Herding with endogeneous timing of decision

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Classical papers - herding with exogenous order

- "A Theory of Fads, Fashion, Custom, and Cultural Change as Information Cascades" Bikhchandani, Hirshleifer and Welch, Journal of Political Economy 1992
- "A Simple Model of Herd Behavior", Abhijit Banerjee, Quarterly Journal of Economics 1992

Main Result: it is rational for agents to ignore their own information, after observing a finite number of other decisions and follow the herd.
Why? - when there is incomplet information about the state of nature an agent's action can reveal his private information to the others.
The belief of the followers on the state of the nature is updated and as a result at one point the followers will ignore their own information and mimic the others.

Herding with endogenous timing of decision:

- “Information Revelation and Strategic Delay in a Model of Investment, Chamley and Gale, Econometrica” 1994
- “Strategic Delay and the Onset of Investment Cascades” Zhang J, RAND Journal of Economics 1997

Motivating example: in recesions everyone is waiting for a signal before making a decision (everyone is waiting to see what everyone else is doing)

“Information Revelation and Strategic Delay in a Model of Investment" Chamley and Gale, Econometrica, 1994

Set up: Investment model with no preset order of action - agents will wait and see what others are doing
Main results:When the decision period is small there is herding or informational cascades as in BHW or AB

Informational value of delay

- N agents out of which n (random number) have an *investment opportunity* (real option) which is *private information*
- Agent's type is $t_i \in \{0, 1\}$
- The random variables$(t_1, ..., t_N)$ are exchangeable
  If π is a permutation of $\{1, ...N\}$ then:

$$\Pr(t_1, ..., t_N) = \Pr\left[\left(t_{\pi(1)}, ..., t_{\pi(N)}\right)\right]$$

  Which can be seen as a situation in which there are n investment opportunities which happens with probability $g_0(n)$ which are allocated randomly to the N agents.

Conditional on n the probability that an agent receives an option is: $\frac{n}{N}$;
Unconditional probability is $\sum_{n=0}^{N} g_0(n) \frac{n}{N}$
Posterior probability of n given that an agent i received an option:

$$g(n) = \frac{g_0(n)\, n}{\sum_{n=0}^{N} g_0(n')\, n'}$$

Exchangeability implies that a sufficient statistic for the sum of the agents information is the number of option n

- The expected return on investment is $v(n)$ assumed increasing in n (the larger the number of options, the better the prospect for investment)
- The value of investment

$$V \equiv \sum_{n} g(n)\, v(n)$$

Actions:

- Each agent decides to exercise his investment option with probability $\lambda \in (0, 1)$;those without options do nothing
- An observer (agent with an option to invest) of the others actions knows:

  - that the information revealed depends only on the number who invest;

  - the largest the number of investors the larger the probability of players with options.
- Thus, the observer can increase his payoff by conditioning his decision on the number who invest k
- The value to the investment to the observer is:

$$V(k) = \sum_n g(n|k)\, v(n)$$

$.g(n|k) = \frac{b(k;n,\lambda)g(n)}{\sum_n b\left(k;n',k\right)g\left(n'\right)}$ where $b(k; n, \lambda)$ is the probability of k successes in n independent Bernoulli trials with probability of success λ in each trial.

- The observer's maximum payoff is:

$$W(\lambda) = \sum_k p(k) \max \{V(k), 0\}$$

where $p(k) = \sum_n b(k; n, \lambda) g(n)$ is the probability of k investments;
Remember: $V \equiv \sum_n g(n) v(n) \geq 0$ which can be seen as
$V = \sum_k p(k) V(k)$ thus

$$W(\lambda) \geq V$$

- $W(\lambda) > V$ iff $V(k)$ is negative for some values of k

Theorem

1. If $g(n)$ is nondegenerate and $0 < \lambda < 1$, then $V(k)$ is increasing; otherwise $V(k)$ is a constant function. If $W(\lambda) > V \geq 0$ there exists $0 < k^* \leq N$ such that $V(k) < 0$ if $k < k^*$ and $V(k) \geq 0$ if $k \geq k^*$. In particular, the observer will invest only if at least one other agent has invested.
2. $W(\lambda)$ is increasing in λ (in the information revealed)

