Herding in Financial Markets

Theory Reading Group

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Agenda for today

- Avery, C. and P. Zemsky (1998): "Multidimensional Uncertainty and Herd Behaviour in Financial Markets", *American Economic Review*, 88(4), 724-748.
- Park, A. and H. Sabourian (2011): "Herding and Contrarian Behaviour in Financial Markets", forthcoming in *Econometrica*.
- Dasgupta, A.and A. Prat (2008): "Information aggregation in financial markets with career concerns", *Journal of Economic Theory*, 143, 83-113.

Amery-Zemsky (1998) - Model Setup

- Single asset $V \in [0,1]$ can be traded by an **exogeneous sequence** of traders $t = 0, 1, 2....$ Denote by $H_t = \{h_0, ..., h_t\}$ the trading history up to period t.
- Each trader is risk neutral and has the option to buy $(h_t = 1)$/sell $(h_t = -1)$ one unit of asset or refrain from trading $(h_t = 0)$.
- Two classes of traders can operate in every period (indipendently overtime):
  - *Informed trader* with probability μ: receive private information $x_\theta \in [0,1]$, where θ is type of informed trader. Finite number of types θ.
  - *Noise trader* with probability $1 - \mu$: buy/sell/notrade with probability $\gamma = (1 - \mu)/3$
- Informed type θ valuation of the asset is $V_\theta^t(x) = E[V | H_t, x_\theta = x]$.
- Since possibility to trade only once, informed agents actions depend only on their private expectation of the asset and the posted ask\bid prices. **No strategic reasoning**.

Amery-Zemsky (1998) - Model Setup

- Traders face a perfectly competitive market maker. His expectation of the asset value depends only on prior information and trading history, i.e. $V_m^t = E[V|H_t]$.
- Trading structure typical of screening models:
  1. Market maker posts ask $p_t^a(H_{t-1}) = E[V|H_{t-1}, h_t = 1]$ and bid $p_t^b(H_{t-1}) = E[V|H_{t-1}, h_t = -1]$ prices.
  2. Trader buys/sell at ask/bid price or does not trade.
- By Bayes' rule the market maker's valuation can be written recursively as

$$V_m^{t+1} = V_m^t \frac{\Pr\left(h_t | V = 1, H_t\right)}{\Pr\left(h_t, H_t\right)}$$

- *No Cascade Assumption*. If $\Pr\left(V = v | H_t\right) \neq 1$, then there exists at least a θ and a set of signal realizations $R \subset \{0, 1\}$ with $\Pr\left(x_\theta \in R | H_t\right) > 0$ such that $V_\theta^t(x_\theta) \neq V_m^t$ for $x_\theta \in R$. Moreover, if $|V_m^t - V| = \delta > 0$ then for some $\varepsilon(\delta) > 0$, $|V_\theta^t(x_\theta) - V_m^t| > \varepsilon(\delta)$.

Amery-Zemsky (1998) - Definitions

- *Definition.* An **informational cascade** occurs in period t if $\Pr(h_t | V, H_{t-1}) = \Pr(h_t | H_{t-1}) \; \forall V, h_t$.
  - No new information reaches the market because distribution over observable actions is independent of the state of the world.
- *Definition.* A trader with private information x_θ engages in **herd behaviour** at time t if (i) $V_\theta^t(x_\theta) > p_t^a(H_{t-1})$, (ii) $V_\theta^0(x_\theta) < p_0^b$ (iii) $V_m^t > V_m^0$ or (i) $V_\theta^t(x_\theta) < p_t^b(H_{t-1})$, (ii) $V_\theta^0(x_\theta) > p_0^a$ (iii) $V_m^t < V_m^0$
  - An agent buy herds if he was initially pessimistic about the value of the asset ($V_\theta^0(x_\theta) < p_0^b$) and after a positive history of trade ($V_m^0 < V_m^t$) he decides to buy ($V_\theta^t(x_\theta) > p_t^a(H_{t-1})$.
- *Definition.* A trader with private information x_θ engages in **contrarian behaviour** at time t if (i) $V_\theta^t(x_\theta) > p_t^a(H_{t-1})$, (ii) $V_\theta^0(x_\theta) < p_0^b$, (iii) $V_m^t < V_m^0$ or (i) $V_\theta^t(x_\theta) < p_t^b(H_{t-1})$, (ii) $V_\theta^0(x_\theta) > p_0^a$ (iii) $V_m^t > V_m^0$.

