Asset Pricing with Price Levels

Thummim Cho and Christopher Polk*

London School of Economics

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*Cho: Department of Finance, London School of Economics, London WC2A 2AE, UK. Email: t.cho@lse.ac.uk. Phone: +44 (0)20 7107 5017. Polk: Department of Finance, London School of Economics, London WC2A 2AE, UK and CEPR. Email: c.polk@lse.ac.uk. Phone: +44 (0)20 7849 4917. We thank Tony Berrada (HEC-McGill Winter Finance discussant), John Campbell, Georgy Chabakauri, Ben Hebert, Christian Julliard, Lukas Kremens, Dmitry Livdan (UBC Winter Finance discussant), Dong Lou, Hanno Lustig, Ian Martin, Cameron Peng, Lukas Schmid, Ran Shi, Argyris Tsiaras, Dimitri Vayanos, Laura Veldkamp, Tuomo Vuolteenaho, and seminar participants at the LSE, Swiss Finance Institute - Lugano, UBC Winter Finance Conference, HEC-McGill Winter Finance Workshop for useful comments and discussions. Amirabas Salarkia provided superb research assistance.
Asset Pricing with Price Levels

Abstract

We propose a novel way to study asset prices based on asset price distortions rather than abnormal returns. We derive the correct expression linking current mispricing to subsequent abnormal returns. Unlike existing measures of mispricing, our expression appropriately reflects that not only the magnitude but also the time and state in which abnormal returns occur determine ex-ante mispricing. Our expression also implies a "price-level" analogue of the relations among risk, expected return, and abnormal return: mispricing is an error term in the relation between price level, which summarizes the entire term structure of discount rates, and price-level risk. Applying our identity to data, we document three empirical facts about price levels of stocks: (1) The CAPM does a relatively good job describing the cross-section of average price levels, despite its poor ability to explain short-horizon returns. (2) Positive (negative) short-horizon abnormal returns do tend to imply underpricing (overpricing), but momentum is an important exception. (3) Portfolios sorted on investment, beta, net equity issuance, and the interaction of value and quality are important price-level anomalies that both long-term buy-and-hold investors and researchers disciplining models from the price-level perspective should prioritize.

Keywords: price level, long-horizon returns, mispricing metric, stochastic discount factor, CAPM

JEL classification: G12, G14, G32
We propose a novel way to study asset prices. Our focus is on distortions in asset prices, taking a sharp break from asset pricing’s traditional emphasis on measuring expected return distortions, i.e. “abnormal returns.”

While abnormal return is certainly an important barometer of capital market efficiency (Fama (1970)), price distortions can have more important real economic consequences as they drive the financing and investment decisions of firms (e.g., Caballero et al. (2006), Cohen et al. (2009), Campello and Graham (2013), van Binsbergen and Opp (2019), and Whited and Zhao (2019)). Furthermore, price distortions matter more than short-horizon abnormal returns for investors who commit their capital over a long investment horizon, such as firm managers, policy makers, and other long-term buy-and-hold investors (Cohen et al. (2009)).

Price and expected return are synonymous in a model with constant expected returns. In this case, \( P_t = \sum_{j=1}^{\infty} E_t \left[ D_{t+j} \right] / (1 + R)^j \) with \( D \) denoting cash flows (dividends), so a distortion in price \( (P) \) implies a distortion in expected return \( (R) \), and vice versa.\(^1\) However, expected returns are almost certainly time-varying (Campbell and Shiller (1988), Fama and French (1989), Cochrane (1992), Cochrane (2008), and van Binsbergen et al. (2012)). Therefore, ex-ante price distortions should be a function of the entire term structure of subsequent abnormal returns, not just the one-period abnormal return.

But is there an exact analytical way to aggregate subsequent abnormal returns over the long run correctly to arrive at ex-ante price distortion? Could such a formula be applied to data to reveal important facts about the price levels of assets?

Our paper makes two contributions. First, we show that the answer to the first question is a surprising yes: there is a simple analytical expression relating ex-ante price distortion to subsequent abnormal returns. By correctly aggregating subsequent abnormal returns in order to measure ex-ante mispricing, our exact identity contrasts sharply with ad-hoc ways to aggregate abnormal returns over time, such as the cumulative abnormal return (CAR) measure widely used in empirical studies. Our identity also implies that ex-ante mispricing can be interpreted as an error term in the relation between an intuitive measure of price level and price-level risk.

\(^1\)Shiller (1984) and Summers (1986) argue that even in this case, persistent price distortion may not generate statistically discernible patterns in expected returns.
Our second contribution is to apply our identity to data to document two important facts about the price levels of stocks. First, risk measured with respect to the Capital Asset Pricing Model (CAPM) explains cross-sectional variation in price levels fairly well, despite its poor ability to explain the cross-section of short-horizon returns (Black et al. (1972) and Fama and French (1992)). We make this point using stocks sorted on quality (Graham (1973), Grantham (2004), and Asness et al. (2019)) or book-to-market ratio (Rosenberg et al. (1985) and Fama and French (1992)), which we argue are natural characteristics with which to sort stocks in a price-level analysis. Second, we show that firms with low (high) investment, net issuance, beta, or the interaction between value and quality appear the most underpriced (overpriced) relative to their CAPM-implied intrinsic value and hence serve as the primary price-level anomalies that multifactor models of price levels should prioritize in explaining.\(^2\)

When presenting our results, we are careful to recognize that the mispricing we detect could signal either a misspecification of our model of risk or fundamental mispricing due to limits of arbitrage, as is always the case when interpreting abnormal returns.\(^3\) Our goal is not to resolve the debate on market efficiency—whether a given “return” anomaly represents mispricing or misspecification of risk—but to carry this debate over to the “price” dimension: Is a particular return anomaly also a price-level anomaly or is it a purely transitory phenomenon? Can a model of risk explain the cross-section of price levels relatively well even if it cannot explain the cross-section of expected returns?

The theoretical part of this paper derives our identity and discusses its implications. Under the mild assumption of no explosive bubbles in prices, “delta” (δ) defined as a percentage deviation of price from value equals the expected sum of subsequent abnormal returns or “alpha” (α), discounted by the cumulative price-adjusted stochastic discount factor (SDF). In particular,

\[ \delta_t = \frac{(P_t - V_t)}{P_t} = -\sum_{j=1}^{\infty} E_t \left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} R_{t+j}^e \right] \]

\[ =: \sum_{j=1}^{\infty} E_t \left[ M_{t,t+j-1} \frac{P_{t+j-1}}{P_t} \frac{\alpha_{t+j}}{1 + R_{f,t+j}} \right], \]

\(^2\)“Focusing on expected returns and betas rather than prices and discounted cashflows . . . makes much less sense in a world with time-varying discount rates” (Cochrane (2011), pp.1063–1064).

\(^3\)This issue is, of course, the famous joint hypothesis problem emphasized in Fama (1970).
where \( V_t = E_t \left[ \sum_{j=1}^{\infty} M_{t,t+j} D_{t+j} \right] \) is the asset’s intrinsic value defined as the present value of cash flows with respect to the model of risk (e.g., CAPM) prescribing the cumulative candidate SDF \( M_{t,t+j}, D \) is dividend, \( P \) is price, and \( R_{f,t+j}, R_{e,t+j} \), and \( \alpha_{t+j} \equiv (1 + R_{f,t+j}) E_{t+j-1} \left[ M_{t+j} R_{e,t+j} \right] \) are risk-free rate of return, excess return (above the risk-free rate), and conditional abnormal return from \( t + j - 1 \) to \( t + j \), respectively.\(^4\) The assumption of no explosive bubbles is not particularly restrictive, as it allows for most types of price deviation from value, including permanent mispricing (e.g., \( \delta_{t+j} = \delta \neq 0 \forall j \)), which our identity can correctly detect.

Our identity in equation (2) implies that not just the magnitude but also the time and state in which abnormal returns occur matter for an ex-ante price-level measure of mispricing. Abnormal returns occurring in a more distant future matter less for ex-ante mispricing, since these abnormal returns are earned on the portion of the initial asset that excludes the market value of dividend payouts up until that point in the future. In the formula, this conclusion follows from \( E_t \left[ M_{t,t+j} P_{t+j}/P_t \right] \) converging to zero as \( j \to \infty \) under the no-explosive-bubble condition. Furthermore, abnormal returns occurring after states with either high state price \( M \) or high cumulative capital gain at time \( t + j - 1 \) matter more at time \( t \) because in such cases, the abnormal return is earned on a time \( t + j - 1 \) price that matters more for \( P_t \). In contrast, by putting equal weights on abnormal returns occurring in all states and periods, ad-hoc measures such as the CAR can lead to substantial estimation errors.\(^5\)

The identity also shows that mispricing \( \delta \) has an intuitive interpretation as an error term in the relation between price level and price-level risk; hence, variation in \( \delta \) across assets reveals how well risk explains cross-sectional variation in our price-level measure. Using the covariance identity to decompose the infinite sum of expectations in equation (1),

\[
\delta_t = -\sum_{j=1}^{\infty} E_t \left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} \right] E_t \left[ R_{e,t+j}^e \right] + \sum_{j=1}^{\infty} Cov_t \left( -M_{t,t+j} \frac{P_{t+j-1}}{P_t}, R_{e,t+j}^e \right). 
\]

\(^4\)\( M_{t,t+j} \) is the ratio of marginal utilities in periods \( t \) and \( t + j \) in consumption-based asset pricing models with time-separable utility. The Capital Asset Pricing Model (CAPM) implies \( M_{t,t+j} = \Pi_{s=1}^j (b_0 - b_1 R_m^e,t+s) \) where \( R_m^e \) is market return in excess of the risk-free interest rate and \( (b_0, b_1) \) are parameters.

\(^5\)Appendix B illustrates how CAR is an empirically poor proxy for mispricing in addition to being theoretically flawed.
To see the analogy between this price-level equation and the conventional return pricing equation, replace the infinite sum with a sum over some arbitrary $J$ post-formation periods: 

$$
\delta_t = -\sum_{j=1}^{J} \mathbb{E}_t \left[ M_{t,t+j} \frac{R_{t+j}}{R_t} \right] \mathbb{E}_t \left[ R_{t+j}^e \right] + \sum_{j=1}^{J} \text{Cov}_t \left( -M_{t,t+j} \frac{R_{t+j}}{R_t}, R_{t+j}^e \right).
$$

When $J = 1$, the expression reduces to the conventional return pricing equation: 

$$
\alpha_t \equiv -\frac{\delta_t}{\mathbb{E}_t \left[ M_{t+1} \right]} = \mathbb{E}_t \left[ R_{t+j}^e \right] - \text{Cov}_t \left( \frac{-M_{t+1}}{\mathbb{E}_t \left[ M_{t+1} \right]}, R_{t+j}^e \right).
$$

As $J \to \infty$, it approaches our price-level equation.

In equation (3), “price level” summarizes the term structure of discount rates on an asset, $$\{ \mathbb{E}_t \left[ R_{t+j}^e \right] \}_{j=1}^{\infty}$$, and is lower when discount rates are higher. “Price-level risk” summarizes the cash-flow risk of the asset perceived by a hypothetical buy-and-hold investor with the model of risk described by the SDF, $M$. In contrast to (unscaled) price, which cannot be compared across different assets and time periods, price level is stationary and can be compared across both of these important dimensions. And unlike a scaled price measure such as the market-to-book ratio, whose cross-sectional variation captures differences in future earnings growth more than those in discount rates (Cohen et al. (2003)), price level only reflects the term structure of discount rates. Price-level risk recognizes that risk perceived by a buy-and-hold investor depends not only on the state a cash flow at time $t+j$ occurs relative to the time $t+j-1$ state, but also depends on the cumulative realization of states from $t$ to $t+j-1$; e.g., a buy-and-hold investor can perceive a cash flow as risky if it tends to be low following a series of “bad” events, even if the low payoff tends to occur in a “good” state relative to the prior period.

The empirical part of our paper studies to what extent risk implied by the CAPM explains cross-sectional variation in the price levels of stocks. We use the CAPM as the candidate SDF not because we believe the CAPM to be the best model of risk, but because (i) its simplicity makes it easier to see how asset pricing with price levels differs from the conventional analysis of returns and (ii) it provides a foundation for multifactor refinements of the model based on price-level analysis. Our econometric approach closely follows that of conventional return analysis based on time-series regressions. We use the generalized method of moments (GMM) to estimate a portfolio’s unconditional average $\delta$, requiring the SDF parameters to explain the price levels of the market portfolio perfectly. When estimating $\delta$, we proxy for the infinite sum

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6In particular, a natural next step is to consider the price-level patterns associated with specifications of the intertemporal CAPM. Campbell et al. (2018) document that incorporating stochastic volatility into the ICAPM framework of Campbell and Vuolteenaho (2004) significantly reduces the pricing errors relative to the CAPM in standard SDF return tests.
in equation (1) with 15 years of monthly returns on a portfolio without rebalancing and take an average over all portfolio sorting periods.\(^7\) Since mispricing $\delta$ depends on the ratio of value to price, sorting stocks on the value-to-book ratio—proxied by a composite metric dubbed quality (Asness et al. (2019))—and the book-to-market ratio should lead to a powerful test. Our primary analysis therefore studies quality- and book-to-market-sorted portfolios, first in isolation and then in tandem via a double sort.

\(^7\) Appendix C shows that no rebalancing—or equivalently, rebalancing based on cumulative capital gains—is the correct way to compute post-formation returns on a portfolio. Appendix D confirms that in our dataset, a long-horizon sum over 15 years proxies well for the infinite sum.
We find that CAPM explains the cross-section of price levels well despite its poor ability to explain returns. Post-formation single-month returns on quality- or book-to-market-sorted decile portfolios are anomalously negatively related to risk measured by CAPM beta (left panel of Figure 1). In contrast, for both portfolios, the slope of the cross-sectional relation between risk and discount rates changes from negative to positive as we go from returns to price levels. CAPM-implied risk does a particularly good job explaining the cross-section of price levels of quality-sorted portfolios and an adequate job describing that of book-to-market-sorted portfolios (right panel of Figure 1). The former finding runs contrary to Asness et al. (2019)’s claim that stock prices do not fully reflect variation in quality while the latter finding is consistent with Cohen et al. (2009). An analysis at the individual stock level in Appendix E shows the same stark contrast between the return and price-level performance of the CAPM, implying that the finding is not simply a consequence of the particular quality- and book-to-market-sorted portfolios studied in Figure 1.

