Asset Pricing with Price Levels

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First draft: November 2019
This version: August 2020

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Abstract

We propose a novel way to study asset prices based on price distortions rather than abnormal returns. We derive the correct identity linking current mispricing to subsequent returns, generating a price-level analogue to the fundamental asset pricing equation, $E[M R^e] = 0$, used to study returns. Our GMM test reveals that the CAPM describes the cross-section of prices noticeably better than it describes expected short-horizon returns. Despite the improvement, significant mispricing remains. We show that book-to-market, quality, and size provide a parsimonious model of CAPM mispricing that both long-term buy-and-hold investors and researchers disciplining models from the price perspective should prioritize.

*Keywords: price level, long-horizon returns, mispricing metric, stochastic discount factor, CAPM*

*JEL classification: G12, G14, G32*
We propose a novel way to study models of asset market equilibrium. Our focus is on distortions in prices, taking a sharp break from the traditional emphasis on measuring expected return distortions, i.e. “abnormal returns.”

While abnormal return is certainly an important barometer of capital market efficiency (Fama (1970)), price distortions likely have more important real economic consequences as they drive the financing and investment decisions of firms.\(^1\) Furthermore, price distortions matter more than short-horizon abnormal returns for investors who commit their capital over a long investment horizon, such as firm managers, policy makers, and other long-term buy-and-hold investors (Cohen, Polk, and Vuolteenaho (2009)).\(^2\)

Price and expected return are synonymous in a model with constant expected returns. In that case, \(P_t = \sum_{j=1}^{\infty} E_t \left[ D_{t+j} \right] / (1 + R)^j\) with \(D\) denoting cash flows (dividends), so a distortion in expected one-period return \((R)\) is sufficient to infer a distortion in price \((P)\).\(^3\) However, expected returns are almost certainly time-varying (Campbell and Shiller (1988), Fama and French (1989), Cochrane (1992), Cochrane (2008), and van Binsbergen, Brandt, and Koijen (2012)). Therefore, ex-ante price distortions should be a function of the entire term structure of subsequent abnormal returns, not just the one-period abnormal return.

But is there an exact analytical way to aggregate subsequent abnormal returns over the long run correctly to arrive at ex-ante price distortion? Could such a formula be applied to data to reveal important facts about the cross-section of asset prices?

Our paper makes two contributions. First, we show that the answer to the first question is a surprising yes: there is a simple analytical expression relating ex-ante price distortion to subsequent abnormal returns. By correctly aggregating subsequent abnormal returns in order to measure ex-ante mispricing, our exact identity contrasts sharply with ad-hoc ways to aggregate abnormal returns over time, such as the cumulative abnormal return (CAR) measure widely used in em-

\(^1\)Papers studying the link between mispricing and firm activity include Stein (1996), Baker and Wurgler (2002), Baker, Stein, and Wurgler (2003), Shleifer and Vishny (2003), Cohen, Polk, and Vuolteenaho (2009), Polk and Sapienza (2009), van Binsbergen, Brandt, and Koijen (2012), and Whited and Zhao (2019).

\(^2\)See also Cochrane (2011): “Focusing on expected returns and betas rather than prices and discounted cashflows . . . makes much less sense in a world with time-varying discount rates” (pp.1063–1064).

\(^3\)That is, up to the asset duration that could differ across assets. Shiller (1984) and Summers (1986) argue that even in this case, persistent price distortion may not generate statistically discernible patterns in expected returns.
pirical studies. Second, we show how to apply our identity to evaluate an asset pricing model based on its ability to explain asset price levels in a manner analogous to the popular time-series regression approach for returns (Black, Jensen, and Scholes (1972)). The close analogy between our approach to prices and the existing approach to returns is attractive, as it allows one to study prices using the apparatus already developed for returns.

When presenting our results on prices, we are careful to recognize that the mispricing we detect could signal either a misspecification of our model of risk or the deviation of price from the intrinsic value of cash flows due to limits of arbitrage, as is always the case when interpreting abnormal returns.\footnote{This issue is, of course, the famous joint hypothesis problem emphasized in Fama (1970).} Our goal is not to resolve the debate on market efficiency—whether a given “return” anomaly represents mispricing or misspecification of risk—but to carry this debate over to the “price” dimension: Are return anomalies also price anomalies or purely transitory phenomena? Which model of risk explain the cross-section of prices and which characteristics help summarize price deviations from levels implied by the model?

The theoretical part of this paper derives our identity and discusses its implications. We show that “delta” ($\delta$), defined as the percentage deviation of price from value, equals the expected sum of subsequent abnormal returns or “alpha” ($\alpha$), discounted by the cumulative price-adjusted stochastic discount factor (SDF). In particular,

$$\delta_t = \frac{(P_t - V_t)}{P_t} = -\sum_{j=1}^{\infty} E_t \left[ \frac{M_{t,t+j}}{P_t} \frac{P_{t+j-1} R^e_{t+j}}{1 + R_{f,t+j}} \right]$$  \hspace{1cm} (1)$$

$$= -\sum_{j=1}^{\infty} E_t \left[ M_{t,t+j-1} \frac{P_{t+j-1}}{P_t} \frac{\alpha_{t+j}}{1 + R_{f,t+j}} \right]$$  \hspace{1cm} (2)$$

where $V_t = E_t \left[ \sum_{j=1}^{\infty} M_{t,t+j} D_{t+j} \right]$ is the asset’s intrinsic value defined as the present value of cash flows, $M_{t,t+j}$ is the cumulative candidate SDF associated with the model of risk (e.g., CAPM), $D$ is dividend, $P$ is price, and $R_{f,t+j}$, $R^e_{t+j}$, and $\alpha_{t+j}$ are the risk-free rate, excess return (above the risk-free rate), and conditional abnormal return from $t + j - 1$ to $t + j$, respectively.\footnote{$M_{t,t+j}$ is the ratio of marginal utilities in periods $t$ and $t + j$ in consumption-based asset pricing models with time-separable utility. The Capital Asset Pricing Model (CAPM) implies $M_{t,t+j} = \Pi_{j=1}^{\infty} (b_0 - b_1 R^m_{t+j})$ where $R^m$ is market return in excess of the risk-free interest rate and $(b_0, b_1)$ are parameters.} The assumption of no explosive bubbles is not restrictive, as it allows for most types of price deviation from value,
including permanent mispricing (e.g., $\delta_{t+j} = \delta \neq 0 \forall j$), which our identity can correctly detect.

The identity is intuitive. The economic surplus, relative to the SDF $M$, from a buy-and-hold strategy on an asset is the net present value of all subsequent abnormal returns. However, since abnormal return at time $t+j+1$ is a rate of return, we can express it in terms of monetary value at time $t+j$ by rolling it back one period using the risk-free rate and multiplying it by $P_{t+j}$:

$$V_t - P_t = E_t \left[ \sum_{j=0}^{\infty} M_{t,t+j} X_{t+j}^{\text{Abnormal}} \right],$$

where $X_{t+j}^{\text{Abnormal}} = P_{t+j} \frac{\alpha_{t+j+1}}{1+R_{t+j}}$ is the abnormal payoff at time $t+j$. Finally, one divides both sides by $P_t$ and changes sign to arrive at the expression for $\delta_t$ in equation (2).

Equation (1) implies that not just the magnitude, but also the time and state in which abnormal returns occur determine the extent of ex-ante mispricing. Abnormal returns occurring in a more distant future matter less for mispricing, since these abnormal returns are earned on the portion of the initial asset that excludes the market value of dividend payouts up until that point in the future.\footnote{In the formula, this conclusion follows from $E_t \left[ M_{t,t+j} \frac{P_{t+j}}{P_t} \right]$ converging to zero as $j \to \infty$ under the no-explosive-bubble condition.}

Furthermore, abnormal returns occurring after states with either high state price $M$ or high cumulative capital gain at time $t+j-1$ matter more at time $t$ because in such cases, the abnormal return is earned on a time $t+j-1$ price that matters more for $P_t$. In contrast, by putting equal weights on abnormal returns occurring in all states and periods, ad-hoc measures such as the CAR can lead to substantial estimation errors.\footnote{Appendices B and C show that CAR is an empirically and theoretically poor proxy for mispricing.}

Our identity implies a price-level analogue of the restriction we test for one-period returns, $0 = E [MR^e]$. Define $\delta (J)$ as the unconditional mean of the right-hand quantity in equation (1) replaced with a finite sum over $J$ months after portfolio formation:

$$\delta (J) = - \sum_{j=1}^{J} E \left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} R_{t+j}^e \right]$$

Choosing $J = 1$ and $J = \infty$ imply, respectively,

$$\delta (1) = - E \left[ M_{t+1} R_{t+1}^e \right]$$

$$\delta (\infty) = - \sum_{j=1}^{\infty} E \left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} R_{t+j}^e \right]$$
Restricting $\delta(1) = 0$ in equation (4) requires the expected one-month return to equal the risk premium, and a deviation from this relation is measured by the abnormal return $\alpha = -\delta(1)/E[MT_{t+1}]$. Similarly, restricting $\delta(\infty) = 0$ in equation (5) means that price equals intrinsic value, and a deviation from this relation is mispricing $\delta = \delta(\infty)$.

We propose estimating mispricing $\delta$ in the same way one estimates abnormal return $\alpha$ based on the time-series approach, as it allows the same test and SDF specification to apply to both the study of prices and returns. In the context of equation (4), one estimates alpha with respect to a factor model specifying $M$ using the sample analogue of $E[MT_{t+1}R_{t+1}^e]$, prescribing $M$ to yield zero abnormal returns on the factor portfolios in sample. Similarly, one estimates mispricing $\delta$ using the sample analogue of $-\sum_{j=1}^{J} E[MT_{t+j} + \frac{P_{t+j-1}}{P_{t}}R_{t+j}^{e}]$ for some sufficiently large $J$, prescribing $M$ to yield zero factor portfolio $\delta$s in sample. We find that the post-portfolio-formation horizon of $J = 15$ years (180 months) is sufficient to generate an accurate estimate of $\delta$.

The empirical part of the paper studies the cross-sectional variation in stock prices with respect to the CAPM. We use the CAPM as the candidate SDF not because we believe the CAPM to be the best model of risk, but because (i) its simplicity makes it easier to see how asset pricing with price levels differs from the conventional analysis of returns and (ii) it provides a foundation for multifactor refinements of the model based on price-level analysis. Since mispricing $\delta$ depends on the ratio of value to price, sorting stocks on the value-to-book ratio—proxied by a composite metric dubbed quality (Asness, Frazzini, and Pedersen (2019))—and the book-to-market ratio should lead to a powerful test. Our primary analysis therefore studies quality- and book-to-market-sorted portfolios, first in isolation and then in tandem via a double sort.

The CAPM explains the cross-section of prices fairly well despite its poor ability to explain returns. We first show that, consistent with previous findings, post-formation single-month returns on quality- or book-to-market-sorted decile portfolios are anomalously negatively related to risk measured by CAPM beta. In contrast, CAPM-implied risk does a particularly good job explaining the prices of quality-sorted portfolios and an adequate job explaining that of book-to-market-sorted portfolios (Figure 2). The results for the book-to-market are consistent with Cohen, Polk,
and Vuolteenaho (2009) while the results for quality appear to run contrary to Asness, Frazzini, and Pedersen (2019)’s claim that stock prices do not fully reflect variation in quality. Nevertheless, after double sorting stocks on both characteristics to generate 25 value-and-quality-sorted portfolios, we find significant variation in mispricing $\delta$ along both dimensions with a spread in CAPM mispricing across the high-quality, inexpensive (high book-to-market) portfolio and the low-quality, expensive (low book-to-market) portfolio of 57 percentage points. This difference is not only economically large but also statistically significant as the associated $t$-statistic is 3.22. Hence, the double sort generates the sort of variation a buy-and-hold investor should be interested in exploiting and represents a challenge for future asset pricing models that make understanding price-level risk a priority.

Turning to decile portfolios sorted on other characteristics (plotted in Figure 4), we document three main findings. First, one-month abnormal return $\alpha$s and mispricing $\delta$s exhibit the negative relation one might expect; positive abnormal returns typically indicate underpricing (the top-left quadrant of Figure 4) while negative abnormal returns typically indicate overpricing (the bottom-right quadrant). Second, investment, net issuance, and beta-sorted portfolios show the largest estimated $\delta$s, making them the prime price-level anomalies that, along with the aforementioned portfolios that are double-sorted on quality and book-to-market, new models of price-level risk should aim to explain. We also find that extreme accruals (Sloan (1996)) are significantly overpriced, although the magnitude of their mispricing is smaller. Finally, high (low) momentum stocks have large positive (negative) abnormal returns but appear to be overpriced (underpriced), suggesting that momentum profits are often earned from continued overreaction (Lou and Polk (2019)). Statistically, however, momentum’s estimated $\delta$s are marginally insignificant in our sample period. Although several characteristics appear to predict mispricing with respect to CAPM, three characteristics—book-to-market ratio, quality, and size—parsimoniously explain mispricings associated with these characteristics.

In summary, our paper is the first to show how to correctly aggregate abnormal returns into ex-ante price distortion. Empirically, we identify stock characteristics that are associated with the largest price distortions with respect to the CAPM and hence should be characteristics that buy-

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10 An analysis at the individual stock level in Figure B5 shows the same stark contrast between the return and price-level performance of the CAPM, implying that the finding is not simply a consequence of the particular quality-and book-to-market-sorted portfolios studied in Figure 1.
and-hold investors who are primarily concerned with market risk exposure should be interested in exploiting. Thus, our approach provides a new challenge to asset pricing models that are not just concerned with capturing variation in average returns over the short run but that also prioritize minimizing model-implied price distortions.

The organization of our paper is as follows. Section 1 reviews related literature. Section 2 presents our framework to price assets in terms of price levels. Section 3 presents data, econometrics, and our primary results related to quality and value. Section 4 extends our analysis to other characteristic sorts. Section 5 reduces the dimensionality of mispricing with respect to CAPM using three key characteristics. Section 6 concludes.

1 Literature review

The vast majority of prior research uses the abnormal return with respect to a factor model such as the CAPM as the metric with which to evaluate trading strategies. Despite the popularity of the abnormal return metric, a simple example shows that abnormal return could be a poor proxy for a price-level measure of model mispricing. Consider the hedge fund managers that, according to Brunnermeier and Nagel (2004), purchased technology stocks in the early stages of the dot-com boom and sold them before the crash. Though these positions and their associated characteristics may forecast positive abnormal returns relative to existing models such as the CAPM, the purchased stocks are likely to have been already overpriced, not underpriced relative to that model. Thus, a stock that is overpriced relative to the CAPM could generate positive abnormal returns in the short run, and vice versa.

Nevertheless, this illustration suggests that the initial model-specific mispricing of an asset may be recovered from the behavior of subsequent abnormal returns over the long run, since in the subsequent crash, the overpriced stocks would have earned negative abnormal returns. Our measure highlights the correct way to accumulate subsequent abnormal returns to measure initial mispricing.

Of course, there is a long history of accumulating realized abnormal returns in order to proxy for price-level deviations from a benchmark model in the corporate finance literature, namely
the well-known cumulative abnormal return (CAR) and buy-and-hold abnormal return (BHAR) methodologies. Work in this area includes Barber and Lyon (1997), Kothari and Warner (1997), Fama (1998), Lyon, Barber, and Tsai (1999), Brav (2000), and Bessembinder, Cooper, and Zhang (2018), among others. Though our method also accumulates future abnormal returns, by correctly discounting the stochastic payoffs associated with mispricing, our delta measure has the exact interpretation as the price deviation from the intrinsic value of cash flows, or equivalently, mispricing from the perspective of long-term shareholders.

Researchers have introduced alternative price-level measures of mispricing. Lee, Myers, and Swaminathan (1999), and others infer the intrinsic value of the firm from its fundamentals. In stark contrast, our formula is an identity linking mispricing to abnormal returns. Black (1986) comments that markets are efficient if prices are within factor of two of intrinsic value but provides no way in which to carefully measure such deviations. Nevertheless, his prior provides a useful benchmark for the economic significance of the price-level misspecifications we measure. Bai, Philippon, and Savov (2016) show that price has become more informative about intrinsic value over time based on the ability of a scaled price ratio to explain subsequent earnings.

The closest mispricing metric to ours is the price-level alpha construct of Cohen, Polk, and Vuolteenaho (2009) (CPV). Unlike that paper, our mispricing identity begins with a clear definition of mispricing that is linked to a specification of the SDF, does not require unobservable quantities such as risk exposures and volatility in the absence of mispricing, and does not rely on the Campbell and Shiller (1988) approximation. The first two differences ensure that $\delta$ is a correct price-level measure of model misspecification free of biases that arise from the approximation and the assumption about the unobserved second moments, and the last difference increases our statistical power compared to CPV, as we discuss in more detail in the appendix.