Amery-Zemsky (1998) - Results

- *Proposition.* An informational cascade never occurs in equilibrium.
  - It is a direct consequence of the no informational cascade assumption on the presence of informed trading.
- *Proposition.* The bid and ask prices converge almost surely to the true value V.
  - Impossibility of informational cascades implies that each period of trade reveals some information even if there is herd behaviour. Continual flow of information leads to long-run convergence.
- *Proposition.* The variance of price paths is bounded as follows: $\sum_{t=1}^{T} Var(\Delta V_m^t) \leq Var(V)$
  - Expected volatility is bounded by the fundamental uncertainty over V. As a consequence, herd behaviour does **not** explain volatility in excess of fundamental values.

Amery-Zemsky (1998) - Results

- *Definition*. A signal x_θ is monotonic if there exists a function $v(x_\theta)$ such that $V_\theta^t(x_\theta)$ is always between $v(x_\theta)$ and V_m^t for all trading histories H_t.
  - Notice that the definition refers to endogeneous variables.
  - Monotonic signals are pervasive in financial literature. For example, $x_\theta \in \{0,1\}$ with $\Pr(x_\theta = i | V = i) = \rho > 1/2, i = 0, 1$.
- *Proposition.* A trader with monotonic signals never engages in herd behaviour.
  - *Proof.* Suppose a trader with monotonic signal x_θ engages in herd buying at time t. Then $V_\theta^t(x_\theta) > p_t^a(H_{t-1}) \geq V_m^t$. Since the signal is monotonic, this implies $v(x_\theta) > V_t^m$. But then $V_\theta^0(x_\theta) > V_m^0$ and the trader was not originally pessimistic, so contradiction.

Amery-Zemsky (1998) - Event Uncertainty

- Until now uncertainty concerned asset value V.
- *Definition.* There is event uncertainty when $1 > \Pr(V = V_m^0) > 0$.
  - Concept introduced by Easley and O'Hara 1992.
  - Some types of informed traders may know about a shock to the underlying asset value while the market maker has no information.
- *Example.* $V \in \{0, \frac{1}{2}, 1\}$ with initial prior $\Pr(V = 0) = \Pr(V = 1) = \lambda$, $\Pr(V = \frac{1}{2}) = 1 - 2\lambda$
  - $\Pr(x = \frac{1}{2}|V) = \begin{cases} 1 & \text{if } V = \frac{1}{2} \\ 0 & \text{if } V \neq \frac{1}{2} \end{cases}$
  - $\Pr(x = i|V = 1) \begin{cases} \rho & \text{if } V = i \\ 1-\rho & \text{if } V \neq i \end{cases} \quad i = 0, 1 \ \rho \geq \frac{1}{2}$
- Signal structure is non-monotonic for $\lambda < 1\backslash 2$.
- Event uncertainty introduces the possibility to have different interpretations of H_t between informed traders and market maker.

Amery-Zemsky (1998) - Event Uncertainty

- *Proposition*. Let $\pi_v^t = \Pr(V = v|H_t)$. No herding behaviour is possible if $\pi_1^t = \pi_0^t$. For $\pi_1^t \neq \pi_0^t$ there exists a critical value $\bar{\rho}$ such that traders engage in herding behaviour if $\rho < \bar{\rho}$. If $\pi_1^t > \pi_0^t$ then buy herding, while if $\pi_1^t < \pi_0^t$ sell herding. The threshold value $\bar{\rho}$ is increasing in $\frac{\pi_1^t}{\pi_0^t}$ $(\frac{\pi_0^t}{\pi_1^t})$ if trader is buy (sell) herding (holding $\pi_{1/2}^t$ constant).
  - Event uncertainty allows informed trader to adjust their private expectation after history H_t more rapidly than the market maker since they can exclude some events $(V = 1/2)$. As a result, herding is possible.
- *Proposition*. As $\lambda \longrightarrow 0$ the probability that there is herding behaviour in the trading history goes to 1. Moreover, the trading history almost surely takes the following form: (i) a finite, initial period during which herding does not occur; (ii) an arbitrary long period of herd behaviour of one type. This herding behaviour is in the wrong direction with probability $\tau \in \left[\frac{(1-\rho)^2}{\rho^2+(1-\rho)^2}, 1-\rho\right]$. In the limit as $\mu \rightarrow 0$, the probability of herd behaviour in the wrong direction goes to $1-\rho$.

