- Folk Theorems and/or Anti-Folk Theorems

Classical Results (mid '90s): Typical Model

- Prisoner's Dilemma Game
 - Efficient payoff in strictly dominated strategies
- Uniform Random Matching
 - All players are matched in pairs at each repetition
 - URM generally used for simplicity
 - Need random matching and some symmetry to align incentives
- Sequential Interactions
 - Actions are match-specific
 - A single interaction at any decision instance
- Different Degrees of Information
 - Always information about own past interactions
 - Occasionally information about past play of matched players
- Anonymous Societies
 - No information about identity of matched players [Ellison]

Classical Results: Desired Properties

- Sequential Rationality [SR]
 - The matching process is always common knowledge
- Global Stability [GS]
 - Any finite history cannot reduce continuation payoffs by much
- Mistake Stability [MS]
 - Small random mistakes cannot reduce continuation payoffs by much

Contagion equilibria:

- Kandori [ReStud1992]
 - Information: know all the past play of matched players
 - A strategy that sustains cooperation and that satisfies $SR + GS$ exists if δ is sufficiently high
 - If players ignore past play of matched players, a strategy that sustains cooperation and satisfying SR is shown to exist for specific parameter values
- Ellison [ReStud 1994]
 - Information: know nothing about matched players
 - A strategy that sustains cooperation and that satisfies $SR + MS$ exists if δ is sufficiently high

A number of studies has since tried to generalize several aspects:

- Different Types of Games:
 - Variable Stake PD: Nageeb Ali & Miller [2010]
 - Games of Collective Action: Lagunoff & Haag [JET 2007]
 - Public Goods Provision: Wolitzky [2010]
 - General Games: Debb [2008]
 - With Uncooperative players: Ghosh and Ray [1996]
- Different Matching Rules
 - Many-to-Many Deterministic Matching: Nageeb Ali & Miller [2010], Lagunoff & Haag [JET 2007]
- Different Interactions Patterns
 - Actions are not match specific: Ahn [2006]
 - Actions are collective: Wolitzki [2010], Lagunoff Haag JET [2007], Nageeb Ali & Miller [2010]

- Institutions have been added to games:
 - To disclose information: Milgrom [1990], Okuno-Fujiara & Postelwaite [1995], Nowak and Sigmund [1998], Dixit [2003], Takahashi [2009]
 - To allow for communication: Debb [2008]
 - To coordinate behavior with sanctions: Dixit [2004], Grief [2006], Maggi [1999]
- Different Degrees of Information
 - Perfect Monitoring: Lagunoff & Haag [2007], Dal Bo [2007]
 - Local Monitoring: Wolitzki [2010], Nageeb Ali & Miller [2010], Ahn [2006], Rosenthal [1979]
 - Diffusion of Information on Network: Fainmesser-Goldberg [2009], Vega-Redondo [2006]
- Also related to the fast growing literature on private monitoring: Piccione [2002], Ely & Valimaki [2002]...

Recent Results: More Desired Properties

- Sequential Rationality of Cooperation
- Mistake Stability
- Maximal Average Cooperation at finite discount rates
 - Wolitzki [2010], Lagunoff & Haag [2007], Pecorino [1999], Ghosh and Ray [1996]
- Folk Theorems
 - Debb [2008]
- More than Folk Theorems
 - Dal Bo [2007]
- Relationship between Group Size and Efficiency
 - Lagunoff & Haag [2007], Kranton [1996], Grief [1993]

My Interest in the Topic

Our (Michele & I) paper discusses:

- Asymmetric Prisoner's Dilemma Game
- Fixed Many-to-Many Matching
 - Each player is matched to multiple players at each repetition
 - Such players are called neighbors and do not vary over time
- Simultaneous Interactions
 - Each player engages in multiple interactions at each repetition
 - Actions are local and affect all neighbors
- Information Structure
 - Players only know the actions chosen by they neighbors
 - Monitoring network is not common knowledge
- Non-anonymous Societies
 - Players know their neighbors' identities

Want to find a strategy that sustains cooperation and that satisfies:

- Ex-post Rationality [ER]
 - Want strategy to be optimal for any possible beliefs about the graph
- Finite Time Stability [FTS]
 - Reversion to cooperation in a finite time after any deviation history
- Mistake Stability [MS]
 - Small random mistakes cannot reduce continuation payoffs by much

Current Findings

1. A strategy that sustains cooperation and that satisfies:

- ER + FTS + MS exists if $\delta = 1$
- ER + FTS + MS exists if δ is sufficiently high and if only clique acyclic graphs are admissible

These results are robust to:

- heterogeneity of discount rates
- uncertainty about payoffs
- uncertainty about the number of players

2. A strategy that sustains cooperation and that satisfies:

- ER exists if δ is sufficiently high and if players have common preferences in each PD