Our exact framework can be used to revisit prior observations about market efficiency. Fama (1970) defines a market as efficient if “prices ‘fully reflect’ all available information” but goes on to test the efficient market hypothesis using returns, finding that the market is semi-strong form efficient. Shiller (1984) writes, “because real returns are nearly unforecastable, the real price of stocks is close to the intrinsic value . . . is one of the most remarkable errors in the history of economic thought” (pp. 458–459). Summers (1986) provides a numerical example that illustrates
this argument, and Campbell (2017) shows how an expected return that follows a persistent AR(1) process may have low volatility but a large effect on the log dividend-price ratio.

Our identity provides a more sophisticated framework in which to understand Shiller’s point. It shows that mispricing can be large even if each post-formation alpha is small, as long as alphas are persistent. Furthermore, mispricing can be large even if alphas are small on average if alphas tend to comove strongly with $\phi$. The latter channel has been overlooked in the literature but can be quantitatively important: Cho (2020) provides empirical evidence that as arbitrageurs trade away the alphas of equity anomalies such as value and momentum, they can end up exposing these anomalies to the systematic risks they face as arbitrageurs. That is, in the presence of arbitrage with limited capital, initial mispricing in terms of alpha can evolve to become a risk premium associated with an endogenous beta.

Several recent papers tackle different but related topics in the growing literature on asset price distortions and long-term portfolio choice. van Binsbergen and Opp (2019) study a quantitative model of a production economy in which the cost of equity faced by firms may be distorted due to the mispricing and analyze the resulting implications for the real economy. By specifying the exact production technology and frictions in the economy, van Binsbergen and Opp (2019) are able to characterize the impact of expected return distortions on output and perform counterfactuals. In contrast, the novelty of our approach is that it allows the ex-ante price distortion to be estimated without having to specify the process that generates dividends. Hence, our approach is better suited for understanding which firms are mispriced with respect to a candidate SDF as well as providing a specification test of a particular SDF based on price distortions.\textsuperscript{11} Of course, the limitation of our approach relative to van Binsbergen and Opp (2019) is that while we provide an easy way to estimate asset price distortions, we cannot draw implications for more direct measures of allocative efficiency such as output.

Chernov, Lochstoer, and Lundeby (2018) propose a new asset-pricing test requiring the linear SDF specification of a factor model to explain both one-period and multi-period factor returns, documenting that popular linear factor models fail to price returns on their own factors accumulated over long horizons. For example, they show that the average four-year gross returns on

\textsuperscript{11}Appendix B provides further details on how our identity and methodology relates to the approach taken by van Binsbergen and Opp (2019).
the market, profitability, and investment factors have annualized misspecification errors that are roughly 7% in absolute magnitude relative to the prediction of the four-year SDF implied by the Fama-French (2015) five-factor model. Our goal of estimating asset price deviation from intrinsic value by correctly aggregating abnormal returns on a buy-and-hold strategy is distinct from their goal of generating high-power asset pricing tests based on long-horizon restrictions on managed portfolios. Importantly, the mispricing identity we obtain for a candidate SDF by iterating the law of motion for mispricing $\delta$ forward cannot be obtained in their analysis, which derives a long-horizon condition under the restriction that the short-horizon condition $E[M R^e] = 0$ holds every period following portfolio formation.

Cochrane (2014) shows how mean-variance characterizations can be applied to the stream of long-run payoffs or return opportunities even in a dynamic framework. His analysis shows that optimal dividend payoffs follow a relatively simple analytical form for a mean-variance optimizer that uses simple discounting to weight future utilities, even though the dynamic portfolio strategy that supports these payoffs may be complex. Our $\delta$ measure describes the present value of marginal utility gains from adding a particular stream of payoffs and therefore could be useful in describing the optimal portfolio choice for long-term investors.

2 A New Identity: Asset Pricing at Long Horizons

This section defines mispricing as a percentage deviation of price from the intrinsic value of cash flows implied by an asset pricing model: $\delta_t = (P_t - V_t)/P_t$ where $V_t = E_t \left[ \sum_{j=1}^{\infty} M_{t+j} D_{t+j} \right]$. It then presents this paper’s core theoretical result that, under mild assumptions, $\delta_t$ equals the sum of subsequent excess returns, discounted by the price-weighted cumulative SDF (Lemma 1):

$$\delta_t = -\sum_{j=1}^{\infty} E_t \left[ M_{t+j} \frac{R_{t+j}}{P_{t+j-1}} \right]. \quad (6)$$

The unconditional statement of the identity, $\delta = -\sum_{j=1}^{\infty} E \left[ M_{t+j} \frac{R_{t+j}}{P_{t+j-1}} \right]$, is a natural price-level counterpart to the conventional asset pricing equation for abnormal one-period return, $\alpha =$
\[ E \left[ \frac{M_{t+1}}{E[M_{t+1}]} R^e_{t+1} \right], \]

and motivates estimating unconditional mispricing \( \delta \) using the sample analogue

\[ \hat{\delta} = -T^{-1} \sum_{t=1}^{T} \sum_{j=1}^{J} M_{t,t+j} \frac{P_{t+j-1}}{P_t} R^e_{t+j} \]

for \( J = 180 \) months (15 years) in subsequent sections. Doing so under the restriction that \( M \) prices the factor portfolios perfectly in sample makes this estimation approach analogous to the time-series regression for returns in which the factor portfolios are assumed to have zero abnormal returns in sample.

### 2.1 The environment

Consider an asset with dividends (or coupons for bonds) \( \{D_{t+j}\}_{j=1}^{\infty} \). Our goal is to relate the mispricing (pricing error) of the asset at time \( t \) to its subsequent returns. We measure mispricing with respect to the (candidate) stochastic discount factor (SDF) \( \{M_{t+j}\}_{j=1}^{\infty} \), where we use \( M_{t,t+j} = \prod_{s=1}^{j} M_{t+s-1,t+s} \) to denote the cumulative SDF. The SDF we analyze could either be a candidate SDF to be compared to the one implied by market prices in the sense of Hansen and Jagannathan (1991, 1997), or it could be the SDF of a particular investor in a market in which the law of one price fails for some assets due to frictions (e.g., Garleanu and Pedersen (2011) and Geanakoplos and Zame (2014)). Note that the derivations below do not rely on \( M \) being the true SDF.

### 2.2 Intrinsic value and mispricing

The intrinsic value, \( V_t \), of the asset is simply the present value of all future dividends:

\[ V_t = \sum_{j=1}^{\infty} E_t \left[ M_{t,t+j} D_{t+j} \right] . \]  

We define mispricing, \( \delta_t \), as the deviation of price from value as a percentage of the current price:

\[ \delta_t = \frac{P_t - V_t}{P_t} . \]

Hence, \( \delta_t > 0 \) if the asset is overpriced, and \( \delta_t < 0 \) if it is underpriced. \( \delta_t \) can range from \(-\infty\) (if \( V_t > 0 \) and \( P_t = 0 \)) to \( 1 \) (if \( V_t = 0 \) and \( P_t > 0 \)), the opposite of the range for abnormal returns,
[−1, ∞).

2.3 Assumptions

To link \( \delta_t \) to subsequent returns, we make two relatively mild assumptions. The first is the existence of a risk-free asset that satisfies the fundamental asset pricing equation.

Assumption 1. There is a benchmark asset or a trading strategy such that its time \( t + j \) return, denoted \( R_{b,t+j} \), satisfies the fundamental asset pricing equation with respect to the candidate SDF \( M \) at time \( t + j - 1 \):

\[
E_{t+j-1} \left[ M_{t+j} \left( 1 + R_{b,t+j} \right) \right] = 1. \tag{9}
\]

A natural choice for the benchmark asset \( b \) is the risk-free asset proxied by the one-month Treasury bill rate, which the conventional return-based asset pricing literature uses to compute excess returns. The second assumption is a weak form of a no-bubble condition.

Assumption 2. The present value of the deviation of price and value at the limit \( j \to \infty \) is zero:

\[
\lim_{j \to \infty} E_t \left[ M_{t+j} \left( P_{t+j} - V_{t+j} \right) \right] = 0.
\]

This assumption is weaker than having two separate no-bubble conditions on price and value, \( \lim_{j \to \infty} E_t \left[ M_{t+j} P_{t+j} \right] = 0 \) and \( \lim_{j \to \infty} E_t \left[ M_{t+j} V_{t+j} \right] = 0 \), which imply Assumption (2). The assumption is not particularly restrictive, as it allows for most types of price deviation from value, including permanent mispricing (e.g., \( \delta_{t+j} = \delta \neq 0 \forall j \)), which our identity can correctly detect.

2.4 The law of motion for mispricing

Under our definitions and Assumption (1), mispricing follows a simple law of motion. Equation (7) and the law of iterated expectations implies the fundamental asset pricing equation holds for value:

\[
1 = E_t \left[ M_{t+1} \frac{V_{t+1} + D_{t+1}}{V_t} \right]. \tag{10}
\]

\[\text{12}^{12}\]We have also derived the sufficient conditions under which our price-level measure is finite.
Next, use equation (8) to substitute the empirically unobserved quantities $V_t$ and $V_{t+1}$ with $V_t = (1 - \delta_t) P_t$ and $V_{t+1} = (1 - \delta_{t+1}) P_{t+1}$ to obtain,

$$\delta_t = 1 - E_t [M_{t+1} (1 + R_{t+1})] + E_t \left[ M_{t+1} \frac{P_{t+1}}{P_t} \delta_{t+1} \right].$$

(11)

Finally, following Assumption (1), use $1 = E_t [M_{t+1} (1 + R_{bt+1})]$ to express mispricing $\delta_t$ at time $t$ in terms of excess return $R^e_{t+1} = R_{t+1} - R_{bt+1}$ and mispricing $\delta_{t+1}$ at time $t+1$:

$$\delta_t = -E_t [M_{t+1} R^e_{t+1}] + E_t \left[ M_{t+1} \frac{P_{t+1}}{P_t} \delta_{t+1} \right].$$

(12)

The law of motion in equation (12) is intuitive. Since $E_t [M_{t+1} R^e_{t+1}]$ is the conditional abnormal return at time $t+1$ adjusted for the gross risk-free rate ($E_t [M_{t+1} R^e_{t+1}] = (1 + R_{f,t+1})^{-1} \alpha_{t+1}$, where $\alpha_{t+1}$ is the abnormal return conditional on time-$t$ information), equation (12) says that underpricing (overpricing) at time $t$ is either “paid out” as a positive (negative) abnormal return or contributes to the remaining mispricing at time $t+1$. The discount factor on $\delta_{t+1}$ is the SDF times the capital gain, which is intuitive given that $\delta_{t+1}$ is normalized by $P_{t+1}$. Hence, $\delta_{t+1}$ matters more at time $t$ if it arises in a state in which $P_{t+1}$ is high (hence the capital gain term) or has a higher present value (hence the SDF term).

2.5 Relating mispricing to subsequent returns

Iterating the law of motion for mispricing (equation (12)) forward and using Assumption (2) to set $\lim_{j \to \infty} E_t [M_{t+1} D_{t+j}] = 0$ expresses mispricing as a discounted sum of future excess returns.

Lemma 1. (Ex-ante mispricing and subsequent excess returns). Let $V_t = \sum_{j=1}^{\infty} E_t [M_{t+j} D_{t+j}]$ be the intrinsic value of the asset defined as the present value of cash flows with respect to a candidate cumulative SDF $\{M_{t+j}\}$. Under Assumptions (1) and (2), ex-ante mispricing $\delta_t$ defined as the percentage difference between market price $P_t$ and intrinsic value $V_t$ is the negative of the sum of expected subsequent excess returns discounted by the price-weighted SDF:

$$\delta_t = \frac{P_t - V_t}{P_t} = -\sum_{j=1}^{\infty} E_t \left[ M_{t+j} \frac{P_{t+j}}{P_t} \frac{P_{t+j-1}}{P_t} R^e_{t+j} \right].$$

(13)
Note that this formula holds regardless of whether or not \( M \) is the correct SDF. For intuition, take a time \( t + j - 1 \) conditional expectation within the expectation and use \( E_{t+j-1} \left[ M_{t+j} R_{t+j}^e \right] = (1 + R_{f,t+j})^{-1} \alpha_{t+j} \) to write the identity in terms of abnormal returns.

**Corollary 1. (Ex-ante mispricing and subsequent abnormal returns).** Ex-ante mispricing \( \delta_t \) can be expressed using subsequent conditional abnormal returns:

\[
\delta_t = - \sum_{j=1}^{\infty} E_t \left[ \phi_{t,t+j-1} \frac{\alpha_{t+j}}{1 + R_{f,t+j}} \right], \quad \phi_{t,t+j} \equiv M_{t,t+j} \frac{P_{t+1}}{P_t}, \tag{14}
\]

where \( \alpha_{t+1} \) is the time \( t + 1 \) risk-adjusted abnormal return investor expects to earn conditional on time-t information and \( \phi_{t,t+j} \) is a discount factor on abnormal returns.

Several observations about mispricing in the price level \( \delta \) follow from equation (14). First, a simple formula relates ex-ante mispricing \( \delta \) to abnormal returns \( \alpha \).

**Remark 1.** Mispricing \( \delta \) is a stochastically discounted sum of subsequent abnormal returns.

The asset pricing literature recognizes that some aggregation of abnormal returns over a long horizon can proxy for price-level distortions and has used long-run return measures such as CAR or BHAR as that proxy in numerous applications. Our identity in equation (14) shows that though there is indeed a simple, analytical formula relating initial ex-ante mispricing to subsequent abnormal returns, the formula is clearly distinct from those existing long-run return measures. To the best of our knowledge, we are the first to supply the correct formula measuring price-level distortion.

What follows immediately from the summation formula in equation (14) is that the persistence of abnormal returns, not just the magnitude, matters for mispricing.

**Remark 2.** Holding all else fixed, mispricing \( \delta \) is larger if subsequent abnormal returns are persistent.

Others have pointed out that the persistence of abnormal returns should matter for price-level distortion (e.g., Cohen, Polk, and Vuolteenaho (2009), Cochrane (2011), van Binsbergen and Opp (2019)), and we confirm this point in our exact relation between mispricing and abnormal returns.
The last set of remarks highlights the importance of the stochastic discounting of abnormal returns. If $\delta_t$ is finite, the discount factor on the risk-free-rate adjusted abnormal return must fall over time. More formally, since the single-period component of the discount factor on abnormal returns is $M_{t+j}\frac{P_{t+j}}{P_{t+j-1}}$ and the fundamental asset pricing equation requires $E_t \left[ M_{t+j} \left( \frac{P_{t+j}}{P_{t+j-1}} + \frac{D_{t+j}}{P_{t+j-1}} \right) \right] = 1$ in the absence of abnormal returns, $M_{t+j}\frac{P_{t+j}}{P_{t+j-1}}$ in general must have an expected value less than 1 once the firm starts paying out dividends. This fact implies that the horizon at which abnormal returns are earned affects the magnitude of the initial mispricing.

**Remark 3.** Abnormal returns occurring sooner are in general associated with larger mispricing $\delta$.

Intuitively, as is the case with any present value formula, the net present value of a buy-and-hold strategy on the asset as a fraction of the initial price—which is what $\delta$ represents—depends less on abnormal returns earned far into the future. However, this simple logic is missing in long-run return measures that are widely-used; both CAR and BHAR do not distinguish abnormal returns earned in the near future from those earned far into the future.

The presence of the SDF $M$ in equation (14) implies that the state in which the abnormal return is earned matters.

**Remark 4.** Abnormal returns occurring in more valuable states are associated with larger mispricing $\delta$.

This point is perhaps the most novel insight of our identity, as implies that not just the “expectation,” but also the “covariance” matters for how we accumulate subsequent $\alpha$s into the initial $\delta$. For example, if the market factor is a priced factor, abnormal returns earned following a market crash imply a large deviation of intrinsic value from price, since most asset-pricing models intuitively view dividends or capital gains earned in such a state as more valuable. Practically speaking, this point implies that being able to predict an asset’s abnormal returns using past returns on a risk factor would have important implications when quantifying ex-ante mispricing.

Finally, capital gain also matters for the covariance component of mispricing.

**Remark 5.** Abnormal returns occurring after relatively large capital gains are associated with
larger mispricing $\delta$.

As a consequence, one cannot simply discount future abnormal returns with the cumulative SDF alone but instead must use a price-weighted cumulative SDF. Capital gain enters into the formula since the abnormal return at time $t + j$ is earned on the $t + j - 1$ price. Hence, the abnormal return matters more for mispricing today if it expected to be earned on a high future price. Practically speaking, this component of $\phi$ means that abnormal returns linked to long-run reversal (De Bondt and Thaler (1985)) count less towards the initial mispricing. This simple intuition has been largely overlooked in the literature’s search for a link between abnormal returns and mispricing.\footnote{Of course, Campbell and Shiller (1988)’s discount parameter implicitly captures the fact that future returns on high dividend-paying assets are worth less in present-value terms.}

To summarize, our identity in equation (14) highlights that abnormal returns occurring sooner, in more valuable states, or after relatively large capital gains are associated with larger price-level deviations. Specifically, the gross risk-free rate expresses the conditional abnormal return earned at time $t + j$ as a time $t + j - 1$ value, the time $t + j - 1$ price $P_{t+j-1}$ translates that abnormal return into time $t + j - 1$ abnormal cash flow, and the cumulative SDF expresses that abnormal cash flow in today’s value. Finally, the formula normalizes the present value of abnormal cash flows with today’s price $P_t$.