We use this finding to explain why asset pricing with price levels can generate results drastically different from conventional asset pricing with returns. The conventional return test compares the expected one-period excess return to the covariance of realized excess returns with the SDF:

\[ E_t \left[ R_{t+1}^e \right] = Cov_t \left( -M_{t+1}/E_t \left[ M_{t+1} \right], R_{t+1}^e \right). \] (4)

In contrast, the price-level test compares the term structure of expected excess returns captured by price level to price-level risk, which in turn depends not only on the term structure of contemporaneous risks, but also on an intertemporal adjustment component:

\[
\begin{align*}
\sum_{j=1}^\infty E_t \left[ \frac{M_{t,t+j} P_{t+j-1}}{P_t} \right] E_t \left[ R_{t+j}^e \right] & \geq \sum_{j=1}^\infty E_t \left[ \phi_{t,t+j-1} \right] Cov_t \left( -M_{t+j}, R_{t+j}^e \right) \\
-1 \times \text{Price level} & + \sum_{j=1}^\infty E_t \left[ \phi_{t,t+j-1} \right] Cov_t \left( -M_{t+j}, R_{t+j}^e \right) \\
\text{Contemporaneous risk} & - \sum_{j=1}^\infty E_t \left[ (\phi_{t,t+j-1} - E_t \left[ \phi_{t,t+j-1} \right]) M_{t+j} \left( R_{t+j}^e - E_t \left[ R_{t+j}^e \right] \right) \right] \\
\text{Intertemporal adjustment} & \\
\end{align*}
\] (5)

where \( \phi_{t,t+j-1} \equiv M_{t,j-1} P_{t+j-1}/P_t \). Hence, for the price-level analysis to overturn the anomalous negative relation between one-period risk and return, a simple convergence in risks and re-
turns of different portfolios over subsequent periods $\{t + j\}_{j=2}^{\infty}$ is not enough. It instead requires (i) a reversal in subsequent returns on different portfolios, (ii) a reversal in contemporaneous risks, and/or (iii) a contribution from the intertemporal adjustment component of price-level risk. For quality portfolios, the improved price-level performance of the CAPM comes from the positive return spread between high and low quality portfolios turning negative around a year after portfolio formation and from high-quality stocks having a higher intertemporal adjustment. For book-to-market portfolios, the improved performance of the CAPM comes from the positive beta spread between value (high B/M) and growth (low B/M) portfolios turning negative around five years following portfolio formation, making value stocks riskier to long-term buy-and-hold investors than what their low beta at the time of portfolio formation suggests (Cohen et al. (2009)).

Contrary to univariate sorts, double sorting stocks on both quality and the book-to-market ratio to generate 25 portfolios leads to significant variation in mispricing $\delta$. The low-quality, expensive (growth) portfolio is overpriced by 24% while the high-quality, inexpensive (value) portfolio is underpriced by 33%, generating an economically and statistically large variation in $\delta$ across these portfolios of 57 percentage points. Hence, the double sort generates the sort of variation a buy-and-hold investor should be interested in exploiting and represents a challenge for future asset pricing models that make understanding price-level risk a priority.

Next, turning to decile portfolios sorted on other characteristics (plotted in Figure 2), we document three main findings. First, one-month abnormal return $\alpha$s and mispricing $\delta$s exhibit the negative relation one might expect; positive abnormal returns typically indicate underpricing (the top-left quadrant) while negative abnormal returns typically indicate overpricing (the bottom-right quadrant). Second, investment, net issuance, and beta-sorted portfolios show the largest estimated $\delta$s, making them the prime price-level anomalies that, along with the aforementioned portfolios that are double-sorted on quality and book-to-market, new models of price-level risk should aim to explain. We also find that extreme accruals (Sloan (1996)) and high-forecasted-growth portfolios (La Porta (1996) and Bordalo et al. (2019)) are significantly overpriced, although the magnitude of their mispricing is smaller. Finally, high (low) momentum stocks have large positive (negative) abnormal returns but appear to be overpriced (underpriced), suggesting that momentum profits are often earned from continued overreaction (Lou and Polk (2019)). Statistically, however, momentum’s estimated $\delta$s are marginally insignificant in our sample period.
The figure plots the estimated abnormal returns ($\alpha$) and mispricing ($\delta$), with respect to CAPM, of extreme decile portfolios sorted on quality, book-to-market, and eight additional characteristics. It shows that positive abnormal returns do tend to mean underpricing (bottom-right) and negative abnormal returns overpricing (top-left). However, momentum and profitability are important exceptions.

In summary, our paper is the first to show how to correctly aggregate abnormal returns into ex-ante price distortion. Empirically, we identify stock characteristics that are associated with the largest price distortions with respect to the CAPM and hence should be characteristics that buy-and-hold investors who are primarily concerned with market risk exposure should be interested in exploiting. Thus, our approach provides a new challenge to asset pricing models that are not just concerned with capturing variation in average returns over the short run but that also prioritize minimizing model-implied price distortions.

The organization of our paper is as follows. Section 1 reviews related literature. Section 2 presents our framework to price assets in terms of price levels. Section 3 presents data, econometrics, and our primary results related to quality and value. Section 4 extends our analysis to other characteristic sorts. Section 5 concludes.
1 Literature review

The vast majority of prior research uses the abnormal return with respect to a factor model such as the CAPM as the metric with which to evaluate trading strategies. Despite the popularity of the abnormal return metric, a simple example shows that abnormal return could be a poor proxy for a price-level measure of model mispricing. Consider the hedge fund managers that, according to Brunnermeier and Nagel (2004), purchased technology stocks in the early stages of the dot-com boom and sold them before the crash. Though these positions and their associated characteristics may forecast positive abnormal returns relative to existing models such as the CAPM, the purchased stocks are likely to have been already overpriced, not underpriced relative to that model. Thus, a stock that is overpriced relative to the CAPM could generate positive abnormal returns in the short run, and vice versa.

Nevertheless, this illustration suggests that the initial model-specific mispricing of an asset may be recovered from the behavior of subsequent abnormal returns over the long run, since in the subsequent crash, the overpriced stocks would have earned negative abnormal returns. Our measure highlights the correct way to accumulate subsequent abnormal returns to measure initial mispricing.

Of course, there is a long history of accumulating realized abnormal returns in order to proxy for price-level deviations from a benchmark model in the corporate finance literature, namely the well-known cumulative abnormal return (CAR) and buy-and-hold abnormal return (BHAR) methodologies. Work in this area includes Barber and Lyon (1997), Kothari and Warner (1997), Fama (1998), Lyon et al. (1999), Brav (2000), and Bessembinder et al. (2018), among others. Though our method also accumulates future abnormal returns, by correctly discounting the stochastic payoffs associated with mispricing, our approach substantially improves on these long-run return measures.

Researchers have introduced alternative price-level measures of mispricing. Lee et al. (1999), and others infer the intrinsic value of the firm from its fundamentals. In stark contrast, our formula is an identity linking mispricing to abnormal returns. Black (1986) comments that markets are efficient if prices are within factor of two of intrinsic value but provides no way in which to
carefully measure such deviations. Nevertheless, his prior provides a useful benchmark for the economic significance of the price-level misspecifications we measure. Bai et al. (2016) shows that price has become more informative about intrinsic value over time based on the ability of a scaled price ratio to explain subsequent earnings.

The closest mispricing metric to ours is the price-level alpha construct of Cohen et al. (2009), (CPV). Unlike that paper, our mispricing identity begins with a clear definition of mispricing that is linked to a specification of the SDF, does not require unobservable quantities such as risk exposures and volatility in the absence of mispricing, and does not rely on the Campbell and Shiller (1988) approximation. The first two differences ensure that $\delta$ is a correct price-level measure of model misspecification free of biases that arise from the approximation and the assumption about the unobserved second moments, and the last difference increases our statistical power compared to CPV, as we discuss in more detail in the appendix.

Our exact framework can be used to revisit prior observations about market efficiency. Fama (1970) defines a market as efficient if “prices ‘fully reflect’ all available information” but goes on to test the efficient market hypothesis using returns, finding that the market is semi-strong form efficient. Shiller (1984) writes, “because real returns are nearly unforecastable, the real price of stocks is close to the intrinsic value . . . is one of the most remarkable errors in the history of economic thought” (pp. 458–459). Summers (1986) provides a numerical example that illustrates this argument, and Campbell (2017) shows how an expected return that follows a persistent AR(1) process may have low volatility but a large effect on the log dividend-price ratio.

Our identity provides a more sophisticated framework in which to understand Shiller’s point. It shows that mispricing can be large even if each post-formation alpha is small, as long as alphas are persistent. Furthermore, mispricing can be large even if alphas are small on average if alphas tend to comove strongly with $\phi$. The latter channel has been overlooked in the literature but can be quantitatively important: Cho (2019) provides empirical evidence that as arbitrageurs trade away the alphas of equity anomalies such as value and momentum, they can end up exposing these anomalies to the systematic risks they face as arbitrageurs. That is, in the presence of arbitrage with limited capital, initial mispricing in terms of alpha can evolve to become a risk
premium associated with an endogenous beta.

Several recent papers tackle different but related topics in the growing literature on asset price distortions and long-term portfolio choice. van Binsbergen and Opp (2019) study a quantitative model of a production economy in which the cost of equity faced by firms may be distorted due to the mispricing and analyze the resulting implications for the real economy. By specifying the exact production technology and frictions in the economy, van Binsbergen and Opp (2019) are able to characterize the impact of expected return distortions on output and perform counterfactuals. In contrast, the novelty of our approach is that it allows the ex-ante price distortion to be estimated without having to specify the process that generates dividends. Hence, our approach is better suited for understanding which firms are mispriced with respect to a candidate SDF as well as providing a specification test of a particular SDF based on price distortions.\textsuperscript{8} Of course, the limitation of our approach relative to van Binsbergen and Opp (2019) is that while we provide an easy way to estimate asset price distortions, we cannot draw implications for more direct measures of allocative efficiency such as output.

Chernov et al. (2018) propose a new asset-pricing test requiring the linear SDF specification of a factor model to explain both one-period and multi-period factor returns, documenting that popular linear factor models fail to price returns on their own factors accumulated over long horizons. For example, they show that the average four-year gross returns on the market, profitability, and investment factors have annualized misspecification errors that are roughly 7\% in absolute magnitude relative to the prediction of the four-year SDF implied by the Fama-French (2015) five-factor model. Our goal of estimating asset price deviation from intrinsic value by correctly aggregating abnormal returns on a buy-and-hold strategy is clearly distinct from their goal of generating high-power asset pricing tests based on long-horizon restrictions on managed portfolios. Importantly, the mispricing identity we obtain for a candidate SDF by iterating the law of motion for mispricing $\delta$ forward cannot be obtained in their analysis, which proceeds under the null of a true SDF that prices all future dividends correctly.

Cochrane (2014) shows how mean-variance characterizations can be applied to the stream of

\textsuperscript{8}Appendix B provides further details on how our identity and methodology relates to the approach taken by van Binsbergen and Opp (2019).
long-run payoffs or return opportunities even in a dynamic framework. His analysis shows that optimal dividend payoffs follow a relatively simple analytical form for a mean-variance optimizer that uses simple discounting to weight future utilities, even though the dynamic portfolio strategy that supports these payoffs may be complex. Our $\delta$ measure describes the present value of marginal utility gains from adding a particular stream of payoffs and therefore could be useful in describing the optimal portfolio choice for long-term investors.

2 Framework: Asset Pricing with Price Levels

In this section, we define mispricing in the price level as a percentage deviation of price from the intrinsic value of cash flows implied by an asset pricing model:

$$\delta_t = \frac{P_t - V_t}{P_t},$$ (6)

where $V_t = E_t \left[ \sum_{j=1}^{\infty} M_{t,t+j} D_{t+j} \right]$. Under mild assumptions, $\delta_t$ equals the sum of subsequent excess returns, discounted by the price-weighted cumulative SDF:

$$\delta_t = -\sum_{j=1}^{\infty} E_t \left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} R_{t+j}^e \right].$$ (7)

This identity is at the heart of our empirical analysis in subsequent sections.

The right-hand expression of (7) implies an intuitive expression for price level and price-level risk components. To see this, decompose the expected discounted excess returns within the sum into a contribution coming from discounted expected excess returns and a contribution coming from the covariance between the discount factor and excess returns:

$$\delta_t = -\sum_{j=1}^{\infty} E_t \left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} R_{t+j}^e \right] + \sum_{j=1}^{\infty} Cov_t \left( -M_{t,t+j} \frac{P_{t+j-1}}{P_t}, R_{t+j}^e \right).$$ (8)

In subsequent sections, we measure the extent to which variation in price levels lines up with variation in price-level risk using an analogous decomposition for the unconditional average $\delta$. 

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Our price level measure summarizes all future discount rates on the asset into a single expression. The benefit of working with this measure instead of the actual market price is that it can easily be compared across different assets at different times. Moreover, unlike the popular approach of scaling by book equity or some other accounting variable, our price-level measure is not contaminated by factors such as expected future profitability. Price-level risk summarizes the cash-flow risk perceived by a buy-and-hold investor.

2.1 The environment

Consider an asset with dividends (or coupons for bonds) \( \{D_{t+j}\}_{j=1}^{\infty} \). Our goal is to relate the mispricing (pricing error) of the asset at time \( t \) to its subsequent returns. We measure mispricing with respect to the (candidate) stochastic discount factor (SDF) \( \{M_{t+j}\}_{j=1}^{\infty} \), where we use \( M_{t+j} = \Pi_{s=1}^{j} M_{t+s-1,t+s} \) to denote the cumulative SDF. The SDF we analyze could either be a candidate SDF to be compared to the one implied by market prices in the sense of Hansen and Jagannathan (1991, 1997), or it could be the SDF of a particular investor in a market in which the the law of one price fails for some assets due to frictions (e.g., Garleanu and Pedersen (2011) and Geanakoplos and Zame (2014)).

2.2 Intrinsic value and mispricing

The intrinsic value, \( V_t \), of the asset is simply the present value of all future dividends:

\[
V_t = \sum_{j=1}^{\infty} E_t [M_{t+j}D_{t+j}].
\]

(9)

We define mispricing, \( \delta_t \), as the deviation of price from value as a percentage of the current price:

\[
\delta_t = \frac{P_t - V_t}{P_t}.
\]

(10)

Hence, \( \delta_t > 0 \) if the asset is overpriced, and \( \delta_t < 0 \) if it is underpriced. \( \delta_t \) can range from \(-\infty\) (if \( V_t > 0 \) and \( P_t = 0 \)) to \( 1 \) (if \( V_t = 0 \) and \( P_t > 0 \)), the opposite of the range for abnormal returns, \([-1, \infty)\).
2.3 Assumptions

To link $\delta_t$ to subsequent returns, we make two relatively mild assumptions.\(^9\) The first is the existence of a risk-free asset that satisfies the fundamental theorem of asset pricing.

**Assumption 1.** At each $t + j - 1$, there is a risk-free asset with return $R_{f,t+j}$ at time $t+j$ that satisfies the fundamental theorem of asset pricing:

$$E_{t+j-1}[M_{t+j}(1+R_{f,t+j})] = 1.$$  \hspace{1cm} (11)

This assumption does not require that a particular proxy for the risk-free rate (e.g., the one-month Treasury bill rate) satisfies the return pricing equation; it merely states that there exists a correct risk-free rate measure. The second assumption is a weak form of a no-bubble condition.

**Assumption 2.** The present value of the deviation of price and value at the limit $j \to \infty$ is zero:

$$\lim_{j \to \infty} E_t[M_{t+j}(P_{t+j}-V_{t+j})] = 0.$$  \hspace{1cm} (12)

This assumption is weaker than having two separate no-bubble conditions on price and value, $\lim_{j \to \infty} E_t[M_{t+j}P_{t+j}] = 0$ and $\lim_{j \to \infty} E_t[M_{t+j}V_{t+j}] = 0$, which imply Assumption (2).

