A War of Attrition wtih Endogenous Effort Levels

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Introduction

- Classic war of attrition (binary action) does not allow for signalling.
  - Strategic equivalence to all-pay auction.
- Bargaining literature allows for time-varying actions, signalling and learning
  - Shortcoming: refused offers don't affect payoffs (except through delay costs)
- Oligopoly, lobbying, animal fighting
- Present paper investigates robustness of WoA predictions to allowing resource expenditure to vary during the game (jump bids).

Introduction

- All-pay auction in each period. Dynamic nature allows bids to convey information about valuations.
- Relative to classic WoA
  - Unique equilibrium
  - Winner identity may change
  - Shorter delay (substitutability with bids)
  - Less rent dissipation
- Both complete and incomplete information.

Model

- $i \in \{1, 2\}$
- $t \in \{1, 2, ...\infty\}$
- Alternating moves
  - In $t = 1, 3, ...$, player 1 chooses $b_t \geq 0$. Player 2 chooses *quit* or *cover* (pay b_t as well).
  - In $t = 2, 4, ...$, ...
- $\delta < 1$
- Player i's type $v_i \in V = \{1, \lambda\}$, $\Pr(v_i = 1) = \mu_i$, $\lambda < 1$
- If game ends, last bidder gets prize. All bids are sunk.
  - Payoff is difference between discounted valuation (if win) and discounted sum of bids.

Model

Strategies

- H_t^1 is set of histories of length t for player 1
  - $H_1^1 = \emptyset$
  - set of positive $(b_1, b_2, ...b_{t-1})$ if t odd, $(b_1, b_2, ...b_t)$ if t even
- Similar for player 2
  - set of positive $(b_1, b_2, ...b_t)$ if t odd, $(b_1, b_2, ...b_{t-1})$ if t even
- PS for player 1: $\sigma_1 = (b_1, c_1)$
  - $b_1 : V \times \cup_{2\mathbb{N}+1} H_t^1 \longrightarrow \mathbb{R}_+$
  - $c_1 : V \times \cup_{2\mathbb{N}} H_t^1 \longrightarrow \{c, q\}$
- Similar for player 2

- Set of infinite histories H^∞; set of terminal histories $H^\dagger$
- Strategy profile $\sigma = (\sigma_1, \sigma_2)$ defines probability distribution over $H = H^\infty \cup H^\dagger$
- $h \in H^\infty$
  - $V_i\left(h; v_i\right) = \sum_\tau -\delta^{\tau-1} b_\tau\left(h\right)$
- $h \in H^\dagger$
  - t odd: $V_1\left(h; v_1\right) = \delta^{t-1} v_1 - \sum_t \delta^{\tau-1} b_\tau\left(h\right)$, $V_2\left(h; v_2\right) = -\sum_t \delta^{\tau-1} b_\tau\left(h\right)$
  - t even: $V_1\left(h; v_1\right) = -\sum_t \delta^{\tau-1} b_\tau\left(h\right)$, $V_2\left(h; v_2\right) = \delta^{t-1} v_2 - \sum_t \delta^{\tau-1} b_\tau\left(h\right)$
- Players maximize expected discounted payoff

Model

Examples

1. IO – alternating move quantity-setting duopoly
2. Political Economy – two lobbyists vying for influence of politician
3. Biology – male house crickets use at least 13 tactics in fights

Model

Solution Concept

- Complete information: subgame perfection
- Incomplete information: undefeated equilibrium
  - beliefs $\rho_i \colon H_t^i \longrightarrow [0,1]$ probability of $-i$ being high type
  - consider the set of SE s.t. support of players' beliefs is non-increasing along every history
  - Let M be a subset of types. SE is pruned if i has out-of-eqm action s.t. if $-i$ interprets the action as evidence that $i \in M$, all and only types in M prefer to play that action.
  - Recursive

Special case of above: $b_t \in \{c\}$

- Asymmetric eqa where i always matches and $-i$ quits whenever possible
- Eqa with delay
  - Players randomize quitting decisions w/ prob $\beta_i = \frac{c(1+\delta)}{\delta v_{-i}}$
  - Surplus fully dissipated
  - The stronger the opponent, the less likely that a player quits

Always a unique SPE. For large enough δ,

1. $v_1 = v_2 = \lambda$: player 1's initial bid induces player 2 to quit for sure.
    - as $\delta \to 1$, bid (and 1's payoff) tend to $\frac{\lambda}{2}$
2. $v_1 = 1 > v_2 = \lambda$: player 1 bids 0, player 2 quits for sure.
3. $v_1 = \lambda < v_2 = 1$: player 1 bids 0, player 2 covers and bids 0, player 1 quits

When unequal strength, all bids are 0 and "weak" player concedes a.s.a.p.–no rent dissipation.
When equal strength, first mover wins, but bids half valuation–partial dissipation.
No delay–maximum 2 periods.
A stronger opponent means more likely to quit.

1. Suppose $v_1 = v_2 = v$. Let $\bar{V}_i$ ($\underline{V}_i$) denote sup (inf) of i's payoffs over all subgames of all SPE when i makes the offer

Because anything greater than $\delta \bar{V}_i$ is necessarily accepted by player i, and anything less than $\delta \underline{V}_i$ is necessarily rejected, it must be that

$$\bar{V}_i = \underline{V}_i = \frac{v}{1+\delta}$$

so the unique eqm must have players bidding $b^* = \frac{\delta v}{1+\delta}$ and covering if and only if $b < b^*$.