Amery-Zemsky (1998) - Event Uncertainty

- Although event uncertainty may lead to extreme herding behaviour, the effect on price volatility is very limited. Necessary restrictions on μ and ρ to obtain herding are such that V_m^t responds slowly to trade history H_t.
  - In other words, event uncertainty requires to have parameter restrictions such that prices are almost "fixed". As a result, Bikhchandani, Hirshleifer and Welsh (1992) results apply.
- *Proposition.* During any interval of trading in which there is herd behaviour, the movement in asset price is less than $\Delta = \frac{3\mu\left(\rho - \frac{1}{2}\right)}{2+\mu}$.
- However, herding behaviour does not lead to informational cascade! During herding period the market maker learns that an information event has occurred. At one point V_m^t moves by more than the information contained in the last period prior to herding and it stops.

Amery-Zemsky (1998) - Composition Uncertainty

- In order to generate greater price volatility AZ introduce composition uncertainty.
- *Definition.* There is composition uncertainty when the probability of trades of different types, μ_θ, is not common knowledge.
- *Example.* $V \in \{0, 1/2, 1\}$, two types of trader $\theta \in \{H, L\}$.
  - $\Pr(x_\theta = 1 | V = i) = \begin{cases} p_\theta & \text{if } V = i \\ 1 - p_\theta & \text{if } V \neq i \end{cases}$ with $p_H = 1$ and $p_L \in (\frac{1}{2}, 1)$.
  - Informed traders know proportions of H and L traders while market maker does not.
- Composition uncertainty exploit the fact that a sequence of trades does not signal whether the market is well or poorly informed, so market maker has to rely on his own prior.
- Significant short-run deviations can be observed under extreme parameter values.

Amery-Zemsky (1998) - Composition Uncertainty

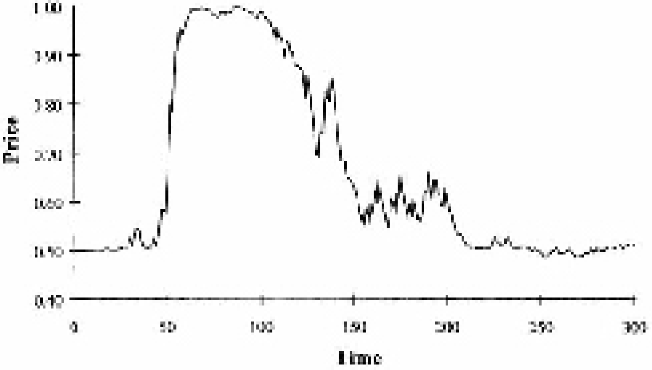

FIGURE 1. AN EXAMPLE OF A PRICE BUBBLE

Park-Sabourian (2011) - Introduction

- Amery and Zemsky (1998)main results are:
  - "Monotonic" signals prevent herding and contrarian behaviour.
  - In a fully rational setup herding can be obtained only if we introduce non-monotonic signals (interpreted as multidimensional uncertainty).
  - Herding results under extreme parameter restrictions.
- Park and Sabourian (2011) claim that these results should be reconsidered since:
  - AZ definition of monotonicity is disputable because it is not based on primitive properties of the signal structure.
  - AZ examples are extreme and may have limited economic relevance.
  - Extreme price movements are possible under not so unlikely situations.

Park-Sabourian (2011) - Model setup

- Single asset $V \in [V_1, V_2, V_3] = [0, V, 2V]$ can be traded by an **exogeneous sequence** of traders. Assume symmetric prior $\Pr(V_1) = \Pr(V_3)$.
- Each trader is risk neutral and has the option to buy $(h_t = 1)$/sell $(h_t = -1)$ one unit of asset or refrain from trading $(h_t = 0)$.
- Two classes of traders can operate in every period (indipendently overtime):
    - *Informed trader* with probability μ. He receives private information about V. Set of possible signals is $S = \{ S_1, S_2, S_3\}$ with $\{\Pr(S_i|V_j)\}_{i,j=1,2,3}$.
    - *Noise trader* with probability $1-\mu$: buy/sell/notrade with probability $\gamma = (1-\mu)/3$.
- Informed type valuation of the asset is $E[V|H_t, S]$.
- Since possibility to trade only once, informed agents actions depend only on their private expectation of the asset and the posted ask\bid prices. **No strategic reasoning**.

Park-Sabourian (2011) - Model setup

- Traders face a perfectly competitive market maker. His expectation of asset value depends only on his prior information and trading history, i.e. $V_m^t = E[V|H_t]$.
- Trading structure typical of **screening** models:
  1. Market maker posts ask $p_t^a(H_{t-1}) = E[V|H_{t-1}, h_t = 1]$ and bid $p_t^b(H_{t-1}) = E[V|H_{t-1}, h_t = -1]$ prices.
  2. Trader buys/sell at ask/bid price or does not trade.
- No Cascade Condition is assumed.