2.4 The law of motion for mispricing

Under our definitions and Assumption (1), mispricing follows a simple law of motion. Equation (9) and the law of iterated expectations implies the fundamental theorem of asset pricing holds for value:

$$1 = E_t\left[M_{t+1}\frac{V_{t+1}+D_{t+1}}{V_t}\right].$$  \hspace{1cm} (12)

Next, use equation (10) to substitute the empirically unobserved quantities $V_t$ and $V_{t+1}$ with

\(^9\)We have also derived the sufficient conditions under which our price-level measure is finite.
\( V_t = (1 - \delta_t) P_t \) and \( V_{t+1} = (1 - \delta_{t+1}) P_{t+1} \) to obtain,

\[
\delta_t = 1 - E_t \left[ M_{t+1} (1 + R_{t+1}) \right] + E_t \left[ M_{t+1} \frac{P_{t+1}}{P_t} \delta_{t+1} \right].
\] (13)

Finally, following Assumption (1), use \( 1 = E_t \left[ M_{t+1} (1 + R_{f,t+1}) \right] \) to express mispricing \( \delta_t \) at time \( t \) in terms of excess return \( R_{t+1}^e \) and mispricing \( \delta_{t+1} \) at time \( t+1 \):

\[
\delta_t = -E_t \left[ M_{t+1} R_{t+1}^e \right] + E_t \left[ M_{t+1} \frac{P_{t+1}}{P_t} \delta_{t+1} \right].
\] (14)

The law of motion in equation (14) is intuitive. Since \( E_t \left[ M_{t+1} R_{t+1}^e \right] \) is the conditional abnormal return at time \( t+1 \) adjusted for the gross risk-free rate \( (E_t \left[ M_{t+1} R_{t+1}^e \right] = (1 + R_{f,t+1})^{-1} \alpha_{t+1} \), where \( \alpha_{t+1} \) is the abnormal return conditional on time-\( t \) information), equation (14) says that underpricing (overpricing) at time \( t \) is either “paid out” as a positive (negative) abnormal return or contributes to the remaining mispricing at time \( t+1 \). The discount factor on \( \delta_{t+1} \) is the SDF times the capital gain, which is intuitive given that \( \delta_{t+1} \) is normalized by \( P_{t+1} \). Hence, \( \delta_{t+1} \) matters more at time \( t \) if it arises in a state in which \( P_{t+1} \) is high (hence the capital gain term) or has a higher present value (hence the SDF term).

### 2.5 Relating mispricing to subsequent returns

Iterating the law of motion for mispricing (equation (14)) forward and using Assumption (2) to set \( \lim_{j \to \infty} E_t \left[ M_{t+j} \frac{P_{t+j}}{P_t} \delta_{t+j} \right] = 0 \) expresses mispricing as a discounted sum of future excess returns:

\[
\delta_t = -\sum_{j=1}^{\infty} E_t \left[ M_{t+j} \frac{P_{t+j-1}}{P_t} R_{t+j}^e \right].
\] (15)

This formula motivates our measures of price level and price-level risk. For intuition, take a time \( t+j-1 \) conditional expectation within the expectation and use \( E_{t+j-1} \left[ M_{t+j} R_{t+j}^e \right] = (1 + R_{f,t+j})^{-1} \alpha_{t+j} \) to write,

\[
\delta_t = -\sum_{j=1}^{\infty} E_t \left[ \phi_{t,j} \frac{\alpha_{t+j}}{1 + R_{f,t+j}} \right], \quad \phi_{t,j} \equiv M_{t+j} \frac{P_{t+j}}{P_t}
\] (16)
where $\alpha_{t+1}$ is the time $t+1$ risk-adjusted abnormal return investor expects to earn conditional on time-$t$ information and $\phi_{t+j}$ is a discount factor on abnormal returns.

Several observations about mispricing in the price level $\delta$ follow from equation (16). First, a simple formula relates ex-ante mispricing $\delta$ to abnormal returns $\alpha$.

**Remark 1.** Mispricing $\delta$ is a stochastically discounted sum of subsequent abnormal returns.

The asset pricing literature understands that some aggregation of abnormal returns over a long horizon can proxy for mispricing in the price level and has used long-run return measures such as CAR or BHAR as that proxy in numerous applications. Our identity in equation (16) shows that although there is indeed a simple, analytical formula relating initial ex-ante mispricing to subsequent abnormal returns, the formula is clearly distinct from those existing long-run return measures. To the best of our knowledge, we are the first to supply the correct formula for the price level measure of mispricing.

What follows immediately from the summation formula in equation (16) is that the persistence of abnormal returns, not just the magnitude, matters for mispricing.

**Remark 2.** Holding all else fixed, mispricing $\delta$ is larger if subsequent abnormal returns are persistent.

Others have pointed out that the persistence of abnormal returns should matter for price-level distortion (e.g., Cohen et al. (2009), Cochrane (2011), van Binsbergen and Opp (2019)), and we confirm this point in our exact relation between mispricing and abnormal returns.

The last set of remarks highlights the importance of the stochastic discounting of abnormal returns. If $\delta_t$ is finite, the discount factor on the risk-free-rate adjusted abnormal return must fall over time. More formally, since the single-period component of the discount factor on abnormal returns is $M_{t+j} \frac{P_{t+j}}{P_{t+j-1}}$ and the fundamental theorem of asset pricing requires $E_t \left[ M_{t+j} \left( \frac{P_{t+j}}{P_{t+j-1}} + \frac{D_{t+j}}{P_{t+j-1}} \right) \right] = 1$ in the absence of abnormal returns, $M_{t+j} \frac{P_{t+j}}{P_{t+j-1}}$ in general must have an expected value less than 1 once the firm starts paying out dividends. This fact implies that the horizon at which abnormal returns are earned affects the magnitude of the initial mispricing.
Remark 3. Abnormal returns occurring sooner are in general associated with larger mispricing $\delta$.

Intuitively, as is the case with any present value formula, the net present value of a buy-and-hold strategy on the asset as a fraction of the initial price—which is what $\delta$ represents—depends less on abnormal returns earned far into the future. However, this simple logic is missing in the widely-used long-run return measures; both CAR and BHAR do not distinguish abnormal returns earned in the near future from those earned far into the future.

The presence of the SDF $M$ in equation (16) implies that the state in which the abnormal return is earned matters.

Remark 4. Abnormal returns occurring in more valuable states are associated with larger mispricing $\delta$.

This point is perhaps the most novel insight of our identity, as implies that not just the “expectation,” but also the “covariance” matters for how we accumulate subsequent $\alpha$s into the initial $\delta$. For example, if the market factor is a priced factor, abnormal returns earned following a market crash imply a large deviation of intrinsic value from price, since most asset-pricing models intuitively view dividends or capital gains earned in such a state as more valuable. Practically speaking, this point implies that being able to predict an asset’s abnormal returns using past returns on a risk factor would have important implications when quantifying ex-ante mispricing.

Finally, capital gain also matters for the covariance component of mispricing.

Remark 5. Abnormal returns occurring after relatively large capital gains are associated with larger mispricing $\delta$.

As a consequence, one cannot simply discount future abnormal returns with the cumulative SDF alone but instead must use a price-weighted cumulative SDF. Capital gain enters into the formula since the abnormal return at time $t + j$ is earned on the $t + j - 1$ price. Hence, the abnormal return matters more for mispricing today if it expected to be earned on a high future price. Practically speaking, this component of $\phi$ means that abnormal returns due to long-run reversal (De Bondt and Thaler (1985)) count less toward the initial mispricing. This simple intuition has been largely
overlooked in the literature’s search for a link between abnormal returns and mispricing.\textsuperscript{10}

To summarize, our identity in equation (16) highlights that abnormal returns occurring sooner, in more valuable states, or after relatively large capital gains are associated with larger price-level deviations. Specifically, the gross risk-free rate expresses the conditional abnormal return earned at time $t + j$ as a time $t + j - 1$ value, the time $t + j - 1$ price $P_{t+j-1}$ translates that abnormal return into time $t + j - 1$ abnormal cash flow, and the cumulative SDF expresses that abnormal cash flow in today’s value. Finally, the formula normalizes the present value of abnormal cash flows with today’s price $P_t$.

### 2.6 Price levels and price-level risks

Consider abnormal return $\alpha$, the return-based measure of model misspecification:

$$\alpha_{t+1} = E_t \left[ \frac{M_{t+1}}{E_t[M_{t+1}]} R^e_{t+1} \right].$$  \hspace{1cm} (17)

Decomposing the right-hand expression using the covariance identity $E_t \left[ \frac{M_{t+1}}{E_t[M_{t+1}]} R^e_{t+1} \right] = E_t [R^e_{t+1}] + Cov_t \left( \frac{M_{t+1}}{E_t[M_{t+1}]}, R^e_{t+1} \right)$ implies

$$\alpha_{t+1} = E_t [R^e_{t+1}] + Cov_t \left( \frac{M_{t+1}}{E_t[M_{t+1}]}, R^e_{t+1} \right),$$  \hspace{1cm} (18)

which allows us to understand abnormal return as a deviation of expected excess return from the risk premium implied by an asset pricing model.

In a similar fashion, we decompose the right-hand expression of (15), $- \sum_{j=1}^{\infty} E_t \left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} R^e_{t+j} \right]$, to show that $\delta_t$ represents the deviation of an expression for price level from a price-level risk measure. The former is the infinite sum of expected excess returns discounted by a non-stochastic (as of $t$) term while the latter is the infinite sum of covariances that captures long-horizon risk:

$$\delta_t = - \sum_{j=1}^{\infty} E_t \left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} R^e_{t+j} \right] E_t [R^e_{t+j}] + \sum_{j=1}^{\infty} Cov_t \left( -M_{t,t+j} \frac{P_{t+j-1}}{P_t}, R^e_{t+j} \right).$$  \hspace{1cm} (19)

\textsuperscript{10}Of course, Campbell and Shiller (1988)’s discount parameter implicitly captures the fact that future returns on high dividend-paying assets are worth less in present-value terms.
The negative sign in the definition of price-level risk ensures that a larger value of price-level risk indicates higher long-horizon risk. Although we decompose the conditional $\delta_t$ here, our empirical analysis in the subsequent sections uses the decomposition of its unconditional average, $\delta = E[\delta_t]$; the unconditional decompositions hold analogously to the conditional ones here.

As suggested above, ultimately our paper is concerned with understanding variation in price levels, which requires a measure of price level that can be compared across time and across assets and is also preferably less contaminated by factors such as expected future profitability than a scaled price ratio such as the market-to-book ratio. Our price-level measure, driven only by discount-rate variation, achieves just that.

To see why Eq. (19) is a natural infinite-horizon counterpart to the familiar one-period return pricing equation in Eq. (18), replace the infinite sum with a finite sum over $J$ post-formation periods: $\delta_t = -\sum_{j=1}^{J} E_t \left[ M_{t+j} \frac{P_{t+j-1}}{P_t} R^e_{t+j} \right] + \sum_{j=1}^{J} Cov_t \left( -M_{t+j} \frac{P_{t+j-1}}{P_t}, R^e_{t+j} \right)$. When $J = 1$, defining $\alpha_t \equiv \delta_t / E_t [M_{t+1}]$ gives the conventional return pricing equation in Eq. (18). As $J \rightarrow \infty$, on the other hand, the equation approaches the price-level equation in Eq. (19).\(^{11}\)

Finally, we note that price-level risk depends not only on the contemporaneous covariance between $M_{t+j}$ and $R^e_{t+j}$, but also on the intertemporal covariance between $\phi_{t+j-1} = M_{t+j-1}P_{t+j-1}/P_t$ and $R^e_{t+j}$. Hence, it is sometimes useful to decompose our price-level risk measure into that coming from contemporaneous beta exposures to the SDF and an intertemporal component:

$$\sum_{j=1}^{\infty} Cov_t \left( -M_{t+j} \frac{P_{t+j-1}}{P_t}, R^e_{t+j} \right) = \sum_{j=1}^{\infty} E_t \left[ \phi_{t+j-1} \right] Cov_t \left( -M_{t+j}, R^e_{t+j} \right)$$

(20)

The first component is a simple discounted sum of future contemporaneous risk premia, whereas the second component corrects the first for the fact that future risk premia can covary with past $\phi$. That is, for a long-term investor, an asset is risky not only if it has a contemporaneous negative

\(^{11}\)Although they are infinite series, we expect both price level and price-level risk to be finite for price processes with a stationary dividend-to-price ratio.
covariance with the SDF, but also if it covaries negatively with the cumulative SDF.

### 3 Asset Pricing Tests on Price Levels

Section 2 implies that a discounted sum of expected excess returns that we call “price level,”

\[
- \sum_{j=1}^{\infty} E \left[ M_{t,t+j} \frac{P_{t+j-1}}{P_t} \right] E \left[ R^e_{t+j} \right],
\]

has a component driven by risk (“price-level risk”) and a component due to the average percentage deviation of price from the intrinsic value of cash flows (“price-level error”) \( \delta = E[\delta_t] = E[(P_t - V_t) / P_t] \). Hence, given an asset pricing model prescribing a SDF, we can empirically disentangle the cross-sectional difference in price levels into that coming from price-level risk and from price-level error, as we would disentangle the cross-sectional difference in expected excess returns into a component driven by risk and remaining abnormal return.

This section examines how well price-level risk perceived by a mean-variance investor who holds the market (CAPM) explains the price levels of portfolios sorted by our proxy for the (intrinsic) value-to-book ratio \((V/B)\) or the book-to-market ratio \((B/M)\), which we argue are natural ways to sort portfolios for price-level tests. Compared to how poorly it explains the corresponding cross-section of returns, the CAPM does a surprisingly better job explaining the cross-section of price levels of portfolios based on a univariate sort on either ratio. We use this result to illustrate how asset pricing with price levels works and why it can generate drastically different results from asset pricing with returns. We then show that a double sort on \(V/B\) and \(B/M\) nonetheless leads to larger price-level errors.

#### 3.1 Data

We combine monthly stock price data from the Center for Research in Security Prices (CRSP) and annual accounting data from CRSP/Compustat Merged (CCM) to create a basic merged dataset. We use one-month Treasury bill rates from Kenneth French’s data library (originally from Ibbotson Associates) as the risk-free rate and the market excess return from the same data library as the market factor.
To estimate price-level errors of characteristic-sorted portfolios, we need post-formation returns over a long horizon of 180 months (15 years) after the initial portfolio formation. That is, we form value-weighted decile portfolios each month at $t$ based on the NYSE decile cut-offs and compute the post-formation returns on these portfolios over $t + 1, \ldots, t + 180$. Post-formation returns at $t + j$ for the portfolio formed at $t$ are buy-and-hold returns that do not reinvest dividends into the same or different stocks.\textsuperscript{12} In summary, our data is three dimensional, as we illustrate in Table B1: we have post-formation returns for 10 different portfolios (or 25 for two-way sorted portfolios), for $T$ different portfolio formation periods, and for $J$ different post-formation periods. Our baseline data use 1957m6–2004m12 as the portfolio formation periods ($T = 558$ months) and $J = 180$ post-formation months, which imply post-formation returns over 1957m7–2019m12. Our portfolio sort begins in 1957m6, since this is when we can first compute all accounting-based characteristics based on the annual Compustat dataset.

The decile portfolios we form are labeled so that decile 10 (1) represents the one with the highest (lowest) abnormal returns according to the existing literature. Hence, for the book-to-market sort, 10 represents the extreme value portfolio and 1 the extreme growth portfolio, while for the size sort, 10 represents the portfolio of the smallest stocks and 1 represents the portfolio of the largest stocks. Table 1 provides descriptive statistics for the portfolios formed from a univariate sort on each of the ten characteristics we consider in the rest of the paper: quality, book-to-market ratio, profitability, investment, momentum, size, beta, net issuance, accruals, and analyst forecasts of long-term growth. Appendix A provides further details on the data construction.

