Haag and Lagunoff, I. (2007) "On the size and structure of group cooperation"

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Outline

Preview of results

- Main idea:
  - Mancur Olson claimed that cooperation decreases with group size
  - In the context of infinitely repeated PD, Olson's Conjecture might be reverted
  - In particular, with heterogeneous δ's and n large enough, increases on n increase cooperation.

Outline

Definitions

- Let p_i describe the cooperation level of player i
- Let $\delta = (\delta_1, ..., \delta_n)$
- Let $E(\delta)$ be the set of equilibria of the infinitely RG
- The maximum average cooperation (MAC) is defined by

$$\max_{p \in E(\delta)} \frac{1}{n} \sum_{i=1}^{n} p_i \tag{1}$$

Prisoner's Dilemma for $n = 2$

	C	D
C	c, c	$-l, d$
D	$d, -l$	$0, 0$
- where
  - $d > c > 0 > -l$
  - $2d > d - l > c$

Prisoner's Dilemma for $n = 2$

- Payoff function

$$\pi_i(p) = p_i(p_j c - (1 - p_j)l) + (1 - p_i)p_j d \tag{2}$$

- If $P \equiv p_1 + p_2$ we can rewrite $\pi_i(p)$ as

$$\pi(p_i, P) = (P - p_i)d - p_i(P - p_i)(d - c - l) - p_i(2 - 1)l \tag{3}$$

Prisoner's Dilemma for any n

- Payoff function

$$\pi_i(p) = \sum_{j \neq i} p_i(p_j c - (1 - p_j)l) + (1 - p_i)p_j d \qquad (4)$$

- If $P \equiv \sum_i p_i$ we can rewrite $\pi_i(p)$ as

$$\pi(p_i, P) = (P - p_i)d - p_i(P - p_i)(d - c - l) - p_i(n - 1)l \quad (5)$$

Outline

- What's the effect of a mean preserving effect of δ on the MAC?

Numerical example

- Consider

	C	D
C	$2, 2$	$-1, 4$
D	$4, -1$	$0, 0$

Numerical example

- Any equilibrium cooperation levels (p_1, p_2) in the infinitely RG have to satisfy

$$p_i(p_j 2 - (1 - p_j)1) + (1 - p_i)p_j 4 \geq (1 - \delta_i)p_j 4 \text{ for } i = 1, 2 \quad (6)$$

Numerical example

- If $\delta_1' = \delta_2' = 1/2$ then $p_1^*(\delta') = p_2^*(\delta') = 1$
- What if $\delta_1'' = 3/4$ and $\delta_2'' = 1/4$?
- Clearly p_1^* can't increase, so MAC decreases
- In particular $p_1^*(\delta'') = 1$ and $p_2^*(\delta'') = 1/2$

Example

- The MAC can be proved to be non increasing in the heterogeneity of δ
- The result can be generalized to any **collective action** games
- Preference for homogeneous populations?

Outline

- What's the effect of n on the MAC?
- Take the previous numerical example and double the population
- Distribution of δ is replicated, i.e.,

$$\delta = (\delta_1, \delta_2, \delta_1, \delta_2) \tag{7}$$

- Assume that for $n = 2$, $\delta_L < \delta_H$
- Assume that incentive compatibility constraints (6) bind
- Therefore

$$p_i = \frac{\delta_i}{\frac{1}{2} + \frac{1}{4}\frac{n-1-P_{-i}}{P_{-i}}} \tag{8}$$

- i's cooperation is increasing on others' cooperation ratio $\frac{P_{-i}}{n-1}$
- For $i = H$, cooperation ratio increases from $\frac{p_L}{1}$ to $\frac{2p_L+p_H}{3}$
- For $i = L$, cooperation ratio decreases from $\frac{p_H}{1}$ to $\frac{2p_H+p_L}{3}$
- So H's cooperation increases whereas L's cooperation decreases
- The first effect dominates because the negative effect on L's cooperation is damped by $\delta_L < \delta_H$
- Thus the MAC increases