### 2.6 Long-horizon asset pricing equations

The identity allows us to develop asset pricing tests that apply to both prices and returns. Define $\delta(J)$ as the unconditional expectation of the right-hand quantity in equation (13) where the infinite sum is replaced with a finite sum over $J$ periods (months) after portfolio formation:

$$
\delta(J) = -\sum_{j=1}^{J} E \left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} R_{t+j}^e \right].
$$

(15)

This equation is a long-horizon generalization of the asset pricing equation for one-period returns. Using the covariance identity to decompose the right-hand side, we can also write

$$
\sum_{j=1}^{J} E \left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} \right] E \left[ R_{t+j}^e \right] = -\delta(J) + \sum_{j=1}^{J} \text{Cov} \left( -M_{t,t+j} \frac{P_{t+j-1}}{P_t}, R_{t+j}^e \right),
$$

(16)
where the quantities $\sum_{j=1}^{J} E\left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} R_{t+j}^e \right]$ and $\sum_{j=1}^{J} \text{Cov}\left( -M_{t,t+j} \frac{P_{t+j-1}}{P_t}, R_{t+j}^e \right)$ can be interpreted as the long-horizon discount rate and long-horizon risk.

To see why equation (15) is a natural long-horizon counterpart to the familiar one-period return pricing equation, take equation (16) for $J = 1$, divide both sides by $E[M_{t+1}]$, and recognize $\alpha = -\delta(1)/E[M_{t+1}]$ as the unconditional abnormal return to write

$$
E[R_{t+1}^e] = \alpha + \text{Cov}\left( -\frac{M_{t+1}}{E[M_{t+1}]}, R_{t+1}^e \right),
$$

which is the conventional asset pricing equation for expected one-period returns. As $J \to \infty$, on the other hand, equation (15) converges to the identity in Lemma 1 that expresses ex-ante mispricing in terms of subsequent returns over an infinite horizon. For intermediate values of $J$, the equation expresses the negative of the net present value, per dollar invested, of a strategy that buys the asset today and sell it after $J$ periods. Thus, equation (15) can be used to test the efficiency of asset prices for a multi-period investment horizon.

One can estimate $\delta(J)$ using the sample analogue of the right-hand side of equation (15):

$$
\hat{\delta}(J) = -T^{-1} \sum_{t=1}^{T} \sum_{j=1}^{J} M_{t,t+j} \frac{P_{t+j-1}}{P_t} R_{t+j}^e \equiv -\sum_{j=1}^{J} E_T \left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} R_{t+j}^e \right],
$$

where $E_T$ denotes the sample mean over $T$ portfolio formation months. Thus, to estimate mispricing $\delta = \delta(\infty)$, the variable of interest, we need to track post-formation capital gains and returns over an infinite horizon, which of course is not feasible in practice. Nevertheless, $\delta(180)$ based on a 15-year horizon post formation provides a good approximation of the infinite sum, since both the discount factor $M_{t,t+j-1} \frac{P_{t+j-1}}{P_t}$ and the conditional abnormal return $E_{t+j-1} \left[ M_{t+j} R_{t+j}^e \right]$ are small after 15 years for any portfolio formed at time $t$ (Appendix C).

In a time-series return regression, the estimated abnormal return is the difference between

\[ \hat{\delta}(180) \]

Specifically, Figure B1 simulates a linearity-generating process (Gabaix (2007)) for intrinsic value and price to show that the finite sum over 15 years is a near-perfect proxy for the true delta. Figure B4 shows that the finite sum $-\sum_{j=1}^{J} E_T \left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} R_{t+j}^e \right]$ indeed plateaus around $J = 15$ years in the data, implying that the remaining terms in the infinite sum are likely to make little difference to the infinite sum.
realized mean excess return and estimated risk premium. Similarly, it is useful to think of the estimated delta as the error term in the relation between discount rate and risk. Hence, we compute the sample analogues of the long-horizon discount rate and long-horizon risk quantities in equation (16) for \( J = 180 \) months and call them “price level” and “price-level risk”:

\[
\sum_{j=1}^{180} E_T \left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} \right] E_T \left[ R_{t+j}^e \right] = -\hat{\delta} \left( 180 \right) + \sum_{j=1}^{180} Cov_T \left( -M_{t,t+j} \frac{P_{t+j-1}}{P_t}, R_{t+j}^e \right),
\]

where the \( T \) subscript continues to indicate a sample moment. The price level summarizes all future discount rates on the asset over the next 15 years into a single expression. Ultimately our paper is concerned with understanding variation in prices and the extent to which observed risk explains this variation. Our price level measure, which can be compared across time and across assets and is less contaminated by factors such as expected future profitability than a scaled price ratio such as the market-to-book ratio, allows us to conceptualize and visualize this objective.

When plotting price level and price-level risk in a graph, we divide both by the estimated \( \sum_{j=1}^{180} E_t \left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} \right] \) of the market portfolio (and multiply price level by \(-1\)), which scales price level and price-level risk to have the same unit as one-month returns. This way, the price-level figures can be compared easily to the expected return figures.

Finally, it is useful to decompose price-level risk into the component arising from contemporaneous covariance with the SDF and the remaining intertemporal component:

\[
\sum_{j=1}^{180} Cov_T \left( -M_{t,t+j} \frac{P_{t+j-1}}{P_t}, R_{t+j}^e \right) = \sum_{j=1}^{180} E_T \left[ \phi_{t,t+j-1} \right] Cov_T \left( -M_{t+j}, R_{t+j}^e \right) - \sum_{j=1}^{180} E_T \left[ \left( \phi_{t,t+j-1} - E \left[ \phi_{t,t+j-1} \right] \right) M_{t+j} \left( R_{t+j}^e - E \left[ R_{t+j}^e \right] \right) \right],
\]

The first component is a simple discounted sum of future contemporaneous risk premia, and the

\[15\text{We prefer to think of the negative of } \sum_{j=1}^{180} E_t \left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} \right] E_T \left[ R_{t+j}^e \right] \text{ as price level so that price level falls as the discount rate rises, but the sign is not important in subsequent analysis.}\]
second component corrects the first for the fact that future risk premia can covary with past $\phi$. That is, for a long-term investor, an asset is risky not only if it has a contemporaneous negative covariance with the SDF, but also if it covaries negatively with the cumulative SDF.

### 2.7 The generalized method of moments estimator

We estimate mispricing $\delta$ with the 180-month sample analogue $\hat{\delta} (180)$ in GMM. For comparison, we also estimate $\hat{\delta} (1)$ for 1-month returns, which measures the extent to which expected one-month returns are abnormal.\(^{16}\) Our GMM moment for each test portfolio is

$$
\delta (b) = - \sum_{j=1}^{J} E_T \left[ M_{t+j} (b) \frac{P_{t+j-1} R_{t+j}}{P_t} \right],
$$

where $b$ denotes the loadings that candidate risk factors have on the SDF.

To be consistent with the implicit assumption in conventional asset pricing regressions with returns, we model the SDF as having constant coefficients on linear factors $f: M_{t+j} = b_0 - b_1 f_{t+j}$. We estimate the SDF parameters $b$ by requiring the SDF to price the factors perfectly in sample, which is analogous to a time-series asset pricing regression using returns where the factor portfolios have zero abnormal returns by definition.

Once $b$ is estimated, we use the asymptotic distribution of GMM errors $\delta (\hat{b})$ to obtain standard errors and test statistics. We estimate the spectral density matrix using the Bartlett kernel used by Newey and West (1987) with a bandwidth of $J$ months to account for serial correlation due to overlapping samples as well as contemporaneous correlation across the portfolios. The variance-covariance matrix of pricing errors is given by

$$
V_\delta = \frac{1}{T} \left( I - D (D'D)^{-1} D' \right) S \left( I - D (D'D)^{-1} D' \right),
$$

where $S$ is the estimated spectral density matrix and $D$ is the matrix representing the estimated derivative of the moments with respect to changes in the parameters. The variance-covariance matrices of other statistics are obtained analogously.

\(^{16}\) $\hat{\delta} (1)$ estimates $\delta (1)$, a scaled multiple of the estimated abnormal return $\alpha$ in a conventional return regression: $\delta (1) = E [M_{t+1}] \alpha$. 
We use five statistics to assess the ability of the asset pricing model to explain price levels.

1. **Hi - Lo**: Estimated difference in $\delta$s between the two extreme decile portfolios, which we use as the primary test statistic with which to reject or not reject an asset pricing model.

2. **Slope**: The slope of the estimated relation between estimated price level and price-level risk. We compute the $p$-value against the null that there is a 45-degree line relation between price level times $-1$ and price-level risk; the sign change on price level is necessary, since according to our definition higher price-level risk should imply lower price level.

3. **$\Delta$**: Weighted mean absolute (pricing) error, $\Delta \equiv \sum_{i=1}^{n} w_i \left| \hat{\delta}_i \right|$, where $w_i$ is the average market capitalization weight of portfolio $i$ in the market, measures the percentage of capital in the stock market that is being misallocated to the mispricing component of prices. To see this, simply write the time-specific $\Delta$ as the sum of the absolute differences between the firm’s intrinsic value and market value ($\sum_{i=1}^{n} |P_{i,t} - V_{i,t}|$) divided by the sum of the market values ($\sum_{i=1}^{n} P_{i,t}$):

$$\Delta_t = \frac{\sum_{i=1}^{n} |P_{i,t} - V_{i,t}|}{\sum_{i=1}^{n} P_{i,t}} = \sum_{i=1}^{n} \frac{P_{i,t}}{\sum_{i=1}^{n} P_{i,t}} |P_{i,t} - V_{i,t}| = \sum_{i=1}^{n} w_{i,t} |\delta_{i,t}|,$$

where $P_{i,t}$ and $V_{i,t}$ respectively measure the total market capitalization and total intrinsic value of the portfolio and $n$ is the number of stocks or portfolios that comprises the market. Hence, $\Delta$ equals 0 under the joint hypothesis of the correct asset pricing model with no mispriced capital at the portfolio level.\(^{17}\)

4. **MSE**: Mean squared (pricing) error, $N^{-1} \sum_{i=1}^{N} \hat{\delta}_i^2$, a measure that is analogous to the Gibrans, Ross, and Shanken (1989) statistic in time-series return regressions.

5. **MAE**: Mean absolute (pricing) error, $N^{-1} \sum_{i=1}^{N} \left| \hat{\delta}_i \right|$, a measure that is similar to the weighted mean absolute error $\Delta$ above but does not discriminate among the test assets based on their economic importance.

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\(^{17}\) We calculate the appropriate $p$-values for $\Delta$ by simulating $\delta$s under the null (based on the asymptotic distribution given above by $V_{\delta}$).
3 Asset Pricing Tests Using Price Levels

We examine how well the simple unconditional CAPM explains the prices of stock portfolios sorted on two natural characteristics for price-level tests: the book-to-market ratio ($B/M$) and quality. The former directly measures book-to-price while the latter is a proxy for the (intrinsic) value-to-book ratio ($V/B$). Compared to how poorly it explains the cross-section of returns of portfolios based on a univariate sort on either ratio, The CAPM does a surprisingly better job explaining the corresponding cross-section of prices. We use this result to illustrate how asset pricing with price levels works and why it can generate drastically different results from asset pricing with returns. We then show that a double sort on $B/M$ and our proxy for $V/B$ nonetheless leads to larger price-level errors.

3.1 Data

We combine monthly stock price data from the Center for Research in Security Prices (CRSP) and annual accounting data from CRSP/Compustat Merged (CCM) to create our basic merged dataset. We use one-month Treasury bill rates from Kenneth French’s data library (originally from Ibbotson Associates) as the risk-free rate and the market excess return from the same data library as the market factor.

To estimate price-level errors of characteristic-sorted portfolios, we need post-formation returns over a long horizon of 180 months (15 years) after the initial portfolio formation. That is, we form value-weight decile portfolios each month at $t$ based on the NYSE decile cutoffs and compute the post-formation returns on these portfolios over $t+1, \ldots, t+180$. Post-formation returns at $t+j$ for the portfolio formed at $t$ are buy-and-hold returns that do not reinvest dividends into the same or different stocks. In summary, our data is three dimensional, as we illustrate in Table B1: we have post-formation returns for 10 different portfolios (or 25 for two-way sorted portfolios), for $T$ different portfolio formation periods, and for $J$ different post-formation periods. Our baseline data use 1957m6–2004m12 as the portfolio formation periods ($T = 558$ months).

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18These $t+j$ returns are equivalent to returns earned by forming a new portfolio every month at $t+j−1$ based on initial time-$t$ weights adjusted by the cumulative capital gain from time $t$ to time $t+j−1$. The rebalancing based on cumulative capital gain is the correct approach for our purpose of inferring the initial price level of the portfolio based on subsequent returns, since it mirrors how the returns earned by investing the dividend payments for an individual asset do not enter into our formula. See the exact argument in Appendix C.
and $J = 180$ post-formation months, which imply post-formation returns over 1957m7–2019m12. Our portfolio sort begins in 1957m6, since this is when we can first compute all accounting-based characteristics based on the annual Compustat dataset.

The decile portfolios we form are labeled so that decile 10 (1) represents the one with the highest (lowest) abnormal returns according to the existing literature. Hence, for the book-to-market sort, 10 represents the extreme value portfolio and 1 the extreme growth portfolio, while for the size sort, 10 represents the portfolio of the smallest stocks and 1 represents the portfolio of the largest stocks. Table 1 provides descriptive statistics for the portfolios formed from a univariate sort on each of the nine characteristics we consider in the rest of the paper: book-to-market ratio, quality, size, momentum, profitability, investment, beta, net issuance, and accruals. Appendix A provides further details on the data construction.

### 3.2 Basic risk adjustment with the market factor

We use the CAPM to model the SDF, as it provides the basic risk adjustment upon which multifactor models are built. Since the monthly risk-free rate and the square of the monthly Sharpe ratio are both small, the intercept term of the CAPM SDF is approximately one (e.g., Cochrane (2005)).

$$M_{t+j} = 1 - b_1 R_{m,t+j}^e$$

This assumption reduces our estimation problem to choosing the single parameter $b_1$ that makes the price-level error of the market portfolio zero in sample.

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19 Our analysis in subsequent chapters will provide a direction for future research on multifactor models of price levels. In addition, it would be natural to consider the intertemporal CAPM specification of Campbell, Giglio, Polk, and Turley (2018), which incorporates stochastic volatility into the ICAPM framework of Campbell and Vuolteenaho (2004) to significantly reduce the pricing errors relative to the CAPM in standard SDF return tests.

20 If the CAPM prices both the market portfolio and the risk-free asset, $b_0 = \frac{1}{1 + R_f} + \frac{S_m}{1 + R_f}$, where $S_m$ is the Sharpe ratio. Hence $\frac{1}{1 + R_f}$ is slightly below 1 and $\frac{S_m}{1 + R_f}$ slightly above 0, giving us $b_0 \approx 1$.

21 The advantage of this parameter reduction is that it prevents us from using another asset such as a proxy for the risk-free interest rate to pin down the intercept term in the SDF. Estimating the additional intercept term by requiring the SDF to price the 1-month Treasury rate perfectly in sample leads to similar results, however.
3.3 Quality and B/M as the primary sorting characteristics

A powerful test on price levels requires test assets that a priori are likely to exhibit large variation in mispricing $\delta$. To do this, recall that $\delta$ measures the percentage deviation of value from price, which can be rewritten in terms of intrinsic value over book equity $V/B$ and book equity over market price $B/M$ (where $V$, $M$, and $B$ are measured per share so that $M = P$):

$$\delta_t = 1 - \frac{V_t}{P_t} = 1 - \frac{V_t}{B_t} \times \frac{B_t}{M_t}. \quad (25)$$

Hence, holding the other ratio fixed, a variation in either $V/B$ or $B/M$ implies variation in $\delta$. This motivates us to sort stocks based on these two ratios.

Since the value-to-book ratio $V/B$ is unobserved, we follow Asness, Frazzini, and Pedersen (2019) to use a composite z-score measure called quality as a proxy for $V/B$. Rewriting Gordon’s growth model in the absence of mispricing as

$$\frac{V}{B} = \frac{profitability \times payout ratio}{required returns - growth}, \quad (26)$$

they use quality measured by a z-score that rewards profitable, fast-growing, safe, and high-payout stocks to proxy for the value-to-book ratio:

$$quality \equiv z(profitability, growth, safety, payout ratio) \propto \frac{V}{B} + noise \quad (27)$$

The book-to-market ratio $B/M$ may contain different information from the value-to-book ratio $V/B$, since a distortion in the discount rate due to abnormal returns affects $B/M$ but not $V/B$:

$$\frac{B}{M} = \left( \frac{profitability \times payout ratio}{abnormal returns + required returns - growth} \right)^{-1}. \quad (28)$$

Since the $B/M$ ratio accounts for variation in long-horizon discount rates due to abnormal returns whereas the $V/B$ ratio does not, double sorting on quality and $B/M$ should lead to larger $\delta$ variation and hence an even more powerful test. In particular, as we illustrate graphically in

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Note that when writing equation (26), Asness, Frazzini, and Pedersen (2019) assume growth to be exogenous and thus independent of the payout ratio.
low-quality stocks that are expensive (low quality and $B/M$) are likely to be overpriced and have a positive $\delta$, whereas high-quality stocks that are inexpensive (high quality and $B/M$) are likely to be underpriced with a negative $\delta$.