### 3.2 Methodology

We use the generalized method of moments (GMM) to estimate the price-level errors of portfolios with respect to an asset pricing model. Our identity in (15) shows that the unconditional price-level error $\delta \equiv E[\delta_t] = E[(P_t - V_t)/P_t]$ with respect to an SDF $M$ equals

\textsuperscript{12}These $t + j$ returns are equivalent to returns earned by forming a new portfolio every month at $t + j - 1$ based on initial time-$t$ weights adjusted by the cumulative capital gain from time $t$ to time $t + j - 1$. The rebalancing based on cumulative capital gain is the correct approach for our purpose of inferring the initial price level of the portfolio based on subsequent returns, since it mirrors how the returns earned by investing the dividend payments for an individual asset do not enter into our formula. See the exact argument in Appendix C.
\[ \delta = - \sum_{j=1}^{\infty} E \left[ M_{t+j} \frac{P_{t+j-1} - P_t}{P_t} R_{t+j}^e \right], \]  
\text{(22)}

where \( \delta = 0 \) under the null of no average price-level error. To translate the right-hand expression into a GMM moment that we can estimate, we replace the infinite sum with a finite sum up to \( J = 180 \) months (15 years) and replace the population mean with a sample mean. Our GMM moment is thus,

\[ \delta(b) = - \sum_{j=1}^{J} E_T \left[ M_{t+j} \frac{P_{t+j-1} - P_t}{P_t} R_{t+j}^e \right], \]  
\text{(23)}

where \( E_T \) denotes the sample mean over \( T \) portfolio formation months, \( t \) the portfolio formation month, and \( j \) the number of post-formation months over which we compute the sum.

The finite sum over 15 years provides a good approximation of the infinite sum, since both the discount factor \( M_{t+j-1} \frac{P_{t+j-1}}{P_t} \) and the conditional abnormal return \( E_{t+j-1} \left[ M_{t+j} R_{t+j}^e \right] \) are small after 15 years for any portfolio formed at time \( t \). Appendix D shows that the finite sum \( - \sum_{j=1}^{J} E_T \left[ M_{t+j} \frac{P_{t+j-1} - P_t}{P_t} R_{t+j}^e \right] \) plateaus around \( J = 15 \) years, implying that the remaining terms in the infinite sum are likely to make little difference to the infinite sum.

To be consistent with the implicit assumption in conventional asset pricing regressions with returns, we model the SDF as having constant coefficients on linear factors \( f : M_{t+j} = b_0 - b_1 f_{t+j} \). We estimate the SDF parameters \( b \) by requiring the SDF to price the factors perfectly in sample, which is analogous to the time-series asset pricing regression with returns in which the factor portfolios are required to have zero abnormal returns in sample. We use the CAPM to model the SDF, as it provides the basic risk adjustment upon which multifactor models are built. Since the monthly risk-free rate and the square of the monthly Sharpe ratio are both small, the intercept term of the CAPM SDF is approximately one (e.g., Cochrane (2005)):

\[ M_{t+j} = 1 - b_1 R_{m,t+j}^e. \]  
\text{(24)}

This assumption reduces our estimation problem to choosing the single parameter \( b_1 \) that makes

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\[ ^{13} \text{If the CAPM prices both the market portfolio and the risk-free asset, } b_0 = \frac{1}{1+R_f} + \frac{S_m}{1+R_f}, \text{ where } S_m \text{ is the Sharpe ratio. Hence } \frac{1}{1+R_f} \text{ is slightly below } 1 \text{ and } \frac{S_m}{1+R_f} \text{ slightly above } 0, \text{ giving us } b_0 \approx 1. \]
the price-level error of the market portfolio zero in sample.\textsuperscript{14}

Once $\mathbf{b}$ is estimated, we use the asymptotic distribution of GMM errors $\mathbf{\delta}(\hat{\mathbf{b}})$ to obtain standard errors and test statistics. We estimate the spectral density matrix using the Bartlett kernel used by \textit{Newey and West (1987)} with a bandwidth of $J$ months to account for serial correlation due to overlapping samples as well as contemporaneous correlations across the portfolios. The variance-covariance matrix of pricing errors is given by

$$V_{\mathbf{\delta}} = \frac{1}{T} \left( I - D (D'\hat{D})^{-1} D' \right) S \left( I - D (D'\hat{D})^{-1} D' \right),$$

(25)

where $S$ is the estimated spectral density matrix and $D$ is the matrix representing the estimated derivative of the moments with respect to changes in the parameters. The variance-covariance matrices of other statistics are obtained analogously.

We use five statistics to assess the ability of the asset pricing model to explain price levels.

1. Estimated difference in $\delta$s between the two extreme decile portfolios, which we use as the primary test statistic with which to reject or not reject an asset pricing model.

2. The slope of the estimated relation between price level and price-level risk. We compute the $p$-value against the null that there is a 45-degree line relation between price level times $-1$ and price-level risk; the sign change on price level is necessary, since according to our definition higher price-level risk should imply lower price level.

3. Weighted mean absolute (pricing) error, $\Delta \equiv \sum_{i=1}^{n} w_i \mid \hat{\delta}_i \mid$, where $w_i$ is the average market capitalization weight of portfolio $i$ in the market, measures what percentage of capital in the stock market that is being misallocated to the mispricing component of prices. To see this, simply write the time-specific $\Delta$ as the sum of the absolute differences between the firm’s intrinsic value and market value ($\sum_{i=1}^{n} |P_{i,t} - V_{i,t}|$) divided by the sum of the market

---

\textsuperscript{14}The advantage of this parameter reduction is that it prevents us from using another asset such as a proxy for the risk-free interest rate to pin down the intercept term in the SDF. Estimating the additional intercept term by requiring the SDF to price the 1-month Treasury rate perfectly in sample leads to similar results, however.
values (Σᵢ₌₁ⁿ Pᵢₜ):

\[ \Delta_t = \frac{\sum_{i=1}^{n} |P_{i,t} - V_{i,t}|}{\sum_{i=1}^{n} P_{i,t}} = \sum_{i=1}^{n} \frac{P_{i,t} - V_{i,t}}{P_{i,t}} = \sum_{i=1}^{n} \frac{P_{i,t}}{P_{i,t}} \left| \delta_{i,t} \right|, \quad \text{(26)} \]

where \( P_{i,t} \) and \( V_{i,t} \) respectively measure the total market capitalization and total intrinsic value of the portfolio and \( n \) is the number of stocks or portfolios that comprises the market. Hence, \( \Delta \) equals 0 under the joint hypothesis of the correct asset pricing model with no mispriced capital at the portfolio level.\(^{15}\)

4. Mean squared (pricing) error, \( N^{-1} \sum_{i=1}^{N} \hat{\delta}_{i}^2 \), a measure similar to the Gibbons et al. (1989) statistic in time-series return regressions.

5. Mean absolute (pricing) error, \( N^{-1} \sum_{i=1}^{N} \left| \hat{\delta}_{i} \right| \), a measure that does not discriminate among the test assets in terms of their economic importance.

### 3.3 Quality and B/M as the primary sorting characteristics

A powerful test on price levels requires test assets that a priori are likely to exhibit large variation in mispricing \( \delta \). To do this, recall that \( \delta \) measures the percentage deviation of value from price, which can be rewritten in terms of the intrinsic value to book equity \( V/B \) and book equity to market price \( B/M \) (where \( V, M, \) and \( B \) are measured per share so that \( M = P \)):

\[ \delta_{i} = 1 - \frac{V_{i}}{P_{i}} = 1 - \frac{V_{i}}{B_{i}} \times \frac{B_{i}}{M_{i}}. \quad \text{(27)} \]

Hence, holding the other ratio fixed, a variation in either \( V/B \) or \( B/M \) implies variation in \( \delta \). This motivates us to sort stocks based on these two ratios.

Since the value-to-book ratio \( V/B \) is unobserved, we follow Asness et al. (2019) to use a composite z-score measure called \textit{quality} as a proxy for \( V/B \). Rewriting Gordon’s growth model in the absence of mispricing as

\[ \frac{V}{B} = \frac{\text{profitability} \times \text{payout ratio}}{\text{required returns} - \text{growth}}; \quad \text{(28)} \]

\(^{15}\)We calculate the appropriate p-values for \( \Delta \) by simulating \( \hat{\delta} \)s under the null (based on the asymptotic distribution given above by \( V_{\delta} \)).
they use *quality* measured by a z-score that rewards profitable, fast-growing, safe, and high-payout stocks to proxy for the value-to-book ratio:

\[
\text{quality} \equiv z(\text{profitability, growth, safety, payout ratio}) \propto \frac{V}{B} + \text{noise} \tag{29}
\]

The book-to-market ratio \(B/M\) may contain different information from the value-to-book ratio \(V/B\), since a distortion in the discount rate due to abnormal returns affects \(B/M\) but not \(V/B\):

\[
\frac{B}{M} = \left( \frac{\text{profitability} \times \text{payout ratio}}{\text{abnormal returns} + \text{required returns} - \text{growth}} \right)^{-1}. \tag{30}
\]

Since the \(B/M\) ratio accounts for variation in long-horizon discount rates due to abnormal returns whereas the \(V/B\) ratio does not, double sorting on *quality* and \(B/M\) should lead to larger \(\delta\) variation and hence an even more powerful test. In particular, as we illustrate graphically in Figure 3, low-quality stocks that are expensive (low *quality* and \(B/M\)) are likely to be overpriced and have a positive \(\delta\), whereas high-quality stocks that are inexpensive (high *quality* and \(B/M\)) are likely to be underpriced with a negative \(\delta\).

We note in passing that, in the context of Gordon’s model, \(B/M\) depends not only on discount rates, but also on factors such as growth and profitability. Indeed, Cohen et al. (2003) show that roughly 80% of the cross-sectional variation in book-to-market ratios reflects predictable differences in expected future profitability. This fact makes \(B/M\) or its reciprocal \(M/B\) a less appealing measure of price level than our identity-motivated measure of price level in (21).

### 3.4 Explaining the price levels of portfolios sorted by quality or B/M

To what extent does the CAPM explain the cross-section of price levels of portfolios formed by a univariate sort on either quality or the book-to-market ratio? Specifically, does the CAPM explain price levels better than it does for returns, and if so, why?

To provide a reference point, we begin with conventional asset pricing tests using returns. The left two panels in Figure 1 show that the CAPM does a very poor job explaining the cross-section

\footnote{Note that when writing equation (28), Asness et al. (2019) assume growth to be exogenous and thus independent of the payout ratio.}
of returns on decile portfolios sorted on quality or book-to-market ratio. High quality stocks earn higher returns than low quality stocks despite having a lower market beta (Asness et al. (2019)). Similarly, High $B/M$ or “value” stocks earn higher returns than low $B/M$ or “growth” stocks despite having a lower market beta (e.g., Fama and French (1992)). Hence, both quality and $B/M$ sorts lead to a cross-sectional relation between risk and returns that is opposite to what the CAPM predicts.

Asset pricing tests with price levels generate meaningfully different results. The right two panels in Figure 1 plot the cross-sectional relation between price level and price-level risk but with a scaling factor that divides both quantities by the value of $\sum_{j=1}^{180} E \left[ M_{t,j} P_{t+j-1}/P_t \right]$ for the market and multiplying the price level by $-1$ for an easier comparison with the left two panels. Long-horizon discount rates and long-horizon risk that scaled price level and scaled price-level risk respectively capture display a positive cross-sectional relation with a fitted line reasonably close to the 45 degree line, suggesting that the CAPM does a decent job describing these portfolios’ price-level risk that buy-and-hold investors would care about.

Tables 2 and 3 report the estimated $\delta$s and test statistics along with standard errors (in parentheses) and $p$-values (in square brackets) for $J$: \{1mo, 1yr, 3yrs, 5yrs, 10yrs, 15yrs\} with $J = 1$ month generating conventional time series return regression results and $J = 15$ years (180 months) proxying price-level results given by $J \to \infty$. The intermediate values of $J$ allow us to see how the performance of the asset pricing model changes as the return horizon increases gradually from 1 month to 15 years.

Table 2 shows that high-quality stocks are undervalued and low-quality stocks are overvalued from the perspective of CAPM investors with a short investment horizon of $J = 1$ month or 1 year. However, for $J = 10$ or 15 years, the estimated $\delta$s are essentially zero for all quality-sorted portfolios and imply that the market price correctly accounts for the quality difference, contrary to the observation that Asness et al. (2019) draw from cumulative returns up to 5 years’ horizon and a cross-sectional regression of $M/B$ ratio on quality rather than an identity likes ours that gives an exact expression for ex-ante mispricing. All test statistics we consider show that price-level risk with respect to CAPM does a reasonable job explaining price-level variation in quality-sorted portfolios, with the slope coefficient in particular being fairly close to 1.
Table 3 reports the result for $B/M$-sorted portfolios. Compared to quality-sorted portfolios, $B/M$ portfolios show larger price-level errors with respect to the CAPM. Growth stocks are estimated to be 31.4 percentage points more overpriced than value stocks, and this difference is statistically significant at the 5% level, though marginally. However, it is obvious from several metrics that again price levels look less anomalous than returns. The cross-sectional slope coefficient changes from $-2.399$ to $0.543$ as we go from returns to price levels and is not statistically different from one. Mean squared error and mean absolute value of errors are large enough to reject the null of a correct model at the 5% significance level but cannot be rejected at the stricter 1% level, contrary to these statistics for the return-based tests that are highly statistically significant.

The mechanism: Why does asset pricing with price levels look different?

Even if both returns and betas of extreme decile portfolios converge over time (Keloharju et al. (2019)), if asset pricing with price levels simply amounts to comparing a simple discounted sum of post-formation returns to that of post-formation betas, we should continue to see the negative cross-sectional relation between the discounted sums of risks and returns that both quality- and $B/M$-sorted portfolios exhibit in the return regression (left two panels in Figure 1). That is, a mere convergence in returns and risks after portfolio formation cannot explain our results. In order for the sign of the negative cross-sectional relation to flip from negative to positive as we move from the return to the price-level perspective, at least one of the following should occur for the extreme decile portfolios: the return spread crosses (flips sign) at some point after portfolio formation; the beta spread crosses; or the intertemporal adjustment component of price-level risk in equation (5) shows an opposite cross-sectional pattern to the contemporaneous risk component, thereby undoing the puzzling pattern that the high-return decile 10 has a lower market beta than the low-return decile 1 immediately following portfolio formation.

The primary reason why price-level risks of quality-sorted portfolios explain their price levels well is that the return spread between extreme decile portfolios crosses. Figure 4 shows that high (low) returns earned by high (low) quality stocks are highly transitory and that the high-minus-low quality return spread turns negative from around one year following the portfolio formation.
and stays negative except for the blip around year 9. Another important contribution comes from
the intertemporal adjustment. Figure 5 shows that although the contemporaneous risk component
of price-level risk is substantially larger in low-quality stocks, the intertemporal component of
price-level risk is larger in high-quality stocks than in low-quality stocks and partly offsets the
contemporaneous component of risk. That is, high-quality stocks are riskier for investors with
a long investment horizon, since their returns tend to be low after the stock market experiences
a series of negative shocks; this would be the case, for instance, if high quality stocks’ market
beta tends to come from the exposure of their cash flow news to market-level cash flow news
(Campbell et al. (2010)).