Park-Sabourian (2011) - Signal distributions

- Result on existence of herding crucially depend on signal distribution.
  - Increasing: $\Pr(S|V_1) \leq \Pr(S|V_2) \leq \Pr(S|V_3)$
  - Decreasing: $\Pr(S|V_1) \geq \Pr(S|V_2) \geq \Pr(S|V_3)$
  - U-shaped: $\Pr(S|V_i) > \Pr(S|V_2)$, $i = 1, 3$
  - Hill-shaped: $\Pr(S|V_i) < \Pr(S|V_2)$, $i = 1, 3$
- Signal bias: $\Pr(S|V_3) - \Pr(S|V_1) \lessgtr 0$. Let nU-shaped (pU-shaped) be a negative (positive) biased U-shaped signal distribution. Similar definition for Hill-shaped signals.

Park-Sabourian (2011) - Results

Theorem

HERDING. (i) Necessity: if type S herds, then S is U-shaped with a non-zero bias. (ii) Sufficiency: if there is a U-shaped type with a non-zero bias, there exists a $\mu_h \in (0,1]$ *such that some informed type herds when* $\mu < \mu_h$.

Theorem

CONTRARIANISM. (i) Necessity: if type S acts as a contrarian, then S is Hill-shaped with a non-zero bias. (ii) Sufficiency: if there is a Hill shaped type with a non-zero bias, there exists a $\mu_c \in (0,1]$ *such that some informed type acts as a contrarian if* $\mu < \mu_c$

Park-Sabourian (2011) - Necessity

1. For any signal S, $E(V|S)$ is less (more) that $E(V) = V$ if and only if S has negative (positive) bias.
2. If $E(V|H_t) > E(V)$ then $\Pr(V_3|H_t) > \Pr(V_1|H_t)$ and if $E(V|H_t) < E(V)$ then $\Pr(V_3|H_t) < \Pr(V_1|H_t)$.
3. Notice that if S is decreasing (increasing) then type S does not buy (sell) at any history. As a result, these types never engage in herding or contrarian behaviour.
4. If type S buy (sell) herds then his initial valuation was below (above) $E(V)$. By 1. this implies that S is negatively (positively) biased. The type is either n-U or n-Hill (p-U or p-Hill) shaped.
5. Agent can not be n-Hill (p-Hill) shaped because in forming his belief he assigns less weight on the tails of his belief (and thus more on the center) relative to the public belief. Because of negative bias the shift towards the center is more for value V_3 than for V_1. By 2. $E(V|H_t)$ attaches more weight to V_3 relative to V_1. Such redistribution of probability mass ensures that $E(V|H_t, S) < E(V|H_t)$.

Park-Sabourian (2011) - Sufficiency

- Suppose $\Pr(V = V_1|H_t) \approx 0$ then there are effectively two states of the world: V_2 and V_3. Then sign of $E(V|S, H_t) - E(V|H_t)$ has the same sign of $\Pr(S|V_3) - \Pr(S|V_2)$. By assumption this difference is positive if S is U-shaped and negative if S is Hill-shaped.
- The probability of noise trader $1 - \mu$ should be significanlty high such that the bid ask-spread is not too wide at every trades history. In other words, we try to keep the cost for informed traders "sufficiently fixed".
- The construction of trading path such that $\Pr(V = V_1|H_t) \approx 0$ (or $\Pr(V = V_3|H_t) \approx 0$) is not trivial and it may require additional assumptions on the evolution of the likelihood of different trading actions.

Park-Sabourian (2011) - MRLP

- Assuming MLRP does not prevent signal structure to be U-shaped or Hill shaped.
  - S_1 types always sell, S_3 types always buy. No restriction on S_2.
  - MLRP is consistent with middle signal S_2 that is decreasing, increasing, U-shaped or Hill-shaped.
- As a result, herding and contrarian behaviours are possible for S_2 types.
- MLRP signals greatly simplify the characterization of histories where herding is possible, i.e. $\Pr(V = V_1|H_t) \approx 0$ or $\Pr(V = V_3|H_t) \approx 0$. This is due to the fact that with MLRP the probability of a buy (sell) is uniformly increasing (decreasing) in the liquidation value.

Park-Sabourian (2011) - Results

Assume MLRP holds. Then:

- **Herding is resilient**: during a buy (sell) herding episode as the number of buys increase, it takes more sales to break the herd.
- **Contrarianism is self defeating**: buy (sell) contrarianism persists if and only if the number of buys is not too large.
- **Large price movements** are consistent with both herding and contrarianism.
  - Under MLRP buys increase prices, sales decrease prices. This is the case because S_3 always buys and S_1 always sells. As a result, trading has always the possibility to be informative in both directions.
  - When buy herding starts a large number of buys induce prices to rise to levels arbitrarily close to V_3. Since herding resilient increase is self-sustaining.
  - When buy contrarian starts large number of sales can lead price to be close to V_1 without ending buy contrarianism.