We note in passing that, in the context of Gordon’s model, $B/M$ depends not only on discount rates, but also on factors such as growth and profitability. Indeed, Cohen, Polk, and Vuolteenaho (2003) show that roughly 80% of the cross-sectional variation in book-to-market ratios reflects predictable differences in expected future profitability. This fact makes $B/M$ or its reciprocal $M/B$ a less appealing measure of price level than our identity-motivated measure of price level or mispricing.

### 3.4 Explaining the prices of portfolios sorted by quality or B/M

To what extent does the CAPM explain the cross-section of prices of portfolios formed by a univariate sort on either quality or the book-to-market ratio? Specifically, does the CAPM explain price levels better than it explains returns, and if so, why?

To provide a reference point, we begin with conventional asset pricing tests using returns. The left two panels in Figure 2 show that the CAPM does a very poor job explaining the cross-section of returns on decile portfolios sorted on quality or book-to-market ratio. High quality stocks earn higher returns than low quality stocks despite having a lower market beta (Asness, Frazzini, and Pedersen (2019)). Similarly, High $B/M$ or “value” stocks earn higher returns than low $B/M$ or “growth” stocks despite having a lower market beta (e.g., Fama and French (1992)). Hence, both quality and $B/M$ sorts lead to a cross-sectional relation between risk and returns that is opposite to what the CAPM predicts.

Asset pricing tests with price levels generate meaningfully different results. The right two panels in Figure 2 plot the cross-sectional relation between price level and price-level risk but with a scaling factor that divides both quantities by the value of $\sum_{j=1}^{180} E \left[ M_{t+j} P_{t+j-1}/P_t \right]$ for the market and multiplying the price level by $-1$ for an easier comparison with the left two panels. Long-horizon discount rates and long-horizon risk that scaled price level and scaled price-level risk respectively capture display a positive cross-sectional relation with a fitted line reasonably close to the 45 degree line, suggesting that the CAPM does a decent job describing these portfolios’
price-level risk that buy-and-hold investors would care about.

Tables 2 and 3 report the estimated $\delta$s and test statistics along with standard errors (in parentheses) and $p$-values (in square brackets) for $J$: \{1mo, 1yr, 3yrs, 5yrs, 10yrs, 15yrs\} with $J = 1$ month generating conventional time series return regression results and $J = 15$ years (180 months) proxying price-level results given by $J \to \infty$. The intermediate values of $J$ allow us to see how the performance of the asset pricing model changes as the return horizon increases gradually from 1 month to 15 years.

Table 2 shows that high-quality stocks are undervalued and low-quality stocks are overvalued from the perspective of CAPM investors with a short investment horizon of $J = 1$ month or 1 year. However, for $J = 10$ or 15 years, the estimated $\delta$s are essentially zero for all quality-sorted portfolios and imply that the market price correctly accounts for the quality difference. Our conclusion, based on an identity that gives an exact expression for ex-ante mispricing, is contrary to the conclusion drawn by Asness, Frazzini, and Pedersen (2019) based on the analysis of cumulative five-year returns as well as a cross-sectional regression of the $M/B$ ratio on quality. All test statistics we consider show that price-level risk with respect to CAPM does a reasonable job explaining price-level variation in quality-sorted portfolios, with the slope coefficient in particular being fairly close to 1.

Table 3 reports the result for $B/M$-sorted portfolios. Compared to quality-sorted portfolios, $B/M$ portfolios show larger price-level errors with respect to the CAPM. Growth stocks are estimated to be 31.4 percentage points more overpriced than value stocks, and this difference is statistically significant at the 5% level, though marginally so. However, it is obvious from several metrics that again price levels look less anomalous than returns. The cross-sectional slope coefficient changes from $-2.399$ to $0.543$ as we go from returns to price levels and is not statistically different from one. Mean squared error and mean absolute value of errors are large enough to reject the null of a correct model at the 5% significance level but cannot be rejected at the stricter 1% level, contrary to these statistics for the return-based tests that are highly statistically significant.

The mechanism: Why does asset pricing with price levels look different?

Even if both returns and betas of extreme decile portfolios converge after portfolio formation
(Keloharju, Linnainmaa, and Nyberg (2019)), we should continue to see a negative cross-sectional relation between the discounted sums of risks and returns that both quality- and $B/M$-sorted portfolios exhibit in the return regression (left two panels in Figure 2). That is, a mere convergence in returns and risks after portfolio formation cannot explain our results. In order for the sign of the negative cross-sectional relation to flip from negative to positive as we move from the return to the price-level perspective, at least one of the following should occur for the extreme decile portfolios. Either the return spread crosses (flips sign) at some point after portfolio formation; the beta spread crosses; or the intertemporal adjustment component of price-level risk in equation (B2) has an opposite cross-sectional pattern to the contemporaneous risk component, thereby undoing the puzzling pattern that the high-return decile 10 has a lower market beta than the low-return decile 1 immediately following portfolio formation.

The primary reason why price-level risks of quality-sorted portfolios explain their price levels well is that the return spread between extreme decile portfolios crosses. Figure 3 shows that high (low) returns earned by high (low) quality stocks are highly transitory and that the high-minus-low quality return spread turns negative from around one year following the portfolio formation and stays negative except for the blip around year 9. Another important contribution comes from the intertemporal adjustment. Figure B2 shows that although the contemporaneous risk component of price-level risk is substantially larger in low-quality stocks, the intertemporal component of price-level risk is larger in high-quality stocks than in low-quality stocks and partly offsets the contemporaneous component of risk. That is, high-quality stocks are riskier for investors with a long investment horizon, since their returns tend to be low after the stock market experiences a series of negative shocks; this would be the case, for instance, if high quality stocks’ market beta tends to come from the exposure of their returns to market-level cash flow news. For example, Campbell, Polk, and Vuolteenaho (2010) shows that accounting variables that are often associated with quality forecast variation in this component of beta.

In contrast, price-level risks of $B/M$-sorted portfolios explain their price levels reasonably well, as the beta spread between high- and low-$B/M$ portfolios crosses. Figure 3 shows that the market beta of the high-$B/M$ portfolio is low at the time of the portfolio formation but rises above that of the low-$B/M$ portfolio, which steadily declines over the post-formation periods. This fact, first documented in Cohen, Polk, and Vuolteenaho (2009), makes value stocks riskier for a long-term
Quality and B/M double sort

As anticipated, double sorting stocks based on quality and B/M to generate 25 portfolios leads to larger variation in \( \delta \)s than a univariate sort (Table 4). Furthermore, the variation in \( \delta \) across the two dimensions of the table is consistent with our conjecture in Figure 1.

Price-level error declines as we move from top to bottom or left to right, which amounts to holding one characteristic fixed while varying the other. We estimate expensive low-quality stocks (top left) to be overpriced by 24\% (\( t \)-statistic of 3.22) and cheap high-quality stocks to be underpriced by 33\% (\( t \)-statistic of \(-2.91\)) with respect to CAPM. Moving diagonally from the top left to the bottom right generates the largest variation in \( \delta \)s. We estimate low-quality, low-B/M stocks to be 57 percentage point more overpriced than high-quality, high-B/M stocks with a \( t \)-statistic of 3.22.

All test statistics strongly reject the null hypothesis, and the mean squared error and mean absolute error are now significant at the 1\% level. Indeed, the price-level variation of Table 2 is exactly the sort of variation a long-horizon investor should be interested in exploiting and represents a challenge for future asset pricing models that make understanding price-level risk a priority.

4 Are Return Anomalies Price-level Anomalies?

We next study the extent to which the CAPM explains price-level variation associated with eight additional characteristics known to be associated with cross-sectional variation in average returns: size, momentum, profitability, investment, beta, net issuance, and accruals. The first four represent a set of prominent return anomalies, and the latter three are chosen for their potential conceptual link to price-level distortions. As noted in Section 3, the CAPM tends to explain price-level variation better than it explains short-horizon return variation. Among the seven characteristics studied, investment, beta, and net issuance appear to be price-level anomalies. We find that size is also associated with somewhat large price-level errors despite it being a less important characteristic.

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\(^{23}\) We discuss in Appendix B how our mispricing metric \( \delta \) improves on the mispricing metric of Cohen, Polk, and Vuolteenaho (2009).
when predicting returns. We also highlight ways in which our price-level results are (in)consistent with conventional interpretations of these return anomalies. Before discussing each characteristic in greater detail, Figure 4 shows that anomalies with positive (negative) CAPM alphas do tend to be underpriced (overpriced) with respect to the CAPM.

4.1 Prominent return anomalies

We begin with the four prominent return anomalies. Fama and French (2015) argue that profitability, investment, and size are characteristics that are important in summarizing the cross-section of returns, and price momentum has been a prominent return anomaly since Jegadeesh and Titman (1993). The panels on the left side of Figure 5 show that all but the size sort generate a negative cross-sectional relation between average return and market beta, implying that these characteristics are associated with cross-sectional variation in average return that cannot be explained by market beta. To what extent are these prominent return characteristics associated with variation in price levels unrelated to CAPM price-level risk?

Size and momentum

Size and momentum are interesting to study from the price-level perspective, given that momentum strongly predicts the cross-section of average returns but is a rather transitory firm characteristic while size weakly predicts the cross-section of average returns but is a rather persistent firm characteristic. In particular, Cohen, Polk, and Vuolteenaho (2009) highlight that signal persistence is an important consideration when moving from the conventional return perspective to the price-level perspective, a point that Cochrane (2011) subsequently emphasizes.

“For example, since momentum amounts to a very small time-series correlation and lasts less than a year, I suspect it has little effect on long-run expected returns and hence the level of stock prices. Long-lasting characteristics are likely to be more important. Conversely, small transient price errors can have a large impact on return measures” (p.1064).

Figure 5 confirms Cochrane’s conjecture, as the economic magnitudes of the variation in both price levels and 8s associated with momentum are small due to its transient nature. Somewhat
surprisingly, however, the small standard errors associated with momentum portfolio δs nevertheless result in some test statistics rejecting the CAPM (Table 5). Consistent with Cochrane’s view, the persistence of the size characteristic does generate large price-level variation as well as statistically significant mean squared and mean absolute error. However, price-level errors associated with size tend to occur mostly in small portfolios, resulting in a size-weighted mean absolute error that is statistically insignificant.

**Profitability and investment**

Table 5 documents that profitability-sorted portfolios are associated with only small price-level errors. In addition, we find a near-45-degree cross-sectional relation between price levels and price-level risks (Figure 5) that contrasts sharply with the corresponding negative cross-sectional relation found in returns. Test statistics indicate that the CAPM does a good job explaining the price levels of stocks with different degrees of profitability.

Why do profitability-sorted portfolios behave well in our price-level analysis, when they are known to be anomalous in conventional return regressions? Not surprisingly, the explanation for the CAPM’s success in pricing the price levels of profitability portfolios is the same as that for portfolios sorted on quality, since quality is a composite metric that depends importantly on profitability. Specifically, the returns of profitability portfolios reverse soon after portfolio formation. High (low) returns earned by more (less) profitable firms are highly transitory, and more profitable firms begin earning lower returns than less profitable firms soon after portfolio formation, consistent with the difference in their market betas. As a consequence, the behavioral interpretation of abnormal returns earned on profitability-based trades is more consistent with investor overreaction to news about profitability rather than fundamental undervaluation (overvaluation) of profitable (unprofitable) companies (Novy-Marx (2013) and Asness, Frazzini, and Pedersen (2019)). The result is also consistent with the observation that cross-sectional variation in the marginal product of capital, which our profitability measure could proxy for, does not necessarily imply misallocation of capital (David, Schmid, and Zeke (2019)).

Price-level variation is more anomalous for investment-sorted portfolios. Price levels of portfolios sorted by asset growth cannot be explained by variation in price-level risk, and we estimate
high-investment firms to be 19.8% more overpriced relative to low-investment firms (Figure 5 and Table 5). Furthermore, out of the four prominent return anomalies we consider, investment is the only characteristic featuring a statistically significance difference in δs across the extreme decile portfolios. Moreover, among the characteristics studied here, the investment sort generates the lowest p-value for all of the test statistics except for mean absolute error and has the largest size-weighted mean absolute error ∆.

Hence, between profitability and investment, the latter is by far the more important price-level anomaly that demands a more sophisticated SDF than the unconditional CAPM considered here. Our finding also echoes the conclusion of Polk and Sapienza (2009) that the investment characteristic is an important signal of mispricing for stocks with respect to the CAPM.

4.2 Characteristics conceptually related to mispricing

Other stock characteristics are interesting to analyze using our mispricing measure either due to their mechanical link to price-level risk (beta) or their conceptual association with mispricing (net issuance and accruals) vis-à-vis the endogenous choices of managers. We explain the conceptual link that each characteristic has to mispricing δ and study the extent to which the characteristic is associated with price-level variation that cannot be explained by exposure to market risk that the CAPM captures.

**Beta**

Equation (20) shows that price-level risk can be decomposed into two terms, i) a discounted sum of contemporaneous covariances between the SDF M and returns and ii) the sum of intertemporal covariances between the price-adjusted SDF φ and returns. The persistence of market beta implies that market beta sorts have the potential to generate large variation in the first of these two terms, the contemporaneous risk component of CAPM price-level risk. In particular, if the resulting variation in price-level risk is not compensated with corresponding variation in price levels, large δs could arise.

Figure 5 shows that the large variation in risk generated by the beta sort does not lead to a correspondingly large variation in price levels. Hence, the low beta anomaly in returns (Black,
Jensen, and Scholes (1972); Frazzini and Pedersen (2014)) is also a price-level phenomenon, not just a temporary distortion that is offset by a subsequent reversion in risk or return. Table 5 shows an estimated difference in $\delta$s of 28.7 percentage points across the high- and low-beta portfolios. The size-weighted absolute error $\Delta$ is large, and all test statistics are highly significant with $p$-values lower than 0.1%. Overall, these results suggest that the low-beta anomaly is an important price-level phenomenon that needs to be explained by new models of price-level risk that add additional or alternative risk factors to the SDF.

*Net share issuance*

A series of papers argue that share repurchase (issuance) indicates undervaluation (overvaluation) as perceived by firm managers (Loughran and Ritter (1995); Ikenberry, Lakonishok, and Vermaelen (1995)). Thus, to the extent that firm managers are long-term investors in the firm, net issuance could be a useful proxy for mispricing with respect to the CAPM.

Figure 5 reproduces the previous finding that low net issuance generates higher returns than high net issuance and that market beta is actually lower for low net issuance stocks than high net issuance stocks, resulting again in the anomalous negative relation between risk and returns seen in the beta sort. In fact, this slope is negative for all four characteristics considered in this part of the analysis. For a buy-and-hold mean-variance investor, low (negative) issuance firms are slightly riskier than high issuance firms, restoring a positive slope. Nevertheless, the mispricing $\delta$s are large: Table 5 shows that high issuance firms are estimated to be modestly overpriced by 7.7%, but low (negative) issuance firms are estimated to be underpriced by 21.5%. That is, share repurchase may be a stronger signal for underpricing than share issuance is for overpricing.

*Accruals*

Earnings management proxied by accruals (Sloan (1996)) is an interesting phenomenon to analyze with our price-level identity, as it is typically motivated as being the result of companies with adverse operating results managing earnings to inflate the value of their firm as perceived by outsiders. Thus, if the firms are successful in managing earnings, high accruals may proxy for the
gap between the market price set by investors and the intrinsic value perceived by firm managers. Table 5 shows results consistent with this interpretation of accruals: high accruals firms are estimated to be overpriced by 8.7%. Interestingly, firms with low accruals are also a bad investment for long-term investors, as they are estimated to be overpriced by 6.5%. However, the relatively small Δ implies that the aggregate mispricing of capital with respect to the CAPM associated with the accruals phenomenon is small, as that mispricing more strongly affects smaller firms.

**Takeaway**

Overall, we find that the return anomalies that the extant literature typically motivates as representing significant price-level distortion do tend to be associated with significant δs, at least in one of the extreme decile portfolios. However, the magnitudes and the economic importance of mispricing associated with accruals are smaller than those associated with market beta and net share issuance.

## 5 Parsimonious CAPM Mispricing: B/M, Quality, and Size

A natural question to ask is the extent to which the anomaly-sorted portfolios in Table 5 provide incremental information about price-level mispricing relative to the variation that we have already identified in Table 4. Though sorts are a simple non-parametric way to measure variation in average returns linked to one or two characteristics, they become less useful as the number of characteristics grows. As a consequence, we first introduce a firm-level measure of delta and then exploit cross-sectional regressions in order to carefully disentangle the incremental contribution of various anomalies relative to our book-to-market and quality measures.