Model

- N agents, n have an investment option
- the option can be exercised at any of the countable dates $t = 1, 2, ..., \infty$
- payoff of a player that invested at time $t : \delta^{t-1} v(n)$; if he never invested the payoff is 0
- Actions are publicly observed:

$$\begin{aligned} x_{it} &= 1 \text{ if player } i \text{ invests at } t \\ &= 0 \text{ if player } i \text{ does not invest at } t \end{aligned}$$

- $x_t = (x_{1t}, ..., x_{Nt})$ outcome at date t

- History of the game at $t : h_t = (x_1, ..., x_{t-1})$;
- the set of all possible histories at t is H_t;
- $H_1 = \{\emptyset\}$ is the initial history;
- $H = \bigcup_{t=1}^{\infty} H_t$ the set of all histories
- In a symmetric equilibrium it is only the number of who invest at each date that matters, so wlog we can write a history $h \in H_t$ as a sequence $(k_1, ..., k_{t-1})$
  The players without options are passive so we don't need to describe the strategies and beliefs for the players with options

- For any h, let $\lambda(h)$ denote the probability that a player who has not yet exercised his option does so after observing history h;so λ is a *behavioral strategy*
- Player's belief: is a function $\mu : H \times N \rightarrow [0,1]$ where $\mu(n|h)$ is the probability that n players have options conditional on the history h.

PBE: (λ, μ) such that i) each playe's strategy is best response at every information set and ii) the probability assesment are consistent with Bayes' rule at every information set that is reached with positive probability.

- In order to find an equilibrium path, the authors use *One Step Property*

- $V(h)-$ the payoff from immediate investment at information set h
- $W^*(h)$ the equilibrium payoff from waiting at the information set h
- $W(\varepsilon, h)$ - the undiscounted payoff from waiting one period at the information set h and then making an irrevocable decision when other players invest with probability ε
- (h, k) - information set reached if k players invest after history h
- If a player waits at h and makes a one-for-all decision to invest at (h, k) or never to invest then his payoff at (h, k) is $\max\{V(h, k), 0\}$.
- The payoff from waiting: $\sum_k p(k|h) \max\{V(h,k), 0\}$ where $p(k|h)$ is.this particular player's probability assessment that k others will invest at h

Definition

One Step Property (OSP) is satisfied at h if:

$$W^*(h) = W(\lambda(h), h) \equiv \underbrace{\sum_k p(k|h) \max\{V(h,k), 0\}}_{payoff \;\; from \;\; waiting}$$

which means that it is optimal for a player who waits at h to make a once-for -all decision at (h, k).

Theorem

For any fixed but arbitrary symmetric PBE (λ, μ) the OSP holds at any information set $h \in E$ (E is an equilibrium path)

Using OSP we can calculate the equilibrium path:

$$\begin{aligned} W(\varepsilon, h) &= \sum_{k=0}^{N-1} p(k|h,\varepsilon) \max\{V(h,k), 0\} \\ &= \sum_{k=0}^{N-1} p(k|h,\varepsilon) \max\left\{\textstyle\sum_{n=0}^{N} p(n|k,h,\varepsilon)\, v(n)), 0\right\} \\ &= \sum_{k=0}^{N-1} p(k|h,\varepsilon) \max\left\{\textstyle\sum_{n=0}^{N} \dfrac{p(n,k|h,\varepsilon)\, v(n)}{p(k|h,\varepsilon)}), 0\right\} \end{aligned}$$

$$= \sum_{k=0}^{N-1} \max \left\{ \sum_{n=0}^{N} p\left(n, k \middle| h, \varepsilon\right) v\left(n\right), 0 \right\}$$

$$= \sum_{k=0}^{N-1} \max \left\{ \sum_{n=0}^{N} b\left(k; n - K\left(h\right) - 1\right) \mu\left(n \middle| h\right) v\left(n\right), 0 \right\}$$

$\sum_{n=0}^{N} p\left(n, k \middle| h, \varepsilon\right)$ is the player's probability assessment of k investments and n options, given the history h

$K\left(h\right)$ the number of players who have invested previously.