On the other hand, price-level risks of $B/M$-sorted portfolios explain their price levels reason-
ably well, as the beta spread between high- and low-$B/M$ portfolios crosses. Figure 4 shows that
the market beta of the high-$B/M$ portfolio is low at the time of the portfolio formation but rises
above that of the low-$B/M$ portfolio, which steadily declines over the post-formation periods.
This fact, first documented in Cohen et al. (2009), makes value stocks riskier for a long-term
investor. \footnote{We discuss in Appendix B how our mispricing measure $\delta$ improves on the mispricing metric of Cohen et al. (2009).}

\textit{Quality and $B/M$ double sort}

As we anticipated earlier, double sorting stocks based on quality and $B/M$ to generate 25 portfo-
lios leads to larger variation in $\delta$s than a univariate sort (Table 4). Furthermore, the variation in
$\delta$ across the two dimensions of the table is consistent with our conjecture in Figure 3.

Price-level error declines as we move from top to bottom or left to right, which amounts
to holding one characteristic fixed while varying the other. We estimate expensive low-quality
stocks (top left) to be overpriced by 24\% ($t$-statistic of 3.22) and cheap high-quality stocks to
be underpriced by 33\% ($t$-statistic of $-2.91$) with respect to CAPM. Moving diagonally from
the top left to the bottom right generates the largest variation in $\delta$s. We estimate low-quality,
low-$B/M$ stocks to be 57 percentage point more overpriced than high-quality, high-$B/M$ stocks
with a $t$-statistic of 3.22.
All test statistics strongly reject the null hypothesis, and the mean squared error and mean absolute error are now significant at the 1% level. Indeed, the price-level variation of Table 2 is exactly the sort of variation a long-horizon investor should be interested in exploiting and represents a challenge for future asset pricing models that make understanding price-level risk a priority.

4 Are Return Anomalies Price-level Anomalies?

Next, we study the extent to which the CAPM explains price-level variation associated with eight additional characteristics known to be associated with cross-sectional variation in average returns: profitability, investment, momentum, size, beta, net issuance, accruals, and analysts’ long-term growth forecasts. The first four represent a set of prominent return anomalies, and the latter four are chosen for their potential conceptual link to price-level distortions. As noted in Section 3, CAPM tends to explain price-level variation better than it explains short-horizon return variation. Among the eight characteristics, investment, beta, and net issuance appear to be price-level anomalies, and size is also associated with somewhat large price-level errors despite it being a relatively unimportant characteristic for returns. We also highlight ways in which our price-level results are (in)consistent with conventional interpretations of these return anomalies.

4.1 Prominent return anomalies

We begin with the four prominent return anomalies. Fama and French (2015) argue that profitability, investment, and size are characteristics that are important in summarizing the cross-section of returns, and price momentum has been one a prominent return anomaly since Jegadeesh and Titman (1993) The panels on the left side of Figure 6 show that all but the size sort generate a negative cross-sectional relation between average return and market beta, implying that these characteristics are associated with cross-sectional variation in average return that cannot be explained by market beta. To what extent are these prominent return characteristics associated with variation in price levels unrelated to CAPM price-level risk?

*Profitability and investment*
Profitability-sorted portfolios are associated with only small price-level errors (Table 5, and they show a near-45-degree cross-sectional relation between price levels and price-level risks (Figure 6) that contrasts sharply with the negative cross-sectional relation in returns. The test statistics all indicate that the CAPM does a good job explaining the price levels of stocks with different degrees of profitability.

Why do profitability-sorted portfolios behave well in our price-level analysis, when they are known to be anomalous in conventional return regressions? Similarly to portfolios sorted on quality, a composite metric that depends importantly on profitability, return reversal after portfolio formation explains the improved price-level fit of profitability portfolios. High (low) returns earned by more (less) profitable firms are highly transitory, and more profitable firms begin earning lower returns than less profitable firms soon after portfolio formation, consistent with the difference in their market betas. Hence, it appears that abnormal returns earned on profitability-based trades are more consistent with investor overreaction to news about profitability rather than fundamental undervaluation (overvaluation) of profitable (unprofitable) companies (Novy-Marx (2013) and Asness et al. (2019)). The result is also consistent with the observation that cross-sectional variation in marginal product of capital, which our profitability measure could proxy for, does not necessarily imply misallocation of capital (David et al. (2019)).

Price-level variation is more anomalous for investment-sorted portfolios. Price levels of portfolios sorted by asset growth cannot be explained by variation in price-level risk, and we estimate high-investment firms to be 19.8% more overpriced relative to low-investment firms (Figure 6 and Table 5). Furthermore, out of the four prominent return anomalies we consider, investment is the only characteristic featuring a statistically significance difference in $\delta$s across the extreme decile portfolios. Moreover, among the characteristics studied here, the investment sort generates the lowest $p$-value for all of the test statistics except for mean absolute error and has the largest size-weighted mean absolute error $\Delta$.

Hence, between profitability and investment, the latter is by far the more important price-level anomaly that demands a more sophisticated SDF than the unconditional CAPM considered here. Our finding also echoes the conclusion of Polk and Sapienza (2008) that the investment characteristic is an important signal of mispricing for stocks with respect to the CAPM.
Momentum and size

Momentum and size are interesting to study from the price-level perspective, given that momentum is a strong return anomaly that is transient, and size is a relatively weak return anomaly that is persistent. Moreover, signal persistence is an important aspect to consider when one moves from the conventional return perspective to the price-level perspective, as highlighted in Cohen et al. (2009). Indeed, Cochrane (2011) subsequently emphasized this point.

“For example, since momentum amounts to a very small time-series correlation and lasts less than a year, I suspect it has little effect on long-run expected returns and hence the level of stock prices. Long-lasting characteristics are likely to be more important. Conversely, small transient price errors can have a large impact on return measures” (p.1064).

Figure 6 confirms Cochrane’s conjecture, as the economic magnitudes of the variation in both price levels and $\delta$s associated with momentum are small due to its transient nature. Somewhat surprisingly, however, the small standard errors associated with momentum portfolio $\delta$s actually result in some test statistics rejecting the CAPM (Table 5). Consistent with Cochrane’s view, the persistence of the size characteristic does generate a large price-level variation as well as statistically significant mean squared and mean absolute error. However, price-level errors associated with size tend to occur mostly in small portfolios, resulting in a size-weighted mean absolute error that is statistically insignificant.

4.2 Characteristics conceptually related to mispricing

Some other stock characteristics are interesting to analyze based on our price level identity either due to their mechanical link to price-level risk (beta) or their conceptual association with mispricing (net issuance, accruals, and analyst forecasts). We explain the conceptual link that each characteristic has to mispricing $\delta$ and study the extent to which the characteristic is associated with price-level variation that cannot be explained by exposure to market risk that the CAPM captures.

Beta
Equation (20) shows that price-level risk can be decomposed into a discounted sum of contemporaneous covariances between the SDF $M$ and returns and the sum of intertemporal covariances between the price-adjusted SDF $\phi$ and returns. Since market beta is a persistent characteristic, in the context of CAPM, this persistence implies that sorts based on estimates of market beta have the potential to generate large variation in price-level risk through the contemporaneous risk component of price-level risk. If this variation in price-level risk is then not compensated with corresponding variation in price levels, large $\delta$s could arise.

Figure 7 shows that the large variation in risk generated by the beta sort does not lead to a correspondingly large variation in price levels. Hence, the low beta anomaly in returns (Black et al. (1972); Frazzini and Pedersen (2014)) is also a price-level phenomenon, not just a temporary distortion that is offset by a subsequent reversion in risk or return. Table 6 shows an estimated difference in $\delta$s of 28.7 percentage points across the high- and low-beta portfolios. The size-weighted absolute error $\Delta$ is large, and all test statistics are highly significant with $p$-values lower than 0.1%. Overall, these results suggest that the low-beta anomaly is an important price-level phenomenon that needs to be explained by new models of price-level risk that add additional or alternative risk factors to the SDF.

Net share issuance

A series of papers argue that share repurchase (issuance) indicates undervaluation (overvaluation) as perceived by firm managers (Loughran and Ritter (1995); Ikenberry et al. (1995)) Thus, to the extent that firm managers are long-term investors in the firm, net issuance could be a good proxy for mispricing with respect to the CAPM.

Figure 7 reproduces the previous finding that low net issuance generates higher returns than high net issuance and that market beta is actually lower for low net issuance stocks than high net issuance stocks, resulting again in the anomalous negative relation between risk and returns seen in the beta sort. In fact, this slope is negative for all four characteristics considered in this part of the analysis. For a buy-and-hold mean-variance investor, low (negative) issuance firms are slightly riskier than high issuance firms, restoring a positive slope. Nevertheless, the mispricing $\delta$s are large: Table 6 shows that high issuance firms are estimated to be modestly overpriced by
7.7%, but low (negative) issuance firms are estimated to be underpriced by 21.5%. That is, share repurchase may be a stronger signal for underpricing than share issuance is for overpricing.

**Accruals**

Earnings management proxied by accruals (Sloan (1996)) is an interesting phenomenon to analyze with our price-level identity, as it is typically motivated as being the result of companies with adverse operating results managing earnings to inflate the value of their firm as perceived by outsiders. Thus, if the firms are successful in managing earnings, high accruals may proxy for the gap between the market price set by investors and intrinsic value perceived by firm managers. Table 6 shows results consistent with this interpretation of accruals: high accruals firms are estimated to be overpriced by 8.7%. Interestingly, firms with low accruals are also a bad investment for long-term investors, as they are estimated to be overpriced by 6.5%. However, the relatively small $\Delta$ implies that the aggregate mispricing of capital with respect to the CAPM associated with the accruals phenomenon is small, as that mispricing more strongly affects smaller firms.

**Analyst long-term growth forecasts**

La Porta (1996) and Bordalo et al. (2019) show that stocks with optimistic (pessimistic) analyst forecasts deliver poor (superior) returns and interpret this finding as confirming that investors overvalue (undervalue) stocks with optimistic (pessimistic) forecasts. We find this to be true to some extent, although the magnitude of mispricing is small. Figure 7 shows that the price levels of portfolios sorted by analyst forecasts line up reasonably well with price-level risk. In particular, stocks with optimistic forecasts are not significantly overpriced, but stocks with pessimistic forecasts are slightly underpriced by 5.8% with a $t$-statistic of 3.20 (Table 6).

**Takeaway**

Overall, we find that the return anomalies that the extant literature typically motivates as representing significant price-level distortion do tend to be associated with significant $\delta$s, at least in
one of the extreme decile portfolios. However, the magnitudes and the economic importance of mispricing associated with accruals and analyst forecasts are smaller than those associated with market beta and net share issuance.

5 Conclusion

Our novel model misspecification measure, delta, precisely links future alpha to current price-level deviations. In stark contrast to existing measures, our approach correctly recognizes that abnormal returns occurring sooner, in more valuable states, or after relatively large capital gains should be associated with larger price-level distortions. Our primary tests reveal that though the CAPM does a relatively good job describing the cross-section of average price levels of both book-to-market- and quality-sorted portfolios in isolation, portfolios formed from double sorts on these two variables generate significant variation in mispricing. We also show that investment, beta, and net equity issuance sorts also produce significant price-level distortions relative to the CAPM while size-, momentum-, profitability-, accruals-, and analyst-growth-forecast-sorted portfolios present much less of a challenge.

As a consequence, our novel mispricing measure and the associated results provide better identification of the stocks that a buy-and-hold mean-variance investor should find attractive/unattractive. Moreover, our approach highlights where new models that aim to explain both short- and long-run patterns in markets should focus. Indeed, by providing an exact metric of the extent to which a candidate asset-pricing model is approximately right, we hope to help future research distinguish among risk-based, behavioral-based, and institutional-friction-based explanations for well-known empirical patterns in markets.
References


Tables and Figures

Table 1: **Descriptive Statistics**

The table describes the 10 stock characteristics used to study mispricing with respect to the CAPM. Columns 2–3 and 4–5 describe the two value-weighted extreme decile portfolios formed based on the characteristic. $\overline{R}$ ($\alpha_{\text{CAPM}}$) is the average return in excess of the risk-free rate (the estimated abnormal return with respect to the CAPM) over one month following the portfolio formation. In the parentheses are heteroskedasticity-robust standard errors. Persistence is the value-weighted probability that the characteristic decile of a stock in the portfolio changes after a year. The sample period is 1957m6–2004m12 except for the analyst forecasts of long-term growth (LTG) variable, which has a sample period of 1981m12–2004m12. See Appendix A for further description of the 10 characteristics.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Decile 1</th>
<th>Decile 10</th>
<th>Persistence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\overline{R}$</td>
<td>$\alpha_{\text{CAPM}}$</td>
<td>$\overline{R}$</td>
</tr>
<tr>
<td>Quality</td>
<td>0.34</td>
<td>-0.24</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
<td>(0.10)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>Book-to-market</td>
<td>0.37</td>
<td>-0.18</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.08)</td>
<td>(0.21)</td>
</tr>
<tr>
<td>Profitability</td>
<td>0.39</td>
<td>-0.21</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(0.11)</td>
<td>(0.20)</td>
</tr>
<tr>
<td>Investment</td>
<td>0.29</td>
<td>-0.36</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(0.09)</td>
<td>(0.21)</td>
</tr>
<tr>
<td>Momentum</td>
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<td>-0.83</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(0.17)</td>
<td>(0.25)</td>
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<td>Size</td>
<td>0.44</td>
<td>-0.03</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td>(0.04)</td>
<td>(0.26)</td>
</tr>
<tr>
<td>Beta</td>
<td>0.40</td>
<td>-0.38</td>
<td>0.51</td>
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<td></td>
<td>(0.32)</td>
<td>(0.13)</td>
<td>(0.14)</td>
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<tr>
<td>Net issuance</td>
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<td>0.93</td>
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<td>(0.23)</td>
<td>(0.08)</td>
<td>(0.19)</td>
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<tr>
<td>Accruals</td>
<td>0.22</td>
<td>-0.43</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>(0.26)</td>
<td>(0.11)</td>
<td>(0.23)</td>
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<td>Analyst forecasts of long-term growth (LTG)</td>
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<td>-0.35</td>
<td>0.82</td>
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<td></td>
<td>(0.47)</td>
<td>(0.22)</td>
<td>(0.22)</td>
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Table 2: Explaining Price Levels of Stocks Sorted on Quality

The table shows that the CAPM does a good job describing the cross-section of price levels of portfolios sorted on quality (last two rows) but a poor job describing the cross-section of one-month returns (first two rows). Quality is the composite metric suggested by Asness et al. (2019) to proxy for the ratio of intrinsic value $V$ to book value $B$.

- In the last two “price level” rows, $\delta$ reports the estimated average ex-ante mispricing defined as

$$\delta \equiv E[(P_t - V_t) / P_t] = -E \left[ \sum_{j=1}^{180} M_{t+j} P_{t+j-1} R_{t+j}^s \right] = -E \left[ \sum_{j=1}^{180} M_{t+j} P_{t+j-1}^{-1} R_{t+j}^s \right] = \delta_{15\text{yrs}},$$

where $t$ is the portfolio formation month, $j$ is the number of post-formation months, $P_t$ is price, $V_t = \sum_{j=1}^\infty E_t [M_{t+j} D_{t+j}]$ is the intrinsic value of dividends, $M_{t+j} = \Pi_{t=1}^j (1 - b_t R_{t+j}^s)$ is the cumulative stochastic discount factor prescribed by the unconditional CAPM, and $R_{t+j}^s$ is one-month return in excess of the risk-free rate. The finite sum over 180 months (15 years) proxies for the infinite sum. Here, positive (negative) $\delta$ means overpricing (underpricing) relative to the present value of cash flows.