Our measure of firm-level mispricing follows naturally from the way we measure portfolio-level delta in the previous sections. In particular, we calculate \( \delta_{i,t}(b) = -\sum_{j=1}^{180} E_T \left[ M_{t,t+j} \frac{P_{i,t+j-1} - R^e_{i,t+j}}{B_{t,j}} \right] \)

where \( M_{t+j} = 1 - b_1 R^e_{m,t+j} \) with \( b_1 \) chosen to price the market portfolio in sample. Of course, firm-level delta observations at a particular time \( t \) do not include all of the terms in the 15-year sum as the lifespan of the typical publicly-traded firm is relatively short. However, to ensure that there

\[ \text{Conceptually, after a firm delists, its alphas are effectively set to zero for the remainder of the 15-year window.} \]
is no bias to our results because of firms leaving the sample, we include delisting returns using the approach suggested by Shumway (1997) and Shumway and Warther (1999).

We then forecast variation in firm-level delta using a monthly panel regression with time fixed effects in order to isolate cross-sectional variation in CAPM mispricing linked to the characteristics we study.\(^{25}^{26}\) This approach also allows us to exploit the cardinal aspect of the characteristics. However, to ensure that our results are not driven by outliers, we winsorize the data at the 2\(^{nd}\) and 98\(^{th}\) percentiles as well as log transform the variables when appropriate (following previous research, e.g. Fama and French (1992)). Finally, we normalize all variables using their full-sample standard deviation to aid in interpretation. We use a panel approach rather than Fama and MacBeth (1973) regressions so that we can easily adjust our standard errors for both cross-sectional and time-series correlation in the regression residuals, as the latter arises primarily because of the overlapping nature of \(\delta\)’s for a particular firm.

We first confirm the results in Table 4 that book-to-market and quality together describe variation in \(\delta\). Columns (1) and (2) in Table 6 show that the link between each of these characteristics and mispricing is weaker when measured separately than when both are included in the regression together in column (3). Indeed, quality has no ability to forecast \(\delta\) without controlling for book-to-market; the \(t\)-statistic associated with quality increases from −0.34 to −2.72 once the book-to-market ratio is included in the regression.

We then add size to the specification, first excluding and then including quality in columns (3) and (4) respectively. In either case, size helps explain variation in CAPM mispricing. Moreover, the results confirm that even in the presence of the size characteristic, including quality strengthens the link between value and \(\delta\) as the coefficient on the log book-to-market equity ratio more than

\(^{25}\)We confirm in Figure B5 that the stark contrast between the return and the price-level performance of CAPM seen in portfolios sorted on characteristics extends to individual stocks.

\(^{26}\)As \(\delta\) is the time-\(t\) sum of expected subsequent excess returns discounted by the price-weighted SDF, we denote it as a time-\(t\) measure. Nevertheless, in these regressions, we are effectively use time-\(t\) characteristics to forecast realized delta which explicitly depends on the \(t+1:t+180\) realizations of not only the returns / capital gains on the stock in question but also the returns on both the market portfolio and the risk-free rate. This forecasting exercise is not truly out-of-sample for two reasons. For one thing, the first-stage of our analysis uses the full sample to estimate the parameter \(b_1\) that determines exactly the way market returns enter the CAPM SDF. Moreover, by estimating the relation between \(\delta\) and time-\(t\) characteristics in a panel regression, the resulting estimates rely on the full sample as well. However, in unreported analysis that measures the link between \(\delta\) and firm characteristics in a truly out-of-sample way, we continue to find that value, quality, and size forecast \(\delta\).
doubles in absolute magnitude.

One potential concern with our finding that the book-to-market ratio and quality are particularly effective in identifying mispricing with respect to the CAPM is that our results may be driven by the historic boom and bust in prices that occurred in the late 1990s and early 2000s. Certainly, the tech stocks of the late 1990s are the quintessential high valuation / low quality firm that our analysis indicates is overpriced. As a response to this concern, column (6) in Table 6 excludes the years 1998–2001 from the regression. Our estimates barely change indicating that this subperiod does not drive our analysis.

In the rest of the columns in Table 6, we add the remaining anomalies considered in Table 5. We add these variables one by one to avoid any multicollinearity issues that might arise from the fact that quality is a composite variable. These regressions reveal that the book-to-market ratio, quality, and size provide the most parsimonious model of firm-level mispricing as these three variables subsume the ability of any of the non-size anomalies studied to explain cross-sectional variation in $\delta$. In particular, none of the variables under consideration are statistically significant at conventional levels, and only momentum and net issuance have $t$-statistics greater than 1.7 (1.84 and 1.71 respectively). Moreover, the economic significance of the point estimates is always lower than any of the point estimates associated with the book-to-market, quality, and size characteristics. Table B3 in the appendix repeats the analysis of Table 6 using the decile number as the explanatory variable and confirms that all of our conclusions are robust to this approach.

6 Conclusion

Our novel model misspecification measure, delta, precisely links future alpha to current price-level deviations. In stark contrast to existing measures, our approach correctly recognizes that abnormal returns occurring sooner, in more valuable states, or after relatively large capital gains should be associated with larger price-level distortions. Our primary tests reveal that though the CAPM does a relatively good job describing the cross-section of average price levels of both

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27 Note that we only exclude these years in the second stage of our analysis as we naturally include all data when estimating each firm’s realized $\delta$ in the first stage.

28 The coefficient on the momentum characteristic is positive, implying that high (low) momentum stocks could be overpriced (underpriced), despite having positive (negative) short-horizon abnormal returns with respect to CAPM.
book-to-market- and quality-sorted portfolios in isolation, portfolios formed from double sorts on these two variables generate significant variation in mispricing. We show that investment, beta, and net equity issuance sorts also produce significant price-level distortions relative to the CAPM while momentum-, profitability-, and accruals-sorted portfolios present much less of a challenge. A regression analysis reveals that a three-characteristic model containing book-to-market, quality, and size provides a parsimonious description of the cross-section of firm-level CAPM mispricing.

As a consequence, our novel mispricing measure and the associated results provide better identification of the stocks that a buy-and-hold mean-variance investor should find attractive/unattractive. Moreover, our approach highlights where new models that aim to explain both short- and long-run patterns in markets should focus. Indeed, by providing an exact metric of the extent to which a candidate asset-pricing model explains variation in prices, we aim to advance future research in both asset pricing and corporate finance. For the former, our economically-important price-level metric could provide a more useful lens through which to distinguish among risk-based, behavioral-based, and institutional-friction-based explanations for well-known empirical patterns in markets. For the latter, our measure of mispricing with respect to a risk model may refine the results of a large literature (e.g. Baker and Wurgler (2002) and Shleifer and Vishny (2003)) that aims to link a firm’s investment and financing decisions to mispricing but often instead simply assumes that the book-to-market equity ratio reflects mispricing.
References


Tables and Figures

Table 1: Descriptive Statistics

The table describes key aspects of portfolios formed on the nine stock characteristics we use to study CAPM mis-pricing. Columns 2–3 and 4–5 describe the two extreme-decile value-weight portfolios for each characteristic where a larger decile number means a larger value of the characteristic. For each of these dynamic portfolios, we calculate $R_e$, the average monthly return in excess of the risk-free rate, and $\alpha_{CAPM}$, the corresponding CAPM abnormal return. We report heteroskedasticity-robust standard errors in parentheses. Column 6 details Persistence, the value-weighted probability that the characteristic decile of a stock in the portfolio changes after a year. The sample period is 1957m6–2004m12. See Appendix A for a detailed description of the way we construct these nine characteristics.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Decile 1 (Low)</th>
<th>Decile 10 (High)</th>
<th>Persistence</th>
</tr>
</thead>
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<tr>
<td></td>
<td>$R_e$</td>
<td>$\alpha_{CAPM}$</td>
<td>$R_e$</td>
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<tr>
<td>Quality</td>
<td>0.34</td>
<td>-0.24</td>
<td>0.70</td>
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<tr>
<td></td>
<td>(0.25)</td>
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<td>(0.19)</td>
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<tr>
<td>Book-to-market</td>
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<td>0.90</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.08)</td>
<td>(0.21)</td>
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<tr>
<td>Profitability</td>
<td>0.39</td>
<td>-0.21</td>
<td>0.61</td>
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<tr>
<td></td>
<td>(0.25)</td>
<td>(0.11)</td>
<td>(0.20)</td>
</tr>
<tr>
<td>Investment</td>
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<td>0.16</td>
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</tr>
<tr>
<td></td>
<td>(0.21)</td>
<td>(0.09)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>Momentum</td>
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<td>-0.83</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(0.17)</td>
<td>(0.25)</td>
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<tr>
<td>Size</td>
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<tr>
<td></td>
<td>(0.26)</td>
<td>(0.14)</td>
<td>(0.18)</td>
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<tr>
<td>Beta</td>
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<td>(0.11)</td>
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<td>Net issuance</td>
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<td>0.47</td>
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<tr>
<td></td>
<td>(0.19)</td>
<td>(0.09)</td>
<td>(0.23)</td>
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<tr>
<td>Accruals</td>
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<td>0.08</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
<td>(0.10)</td>
<td>(0.26)</td>
</tr>
</tbody>
</table>
Table 2: Pricing Quality-sorted Portfolios: Returns vs. Prices

The table shows that the CAPM does a good job describing the cross-section of prices of portfolios sorted on quality (the last row) but a poor job describing the cross-section of one-month returns (the first row). Quality is the composite metric introduced by Asness, Frazzini, and Pedersen (2019) to proxy for the ratio of intrinsic value-to-book ratio (V/B). We form ten value-weighted portfolios by sorting stocks based on quality. We form these portfolios in 1957m6–2004m12 and track post-formation returns up to 15 years, covering 1957m7-2019m12. In the first “return” row, $\delta$ measures $-1$ times the average one-month abnormal return (over the gross one-month risk-free rate):

$$\delta(1) = -E[M_{t+1}R_{t+1}].$$

Positive (negative) $\delta$ here means negative (positive) one-month abnormal return. The reported $\delta$s in the last row are estimated values of mispricing defined as

$$\delta = E\left[\frac{n-Y_t}{T}\right] \approx \delta(180) = -E\left[\sum_{j=1}^{180} M_{t+j} \frac{R_{t+j} - R_{t}}{T} \right],$$

where $t$ denotes the portfolio formation month and $j$ denotes the number of post-formation months. In the remaining rows, the reported $\delta$ estimates illustrate how the estimated $\delta(J) = -E\left[\sum_{j=1}^{J} M_{t+j} \frac{R_{t+j} - R_{t}}{T} \right]$ changes as $J$ takes values less than 180. We use the SDF implied by the unconditional CAPM, $M_{t+j} = \Pi_{s=1}^{j} (1 - b_1 R_{t+s})$, where $b_1$ is chosen such that the market portfolio has a zero in-sample $\delta$ at the horizon being studied. This estimate automatically implies $\delta$ estimates for the decile portfolios which are reported in Columns 2–6 for the lowest (“Lo”), second lowest (“2”), second highest (“9”), and highest (“Hi”) quality stock portfolios as well as the difference in $\delta$s between the two extreme deciles. The last column reports the estimate of $b_1$ along with the associated monthly Sharpe Ratio of the market. We report in parentheses standard errors estimated based on the Bartlett kernel used by Newey and West (1987) with a bandwidth of $J$ months. The test statistics in Columns 7–10 are the slope of the line through price levels (times $-1$) and price-level risks of the decile portfolios (slope = 1 under the null), size-weighted mean absolute error, mean squared error, and equal-weighted mean absolute error. We report $p$-values in square brackets.

<table>
<thead>
<tr>
<th>$J$</th>
<th>100 × $\delta$ (stderr)</th>
<th>Test statistics [p-value]</th>
<th>Parameter</th>
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<tr>
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<td>1mo (&quot;return&quot;)</td>
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</tr>
<tr>
<td></td>
<td>Lo</td>
<td>2</td>
<td>9</td>
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<td>(0.07)</td>
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<td>(0.74)</td>
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<td>(2.01)</td>
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<td>(1.62)</td>
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</tbody>
</table>
The table shows that the CAPM does an adequate job describing the cross-section of prices of portfolios sorted on the book-to-market equity ratio (the last row) but a poor job describing the cross-section of one-month returns (the first row). We form ten value-weighted portfolios by sorting stocks based on the book-to-market ratio. We form these portfolios in 1957m6–2004m12 and track post-formation returns up to 15 years, covering 1957m7-2019m12. In the first “return” row, \( \delta \) measures \(-1\) times the average one-month abnormal return (over the gross one-month risk-free rate):

\[
\delta (1) = -E \left[ M_{t+1} R_{t+1}^e \right].
\]

Positive (negative) \( \delta \) here means negative (positive) one-month abnormal return. The reported \( \delta \)s in the last row are estimated values of mispricing defined as

\[
\delta = E \left[ \frac{P_t - V_t}{P_t} \right] \approx \delta (180) = -E \left[ \sum_{j=1}^{180} M_{t+j} \frac{P_{t+j} - 1}{P_t} R_{t+j}^e \right],
\]

where \( t \) denotes the portfolio formation month and \( j \) denotes the number of post-formation months. In the remaining rows, the reported \( \delta \) estimates illustrate how the estimated \( \delta (J) = -E \left[ \sum_{j=1}^{J} M_{t+j} \frac{P_{t+j} - 1}{P_t} R_{t+j}^e \right] \) changes as \( J \) takes values less than 180. We use the SDF implied by the unconditional CAPM, \( M_{t+j} = \Pi_{s=1}^{j} \left( 1 - b_1 R_{m,t+s} \right) \), where \( b_1 \) is chosen such that the market portfolio has a zero in-sample \( \delta \) at the horizon being studied. This estimate automatically implies \( \delta \) estimates for the decile portfolios which are reported in Columns 2–6 for the lowest (“Lo”), second lowest (“2”), second highest (“9”), and highest (“Hi”) book-to-market stock portfolios as well as the difference in \( \delta \)s between the two extreme deciles. The last column reports the estimate of \( b_1 \) along with the associated monthly Sharpe Ratio of the market. We report in parentheses standard errors estimated based on the Bartlett kernel used by Newey and West (1987) with a bandwidth of \( J \) months. The test statistics in Columns 7–10 are the slope of the line through price levels (times \(-1\)) and price-level risks of the decile portfolios (slope = 1 under the null), size-weighted mean absolute error, mean squared error, and equal-weighted mean absolute error. We report \( p \)-values in square brackets.

<table>
<thead>
<tr>
<th>( J )</th>
<th>100 × ( \delta ) (stderr)</th>
<th>Test statistics ( [p-value] )</th>
<th>Parameter ( b_1 [S_m] )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lo</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>1mo</td>
<td>0.18</td>
<td>-0.02</td>
<td>-0.40</td>
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<td>1yr</td>
<td>1.97</td>
<td>0.30</td>
<td>-4.43</td>
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<td>3yrs</td>
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<td>-13.44</td>
</tr>
<tr>
<td>10yrs</td>
<td>9.28</td>
<td>4.15</td>
<td>-18.44</td>
</tr>
<tr>
<td>15yrs</td>
<td>12.41</td>
<td>3.61</td>
<td>-19.92</td>
</tr>
</tbody>
</table>
Table 4: Pricing Quality-and-B/M-sorted Portfolios

The table shows that mispricing relative to the CAPM is large for portfolios double-sorted on quality and the book-to-market equity ratio. We form 25 value-weighted portfolios by sorting stocks based on quality within each book-to-market equity quintile. We form portfolios in 1957m6–2004m12 and track post-formation returns up to 15 years, covering 1957m7–2019m12. The reported $\delta$s are estimated values of mispricing defined as $\delta = -E \left[ \sum_{j=1}^{J} M_{t,+j} \frac{P_{t,+j-1}}{P_{t-1}} R_{t+1} \right]$, where $t$ denotes the portfolio formation month, $j$ denotes the number of post-formation months, and $J = 180$ months. We use the SDF implied by the unconditional CAPM, $M_{t,+j} = 1 - b_1 R_{m,t,+j}$, where $b_1$ is chosen such that the market portfolio has a zero in-sample $\delta$. We report in parentheses standard errors estimated based on the Bartlett kernel used by Newey and West (1987) with a bandwidth of $J$ months. The test statistics are the slope of the line through price levels (times $-1$) and price-level risks of decile portfolios (slope = 1 under the null), size-weighted mean absolute error, mean squared error, and equal-weighted mean absolute error. We report $p$-values in square brackets.

