Ahn, I. (1997) "Imperfect information repeated games with a single perfect observer"

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Outline

1. Preview of results
2. Example
3. A FT under the star graph
4. A FT under the more general graphs

Preview of results

- Main idea: Extend the folk theorem (FT) when there is no perfect observability
- Constraint: Focus on repeated games (RG) where there is someone who observes and is observed by everyone

Preview of results

- Good old FT assumes perfect observability (PO)
- Not a good assumption for large populations
- Under PO, it is easy to implement punishments
- Once a deviation is observed, retaliation follows
- Without PO, the following problem arises

Preview of results

- Assume Mary is observed by Philip only and Philip is observed by everyone
- If Philip sees Mary deviating, he has to signal her deviation with an action
- But how the others know that Philip is signaling instead of deviating himself?

Preview of results

- So if signaling Mary turns into punishment against Philip, he will have no incentive to denounce her...
- ... unless Mary can punish Philip for not signaling her!
- In short, extending the FT under IO requires imposing restrictions on the payoff functions.

Outline

Example

- Players: $\{0, 1, 2\}$
- Observability (2 cases):
  - Complete graph (PO):
    - $N_i = S_i = \{0, 1, 2\}$ for $i = 0, 1, 2$
  - Star graph (IO):
    - $N_i = S_i = \{0, 1, 2\}$ for $i = 0$
    - $N_i = S_i = \{0\}$ for $i = 1, 2$

Example

- Nash equilibrium of the stage game induces payoffs $(15, 15, 15)$
- We know how to implement $(30, 30, 30)$ average payoffs in RG under the **complete graph** (PO) case
- Can we do the same in the **star graph case**?

1

0		I	N
	I	30, 30, 30	0, 0, 0
	N	0, 0, 0	10, 10, 10

H

1

0		I	N
	I	40, −10, 40	0, 0, 0
	N	0, 0, 0	15, 15, 15

L

2

Example

- Player 2 has not to have incentives to deviate:
- $30 \geq (1-\delta)40 + \delta V_2 \Rightarrow V_2 < 30$
- But now Player 0 has to have incentives to denounce her by playing N instead of I:
- $30 \leq (1-\delta)0 + \delta V_1 \Rightarrow V_1 \geq 30$

1

0	I	N
I	30, 30, 30	0, 0, 0
N	0, 0, 0	10, 10, 10

H

1

0	I	N
I	40, −10, 40	0, 0, 0
N	0, 0, 0	15, 15, 15

L

2

Example

- Claim: There is no way $V_2 < 30 \leq V_1$
- Therefore average payoffs $(30, 30, 30)$ can not be implemented under the **star graph**

1

0		I	N
	I	30, 30, 30	0, 0, 0
	N	0, 0, 0	10, 10, 10

H

1

0		I	N
	I	40, −10, 40	0 , 0, 0
	N	0, 0, 0	15 , 15 , 15

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A FT under the star graph

(IP) For each player $i \in N$, there exists an action $m_i \in A_i$ *s.t.*

$$u_0(\mathbf{a}^*) > u_0(a_0^*, m_i, (a_j^*)_{j \in N - \{i\}}).$$

Under "Independent punisshment" (IP), the FT can be reintroduced through the *modified finite periods Nash reversion* (MFNR) strategy profile:

A FT under the star graph

- Phase I (Normal)
  - Play a^*
  - If i deviates she and 0 move to phase II
- Phase II (Signaling)
  - Player 0 plays s_0
  - Player i plays m_i as long as a_0^* is observed
- Phase III (Finite Punishment):
  - Play stage NE for K periods once s_0 observed
- Phase IV (Infinte Punishment)
  - Play stage NE forever if neither a_0^* nor s_0 are observed

A FT under the star graph

Note: We also need to specify actions for player 0 in the *multiplayer's* signaling phase (i.e, more than one individual have deviated!)

Outline

A FT under P-0 graphs

- Can we extend the FT to more general graphs where there is still a single perfect observer?
  - Either we impose a "General Punishment" (GP) restriction on payoffs
  - Or we focus on symmetric graphs ($N_i = S_i$ for all i)
- On both cases we need a *extended* MFNR strategy profile