- In the first two “return” rows, $\delta$ measures $-1$ times the average one-month abnormal return (over the gross one-month risk-free rate):

$$\delta_{\text{1mo}} = -E \left[ \alpha_{t+1} / (1 + R_{f,t+1}) \right] = -E \left[ M_{t+1} R_{t+1}^s \right].$$

Here, positive (negative) $\delta$ means negative (positive) one-month abnormal return.

- In the remaining rows, the reported $\delta$ estimates $\delta_j = -E \left[ \sum_{j=1}^{J} M_{t+j} P_{t+j-1}^{-1} R_{t+j}^s \right]$ illustrate how the result changes with the intermediate values of $J$.

To estimate these quantities, we form ten portfolios sorted on quality over 1957m6–2004m12 and track post-formation returns up to 15 years, covering 1957m7–2019m12. In GMM, we choose $b_t$ that makes the market portfolio correctly priced in-sample, which automatically implies $\delta$s for the portfolios. Columns 2-6 report $\delta$ estimates for the lowest (“Lo”), second lowest (“2”), second highest (“9”), and highest (“Hi”) quality stock portfolios as well as the difference in $\delta$s between the two extreme deciles. We report in parentheses standard errors estimated based on the Bartlett kernel used by Newey and West (1987) with a bandwidth of $J$ months. The next four columns report test statistics: $-1$ times the slope of the line through price levels and price-level risks of decile portfolios (hence Slope = 1 under the null), size-weighted mean absolute error ($\Delta$), mean squared error (MSE), and equal-weighted mean absolute error (MAE). In the square brackets are the $p$-values. The last column reports the estimated $b_1$ and the associated monthly Sharpe ratio of the market ($S_m$).

<table>
<thead>
<tr>
<th>$J$</th>
<th>$\delta$ (std.err)</th>
<th>Test statistics [p-value]</th>
<th>Parameter</th>
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<tbody>
<tr>
<td></td>
<td>Lo</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>1mo</td>
<td>0.002</td>
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<td>&quot;return&quot;</td>
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<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>1yr</td>
<td>0.023</td>
<td>0.004</td>
<td>-0.018</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.007)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>3yrs</td>
<td>0.029</td>
<td>0.007</td>
<td>-0.028</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.015)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>5yrs</td>
<td>0.035</td>
<td>0.006</td>
<td>-0.028</td>
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<tr>
<td></td>
<td>(0.025)</td>
<td>(0.018)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>10yrs</td>
<td>0.025</td>
<td>0.010</td>
<td>-0.032</td>
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<tr>
<td></td>
<td>(0.030)</td>
<td>(0.014)</td>
<td>(0.032)</td>
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<tr>
<td>15yrs</td>
<td>0.015</td>
<td>0.012</td>
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<tr>
<td>&quot;price level&quot;</td>
<td>(0.029)</td>
<td>(0.013)</td>
<td>(0.028)</td>
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</table>
Table 3: Explaining Price Levels of Stocks Sorted on Book-to-Market Ratio

The table shows that the CAPM does an adequate job describing the cross-section of price levels of portfolios sorted on book-to-market ratio (last two rows) but a poor job describing the cross-section of one-month returns (first two rows).

- In the last two “price level” rows, $\delta$ reports the estimated average ex-ante mispricing defined as

$$\delta = E \left[ (P_t - V_t) / P_t \right] = -E \left[ \sum_{j=1}^{180} M_{t+j} P_{t+j-1} R^e_{t+j} \right] \approx -E \left[ \sum_{j=1}^{180} M_{t+j} P_{t+j-1} P_t^{-1} R^e_{t+j} \right] = \delta_{15\text{yrs}},$$

where $t$ is the portfolio formation month, $j$ is the number of post-formation months, $P_t$ is price, $V_t = \sum_{j=1}^{\infty} E_t [M_{t+j} D_{t+j}]$ is the intrinsic value of dividends, $M_{t+j} = \Pi_{j=1}^t (1 - b_1 R^e_{m+j})$ is the cumulative stochastic discount factor prescribed by the unconditional CAPM, and $R^e_t$ is one-month return in excess of the risk-free rate. The finite sum over 180 months (15 years) proxies for the infinite sum. Here, positive (negative) $\delta$ means overpricing (underpricing) relative to the present value of cash flows.

- In the first two “return” rows, $\delta$ measures $-1$ times the average one-month abnormal return (over the gross one-month risk-free rate):

$$\delta_{1\text{mo}} = -E \left[ \alpha_{t+1} / (1 + R_{f,t+1}) \right] = -E \left[ M_{t+1} R^e_{t+1} \right].$$

Here, positive (negative) $\delta$ means negative (positive) one-month abnormal return.

- In the remaining rows, the reported $\delta$ estimates $\delta_j = -E \left[ \sum_{j=1}^{J} M_{t+j} P_{t+j-1} P_t^{-1} R^e_{t+j} \right]$ illustrate how the result changes with the intermediate values of $J$.

To estimate these quantities, we form ten portfolios sorted on quality over 1957m6–2004m12 and track post-formation returns up to 15 years, covering 1957m7–2019m12. In GMM, we choose $b_1$ that makes the market portfolio correctly priced in-sample, which automatically implies $\delta$s for the portfolios. Columns 2–6 report $\delta$ estimates for the lowest (“Lo”), second lowest (“2”), second highest (“9”), and highest (“Hi”) quality stock portfolios as well as the difference in $\delta$s between the two extreme deciles. We report in parentheses standard errors estimated based on the Bartlett kernel used by Newey and West (1987) with a bandwidth of $J$ months. The next four columns report test statistics: $-1$ times the slope of the line through price levels and price-level risks of decile portfolios (hence Slope = 1 under the null), size-weighted mean absolute error ($\Delta$), mean squared error (MSE), and equal-weighted mean absolute error (MAE). In the square brackets are the $p$-values. The last column reports the estimated $b_1$ and the associated monthly Sharpe ratio of the market ($S_m$).

<table>
<thead>
<tr>
<th>$J$</th>
<th>Lo</th>
<th>2</th>
<th>9</th>
<th>Hi</th>
<th>Lo - Hi</th>
<th>Test statistics [p-value]</th>
<th>Parameter $b_1[S_m]$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1mo</strong></td>
<td>0.002</td>
<td>-0.000</td>
<td>-0.004</td>
<td>-0.004</td>
<td>0.006</td>
<td>-2.489 [0.002]</td>
<td>0.001 [0.000]</td>
</tr>
<tr>
<td>(“return”)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.006)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>1yr</td>
<td>0.020</td>
<td>0.003</td>
<td>-0.044</td>
<td>-0.050</td>
<td>0.070</td>
<td>-1.872 [0.007]</td>
<td>0.016 [0.009]</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.010)</td>
<td>(0.022)</td>
<td>(0.009)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>3yrs</td>
<td>0.046</td>
<td>0.019</td>
<td>-0.089</td>
<td>-0.119</td>
<td>0.164</td>
<td>-1.187 [0.032]</td>
<td>0.040 [0.013]</td>
</tr>
<tr>
<td></td>
<td>(0.036)</td>
<td>(0.020)</td>
<td>(0.015)</td>
<td>(0.024)</td>
<td>(0.056)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>5yrs</td>
<td>0.061</td>
<td>0.034</td>
<td>-0.134</td>
<td>-0.157</td>
<td>0.218</td>
<td>-0.897 [0.042]</td>
<td>0.057 [0.032]</td>
</tr>
<tr>
<td></td>
<td>(0.057)</td>
<td>(0.034)</td>
<td>(0.025)</td>
<td>(0.031)</td>
<td>(0.086)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>10yrs</td>
<td>0.093</td>
<td>0.041</td>
<td>-0.184</td>
<td>-0.184</td>
<td>0.276</td>
<td>1.181 [0.067]</td>
<td>0.071 [0.085]</td>
</tr>
<tr>
<td></td>
<td>(0.086)</td>
<td>(0.051)</td>
<td>(0.051)</td>
<td>(0.044)</td>
<td>(0.129)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>15yrs</td>
<td>0.124</td>
<td>0.036</td>
<td>-0.199</td>
<td>-0.184</td>
<td>0.308</td>
<td>0.749 [0.057]</td>
<td>0.085 [0.087]</td>
</tr>
<tr>
<td>(“price level”)</td>
<td>(0.108)</td>
<td>(0.054)</td>
<td>(0.054)</td>
<td>(0.038)</td>
<td>(0.144)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
</tbody>
</table>
Table 4: **Mispricings in Stocks Double-Sorted on Quality and B/M**

The table shows that mispricing relative to the CAPM is large for portfolios double-sorted on quality and book-to-market ratio. We form 25 portfolios by sorting stocks based on quality within each book-to-market quintile. We form portfolios in 1957m6–2004m12 and track post-formation returns up to 15 years, covering 1957m7–2019m12. The reported $\delta$s are estimated values of mispricing defined as $\delta = -E\left[\sum_{j=1}^{J} M_{t+j} \frac{P_{t+j}}{P_t} R_{t+j}^e\right]$, where $t$ denotes the portfolio formation month, $j$ denotes the number of post-formation months, and $J = 180$ months. We use the SDF implied by the unconditional CAPM, $M_{t+j} = 1 - b_1 R_{m,t+j}$, where $b_1$ is chosen such that the market portfolio has a zero in-sample $\delta$. We report in parentheses standard errors estimated based on the Bartlett kernel used by Newey and West (1987) with a bandwidth of $J$ months. The test statistics are the slope of the line through price levels (times $-1$) and price-level risks of decile portfolios (slope = 1 under the null), size-weighted mean absolute error, mean squared error, and equal-weighted mean absolute error. We report $p$-values in square brackets.

<table>
<thead>
<tr>
<th>Quality</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>1-5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.235</td>
<td>0.171</td>
<td>0.074</td>
<td>0.060</td>
<td>0.007</td>
<td>0.228</td>
</tr>
<tr>
<td></td>
<td>(0.073)</td>
<td>(0.065)</td>
<td>(0.085)</td>
<td>(0.102)</td>
<td>(0.113)</td>
<td>(0.063)</td>
</tr>
<tr>
<td>2</td>
<td>0.096</td>
<td>0.080</td>
<td>-0.012</td>
<td>-0.107</td>
<td>-0.146</td>
<td>0.243</td>
</tr>
<tr>
<td></td>
<td>(0.036)</td>
<td>(0.050)</td>
<td>(0.042)</td>
<td>(0.024)</td>
<td>(0.027)</td>
<td>(0.045)</td>
</tr>
<tr>
<td>3</td>
<td>0.047</td>
<td>-0.030</td>
<td>-0.087</td>
<td>-0.121</td>
<td>-0.227</td>
<td>0.274</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.021)</td>
<td>(0.013)</td>
<td>(0.012)</td>
<td>(0.032)</td>
<td>(0.041)</td>
</tr>
<tr>
<td>4</td>
<td>-0.038</td>
<td>-0.142</td>
<td>-0.139</td>
<td>-0.167</td>
<td>-0.205</td>
<td>0.166</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.059)</td>
<td>(0.031)</td>
<td>(0.030)</td>
<td>(0.055)</td>
<td>(0.045)</td>
</tr>
<tr>
<td>5</td>
<td>-0.116</td>
<td>-0.132</td>
<td>-0.199</td>
<td>-0.253</td>
<td>-0.332</td>
<td>0.217</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td>(0.026)</td>
<td>(0.049)</td>
<td>(0.061)</td>
<td>(0.114)</td>
<td>(0.139)</td>
</tr>
<tr>
<td>1-5</td>
<td>0.351</td>
<td>0.303</td>
<td>0.273</td>
<td>0.314</td>
<td>0.339</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.071)</td>
<td>(0.088)</td>
<td>(0.132)</td>
<td>(0.159)</td>
<td>(0.225)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>11-55</th>
<th>Slope</th>
<th>$\Delta$</th>
<th>MSE</th>
<th>MAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test statistics:</td>
<td>0.567</td>
<td>0.13</td>
<td>0.11</td>
<td>0.02</td>
<td>0.13</td>
</tr>
<tr>
<td>(stderr)/[$p$-value]</td>
<td>(0.176)</td>
<td>[0.003]</td>
<td>[0.033]</td>
<td>[0.004]</td>
<td>[0.002]</td>
</tr>
</tbody>
</table>
**Table 5: Explaining Price Levels: Prominent Return Anomalies**

The table reports estimated mispricings \((J = 15\text{yrs})\) vs. abnormal returns \((J = 1\text{mo})\) with respect to the CAPM for portfolios sorted on prominent return anomaly characteristics. For each characteristic, we form decile portfolios in 1957m6–2004m12 and track post-formation returns up to 15 years, covering 1957m7–2019m12 (1982m1–2019m12 for analyst forecast). The reported \(\hat{\delta}\)s for \(J = 15\) years are estimated values of mispricing defined as 
\[
\hat{\delta} = -E \left[ \sum_{j=1}^{\infty} M_{t+j} \frac{R_{t+j} - R_{t}}{P_{t+j}} \right],
\]
where \(t\) denotes the portfolio formation month, and \(j\) denotes the number of post-formation months. The reported \(\hat{\delta}\)s for \(J = 1\) month are the estimated errors from a conventional time-series asset pricing regression based on one-month returns. We use the SDF implied by the unconditional CAPM, 
\[
M_{t+j} = 1 - b_1 R_{m,t+j},
\]
where \(b_1\) is chosen such that the market portfolio has a zero in-sample \(\hat{\delta}\). We report in parentheses standard errors estimated based on the Bartlett kernel used by Newey and West (1987) with a bandwidth of \(J\) months. The test statistics are \(-1\) times the slope of the line through price levels and price-level risks of decile portfolios (Slope = 1 under the null), size-weighted mean absolute error (\(\Delta\)), mean squared error (MSE), and equal-weighted mean absolute error (MAE). We report \(p\)-values in square brackets.