<table>
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<th>Quality</th>
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<th>3</th>
<th>4</th>
<th>5</th>
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<td>0.171</td>
<td>0.074</td>
<td>0.060</td>
<td>0.007</td>
<td>0.228</td>
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<td>Book-to-market ratio</td>
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<td>(0.065)</td>
<td>(0.085)</td>
<td>(0.102)</td>
<td>(0.113)</td>
<td>(0.063)</td>
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<tr>
<td>2</td>
<td>0.096</td>
<td>0.080</td>
<td>-0.012</td>
<td>-0.107</td>
<td>-0.146</td>
<td>0.243</td>
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<tr>
<td></td>
<td>(0.036)</td>
<td>(0.050)</td>
<td>(0.042)</td>
<td>(0.024)</td>
<td>(0.027)</td>
<td>(0.045)</td>
</tr>
<tr>
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<td>0.047</td>
<td>-0.030</td>
<td>-0.087</td>
<td>-0.121</td>
<td>-0.227</td>
<td>0.274</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.021)</td>
<td>(0.013)</td>
<td>(0.012)</td>
<td>(0.032)</td>
<td>(0.041)</td>
</tr>
<tr>
<td>4</td>
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<td>-0.142</td>
<td>-0.139</td>
<td>-0.167</td>
<td>-0.205</td>
<td>0.166</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.059)</td>
<td>(0.031)</td>
<td>(0.030)</td>
<td>(0.055)</td>
<td>(0.045)</td>
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<td>-0.199</td>
<td>-0.253</td>
<td>-0.332</td>
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<tr>
<td></td>
<td>(0.044)</td>
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<td>(0.061)</td>
<td>(0.114)</td>
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<tr>
<td>1-5</td>
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<tr>
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<table>
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<th></th>
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<th>MSE</th>
<th>MAE</th>
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<td>0.11</td>
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<tr>
<td>(stderr)/[$p$-value]</td>
<td>(0.176)</td>
<td>[0.003]</td>
<td>[0.033]</td>
<td>[0.004]</td>
<td>[0.002]</td>
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</table>
Table 5: Pricing Anomaly-sorted Portfolios: Returns vs. Prices

The table reports estimated mispricings ($J = 15\text{yrs}$) vs. abnormal returns ($J = 1\text{mo}$) with respect to the CAPM for portfolios sorted on prominent return anomaly characteristics or characteristics conceptually linked to mis-pricing. For each characteristic, we form ten value-weighted portfolios in 1957m6–2004m12 and track post-formation returns up to 15 years, covering 1957m7–2019m12. Decile 10 (1) denotes stocks with the highest (lowest) value of the characteristic. The reported $\delta$s for $J = 15\text{yrs}$ are estimated values of mispricing defined as $\delta = -E \left[ \sum_{j=1}^{\infty} M_{t+j} \frac{P_{t+j}}{P_t} - 1 \right]$, where $t$ denotes the portfolio formation month, and $j$ denotes the number of post-formation months. The reported $\delta$s for $J = 1\text{mo}$ are the estimated errors from a conventional time-series asset pricing regression based on one-month returns. We use the SDF implied by the unconditional CAPM, $M_{t+j} = 1 - b_1 R_{m,t+j}$, where $b_1$ is chosen such that the market portfolio has a zero in-sample $\delta$. We report in parentheses standard errors estimated based on the Bartlett kernel used by Newey and West (1987) with a bandwidth of $J$ months. The test statistics are $-1$ times the slope of the line through price levels and price-level risks of decile portfolios (Slope $= 1$ under the null), size-weighted mean absolute error ($\Delta$), mean squared error (MSE), and equal-weighted mean absolute error (MAE). We report $p$-values in square brackets.

<table>
<thead>
<tr>
<th>Sort</th>
<th>J</th>
<th>100 × $\delta$ (stderr)</th>
<th>Test statistics</th>
<th>[p-value]</th>
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<td></td>
<td></td>
<td>Lo</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>Size</td>
<td>1mo</td>
<td>-0.24</td>
<td>-0.24</td>
<td>0.01</td>
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<td></td>
<td></td>
<td>(0.08)</td>
<td>(0.06)</td>
<td>(0.07)</td>
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<tr>
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<tr>
<td></td>
<td></td>
<td>(4.26)</td>
<td>(3.00)</td>
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<td></td>
<td></td>
<td>(0.14)</td>
<td>(0.10)</td>
<td>(0.09)</td>
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<tr>
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<td></td>
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<td>(4.52)</td>
<td>(2.86)</td>
<td>(2.64)</td>
</tr>
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<td>0.14</td>
<td>-0.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.10)</td>
<td>(0.07)</td>
<td>(0.07)</td>
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<td>-5.67</td>
<td>-8.03</td>
<td>4.08</td>
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<td>(5.07)</td>
<td>(5.86)</td>
</tr>
<tr>
<td>Investment</td>
<td>1mo</td>
<td>-0.15</td>
<td>-0.22</td>
<td>0.11</td>
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<tr>
<td></td>
<td></td>
<td>(0.08)</td>
<td>(0.07)</td>
<td>(0.09)</td>
</tr>
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<td></td>
<td>15yrs</td>
<td>-10.54</td>
<td>-10.87</td>
<td>9.10</td>
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<tr>
<td></td>
<td></td>
<td>(4.38)</td>
<td>(2.86)</td>
<td>(4.01)</td>
</tr>
<tr>
<td>Beta</td>
<td>1mo</td>
<td>-0.24</td>
<td>-0.33</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.10)</td>
<td>(0.09)</td>
<td>(0.07)</td>
</tr>
<tr>
<td></td>
<td>15yrs</td>
<td>-16.19</td>
<td>-17.53</td>
<td>9.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.03)</td>
<td>(3.27)</td>
<td>(2.21)</td>
</tr>
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<td>Net issuance</td>
<td>1mo</td>
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<td>-0.22</td>
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<td>(0.07)</td>
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<td>-21.55</td>
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<td>(6.82)</td>
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<td>(2.26)</td>
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</tr>
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<td></td>
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<td>(0.09)</td>
<td>(0.09)</td>
</tr>
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<td>15yrs</td>
<td>6.46</td>
<td>-2.11</td>
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<tr>
<td></td>
<td></td>
<td>(2.51)</td>
<td>(2.51)</td>
<td>(0.86)</td>
</tr>
</tbody>
</table>
Table 6: **Forecasting Delta with Characteristics: Firm-level Regressions**

The table uses firm-level panel regressions with time fixed effects to show that the book-to-market equity ratio, quality, and size characteristics provide the most parsimonious model of firm-level mispricing as these three variables subsume the ability of any of the non-size anomalies studied to forecast cross-sectional variation in $\delta$. The dependent variable is the estimated firm-specific $\delta$ measured over the subsequent 180 months while the independent variables are a variety of firm stock characteristics. To ensure that our results are not driven by outliers, we winsorize the data at the 2nd and 98th percentiles as well as log transform the variables when appropriate. Specifically, we log transform size, book-to-market, momentum, and investment prior to winsorization. We adjust the regression residual weights so that the cross-sections are equally-weighted across time and the weights within each cross-section are proportional to a stock’s market capitalization. We normalize all variables using their full-sample standard deviation to aid in interpretation. We report $t$-statistics in parentheses that are based on standard errors clustered by stock and time, and we denote significance at the 10%, 5%, and 1% levels using *, **, and ***.

<table>
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<th>(1)</th>
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<th>(5)</th>
<th>(6)</th>
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<td>-0.15***</td>
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<td></td>
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<td>-0.08***</td>
<td>-0.07**</td>
<td>-0.08**</td>
<td>-0.07**</td>
<td>-0.08**</td>
<td>-0.07**</td>
<td>-0.07**</td>
<td>-0.07**</td>
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<td></td>
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<td><strong>Size</strong></td>
<td>0.08***</td>
<td>0.06***</td>
<td>0.06***</td>
<td>0.06***</td>
<td>0.07***</td>
<td>0.06***</td>
<td>0.06***</td>
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<td>0.000</td>
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<td><strong>Exclude 1998–2001</strong></td>
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<td>492564</td>
<td>495584</td>
<td>495466</td>
<td>428195</td>
<td></td>
</tr>
</tbody>
</table>
Figure 1: **Double Sorting on Quality and B/M: Illustration**

This diagram illustrates how a double sort on quality and the book-to-market equity ratio should generate a large cross-sectional variation in mispricing $\delta$ if quality proxies for the intrinsic value-to-book equity ratio. Quality is a composite measure that proxies for the ratio of intrinsic value-to-book ratio.
The left panels report the cross-sectional relation between realized mean excess return and mean excess return predicted by the CAPM for 10 quality- or B/M-sorted sorted portfolios formed in 1957m6–2004m12. The right panels report the cross-sectional relation between estimated price levels and price-level risk with respect to the CAPM. The right panels are scaled to have comparable units to one-month returns: Scaled price level (scaled price-level risk) is \(-1 \times \text{price level (price-level risk)} / \sum_{j=1}^{180} \mathbb{E}_T \left[ R_{t+j} \right] \) of the market portfolio, where price level and price-level risk correspond to the portfolio’s \(-1 \sum_{j=1}^{180} \mathbb{E}_T \left[ M_{t+j} \frac{P_{t+j-1}}{P_t} \right] E_T \left[ R_{t+j}^e \right] \) and \(\sum_{j=1}^{180} \text{Cov}_T \left( -M_{t+j} \frac{P_{t+j-1}}{P_t}, R_{t+j}^e \right) \) such that the estimated mispricing is the sum of the two expressions. The 45-degree dash line indicates the perfect cross-sectional relation between risk and discount rates.

Figure 2: Explaining Returns vs. Prices: Quality and Book-to-Market
Figure 3: **Post-formation Behavior of Return and Risk: Quality and B/M**

The left panel reports the post-formation mean excess returns of the two extreme decile portfolios, and the right panel reports their post-formation market betas. The extreme decile portfolios are formed in 1957m6–2004m12 by sorting stocks based on quality or the book-to-market equity ratio, and we track post-formation returns over the subsequent 15 years (1957m7–2019m12).
Figure 4: Are Return Anomalies Price Anomalies?

The figure plots the estimated abnormal returns ($\alpha$) and mispricing ($\delta$), with respect to the CAPM, of extreme decile portfolios sorted on quality, book-to-market, and seven additional firm characteristics. The figure shows that positive abnormal returns usually indicate underpricing (bottom-right) while negative abnormal returns usually indicate overpricing (top-left). However, momentum and profitability are important exceptions.
Figure 5: Explaining Returns vs. Prices: Other Return Anomalies

The left panels report the cross-sectional relation between realized mean excess return and mean excess return predicted by the CAPM for 10 decile portfolios formed in 1957m6–2004m12. The right panels report the cross-sectional relation between estimated price levels and price-level risk with respect to the CAPM. Scaled price level is higher if long-horizon discount rates are higher, contrary to our original definition of price level. The 45-degree dash line indicates the perfect cross-sectional relation between risk and discount rates. See the description in Figure 2 for further details.
Explaining Returns vs. Prices: Other Return Anomalies

The left panels report the cross-sectional relation between realized mean excess return and mean excess return predicted by the CAPM for 10 decile portfolios formed in 1957m6–2004m12 (1981m12–2004m12 for analyst forecast). The right panels report the cross-sectional relation between estimated price levels and price-level risk with respect to the CAPM. Scaled price level is higher if long-horizon discount rates are higher, contrary to our original definition of price level. The 45-degree dash line indicates the perfect cross-sectional relation between risk and discount rates. See the description in Figure 2 for further details.
Appendix to “Asset Pricing with Price Levels”

A  Data

A.1  Basic data adjustments

We use domestic common stocks (CRSP share code 10 or 11) listed on the three major exchanges (CRSP exchange code 1, 2, or 3). We replace missing prices with the average bid-ask price when available and drop observations with missing share or price information in the previous month. We code missing returns as zero returns and add delisting returns to returns. If delisting returns are missing but the CRSP delisting code is 500 or between 520 and 584, we use \(-35\% \text{ (} -55\% \text{)}\) as the delisting returns for NYSE and AMEX stocks (for NASDAQ stocks) (Shumway (1997) and Shumway and Warther (1999)). To compute capital gains, we use the CRSP split-adjustment factor (CFACSHR) to ensure that capital gains are not affected by split events. The market factor is downloaded from Kenneth French’s website.

A.2  Characteristics and portfolios

One important characteristic with which we sort stocks is the book-to-market-equity (B/M) ratio computed each year in June. B/M ratio is the stock’s book value of equity in the previous fiscal year divided by its market value of equity in December of the previous calendar year. Book value of equity is defined as stockholders’ equity \(SEQ\) (Compustat item 144) plus balance sheet deferred taxes and investment tax credit \(TXDITC\) (item 35) minus book value of preferred stock \(BE = SEQ + TXDITC - BPSTK\). Book value of preferred stock \(BPSTK\) equals the preferred stock redemption value \(PSTKRV\) (item 56), preferred stock liquidating value \(PSTKL\) (item 10), preferred stock \(PSTK\) (item 130), or zero depending on data availability. If \(SEQ\) is unavailable, we set it equal to total assets \(AT\) (item 6) minus total liabilities \(LT\) (item 181). If \(TXDITC\) is unavailable, it is assumed to be zero. We treat zero or negative book values as missing.

Another important characteristic with which we sort stocks is the quality measure of Asness,
Frazzini, and Pedersen (2019) defined as a z-score based on the four characteristics below:

\[ \text{quality} = z(\text{profitability, growth, safety, payout-ratio}). \]

These z scores of different characteristics are in turn obtained as an equal weighted average of different measures of each characteristic. See Asness, Frazzini, and Pedersen (2019) for more details on the measure and its construction.

We also examine portfolios sorted by seven additional characteristics related to the cross-section of returns. We define profitability and investment as in Fama and French (2015). Profitability is computed each year in June. Operating profitability (“profitability”) in calendar year \( y \) is operating profits in fiscal year \( y - 1 \) over book value of equity in fiscal year \( y - 1 \), where operating profits equals sales \( \text{SALE} \) (Compustat item 12) minus cost of goods sold \( \text{COGS} \) (item 41), interest and related expenses \( \text{XINT} \) (item 134) (if available), and selling, general, and administrative expenses \( \text{XSGA} \) (item 132) (if available). Asset growth (“investment”) is also computed each year in June, and investment in calendar year \( y \) is total assets in fiscal year \( y - 1 \) divided by total assets in fiscal year \( y - 2 \). Size is market equity calculated every month. Momentum is calculated every month and is the cumulative gross return over the previous 12 months excluding the month before the portfolio formation. Beta is the trailing 3-year market beta (minimum of 2 years) calculated each month based on overlapping 3-day returns. Net issuance is calculated as of June and is the split-adjusted growth in shares outstanding in the previous year. Accruals measures the degree to which earnings come from non-cash sources and is defined according to Sloan (1996).

For each characteristic, we form 10 value-weight portfolios based on the distribution of the characteristic among NYSE stocks. When doing so, to mitigate the effect of stale prices, we exclude microcaps defined as market equity below the bottom 10% cutoff among NYSE stocks. (Our results are very similar if we instead exclude stocks with levels of Amihud (2002) liquidity that are below the corresponding bottom 10% NYSE cutoff.) Hence, the 10 portfolios sorted by size are only based on the upper 90% size groups. However, to avoid a look-ahead bias, if after portfolio formation a stock’s size falls below the 10 percentile NYSE cutoff, we still keep it in the portfolio. To be consistent, the market portfolio that we require to be correctly priced is also formed after excluding the stocks below the 10% NYSE cutoff.
Comparing $\delta$ to Other Metrics of Price-level Error

Some readers may be interested in how our price-level measure of mispricing, $\delta_t = \frac{P_t - V_t}{P_t}$, compares to existing measures of mispricing or long-term return.

Market-to-book ratio

The market-to-book-equity ratio, closely related to long-run reversal, is one popular measure of mispricing (De Bondt and Thaler (1985), Rosenberg, Reid, and Lanstein (1985), and Lakonishok, Shleifer, and Vishny (1994)). However, the market-to-book-equity ratio is a highly imperfect measure of mispricing, since factors other than mispricing can influence the ratio. The decomposition of Vuolteenaho (2002) and Cohen, Polk, and Vuolteenaho (2003) shows that the log market-to-book-equity ratio $mb_t$ is approximately,

$$mb_t = \sum_{j=1}^{\infty} \rho_j^{-1} E_t [\rho e_{t+j} - \sum_{j=1}^{\infty} \rho_j^{-1} E_t [\alpha_{t+j} + \lambda_{M,t+j} \beta_{M,t+j}] + \frac{1}{2} \sum_{j=1}^{\infty} \rho_j^{-1} Var_t (r_{t+j}), \quad (29)$$

where $\rho e_{t+j}$ is the log return on equity, $\alpha_{t+j}$ is abnormal return, $\lambda_{M,t+j} \beta_{M,t+j}$ is the risk premium implied by the SDF $M$, and $r_{t+j}$ is log return. Hence, besides the distortion in the discount rate due to $\alpha_{t+j}$, other factors such as earnings growth, risk, and volatility can affect cross-sectional and time-series variation in the market-to-book-equity ratio.

