Ahn, I. (1997) "Imperfect information repeated games with a single perfect observer"

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Oliver Pardo Ahn, I. (1997) "Imperfect information repeated games with a sin

Outline



2 Example

- 3 A FT under the star graph
- A FT under the more general graphs

Preview of results

- Main idea: Extend the folk theorem (FT) when there is no perfect observability
- Constraint: Focus on repeated games (RG) where there is someone who observes and is observed by everyone

Preview of results

- Good old FT assumes perfect observability (PO)
- Not a good assumption for large populations
- Under PO, it is easy to implement punishments
- Once a deviation is observed, retaliation follows
- Without PO, the following problem arises

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Preview of results

- Assume Mary is observed by Philip only and Philip is observed by everyone
- If Philip sees Mary deviating, he has to signal her deviation with an action
- But how the others know that Philip is signaling instead of deviating himself?

Preview of results

- So if signaling Mary turns into punishment against Philip, he will have no incentive to denounce her...
- ... unless Mary can punish Philip for not signaling her!
- In short, extending the FT under IO requires imposing restrictions on the payoff functions.

Outline





- 3 A FT under the star graph
- A FT under the more general graphs

Example

- \bullet Players: $\{0,1,2\}$
- Observability (2 cases):
 - Complete graph (PO):
 - $N_i = S_i = \{0, 1, 2\}$ for i = 0, 1, 2
 - Star graph (IO):

•
$$N_i = S_i = \{0, 1, 2\}$$
 for $i = 0$

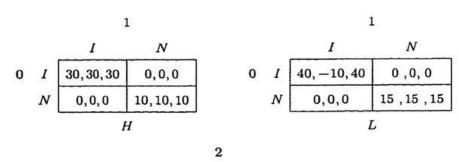
• $N_i = S_i = \{0\}$ for i = 1, 2

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Example

- Nash equilibrium of the stage game induces payoffs (15, 15, 15)
- We know how to implement (30, 30, 30) average payoffs in RG under the **complete graph** (PO) case
- Can we do the same in the star graph case?



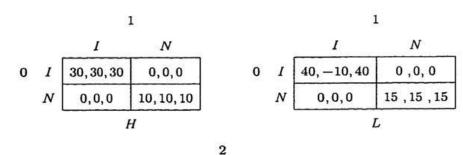
Example

• Player 2 has not to have incentives to deviate:

•
$$30 \ge (1-\delta)40 + \delta V_2 \Rightarrow V_2 < 30$$

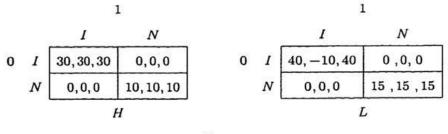
• But now Player 0 has to have incentives to denounce her by playing *N* instead of *I*:

•
$$30 \le (1-\delta)0 + \delta V_1 \Rightarrow V_1 \ge 30$$



Example

- Claim: There is no way $V_2 < 30 \le V_1$
- Therefore average payoffs (30, 30, 30) can not be implemented under the **star graph**



Outline



2 Example



A FT under the more general graphs

A FT under the star graph

(IP) For each player $i \in N$, there exists an action $m_i \in A_i$ s.t.

$$u_0(\mathbf{a}^*) > u_0(a_0^*, m_i, (a_j^*)_{j \in N - \{i\}}).$$

Under "Independent punisshment" (IP), the FT can be reintroduced through the *modified finite periods Nash reversion* (MFNR) strategy profile:

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A FT under the star graph

- Phase I (Normal)
 - Play a*
 - If *i* deviates she and 0 move to phase II
- Phase II (Signaling)
 - Player 0 plays s_0
 - Player *i* plays m_i as long as a_0^* is observed
- Phase III (Finite Punishment):
 - Play stage NE for K periods once s_0 observed
- Phase IV (Infinte Punishment)
 - Play stage NE forever if neither a_0^* nor s_0 are observed

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A FT under the star graph

Note: We also need to specify actions for player 0 in the *multiplayer's* signaling phase (i.e, more than one individual have deviated!)

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Outline



2 Example

- 3 A FT under the star graph
- A FT under the more general graphs

A FT under P-0 graphs

- Can we extend the FT to more general graphs where there is still a single perfect observer?
 - Either we impose a "General Punishment" (GP) restriction on payoffs
 - Or we focus on symmetric graphs $(N_i = S_i \text{ for all } i)$
- On both cases we need a *extended* MFNR strategy profile

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