<table>
<thead>
<tr>
<th>Sort</th>
<th>(J)</th>
<th>1</th>
<th>2</th>
<th>9</th>
<th>10</th>
<th>1 - 10</th>
<th>Slope</th>
<th>(\Delta)</th>
<th>MSE</th>
<th>MAE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Profitability</strong></td>
<td></td>
<td>0.002</td>
<td>0.001</td>
<td>-0.001</td>
<td>-0.001</td>
<td>0.003</td>
<td>-0.361</td>
<td>0.001</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>15yrs</td>
<td>-0.057</td>
<td>-0.080</td>
<td>0.041</td>
<td>0.037</td>
<td>-0.094</td>
<td>0.846</td>
<td>0.035</td>
<td>0.002</td>
<td>0.038</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.052)</td>
<td>(0.051)</td>
<td>(0.059)</td>
<td>(0.073)</td>
<td>(0.120)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Investment</strong></td>
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<td>0.004</td>
<td>0.001</td>
<td>-0.002</td>
<td>-0.002</td>
<td>0.005</td>
<td>-1.050</td>
<td>0.001</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>15yrs</td>
<td>0.092</td>
<td>0.091</td>
<td>-0.110</td>
<td>-0.106</td>
<td>0.198</td>
<td>-0.015</td>
<td>0.061</td>
<td>0.006</td>
<td>0.069</td>
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<tr>
<td></td>
<td></td>
<td>(0.031)</td>
<td>(0.040)</td>
<td>(0.029)</td>
<td>(0.044)</td>
<td>(0.073)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Momentum</strong></td>
<td></td>
<td>0.008</td>
<td>0.003</td>
<td>-0.003</td>
<td>-0.006</td>
<td>0.014</td>
<td>-1.115</td>
<td>0.002</td>
<td>0.000</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>15yrs</td>
<td>-0.065</td>
<td>-0.078</td>
<td>0.027</td>
<td>0.051</td>
<td>-0.116</td>
<td>0.662</td>
<td>0.038</td>
<td>0.002</td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.045)</td>
<td>(0.029)</td>
<td>(0.026)</td>
<td>(0.036)</td>
<td>(0.077)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Size</strong></td>
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<td>0.000</td>
<td>-0.002</td>
<td>-0.002</td>
<td>0.003</td>
<td>3.338</td>
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<td>0.001</td>
</tr>
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<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>15yrs</td>
<td>0.040</td>
<td>-0.033</td>
<td>-0.191</td>
<td>-0.176</td>
<td>0.217</td>
<td>1.774</td>
<td>0.058</td>
<td>0.019</td>
<td>0.121</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.086)</td>
<td>(0.054)</td>
<td>(0.030)</td>
<td>(0.043)</td>
<td>(0.126)</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
Table 6: Explaining Price Levels: Characteristics with Conceptual Links to Price-level Errors

The table reports estimated mispricings \((J = 15\text{yrs})\) vs. abnormal returns \((J = 1\text{mo})\) with respect to the CAPM for portfolios sorted on characteristics conceptually linked to mispricing. For each characteristic, we form decile portfolios in 1957m6–2004m12 (1981m12–2004m12 for analyst forecast) and track post-formation returns up to 15 years, covering 1957m7–2019m12 (1982m1–2019m12 for analyst forecast). The reported \(\delta\)s for \(J = 15\text{ years}\) are estimated values of mispricing defined as 

\[
\delta = -E\left[\sum_{j=1}^{\infty} M_{t+j} \frac{R_{t+j}}{R^e_{t+j}}\right],
\]

where \(t\) denotes the portfolio formation month, and \(j\) denotes the number of post-formation months. The reported \(\delta\)s for \(J = 1\text{ month}\) are the estimated errors from a conventional time-series asset pricing regression based on one-month returns. We use the SDF implied by the unconditional CAPM, 

\[
M_{t+j} = 1 - b_1 R^e_{m,t+j},
\]

where \(b_1\) is chosen such that the market portfolio has a zero in-sample \(\delta\). We report parentheses standard errors estimated based on the Bartlett kernel used by Newey and West (1987) with a bandwidth of \(J\) months. The test statistics are \(-1\) times the slope of the line through price levels and price-level risks of decile portfolios (Slope = 1 under the null), size-weighted mean absolute error (\(\Delta\)), mean squared error (MSE), and equal-weighted mean absolute error (MAE). We report \(p\)-values in square brackets.

<table>
<thead>
<tr>
<th>Sort</th>
<th>(J)</th>
<th>1</th>
<th>2</th>
<th>9</th>
<th>10</th>
<th>1 - 10</th>
<th>Slope</th>
<th>(\Delta)</th>
<th>MSE</th>
<th>MAE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beta</td>
<td>1mo</td>
<td>0.004</td>
<td>0.002</td>
<td>-0.003</td>
<td>-0.002</td>
<td>0.006</td>
<td>-0.433</td>
<td>0.002</td>
<td>0.000</td>
<td>0.002</td>
</tr>
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<td></td>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>[0.007]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td></td>
<td>15yrs</td>
<td>0.125</td>
<td>0.093</td>
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<td>0.287</td>
<td>-0.247</td>
<td>0.090</td>
<td>0.012</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.044)</td>
<td>(0.022)</td>
<td>(0.033)</td>
<td>(0.040)</td>
<td>(0.074)</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>Net issuance</td>
<td>1mo</td>
<td>0.004</td>
<td>0.002</td>
<td>-0.002</td>
<td>-0.005</td>
<td>0.008</td>
<td>-3.162</td>
<td>0.002</td>
<td>0.000</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td></td>
<td>15yrs</td>
<td>0.077</td>
<td>0.045</td>
<td>-0.092</td>
<td>-0.215</td>
<td>0.293</td>
<td>2.887</td>
<td>0.056</td>
<td>0.007</td>
<td>0.058</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.022)</td>
<td>(0.023)</td>
<td>(0.014)</td>
<td>(0.068)</td>
<td>(0.072)</td>
<td>[0.000]</td>
<td>[0.004]</td>
<td>[0.002]</td>
<td>[0.005]</td>
</tr>
<tr>
<td>Accruals</td>
<td>1mo</td>
<td>0.004</td>
<td>0.002</td>
<td>-0.001</td>
<td>-0.001</td>
<td>0.005</td>
<td>-1.206</td>
<td>0.001</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>[0.003]</td>
<td>[0.060]</td>
<td>[0.000]</td>
<td>[0.007]</td>
</tr>
<tr>
<td></td>
<td>15yrs</td>
<td>0.087</td>
<td>0.032</td>
<td>-0.021</td>
<td>0.065</td>
<td>0.023</td>
<td>-0.048</td>
<td>0.037</td>
<td>0.002</td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.011)</td>
<td>(0.008)</td>
<td>(0.025)</td>
<td>(0.025)</td>
<td>(0.030)</td>
<td>[0.000]</td>
<td>[0.004]</td>
<td>[0.001]</td>
<td>[0.001]</td>
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<td>(0.001)</td>
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<td>[0.033]</td>
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</tbody>
</table>
Figure 3: **Double Sorting on Quality and B/M: Illustration**

Assuming that quality proxies for the intrinsic value to book equity ratio, this diagram illustrates how a double sort on quality and the book-to-market ratio should generate a large cross-sectional variation in mispricing $\delta$ that asset pricing models should aim to explain.
Figure 4: **Post-formation Behavior of Return and Risk: Quality and B/M**

The left panel reports the post-formation mean excess returns of the two decile portfolios, and the right panel reports the post-formation market betas. The portfolios are formed in 1957m6–2004m12, and we track post-formation returns over the subsequent 15 years (1957m7–2019m12).
Figure 5: Decomposing Price-level Risk: Quality and B/M

These figures decomposes price-level risk into its contemporaneous and intertemporal components:

$$\sum_{j=1}^{\infty} \text{Cov}_t \left( -M_{t+j} \frac{R_{t+j} - R_{t+j-1}}{P_t}, R_{t+j} \right) = \sum_{j=1}^{\infty} E_t \left[ \phi_{t+j-1} \right] \text{Cov}_t \left( -M_{t+j}, R_{t+j} \right)$$

Contemporaneous risk

$$- \sum_{j=1}^{\infty} E_t \left[ \left( \phi_{t+j-1} - E_t \left[ \phi_{t+j-1} \right] \right) M_{t+j} \left( R_{t+j} - E_t \left[ R_{t+j} \right] \right) \right]$$

Intertemporal adjustment

where $\phi_{t+j} = M_{t+j} P_{t+j-1} / P_t$. Although contemporaneous risk is a large fraction of the overall price-level risk, both the contemporaneous and intertemporal components contribute importantly to the cross-sectional variation in price-level risk.
Figure 6: Explaining Returns vs. Price Levels: Prominent Return Anomalies

The left panel reports the cross-sectional relation between realized mean excess return and mean excess return predicted by the CAPM for 10 decile portfolios formed in 1957m6–2004m12. The right panel reports the cross-sectional relation between estimated price levels and price-level risk with respect to the CAPM. Scaled price level is higher if long-horizon discount rates are higher, contrary to our original definition of price level. The 45-degree dash line indicates the perfect cross-sectional relation between risk and discount rates. See the description in Figure 1 for further details.
Figure 7: Explaining Returns vs. Price Levels: Characteristics Conceptually Linked to Mispricing

The left panel reports the cross-sectional relation between realized mean excess return and mean excess return predicted by the CAPM for 10 decile portfolios formed in 1957m6–2004m12 (1981m12–2004m12 for analyst forecast). The right panel reports the cross-sectional relation between estimated price levels and price-level risk with respect to the CAPM. Scaled price level is higher if long-horizon discount rates are higher, contrary to our original definition of price level. The 45-degree dash line indicates the perfect cross-sectional relation between risk and discount rates. See the description in Figure 1 for further details.
Appendix to “Asset Pricing with Price Levels”

A  Data

A.1  Basic data adjustments

We use domestic common stocks (CRSP share code 10 or 11) listed on the three major exchanges (CRSP exchange code 1, 2, or 3). We replace missing prices with the average bid-ask price when available and drop observations with missing share or price information in the previous month. We code missing returns as zero returns and add delisting returns to returns. If delisting returns are missing but the CRSP delisting code is 500 or between 520 and 584, we use $-35\%$ ($-55\%$) as the delisting returns for NYSE and AMEX stocks (for NASDAQ stocks) (Shumway (1997) and Shumway and Warther (1999)). To compute capital gains, we use the CRSP split-adjustment factor (CFACSHR) to ensure that capital gains are not affected by split events. The market factor is downloaded from Kenneth French’s website.

A.2  Characteristics and portfolios

One important characteristic with which we sort stocks is the book to market (B/M) ratio computed each year in June. B/M ratio is the stock’s book value of equity in the previous fiscal year divided by market value of equity in December of the previous calendar year. Book value of equity is defined as the stockholders’ equity $SEQ$ (Compustat item 144) plus balance sheet deferred taxes and investment tax credit $TXDITC$ (item 35) minus book value of preferred stock ($BE = SEQ + TXDITC - BPSTK$). Book value of preferred stock $BPSTK$ equals the preferred stock redemption value $PSTKRV$ (item 56), preferred stock liquidating value $PSTKLI$ (item 10), preferred stock $PSTK$ (item 130), or zero depending on data availability. If $SEQ$ is unavailable, we set it equal to total assets $AT$ (item 6) minus total liability $LT$ (item 181). If $TXDITC$ is unavailable, it is assumed to be zero. We treat zero or negative book values as missing.

Another important characteristic with which we sort stocks is the quality measure of Asness.
et al. (2019) defined as a z-score based on the four characteristics below:

\[ \text{quality} = z(\text{profitability}, \text{growth}, \text{safety}, \text{payout-ratio}) . \]

These \( z \) scores of different characteristics are in turn obtained as an equal weighted average of different measures of each characteristic. See Asness et al. (2019) for more details on the measure and its construction.

We also look at portfolios sorted by eight additional characteristics related to the cross-section of returns. We define profitability and investment as in Fama and French (2015). Profitability is computed each year in June. Operating profitability ("profitability") in calendar year \( y \) is operating profits in fiscal year \( y - 1 \) over book value of equity in fiscal year \( y - 1 \), where operating profits \( \text{SALE} \) (Compustat item 12) minus cost of goods sold \( \text{COGS} \) (item 41), interest and related expenses \( \text{XINT} \) (item 134) (if available), and selling, general, and administrative expenses \( \text{XSGA} \) (item 132) (if available). Asset growth ("investment") is also computed each year in June, and investment in calendar year \( y \) is total assets in fiscal year \( y - 1 \) divided by total assets in fiscal year \( y - 2 \). Size is market equity calculated every month. Momentum is calculated every month and is the cumulative gross return over the previous 12 months excluding the month before the portfolio formation. Beta is the trailing 3-year market beta (minimum of 2 years) calculated each month based on overlapping 3-day returns. Net issuance is calculated as of June and is the split-adjusted growth in shares outstanding in the previous year. Accruals measures the degree to which earnings come from non-cash sources and is defined according to Sloan (1996). Forecasted long-term growth is downloaded from IBES for the period of 1981m12 and onward.

For each characteristic, we form 10 value-weight portfolios based on the distribution of the characteristic among NYSE stocks. When doing so, to mitigate the effect of stale prices, we exclude microcaps defined as market equity below the bottom 10% cutoff among NYSE stocks. (Our results are very similar if we instead exclude stocks with liquidity level based on Amihud (2002) below the bottom 10% NYSE cutoff.) Hence, the 10 portfolios sorted by size are only based on the upper 90% size groups. However, to avoid a look-ahead bias, if after portfolio formation a stock’s size falls below the 10 percentile NYSE cutoff, we still keep it in the portfolio. To be consistent, the market portfolio that we require to be correctly priced is also formed after
excluding the stocks below the 10% NYSE cutoff.

B Comparing $\delta$ to Other Metrics of Price-level Error

Some readers may be interested in how our price-level measure of mispricing, $\delta_t = \frac{P_t - V_t}{P_t}$, compares to existing measures of mispricing or long-term return.

Market-to-book ratio

The market-to-book ratio, closely related to long-run reversal, is one popular measure of mispricing (De Bondt and Thaler (1985), Rosenberg et al. (1985), and Lakonishok et al. (1994)). However, market-to-book ratio is a highly imperfect measure of mispricing, since many other factors than mispricing can influence the ratio. The decomposition of Vuolteenaho (2002) and Cohen et al. (2003) shows that the log market-to-book ratio $mb_t$ is approximately,

$$mb_t = \sum_{j=1}^{\infty} \rho_j E_t [roe_{t+j}] - \sum_{j=1}^{\infty} \rho_j E_t [\alpha_{t+j} + \lambda_{M,t+j} \beta_{M,t+j}] + \frac{1}{2} \sum_{j=1}^{\infty} \rho_j Var_t (r_{t+j}), \quad (31)$$

where $roe_{t+j}$ is the log return on equity, $\alpha_{t+j}$ is abnormal return, $\lambda_{M,t+j} \beta_{M,t+j}$ is the risk premium implied by the SDF $M$, and $r_{t+j}$ is log return. Hence, besides the distortion in the discount rate due to $\alpha_{t+j}$, other factors such as earnings growth, risk, and volatility can affect the cross-sectional and time-series variation in the market-to-book ratio.

Price-level alpha

Cohen et al. (2009) (CPV) were the first to propose an identity for measuring mispricing in the price level. They define the fundamental value of a stock as the present value of future dividends discounted with the discount factors that would have prevailed in the absence of mispricing:

$$V_t = \sum_{j=1}^{\infty} E_t \left[ \frac{D_{t+j}}{\Pi_{s=1}^{j} (1 + R_{t+s})} \right], \quad (32)$$
where $R_{v,t+j}$ is the return on $V_t$. They then show that price-level alpha $\alpha_{t}^{\text{price}}$, defined as the log deviation of price from value, approximately equals

$$\alpha_{t}^{\text{price}} = -\sum_{j=1}^{\infty} \rho^{j-1} E_t \left[ R_{t+j} - r_{v,t+j} \right]$$

where $r$ denotes log return, $R$ denotes simple return, and $\beta_{M,v,t+j}$ is quantity of risk in the absence of mispricing. Hence, if the distortion in the volatility of log return due to mispricing is small,

$$-\sum_{j=1}^{\infty} \rho^{j-1} E_t \left[ R_{t+j} \right] \approx \alpha_{t}^{\text{price}} - \sum_{j=1}^{\infty} \rho^{j-1} E_t \left[ \lambda_{M,t+j} \beta_{M,v,t+j} \right].$$

CPV use this relation to run a cross-sectional regression that explains the price level based on price-level risk with respect to the CAPM, using cash-flow betas measured by the exposure of a portfolio’s return on equity to the market’s return on equity to estimate market betas in the absence of mispricing. Their regression is useful in rejecting the null of no mispricing, since under that null the volatility distortion should be zero, but using their identity to draw a conclusion beyond the rejection of the null is difficult due to the potentially large volatility distortion along with measurement error introduced from estimating $\beta_{M,v,t+j}$ with their cash-flow beta proxy. Our identity addresses this problem by expressing mispricing in terms of observable quantities (other than the SDF loadings).