Price-level alpha

Cohen, Polk, and Vuolteenaho (2009) (CPV) were the first to propose an identity for measuring price-level distortions. They define the fundamental value of a stock as the present value of future dividends discounted with the discount factors that would have prevailed in the absence of mispricing:

$$V_t = \sum_{j=1}^{\infty} E_t \left[ \frac{D_{t+j}}{\prod_{s=1}^{j} (1 + R_{v,t+s})} \right], \quad (30)$$
where $R_{t+j+s}$ is the return on $V_t$. They then show that price-level alpha $\alpha^{\text{price}}_t$, defined as the log deviation of price from value, approximately equals

$$\alpha^{\text{price}}_t = -\sum_{j=1}^{\infty} \rho^{j-1} E_t \left[ r_{t+j} - r_{v,t+j} \right]$$

where $r$ denotes log return, $R$ denotes simple return, and $\beta_{M,v,t+j}$ is quantity of risk in the absence of mispricing. Hence, if the distortion in the volatility of log return due to mispricing is small,

$$-\sum_{j=1}^{\infty} \rho^{j-1} E_t \left[ R_{t+j} \right] \approx \alpha^{\text{price}}_t - \sum_{j=1}^{\infty} \rho^{j-1} E_t \left[ \lambda_{M,t+j} \beta_{M,v,t+j} \right].$$

(32)

CPV use this relation to run a cross-sectional regression that explains the price level based on price-level risk with respect to the CAPM, using cash-flow betas measured by the exposure of a portfolio’s return on equity to the market’s return on equity to estimate market betas in the absence of mispricing. If the convexity adjustment is done correctly, this approach could be useful in rejecting the null of no mispricing, since under that null the volatility distortion should be zero, but using their identity to draw a conclusion beyond the rejection of the null is difficult due to the potentially large volatility distortion along with measurement error introduced from estimating $\beta_{M,v,t+j}$ with their cash-flow beta proxy. Our identity addresses this problem by expressing mispricing in terms of observable quantities (other than the SDF loadings).

The misspecification metric in van Binsbergen and Opp (2019)

van Binsbergen and Opp (2019) introduce a production economy in which they generate several insights about real distortions arising from abnormal returns. An important quantity in their analysis is log abnormal return defined as $\tilde{\alpha}_t = P_t - E_t \left[ M_{t+1} (D_{t+1} + P_{t+1}) \right]$, where $M_{t+1}$ is a candidate stochastic discount factor. Rewriting this expression, iterating it forward, and imposing a transversality condition implies,

$$P_t = \sum_{j=0}^{\infty} E_t \left[ M_{t+j+1} e^{-\sum_{s=0}^{j} \tilde{\alpha}_s} D_{t+j+1} \right].$$

(33)
This expression, although not the main point of their paper, is useful in our context as follows. If \( P_t \) differs from intrinsic value defined as \( V_t = \sum_{j=0}^{\infty} E_t [M_{t,t+j+1}D_{t+j+1}] \) because of frictions, then the above expression implies that the price deviation from value is an aggregation of abnormal returns:
\[
P_t - V_t = \sum_{j=0}^{\infty} E_t \left[ M_{t,t+j+1}D_{t+j+1} \left( e^{-\sum_{s=0}^{j} \alpha_{t+s}} - 1 \right) \right].
\]
This expression also allows van Binsbergen and Opp (2019) to anticipate that price distortion depends not just on the magnitude of the alphas, but also on their persistence. However, the expression involves the dividend process and is therefore difficult to take to data without putting more structure on how dividends evolve.

**Estimating the fundamental value directly**

Another approach to estimating fundamental mispricing is to estimate the fundamental value directly from cash-flow data:
\[
V_t = \sum_{j=1}^{\infty} E_t [M_{t,t+j}D_{t+j}].
\]
Estimating this quantity unconditionally in the data is problematic for two reasons. First, the predictability of dividends means that conditional covariance is smaller than the unconditional covariance. Second, truncation of the infinite sum at some finite \( J \) may leave out a large fraction of value.\(^{29}\) To estimate the conditional expectation, one can estimate the dynamics of the factors in the SDF and dividends in a VAR and use it to obtain the conditional fundamental values at each given point in time. However, depending on the cash flow, a simple VAR with a few number of lags may not capture the persistent nature of dividend cash flows, and the constant coefficients in the estimated VAR model could understate the extent to which the SDF covaries with cash flows in a crisis scenario.

---

\(^{29}\)For example, suppose \( V_t \) follows a Gordon growth model with expected dividend growth rate \( g \) and constant discount rate \( R \). \( \sum_{j=1}^{\infty} E_t \left[ D_t \left( 1 + g \right)^j / (1 + R)^j \right] = \left( \frac{1 + g}{1 + R} \right)^j E_t \left[ D_t \left( 1 + g \right) / R \right] = \left( \frac{1 + g}{1 + R} \right)^j V_t \). This means, if \( g = 5\% \) and \( R = 10\% \), about 40\% (10\%) of \( V_t \) comes from dividends occurring 20 (50) years after \( t \).
Cumulative abnormal return (CAR)

One popular measure of long-term return is the cumulative abnormal return defined as the simple sum of abnormal returns over a period of time:

$$\text{CAR}_t = -\sum_{j=1}^{\infty} E_t [\alpha_{t+j}]$$

(written with a sign flip so that, like our $\delta_t$, positive abnormal returns means a negative $\text{CAR}_t$).

How well can CAR proxy for ex-ante mispricing of the portfolio?

To see how CAR relates to price-level mispricing $\delta$, rewrite equation (14) as

$$\delta_t = -\sum_{j=1}^{\infty} E_t [w_{t,t+j}] E_t [\alpha_{t+j}] - \sum_{j=1}^{\infty} \text{Cov}_t (w_{t,t+j}, \alpha_{t+j}),$$

where

$$w_{t,t+j} = M_{t,t+j-1} \frac{P_{t+j-1}}{P_t} \frac{1}{1+R_{t,t+j}}.$$  

Hence, CAR is an exact measure of mispricing when $E_t [w_{t,t+j}] = 1$ and $\text{Cov}_t (w_{t,t+j}, \alpha_{t+j}) = 0$. These conditions would approximately hold, for example, if abnormal return tends to be stable (i.e., conditional abnormal return equals unconditional abnormal return), the gross monthly risk-free rate is close to 1, and the portfolio has a very high duration such that cumulative capital gain approximately equals cumulative return (in which case $E_t [M_{t+s} \frac{P_{t+s}}{P_{t+s-1}}] \approx E_t [M_{t+s} (1+R_{t+s})] \approx 1$ when abnormal returns are small and both the SDF and returns exhibit little serial covariance). Nevertheless, Cho (2020) shows that returns on anomaly trading strategies depend importantly on shocks to the capital of arbitrageurs proxied by aggregate funding liquidity and the aggregate arbitrageur’s portfolio, suggesting that these conditions are violated.

“Discounted” CAR (DCAR)

Under slightly less strong assumptions than the ones specified for CAR, mispricing $\delta_t$ is approximately a discounted sum of subsequent abnormal returns with a simple geometric discount factor. To see this, start with Equation (37) and continue to assume $\text{Cov}_t (w_{t,t+j}, \alpha_{t+j}) = 0$. Next, note that if monthly risk-free rates are small and both the SDF and returns exhibit little serial covari-
ance,
\[
E_t \left[ w_{t,t+j} \right] \approx \prod_{s=1}^{j-1} E_t \left[ M_{t+s} \frac{P_{t+s}}{P_{t+s-1}} \right] = \prod_{s=1}^{j-1} E_t \left[ M_{t+s} (1 + R_{t+s}) \frac{1}{1 + \frac{D_{t+s}}{P_{t+s}}} \right].
\] (39)

Then, replace \( \frac{1}{1 + \frac{D_{t+s}}{P_{t+s}}} \) with the Campbell and Shiller (1988) discount factor \( \rho = \frac{1}{1 + D/P} \) (where \( D/P \) is the long-run average of the dividend-price ratio) and assume that abnormal returns are small \( E_t [M_{t+s} (1 + R_{t+s})] \approx 1 \) to obtain
\[
E_t \left[ w_{t,t+j} \right] \approx \rho^{j-1}
\] (40)

Hence, under these strong assumptions, we can write mispricing as a sum of subsequent abnormal returns discounted at a constant rate:
\[
\hat{\delta}_t \approx - \sum_{j=1}^{\infty} \rho^{j-1} E_t \left[ \alpha_{t+j} \right].
\] (41)

We call this discounted CAR and analyze this potential metric together with simple CAR below.

**How well do CAR and DCAR proxy for \( \delta \)? An empirical investigation**

Measures such as the cumulative abnormal returns (CAR),
\[
CAR = - \sum_{j=1}^{J} E_t \left[ \alpha_{t+j} \right],
\]
proxy for the degree to which the assets generate abnormal returns over a long period of time following a corporate or market event of interest but do not have a clear theoretical interpretation. (The convention is to not have a negative sign before the sum, but we put it there for easier comparison to \( \delta \).) Given that our mispricing measure \( \delta_t = (P_t - V_t) / P_t \) also takes the form of a discounted sum of future abnormal returns, how well does CAR proxy for the initial mispricing of the assets?

**Figure B3** plots the estimated \( \hat{\delta}_s \) based on the in-sample value of the SDF loading \( b_1 \) and CARs of B/M sorted portfolios with \( J = 180 \) months. Both for quality and book-to-market sorted portfolios, there is a substantial difference between CARs and \( \delta_s \). In particular, CAR tends to
exaggerate the cross-sectional variation in $\delta$s, possibly due to the equal weighting of all future abnormal returns.

It is interesting to ask whether we can do better by introducing a constant discount factor to the CAR formula to obtain a discounted CAR (“DCAR”) with $\rho = 0.9^{1/12}$,

$$
DCAR = -\sum_{j=1}^{J} \rho^{j-1} E_t [\alpha_{t+j}],
$$

which we show to be a crude approximation of $\delta$ in Appendix B.\(^{30}\) We find that DCAR traces the cross-sectional variation in $\delta$ better, although there is still some meaningful difference between $\delta$s and DCAR (Figure B3). The remaining difference tends to arise from the inability of these simple measures to account for the intertemporal component of price-level risk.

An ex-post identity for mispricing

Our identity uses the SDF to discount future cash flows, so some readers would wonder whether defining ex-post realized returns as the discount factor yields a similar identity. Such an approach yields an identity that holds both ex-ante and ex-post, but it involves the unobserved return in the absence of mispricing:

$$
\delta_t = -\sum_{j=1}^{\infty} \frac{1}{\prod_{s=1}^{j} (1 + R_{v,t+s})} \left( R_{t+j} - R_{v,t+j} \right),
$$

where $R_v$ is the return on fundamental value $V$.

C Quantitative Analysis\(^{31}\)

This section provides a quantitative model of the intrinsic value and price processes. The intrinsic value process follows a linearity-generating process of Gabaix (2007), and the price process also follows a similar form that yields linearity in both the price-dividend and the value-dividend ratios.

\(^{30}\)We find that DCAR does not perform as well when $\rho$ equals the suggested value of 0.95–0.96 (Campbell (2017).

\(^{31}\)For the results in this section, we are indebted to Robert Rogers and Ran Shi for the extraordinary skill with which they provided a framework that formed the basis of our quantitative model. Robert Rogers also provided a groundwork for our simulation results by conducting a thorough research on suitable parameter values and generating the first batch of simulation results.
C.1 A linearity-generating model of price and intrinsic value

Basic structure. The stochastic discount factor (SDF) follows

\[ M_{t+1} = \frac{1 + \varepsilon_{M,t+1}}{1 + R_f}, \tag{43} \]

The asset has a growth rate of cash flows (dividends)

\[ \frac{D_{t+1}}{D_t} = 1 + g_t + \varepsilon_{D,t+1}, \tag{44} \]

where

\[ \pi_t = -Cov_t(\varepsilon_{M,t+1}, \varepsilon_{D,t+1}) \tag{45} \]

is the time-varying risk premium associated with the cash flows. The state variables \((g_t, \pi_t)\) follow an AR(1) process

\[ \tilde{g}_{t+1} = g_{t+1} + g_* = \xi_t \rho_{\tilde{g}} \tilde{g}_t + \varepsilon_{g,t+1} \]
\[ \tilde{\pi}_{t+1} = \pi_{t+1} + \pi_* = \xi_t \rho_{\pi} \tilde{\pi}_t + \varepsilon_{\pi,t+1}, \tag{46} \]

where \(\xi_t\) is the linearity generating “twist” that preserves linearity of the price-dividend process as a function of the state variables:

\[ \xi_t = \frac{1 + g_* - \pi_*}{1 + g_t - \pi_t}. \tag{47} \]

We assume that the shocks \(\varepsilon_{\pi,t+1}\) and \(\varepsilon_{g,t+1}\) have a zero mean and are uncorrelated with

\[ M_{t+1} \frac{D_{t+1}}{D_t}. \tag{48} \]

Mispriicing shocks. We consider a particular form of deviation from the correct price. Suppose that the arbitrageur’s cost of capital varies over time, generating a wedge in the pricing equation denoted by \(x_t\).

\[ 1 - \frac{D_t}{P_t} \frac{1}{1 + R_f} x_t = E_t \left[ M_{t+1} \frac{P_{t+1} + D_{t+1}}{P_t} \right] \tag{49} \]

That is, as in Cho (2020), arbitrageurs do not have unlimited capital and their time-varying capital generates variation in \(x_t\). Here, a positive \(x_t\) means overpricing and a negative \(x_t\) means underpricing.
ing. \( x_t \) follows
\[
\tilde{x}_{t+1} = \xi_t \rho \tilde{x}_t + \epsilon_{x,t+1}
\] (50)
and its exposure to risk is measured by \( \omega_t \):
\[
\text{Cov}_t \left( \frac{M_{t+1} D_{t+1}}{D_t}, \epsilon_{x,t+1} \right) = \frac{\omega_t}{1 + R_f},
\] (51)
where
\[
\tilde{\omega}_{t+1} = \xi_t \rho \omega_t + \epsilon_{\omega,t+1}.
\] (52)
Assume that \( \epsilon_{\omega,t+1} \) is uncorrelated with \( M_{t+1} D_{t+1} / D_t \).

**The price process.** The setting above gives a price-to-dividend process and a value-to-dividend process that are linear in the state variables. To see this, conjecture the following price-to-dividend process:
\[
\frac{P_t}{D_t} = \beta_{p,0} + \beta_{p,g} \tilde{g}_t + \beta_{p,\pi} \tilde{\pi}_t + \beta_{p,x} \tilde{x}_t + \beta_{p,\omega} \tilde{\omega}_t.
\] (53)
Plugging this into both sides of the pricing equation with limited arbitrage (equation (49)) implies
\[
(1 + R_f) \left( \beta_{p,0} + \beta_{p,g} \tilde{g}_t + \beta_{p,\pi} \tilde{\pi}_t + \beta_{p,x} \tilde{x}_t + \beta_{p,\omega} \tilde{\omega}_t \right) - (\tilde{x}_t + x_*)
\]
\[= (\kappa_* + \tilde{g}_t - \tilde{\pi}_t) \left( 1 + \beta_{p,0} \right)
\]
\[+ \kappa_* (\beta_{p,g} \rho_{g} \tilde{g}_t + \beta_{p,\pi} \rho_{\pi} \tilde{\pi}_t + \beta_{p,x} \rho_{x} \tilde{x}_t + \beta_{p,\omega} \rho_{\omega} \tilde{\omega}_t) + \beta_{p,x} (\tilde{\omega}_t + \omega_*),
\] (54)
where
\[
\kappa_* = 1 + g_* - \pi_*
\] (55)
Matching coefficients,
\[
(1 + R_f) \beta_{p,0} - x_* = \kappa_* (1 + \beta_{p,0}) + \beta_{p,x} \omega_*
\]
\[= (1 + R_f) \beta_{p,g} = 1 + \beta_{p,0} + \kappa_* \beta_{p,g} \rho_{g}
\]
\[= (1 + R_f) \beta_{p,\pi} = - \left( 1 + \beta_{p,0} \right) + \kappa_* \beta_{p,\pi} \rho_{\pi}
\]
\[= (1 + R_f) \beta_{p,x} - 1 = \kappa_* \beta_{p,x} \rho_{x}
\]
\[= (1 + R_f) \beta_{p,\omega} = \kappa_* \beta_{p,\omega} \rho_{\omega} + \beta_{p,x}
\] (56)
Solving these equations,

\[
\beta_{p,0} = \frac{\kappa + x_s + \beta_{p,x} \omega_s}{1 + R_f - \kappa_s}, \\
\beta_{p,g} = \frac{x_s + \beta_{p,x} \omega_s}{(1 + R_f - \kappa_s)(1 + R_f - \kappa_s \rho_s)}, \\
\beta_{p,\pi} = -\frac{1}{(1 + R_f - \kappa_s)(1 + R_f - \kappa_s \rho_s)}, \\
\beta_{p,x} = \frac{\beta_{p,x}}{1 + R_f - \kappa_s \rho_x}, \\
\beta_{p,\omega} = \frac{\beta_{p,x}}{1 + R_f - \kappa_s \rho_\omega}
\]

(57)

The intrinsic value process. Under the null of a correct pricing model, \( \bar{x}_t = x_s = \bar{\omega}_t = \omega_s = 0 \). In this case,

\[
\frac{V_t}{D_t} = \beta_{v,0} + \beta_{v,g} \bar{g}_t + \beta_{v,\pi} \bar{\pi}_t,
\]

(58)

where

\[
\beta_{v,0} = \frac{\kappa_s}{1 + R_f - \kappa_s}, \\
\beta_{v,g} = \frac{x_s + \beta_{p,x} \omega_s}{(1 + R_f - \kappa_s)(1 + R_f - \kappa_s \rho_s)}, \\
\beta_{v,\pi} = -\frac{1}{(1 + R_f - \kappa_s)(1 + R_f - \kappa_s \rho_\pi)}.
\]

(59)

The process for \( \Delta_t = P_t - V_t \). It is useful to consider the process for the price deviation from intrinsic value. This is given as the difference between equation (53) and equation (58):

\[
\frac{\Delta_t}{D_t} = \beta_{\Delta,0} + \beta_{\Delta,g} \bar{g}_t + \beta_{\Delta,\pi} \bar{\pi}_t + \beta_{\Delta,x} \bar{x}_t + \beta_{\Delta,\omega} \bar{\omega}_t,
\]

(60)

where

\[
\beta_{\Delta,0} = \frac{x_s + \beta_{p,x} \omega_s}{1 + R_f - \kappa_s}, \\
\beta_{\Delta,g} = \frac{x_s + \beta_{p,x} \omega_s}{(1 + R_f - \kappa_s)(1 + R_f - \kappa_s \rho_s)}, \\
\beta_{\Delta,\pi} = -\frac{1}{(1 + R_f - \kappa_s)(1 + R_f - \kappa_s \rho_\pi)}, \\
\beta_{\Delta,x} = \frac{\beta_{p,x}}{1 + R_f - \kappa_s \rho_x}, \\
\beta_{\Delta,\omega} = \frac{\beta_{p,x}}{1 + R_f - \kappa_s \rho_\omega}
\]

(61)

Delta and alphas. Hence, we also have analytical expressions for the delta and the alphas:

\[
\delta_t = \frac{P_t/D_t - V_t/D_t}{P_t/D_t}.
\]

(62)
To compute conditional alphas, note that by the definition of $x_t$, alpha is simply

$$\alpha_{t+1} = -\frac{x_t}{P_t/D_t}. \quad (63)$$

Use $\delta$ denote the unconditional mean value of $\delta_t$.