We also know that $W\left(\varepsilon, h\right)$ increasing in ε if $W\left(\varepsilon, h\right) > V\left(h\right) > 0$

Theorem

Let (λ, μ) *be a fixed but arbitrary symmetric PBE. After any history* $h \in E$, *one of the following mutually exclusive situations occurs:*
a)$V(h) \leq 0$ *and* $\lambda(h) = 0$
b)$V(h) \geq \delta \sum_n \mu(n|h) \max(v(n), 0) > 0$ *and* $\lambda(h) = 1$
c)Neither a or b apply. In that case, $0 < \lambda(h) < 1$ *is the unique value such that* $V(h) = \delta W(\lambda(h), h) = \delta W^*(h)$

Thus there are three cases:
-for sufficient pessimistic beliefs no one is willing to invest (no information is revealed -this is an absorbing state)
- for sufficient optimistic beliefs all players immediately invest and the game ends.
- for intermediate beliefs players are indifferent between investing and waiting and randomize between the two.
Equlibrium path: given h, get $\mu(h)$ using Bayes rule, then calculate the the value of $V(h)$; use the above theorem to get $\lambda(h)$ uniquely.

The effect of period length on delay

Definition

A symmetric PBE exhibits delay if players who have a positive payoff from investing choose not to invest - i.e. $V(h) > 0$ and $\lambda(h) < 1$ for some history h that is reached with positive probability

Suppose a player could obtain complete information by delaying his investment one period after the beginning of the game, then his equilibrium payoff at the second date must be at least:

$$W(1, \oslash) \equiv \sum_n g(n) \max\{v(n), 0\}$$

where $g(n)$ is the probability assessment of an active player at the first date.

Definition

Define δ_1 by:

$$V(\oslash) = \delta_1 W(1, \oslash)$$

where $V(\oslash) = \sum_n g(n) v(n)$ (value of investment at the first date)

Theorem

Any symmetric PBE must exhibit delay if $\delta > \delta_1$. *No equilibrium will exhibit delay if* $\delta < \delta_1$

Next: write $\delta = e^{-\rho\tau}$ where τ is the length of time period and ρ is the rate of time preference.
If τ_1 corresponds to δ_1 the above theorem says that there will be delay if $0 < \tau < \tau_1$ but not if $\tau > \tau_1$.
However as τ becomes very small the delay becomes also very small

Theorem

In any symmetric PBE all investment ends after at most N periods. Thus as $\tau \to 0$ *the length of the game converges to zero as well.*

Logic of the proof:

- We know that in order to keep the investment going at least one player has to invest at each date. So the game lasts mostly N periods.
- As τ becomes very short players make their decision to invest or not within an arbitrarily short of time after the start of the game
- Since the decision never to invest implies infinite delay - we cannot really say that there is no delay, but the process of investment and information revelation ends almost instantaneously when the period length is vanishing short.
- Result :**Collapse of investment!**
  With positive probability players stop investing even though the true return to investment is positive! Like in Bickhchandani (1992) and Banerjee (1994)
  However when $\tau \longrightarrow \tau_1$ the probability of herd behavior and informational cascades decreases toward zero.
- N goes to infinity:
  - initially there is a period of negligible investement
  - followed either by an investment surge or an investment collapse.

Strategic delay and the onset of investment cascades, Zhang, 1997

- It allows agents to differ in the quality of their private information
- Unique symmetric equilibrium in which
  - the player with the best signal moves first (delay is costly so the highest precision agent has the least to learn from the others; this player has the lowest incentives to wait).
  - the time of waiting depends negatively on the quality of information if nobody moved.
  - if somebody moved the followers can infer correctly that it was the person with the best quality of information thus they will mimic his action $\Longrightarrow$ **Informational Cascades!**