**The misspecification metric in van Binsbergen and Opp (2019)**

van Binsbergen and Opp (2019) introduce a production economy in which they generate several insights about real distortions arising from abnormal returns. An important quantity in their analysis is log abnormal return defined as $\tilde{\alpha}_t \equiv P_t - E_t \left[ M_{t+1} (D_{t+1} + P_{t+1}) \right]$, where $M_{t+1}$ is a candidate stochastic discount factor. Rewriting this expression, iterating it forward, and imposing a transversality condition implies,

$$P_t = \sum_{j=0}^{\infty} E_t \left[ M_{t+j+1} e^{-\sum_{s=0}^{j} \tilde{\alpha}_{t+s} D_{t+j+1}} \right].$$

53
This expression, although not the main point of their paper, is useful in our context as follows. If \( P_t \) differs from intrinsic value defined as \( V_t = \sum_{j=0}^{\infty} E_t \left[ M_{t,t+j+1} D_{t+j+1} \right] \) because of frictions, then the above expression implies that the price deviation from value is an aggregation of abnormal returns:

\[
P_t - V_t = \sum_{j=0}^{\infty} E_t \left[ M_{t,t+j+1} D_{t+j+1} \left( e^{-\sum_{s=0}^{j} \tilde{\alpha}_t - 1} \right) \right]. \tag{36}
\]

This expression also allows van Binsbergen and Opp (2019) to anticipate that price distortion depends not just on the magnitude of the alphas, but also on their persistence. However, the expression involves the dividend process and is therefore difficult to take to data without putting more structure on how dividends evolve.

**Estimating the fundamental value directly**

Another approach to estimating fundamental mispricing is to estimate the fundamental value directly from cash-flow data:

\[
V_t = \sum_{j=1}^{\infty} E_t \left[ M_{t,t+j} D_{t+j} \right]. \tag{37}
\]

Estimating this quantity unconditionally in the data is problematic for two reasons. First, the predictability of dividends means that conditional covariance is smaller than the unconditional covariance. Second, truncation of the infinite sum at some finite \( J \) leaves out a large fraction of the value.\(^{18}\) To estimate the conditional expectation, one can estimate the dynamics of the factors in the SDF and dividends in a VAR and use it to obtain the conditional fundamental values at each given point in time. However, depending on the cash flow, a simple VAR with a few number of lags may not capture the persistent nature of dividend cash flows, and the constant coefficients in the estimated VAR model could understate the extent to which the SDF covaries with cash flows in a crisis scenario.

\(^{18}\)For example, suppose \( V_t \) follows a Gordon growth model with expected dividend growth rate \( g \) and constant discount rate \( R \). \( \sum_{j=J}^{\infty} E_t \left[ D_t \left( 1 + g \right)^j / (1 + R)^j \right] = \left( 1 + g / (1 + R) \right)^J E_t \left[ D_t \frac{1+g}{R-g} \right] = \left( 1 + g / (1 + R) \right)^J V_t. \) This means, if \( g = 5\% \) and \( R = 10\% \), about 40\% (10\%) of \( V_t \) comes from dividends occurring 20 (50) years after \( t \).
Cumulative abnormal return (CAR)

One popular measure of long-term return is the cumulative abnormal return defined as the simple sum of abnormal returns over a period of time:

$$\text{CAR}_t = -\sum_{j=1}^{\infty} E_t[\alpha_{t+j}]$$

(written with a sign flip so that, like our $\delta_t$, positive abnormal returns means a negative $\text{CAR}_t$).

How well can CAR proxy for ex-ante mispricing of the portfolio?

To see how CAR relates to price-level mispricing $\delta_t$, rewrite equation (16) as

$$\delta_t = -\sum_{j=1}^{\infty} E_t[w_{t,t+j}] E_t[\alpha_{t+j}] - \sum_{j=1}^{\infty} \text{Cov}_t\left( w_{t,t+j}, \alpha_{t+j} \right),$$

where

$$w_{t,t+j} = M_{t,t+j-1} \frac{P_{t+j-1}}{P_t} \frac{1}{1+R_{f,t+j}}.$$

Hence, CAR is an exact measure of mispricing when $E_t[w_{t,t+j}] = 1$ and $\text{Cov}_t\left( w_{t,t+j}, \alpha_{t+j} \right) = 0$.

These conditions would approximately hold, for example, if abnormal return tends to be stable (i.e., conditional abnormal return equals unconditional abnormal return), the gross monthly risk-free rate is close to 1, and the portfolio has a very high duration such that cumulative capital gain approximately equals cumulative return (in which case $E_t\left[M_{t,s}\frac{P_{t+s}}{P_{t+s-1}}\right] \approx E_t\left[M_{t,s}(1+R_{t+s})\right] \approx 1 \implies E_t\left[ \prod_{s=1}^{j-1} \left( M_{t,s+1}\frac{P_{t+s}}{P_{t+s-1}} \right) \right] \approx 1$ when abnormal returns are small and both the SDF and returns exhibit little serial covariance). Nevertheless, Cho (2019) shows that returns on anomaly trading strategies depend importantly on shocks to the capital of arbitrageurs proxied by aggregate funding liquidity and the aggregate arbitrageur’s portfolio, suggesting that these conditions are violated.

“Discounted” CAR

Under slightly less strong assumptions than the ones specified for CAR, mispricing $\delta_t$ is approximately a discounted sum of subsequent abnormal returns with a simple geometric discount factor. To see this, start with Equation (39) and continue to assume $\text{Cov}_t\left( w_{t,t+j}, \alpha_{t+j} \right) = 0$. 
Next, note that if monthly risk-free rates are small and both the SDF and returns exhibit little serial covariance,

\[ E_t \left[ w_{t,t+j} \right] \approx \Pi_{s=1}^{j-1} E_t \left[ M_{t+s} \frac{P_{t+s}}{P_{t+s-1}} \right] = \Pi_{s=1}^{j-1} E_t \left[ M_{t+s} (1 + R_{t+s}) \frac{1}{1 + D_{t+s}/P_{t+s}} \right]. \] (41)

Then, replace \( \frac{1}{1 + D_{t+s}/P_{t+s}} \) with the Campbell and Shiller (1988) discount factor \( \rho = \frac{1}{1 + D/P} \) (where \( D/P \) is the long-run average of the dividend-price ratio) and assume that abnormal returns are small \( (E_t [M_{t+s} (1 + R_{t+s})] \approx 1) \) to obtain

\[ E_t \left[ w_{t,t+j} \right] \approx \rho^{j-1} \] (42)

Hence, under these strong assumptions, we can write mispricing as a sum of subsequent abnormal returns discounted at a constant rate:

\[ \delta_t \approx -\sum_{j=1}^{\infty} \rho^{j-1} E_t \left[ \alpha_{t+j} \right]. \] (43)

We call this discounted CAR and analyze this potential metric in together with simple CAR below.

**How well do CAR and DCAR proxy for \( \delta \)? An empirical investigation**

Measures such as the cumulative abnormal returns (CAR),

\[ CAR = - \sum_{j=1}^{J} E_t \left[ \alpha_{t+j} \right], \]

proxy for the degree to which the assets generate abnormal returns over a long period of time following a corporate or market event of interest but do not have a clear theoretical interpretation. (The convention is to not have a negative sign before the sum, but we put it there for easier comparison to \( \delta \).) Given that our mispricing measure \( \delta_t = (P_t - V_t) / P_t \) also takes the form of a discounted sum of future abnormal returns, how well does CAR proxy for the initial mispricing of the assets?

**Figure B1** plots the estimated \( \delta_s \) based on the in-sample value of the SDF loading \( b_1 \) and
CARs of B/M sorted portfolios with \( J = 180 \) months. Both for quality and book-to-market sorted portfolios, there is a substantial difference between CARs and \( \delta \)s. In particular, CAR tends to exaggerate the cross-sectional variation in \( \delta \)s, possibly due to the equal weighting of all future abnormal returns.

It is interesting to ask whether we can do better by introducing a constant discount factor to the CAR formula to obtain a discounted CAR (“DCAR”) with \( \rho = 0.91/12 \),

\[
DCAR = -\sum_{j=1}^{J} \rho^{j-1} E_t [\alpha_{t+j}],
\]

which we show to be a crude approximation of \( \delta \) in Appendix B. We find that DCAR traces the cross-sectional variation in \( \delta \) better, although there is still some meaningful difference between \( \delta \)s and DCAR (Figure B1). The remaining difference tends to arise from the inability of these simple measures to account for the intertemporal component of price-level risk.

**An ex-post identity for mispricing**

Our identity uses the SDF to discount future cash flows, so some readers would wonder whether defining ex-post realized returns as the discount factor yields a similar identity. Such an approach yields an identity that holds both ex-ante and ex-post, but it involves the unobserved return in the absence of mispricing:

\[
\delta_t = -\sum_{j=1}^{\infty} \frac{1}{\prod_{s=1}^{j} (1 + R_{v,t+s})} \left( R_{t+j} - R_{v,t+j} \right), \tag{44}
\]

where \( R_v \) is the return on fundamental value \( V \).

**C Portfolio-level \( \delta \)**

In practice, one would most likely want to estimate the \( \delta \) of a portfolio of stocks, which requires expressing the portfolio \( \delta \) as a function of post-formation capital gains and returns on the portfolio. These capital gains and returns should be those based on a buy-and-hold strategy that does not rebalance the portfolio (or equivalently, use the original weight times the stock’s cumulative
capital gain to rebalance the portfolio every month). If \( w_{i,t} \) is the portfolio weight on security \( i \) at the time of portfolio formation,

\[
\delta_t = \sum_{i=1}^{N} w_{i,t} \delta_i,t
\]

\[
= - \sum_{i=1}^{N} \sum_{j=1}^{\infty} E_t \left[ M_{t,t+j} w_{i,t} \frac{P_{t+j-1}}{P_t} R_{i,t+j}^e \right]
\]

\[
= - \sum_{j=1}^{\infty} E_t \left[ M_{t,t+j} \left( \sum_{i=1}^{N} \frac{w_{i,t} P_{t+j-1}}{P_t} \right) \sum_{i=1}^{N} \left( \frac{w_{i,t} P_{t+j-1}}{P_t} \frac{P_{t+j}}{P_t} R_{i,t+j}^e \right) \right]
\]

so that

\[
w_{i,t+j} = \frac{w_{i,t} P_{t+j}}{\sum_{i=1}^{N} w_{i,t} P_{t+j}}
\]

is the correct post-formation weight on security \( i \), \( P_t = \sum_{i=1}^{N} w_{i,t} P_{t,i} \) is the price of the portfolio, \( P_{t+j} = P_t \sum_{i=1}^{N} w_{i,t} \frac{P_{t+j}}{P_{t,i}} \) is the portfolio’s post-formation price, and \( R_{t+j}^e = w_{i,t+j} R_{i,t+j}^e \) is the portfolio’s post-formation excess return. Delisted securities can be considered to have a zero price afterward.

**D Proxying for the Infinite Sum with a 15-year Sum**

To estimate \( \delta \), we replace the infinite sum in equation (15) with a finite sum over \( J = 180 \) months (15 years). However, since both the discount factor \( M_{t,t+j-1} P_{t+j-1} / P_t \) and the conditional abnormal return \( \alpha_{t+j} \) that appear in the expression are likely to decay over post-formation years, the finite sum over 15 post-formation years provides a good approximation of the infinite sum. In particular, the left panel of Figure B2 shows that the estimated \( \delta \) based on \( J \) post-formaiton periods plateaus around year 15. The right panel of Figure B2, which plots the derivative of the paths plotted in the left panel, confirms that the marginal contribution to our mispricing measure becomes relatively small in post-formation year 13.

**E Evidence from Individual Stocks**

The stark contrast between the return and the price-level performance of CAPM seen in portfolios sorted on single characteristic extends to analysis at the individual stock level, implying that it
is not unique to the portfolios considered above (Figure B3. The return plot compares the average excess returns on an individual stock to its estimated risk measured by \( \text{Cov} \left( -M_{t+1} / E [M_{t+1}], R_{t+1}^e \right) \) where \( M_{t+1} = 1 - b_1 R_{m,t+1}^e \) and \( b_1 \) is the value that makes the market portfolio correctly priced in sample. The price level plot similarly compares estimated price level to estimated price-level risk for individual stocks.

Using individual stocks to perform the analysis has the obvious caveat coming from the unexpected component of returns. It would not be sensible to attribute the high historical average returns on Amazon or Starbucks since their inception to high risk. On the other extreme are numerous stocks with low historical average returns due to their eventual delistings, and it would be equally problematic to attribute these low returns to low risk. Delisting creates another problem, since estimated price level and price-level risk are based on post-formation returns over 15 years, and stocks that get delisted are likely to have artificially high price levels and low price-level risk. Nevertheless, our analysis is still valid if our purpose is to examine the overall pattern across all stocks and there is no obvious reason for a systematic bias. Our approach is unlikely to be introducing a large bias, since unexpected components of returns, by definition, are unlikely to be strongly associated with the riskiness of the stocks.
Table B1: Post-formation Returns: Illustration

The table describes our three-dimensional data structure. Our data consist of overlapping samples of returns over 180 post-formation months on portfolios formed in 559 different months (1957m7–2004m12) for 10 different characteristic deciles.

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<th>Number of post-formation months $j$</th>
<th>Return</th>
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<tr>
<td>BM1</td>
<td>1957m6</td>
<td>1957m8</td>
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<td>...</td>
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Simple CAR

“Discounted” CAR

Figure B1: Alternative Long-run Return Measures

The two figures compare estimated mispricing $\delta$ to the cumulative abnormal return (CAR),

$$\text{CAR} = -\sum_{j=1}^{J} E_t [\alpha_{t+j}],$$

and against the discounted CAR (DCAR),

$$\text{DCAR} = -\sum_{j=1}^{J} \rho^j E_t [\alpha_{t+j}]$$

with $\rho = 0.9^{1/12}$ and $J = 180$ months for twenty portfolios sorted by quality, book-to-market ratio, and eight additional characteristics. These portfolios are formed in 1957m6–2004m12 and the abnormal returns are estimated separately for each post-formation month up to $J = 180$ months. We estimate the expected conditional abnormal returns simply using unconditional abnormal returns, which implicitly assumes that the conditional component of the abnormal return is not contemporaneously correlated with the market factor. The mispricing $\delta$ is estimated so that the market portfolio has an in-sample $\delta$ of zero. The 95% confidence interval is denoted in gray.
Figure B2: Estimated $\delta$ by the Choice of Finite Post-formation Periods $J$

The figures in the left panel plot the values of $\delta$ estimated based on $J$ post-formation years for the extreme decile quality- and book-to-market, sorted portfolios and extreme quality and book-to-market double-sorted portfolios, where $J$ is the horizontal axis. The figures in the right panel plot how much each post-formation month contributes to the estimated $\delta$s. Portfolio formation period spans 1957m6–2004m12 and post-formation period spans 1957m7–2019m12.
Figure B3: Explaining Returns vs. Price Levels: Individual Stocks

The left panel reports the cross-sectional relation between realized mean excess return and mean excess return predicted by the CAPM for one-period returns on individual stocks between 1957m7 and 2005m1. The right panel reports the cross-sectional relation between estimated scaled price level and estimated scaled price-level risk with respect to the CAPM is close to the perfect relation denoted by the dash line. Scaled price level is higher if long-horizon discount rates are higher, contrary to our original definition of price level.