**Finite delta.** In practice, we cannot use the exact identity in equation (2) to estimate $\delta$ and instead use a finite sum over $J$ periods to proxy for the infinite sum. To compute finite delta over $J$ post-formation periods (denoted $\delta(J)$) in this model, note that

$$\delta(J) = \delta - E \left[ M_{t+J} \frac{P_{t+J}}{P_t} \delta_{t+J} \right] = \delta - E \left[ M_{t+J} \frac{D_{t+J} P_{t+J}}{D_t} \frac{1}{P_t/D_t} \frac{V_{t+J}/D_{t+J}}{P_t/D_t} \right]. \quad (64)$$

Note

$$M_{t+j} \frac{D_{t+j}}{D_{t+j-1}} = \frac{1}{1+R_f} \left( 1 + g_{t+j-1} - \pi_{t+j-1} + e_{t+j} \right), \quad (65)$$

where

$$e_{t+j} = \frac{1}{1+R_f} \left[ (1 + g_{t+j-1}) \epsilon_{M,t+j} + \epsilon_{D,t+j} + \epsilon_{M,t+j} \epsilon_{D,t+j} + \pi_t \right] \quad (66)$$

is a mean-zero error:

$$E_{t+j-1} [e_{t+j}] = 0. \quad (67)$$

We choose a constant $\sigma_e$ for our simulations. Assuming that $Cov_{t+j-1} \left( \epsilon_{M,t+j}^2, \sigma_{D,t+j}^2 \right)$, $Cov_{t+j-1} \left( \epsilon_{M,t+j}, \epsilon_{M,t+j} \epsilon_{D,t+j} \right)$, and $Cov_{t+j-1} \left( \epsilon_{D,t+j}, \epsilon_{M,t+j} \epsilon_{D,t+j} \right)$ are all close to zero,

$$Var_{t+j-1} (e_{t+j}) \approx \frac{1}{(1+R_f)} \left[ (1 + g_{t+j-1}) \sigma_M^2 + \sigma_D^2 - 2(1 + g_{t+j-1}) \pi_{t+j-1} + \sigma_M^2 \sigma_D^2 + \pi_{t+j-1}^2 \right]. \quad (68)$$

Plugging in the parameter values and replacing $g_{t+j-1}$ and $\pi_{t+j-1}$ with their steady-state values $g_\ast$ and $\pi_\ast$ allow us to pick a value for $\sigma_e$ that is consistent with the rest of the model.

**CAR and BHAR.**

We compute CAR as the sum of expected alphas over $J$ periods after portfolio formation at time $t$:

$$CAR(J) = \sum_{j=1}^{J} E [\alpha_{t+j}] \quad (69)$$
Similarly, we compute BHAR as the expected cumulative alphas over $J$ periods:

$$BHAR(J) = E \left[ \Pi_{j=1}^{J} (1 + \alpha_{t+j}) - 1 \right]$$  \hspace{1cm} (70)

C.2 How well do finite $\delta$, CAR, and BHAR proxy for the true $\delta$?

The linearity-generating model above can be used to generate different kinds of insights on mispricing and abnormal returns. Here, we focus on what it implies about the relative performance of a finite $\delta$, CAR, and BHAR and use the parameter values specified in Table B2.

Figure B1 shows that a finite $\delta$ based on 180 post-formation months is a near-perfect proxy for the actual $\delta$. This is true even when $\delta$ does not converge to zero but to a nonzero steady-state value due to permanent mispricing associated with either an expected return distortion (Panel B) or a distortion in risk (Panel C). When there is no permanent mispricing, the negative of CAR and BHAR can sometimes be a decent proxy for the true $\delta_t$. However, they deviate substantially from the true $\delta_t$ in the presence of even a small amount of permanent mispricing.

C.3 Empirical support for the 15-year finite $\delta$

Does a finite $\delta$ using post-formation returns over 15 years perform well in practice as it does in simulations of the population of data? Recall that to estimate $\delta$ using 15-year post-formation returns, we replace the infinite sum in equation (13) with a finite sum over $J = 180$ months (15 years). However, since both the discount factor $M_{t,t+j-1}P_{t+j-1}/P_t$ and the conditional abnormal return $\alpha_{t+j}$ that appear in the expression are likely to decay over post-formation years, the finite sum over 15 post-formation years provides a good approximation of the infinite sum. In particular, the left panel of Figure B4 shows that the estimated $\delta$ based on $J$ post-formation periods plateaus around year 15. The right panel of Figure B4, which plots the derivative of the paths plotted in the left panel, confirms that the marginal contribution to our mispricing measure becomes relatively small in post-formation year 13.
D Portfolio-level $\delta$

In practice, one would typically estimate the $\delta$ of a portfolio of stocks, which requires expressing the portfolio $\delta$ as a function of post-formation capital gains and returns on the portfolio. These capital gains and returns should be those based on a buy-and-hold strategy that does not rebalance the portfolio (or equivalently, use the original weight times the stock’s cumulative capital gain to rebalance the portfolio every month). If $w_{i,t}$ is the portfolio weight on security $i$ at the time of portfolio formation,

$$
\delta_t = \sum_{i=1}^{N} w_{i,t} \delta_{i,t}
$$

$$
= -\sum_{i=1}^{N} \sum_{j=1}^{\infty} E_t \left[ M_{t,t+j} w_{i,t} \frac{P_{t+j-1} R^e_{i,t+j}}{P_{t+j-1}} \right] 
$$

$$
= -\sum_{j=1}^{\infty} E_t \left[ M_{t,t+j} \left( \sum_{i=1}^{N} w_{i,t} \frac{P_{t+j-1}}{P_{t+j}} \right) \right] 
$$

so that

$$
w_{i,t+j} = \frac{w_{i,t} P_{t+j}/P_{t}}{\sum_{i=1}^{N} w_{i,t} P_{t+j}/P_{t}} \tag{72}
$$

is the correct post-formation weight on security $i$, $P_t = \sum_{i=1}^{N} w_{i,t} P_{i,t}$ is the price of the portfolio, $P_{t+j} = \sum_{i=1}^{N} w_{i,t} P_{i,t+j} / P_{i,t}$ is the portfolio’s post-formation price, and $R^e_{i,t+j} = w_{i,t+j-1} R^e_{i,t+j}$ is the portfolio’s post-formation excess return. Delisted securities can be considered to have a zero price afterward.
### Additional Tables and Figures

Table B1: Post-formation Returns: Illustration

The table describes our three-dimensional data structure. Our data consist of overlapping samples of returns over 180 post-formation months on portfolios formed in 559 different months (1957m7–2004m12) for 10 different characteristic deciles.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Formation month $t$</th>
<th>Calendar month $t + j$</th>
<th>Number of post-formation months $j$</th>
<th>Return</th>
<th>Capital gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>BM1</td>
<td>1957m6</td>
<td>1957m7</td>
<td>1</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>BM1</td>
<td>1957m6</td>
<td>1957m8</td>
<td>2</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>BM1</td>
<td>1957m6</td>
<td>1972m6</td>
<td>180</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>BM1</td>
<td>1957m7</td>
<td>1957m8</td>
<td>1</td>
<td>...</td>
<td>...</td>
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<tr>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>BM1</td>
<td>2004m12</td>
<td>2019m12</td>
<td>180</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>BM2</td>
<td>1957m6</td>
<td>1957m7</td>
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<tr>
<td>:</td>
<td>:</td>
<td>:</td>
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<td>:</td>
</tr>
<tr>
<td>BM10</td>
<td>2004m12</td>
<td>2019m12</td>
<td>180</td>
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</table>
Table B2: **Quantitative Analysis: Parameter Selection**

The table reports the parameter values used in the quantitative analysis of delta and its approximations. All values are monthly.

<table>
<thead>
<tr>
<th>Panel A. Values that differ across analyses</th>
<th>No permanent mispricing</th>
<th>Permanent return distortion</th>
<th>Permanent risk distortion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_s$</td>
<td>0</td>
<td>$[-0.6, 0.1]$</td>
<td>$[-0.25, 0.7]$</td>
</tr>
<tr>
<td>$\bar{x}_0$</td>
<td>$[-14, 29]$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\omega_s$</td>
<td>0</td>
<td>0</td>
<td>$x_s/20$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B. Common values</th>
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</thead>
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<tr>
<td>$R_f$</td>
<td>0.003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_*$</td>
<td>0.004</td>
<td>$\pi_*$</td>
<td>0.003</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>0.003</td>
<td>$\sigma_\pi$</td>
<td>0.002</td>
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<tr>
<td>$\sigma_x$</td>
<td>0.003</td>
<td>$\sigma_\omega$</td>
<td>$\sigma_x/100$</td>
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<tr>
<td>$\sigma_e$</td>
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<td>$\rho_g$</td>
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<td>$\rho_\pi$</td>
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<td>$\rho_x$</td>
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<td>$\rho_\omega$</td>
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<td>$\tilde{g}_0$</td>
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<td>$\tilde{\pi}_0$</td>
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<tr>
<td>$\tilde{\omega}_0$</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table B3: Forecasting Delta with Characteristics Deciles: Firm-level Regressions

The table uses firm-level panel regressions with time fixed effects to show that the book-to-market-equity ratio, quality, and size provide the most parsimonious model of firm-level mispricing as these three variables subsume the ability of any of the non-size anomalies studied to explain cross-sectional variation in $\delta$. The left-hand side is the estimated firm-specific $\delta$. The analysis here essentially repeats the analysis of Table 6 using the characteristic decile numbers (e.g., the explanatory variable equals 10 for stocks in the highest decile and 1 for stocks in the lowest decile of the characteristic). We adjust the regression residual weights so that the cross-sections are equally-weighted across time and the weights within each cross-section are proportional to the market capitalization. We report $t$-statistics in parentheses that are based on standard errors clustered by stock and time, and we denote significance at the 10%, 5%, and 1% levels using *, **, and ***.

<table>
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<tr>
<th></th>
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<th>(2)</th>
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<td>-0.05***</td>
<td>-0.03***</td>
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<td>0.03***</td>
<td>0.03***</td>
<td>0.03***</td>
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<td>716410</td>
<td>716026</td>
<td>716224</td>
<td>602943</td>
</tr>
</tbody>
</table>
Panel A. Transitory mispricing ($x_s = 0, \omega_s = 0, \tilde{x}_0 \neq 0$)

Finite $\delta$  
CAR  
BHAR

Panel B. Permanent mispricing due to return distortion ($x_s \neq 0$)

Finite $\delta$  
CAR  
BHAR

Panel C. Permanent mispricing due to risk distortion ($\omega_s \neq 0$)

Finite $\delta$  
CAR  
BHAR

Figure B1: Quantitative Comparison of Mispricing Metrics

These figures compare the ability of a finite $\delta$, CAR, and BHAR to proxy for the true $\delta$ using the quantitative model specified in Section C.1 and parameter values in Table B2. Finite $\delta$ based on 180 post-formation months, defined as $\delta(180) = -\sum_{j=1}^{180} E \left[ M_{t+j} \frac{R_{t+j-1}}{R_t} R_{t+j} \right]$ is a near-perfect proxy for the actual $\delta$ defined as $\delta = E \left[ \frac{P_t - V_t}{P_t} \right] = -\sum_{j=1}^{\infty} E \left[ M_{t+j} \frac{R_{t+j-1}}{R_t} R_{t+j} \right]$. This is true even when $\delta$ does not converge to zero but to a nonzero steady-state value due to expected return distortion (Panel B) or risk distortion (Panel C). When there is no permanent mispricing, the negative of CAR and BHAR based on $J$ post-formation months, defined respectively as $-\text{CAR} (J) = -\sum_{j=1}^{J} E \left[ \alpha_{t+j} \right]$ and $-\text{BHAR} (J) = -E \left[ \Pi_{j=1}^{J} (1 + \alpha_{t+j}) - 1 \right]$, can sometimes be a decent proxy for the true $\delta$. However, they deviate substantially from the true $\delta$ in the presence of even a small amount of permanent mispricing.
Figure B2: **Decomposing Price-level Risk: Quality and B/M**

These figures decomposes price-level risk into its contemporaneous and intertemporal components:

\[
\sum_{j=1}^{180} \text{Cov}_T \left( -M_{t+j} \frac{P_{t+j-1}}{P_t}, R_{t+j}^e \right) = \sum_{j=1}^{180} E_T \left[ \phi_{t+j-1} \right] \text{Cov}_T \left( -M_{t+j}, R_{t+j}^e \right) \\
- \sum_{j=1}^{180} E_T \left[ \left( \phi_{t+j-1} - E_t \left[ \phi_{t+j-1} \right] \right) M_{t+j} \left( R_{t+j}^e - E_t \left[ R_{t+j}^e \right] \right) \right],
\]

where \( \phi_{t+j} = M_{t+j} P_{t+j-1}/P_t \). Although contemporaneous risk is a large fraction of the overall price-level risk, both the contemporaneous and intertemporal components contribute importantly to the cross-sectional variation in price-level risk.
Figure B3: Alternative Long-run Return Measures

The two figures compare estimated mispricing $\delta$ to the cumulative abnormal return (CAR),

$$\text{CAR} = -\sum_{j=1}^{J} E_t [\alpha_{t+j}],$$

and against the discounted CAR (DCAR),

$$\text{DCAR} = -\sum_{j=1}^{J} \rho^j E_t [\alpha_{t+j}]$$

with $\rho = 0.9^{1/12}$ and $J = 180$ months for twenty portfolios sorted by quality, the book-to-market-equity ratio, and seven additional characteristics. These portfolios are formed in 1957m6–2004m12 and the abnormal returns are estimated separately for each post-formation month up to $J = 180$ months. We estimate the expected conditional abnormal returns simply using unconditional abnormal returns, which implicitly assumes that the conditional component of the abnormal return is not contemporaneously correlated with the market factor. The mispricing $\delta$ is estimated so that the market portfolio has an in-sample $\delta$ of zero. The 95% confidence interval is denoted in gray.
Figure B4: Estimated $\delta$ by the Choice of Finite Post-formation Periods $J$

The figures in the left panel plot the values of $\delta$ estimated based on $J$ post-formation years for the extreme decile quality- and book-to-market, sorted portfolios and extreme quality and book-to-market double-sorted portfolios, where $J$ is the horizontal axis. The figures in the right panel plot how much each post-formation month contributes to the estimated $\delta$s. Portfolio formation period spans 1957m6–2004m12 and post-formation period spans 1957m7–2019m12.
Figure B5: Explaining Returns vs. Price Levels: Individual Stocks

The left panel reports the cross-sectional relation between realized mean excess return and mean excess return predicted by the CAPM for one-period returns on individual stocks between 1957m7 and 2005m1. The right panel reports the cross-sectional relation between estimated scaled price level and estimated scaled price-level risk with respect to the CAPM is close to the perfect relation denoted by the dash line. The right panel is scaled to have comparable units to one-month returns: Scaled price level (scaled price-level risk) is $-1 \times \text{price level (price-level risk)}$ divided by the estimated $\sum_{j=1}^{180} E_T \left( M_{t+j} \frac{P_{t+j-1}}{P_t} \right)$ of the market portfolio.