Dasgupta-Prat (2008) - Introduction

- Paper brings together literature on dynamic trading models under asymmetric information (Glosten-Milgrom 1985) and career concerns literature on sequential investment decision making (Scharfstein and Stein 1990).
- Reputational concerns by portfolio managers generate herding.
- Prices never converge to true liquidation value, even after an infinite sequence of trades. This result is in sharp contrast to the previous literature on statistical herding.
- Career concerns reduce volatility of prices and increase liquidity.

Dasgupta-Prat (2008) - Model setup

- Single asset $V \in [0,1]$, $\Pr(V=1) = \frac{1}{2}$, can be traded by an **exogeneous sequence** of traders $t = 0, 1, 2....$ Denote by $H_t = \{h_0, ..., h_t\}$ the trading history up to period t.
- Each trader can buy ($h_t = 1$) or sell ($h_t = -1$) the asset.
- Two classes of traders can operate in every period (indipendently overtime):
  - *Noise trader* with probability $1-\mu$: buys or sells with probability $\frac{1}{2}$
  - *Fund manager* with probability μ:
    - Two types $\theta \in \{b, g\}$ with $\Pr(\theta = g) = \gamma$.
    - Private signal $s_t \in \{0,1\}$ with $\Pr(s_t = v | v, \theta) = \sigma_\theta$ where $\frac{1}{2} \leq \sigma_b < \sigma_g \leq 1$.
    - Fund managers do **not** know their type.
- Direct profit $\pi_t(a_t, p_t^a, p_t^b, v) = \begin{cases} v - p_t^a & \text{if } a_t = 1 \\ p_t^b - v & \text{if } a_t = 0 \end{cases}$
- Reputation payoff: $r_t(h_T, v) = \Pr(\theta_t = g | H_T, v)$
- **Fund manager payoff:** $u_t = \beta\pi_t + (1-\beta)r_t$ **with** $\beta \in (0,1)$.

Dasgupta-Prat (2008) - Results

Theorem

For any game with $\beta < 1$*, there exists a* $\bar{p} \in (0, \frac{1}{2})$ *such that in any equilibrium of the game, at all times,* $p_t \in (\bar{p}, 1 - \bar{p})$.

Intuition of the proof.
Three different forces drive the result:

1. Profit motive encourages managers to trade sincerely.
2. When public belief indicates some liquidation value v to be highly probable, career concerns encourage money manager to trade according to the signal that is more likely to arise if $V = v$.
3. When prices become sufficiently **extreme**, thus sufficienlty **precise**, profit motives are low (since public and private beliefs converge) while reputational cost of contrarian action is high.

As a result, herding starts with sufficiently extreme prices and no further information revelation occurs.

Dasgupta-Prat (2008) - Results

Let $\alpha_i^t(H_t, e)$ be the probability of manager with signal i buys in equilibrium $e \in E$.

- *Definition.* The most revealing equilibrium $\bar{e}$ is such that for any t, for any H_t and any other equilibrium $e \in E$, we have $\alpha_1^t(H_t, \bar{e}) \geq \alpha_1^t(H_t, e)$ and $\alpha_0^t(H_t, \bar{e}) \leq \alpha_0^t(H_t, e)$
- *Proposition*. The most revealing equilibrium exists and is unique. In the most revealing equilibrium, play depends on history only through price. It can be expressed as follows:
  - If $p_t \leq \frac{1}{2}$ then $\alpha_0^t(H_t, \bar{e}) = 0$ and $\alpha_1^t(H_t, \bar{e}) = \alpha_1(p_t) = 1\{\Delta u_1(\alpha_0 = 0, \alpha_1 = 1, p) \geq 0\}$
  - If $p_t \geq \frac{1}{2}$ then $\alpha_1^t(H_t, \bar{e}) = 1$ and $\alpha_0^t(H_t, \bar{e}) = \alpha_0(p_t) = 1\{\Delta u_0(\alpha_0, \alpha_1 = 1, p) \geq 0\}\ \forall \alpha_0$

Comparative statics (WLOG case $p_t \leq \frac{1}{2}$):

- *Proposition*. For all $\beta'' > \beta'$:
  - $\alpha_1(\beta'', p_t) \geq \alpha_1(\beta', p_t)$
  - if $\alpha_1(\beta', p_t) \in (0, 1)$ then $\alpha_1(\beta'', p_t) > \alpha_1(\beta', p_t)$