2. Suppose $v_1 = 1 > v_2 = \lambda$. Must have
   i. $\underline{V}_1 \geq 1 - \delta \bar{V}_2$
   ii. $\bar{V}_1 \leq 1 - \delta \underline{V}_2$
   iii. $\bar{V}_2 \leq (\lambda - \delta \underline{V}_1)^+$
   iv. $\underline{V}_2 \geq \lambda - \delta \bar{V}_1$

   which yields $V_1 = 1 \wedge \frac{1-\delta\lambda}{1-\delta^2}$, $V_2 = \left(\frac{\lambda-\delta}{1-\delta^2}\right)^+$

Player i offers $b_i^* = \delta V_{-i}$ and covers if and only if $b_{-i} < \delta V_i$.

One player's valuation is known, the other's is uncertain.

1. Asymmetric eqa in which one player quits immediately
2. Eqa with delay (at most 2 phases):
    1. Initial phase: low type randomizes covering decision, high type covers with prob 1
        - as long as player covers, assign decreasing probability to low type opponent until some threshold unconditional probability reached.
    2. Second phase: low type gives up immediately, high type randomizes

As before, this involves

- multiplicity of eqa–range of payoffs
- delay–expected number of periods until concession grows unboundedly as $\delta \longrightarrow 1$

Wlog, player 1's valuation is known. Two cases:

1. Strong uninformed player: $v_1 = 1$; $v_2 = 1$ with probability $\mu \in [0, 1]$, λ otherwise.
2. Weak uninformed player: $v_1 = \lambda$; $v_2 = 1$ with probability $\mu \in [0, 1]$, λ otherwise.

Types of bids b_i:

1. Fully Deterrent (FD) if $-i$ quits with probability 1, independent of type
2. Partially Deterrent (PD) if $-i$ quits if type is λ, covers if type is 1
3. Zero Deterrent (0D) if $-i$ covers with probability 1, independent of type

A. Separating (S) if $-i$ assigns positive probability to both types, and b_i only submitted by one type
B. Pooling (P) if $-i$ assigns positive probability to both types, and b_i submitted by both types with prob 1
C. Semi-pooling (SP) if $-i$ assigns positive probability to both types, any other bid

One-sided Incomplete Information

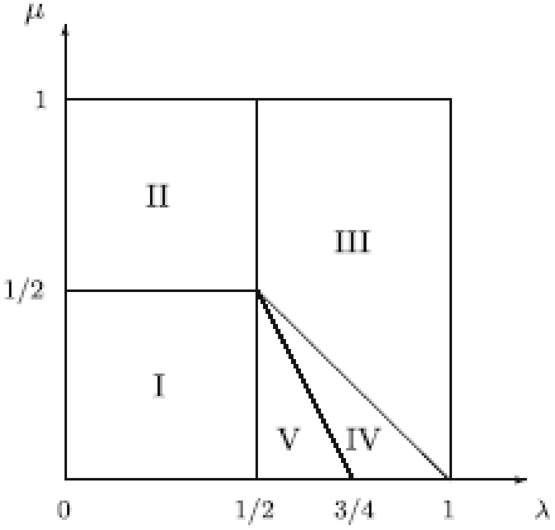
Strong uninformed player

5 classes of eqm strategies (depending on parameter space) for $\delta \to 1$

- Play never lasts more than 2 periods
  - Player 1 at least partially deters
- Possible inefficiency
  - If weak player 2 has the hand, either bids s.t. player 1 concedes, or bids 0 and concedes next period.

As μ and λ increase

- Player 1 more likely to bid high s.t. player 2 concedes (o/w takes chances that 2 is weak)
- Weak player 2 more likely to mimic bid of strong type
- Player 1's payoff decreases
- Player 2's payoff decreases if weak or if doesn't have hand
- Strong player 2's payoff increases with μ, ambiguous effect of λ



I: S, PD
II: S, FD
III: P, FD
IV: P, PD
V: SP, PD

One-sided Incomplete Information

Weak uninformed player

Unique eqm

For δ close to 1, player 1 always bids 0, which both types of player 2 always cover. Then player 2 bids 0, and player 1 concedes with probability 1.

Holds for all $\mu > 0$.

Compare to complete info with weak types: player 2 benefits from uncertainty.

Both valuations unknown.

1. Asymmetric eqa in which one player quits immediately
2. Eqa with delay (up to 3 phases):
    1. Initial phase: low types randomize covering decision, high types always cover
        - as long as player covers, assign decreasing probability to low type opponent until some threshold unconditional probability reached
    2. Second phase: that low type quits immediately, proceed as in one-sided incomplete information

As before, multiplicity, delay, dissipation.

Two-sided Incomplete Information

Variable Effort WoA

Even here the equilibrium is unique.
However, outcome depends intricately on parameters.

- Delay maximum of 3 periods
- Partial rent dissipation

Suppose low μ_1 and/or λ

- Low μ_2: 1 PD, high 2 covers and PD, high 1 deters 2
- Mid μ_2: low 1 randomizes, makes 2 pessimistic so he bids higher
- High μ_2: high 1 FD, low 1 either mimics or randomizes or bids nothing

Suppose high μ_1 and/or λ

- Low μ_2: 1 PD, high 2 covers and PD, high 1 deters 2
- Mid μ_2: high 1 randomizes, makes 2 optimistic so indifferent b/w FD or PD
- High μ_2: high 1 FD, low 1 either mimics or randomizes or bids nothing

Conclusion

Predictions of WoA sensitive to available actions.
When signalling available, equilibrium is unique, rent-dissipation only partial, weaker players concede more quickly than stronger players.
Predictions for jump-bidding in all-pay auctions:

- Jump bids used in unique SP (undefeated) eqm
- Dramatic reduction in auctioneer's expected revenue.
- In line with literature on winner-only pays ascending auctions with costly bidding (Daniel and Hirschleifer 1998).