Persistence of dollarization after price stabilization

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Abstract

Credit contracts in developing countries are often denominated in foreign currencies, even after many of these economies succeeded in controlling inflation. This paper proposes a new interpretation of this apparent puzzle based on the demand for insurance against real shocks: the fact that devaluations occur more frequently in adverse states of the world provides a motive for holding dollar assets. This approach implies a complementarity between the optimal monetary policy and the currency denomination of contracts. When a large proportion of liabilities is denominated in a foreign currency, the optimal exchange rate volatility is low, which reinforces the demand for dollar assets.

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1 Introduction

When analyzing financial arrangements in emerging economies, the literature points out the risks associated with a corporate sector overly exposed to depreciations in the exchange rate.[1] Indeed, a large share of credit contracts among residents in developing countries is denominated in foreign currencies, mainly dollars. Consequently, these economies are often trapped in suboptimal monetary policies, pegs or regimes with reduced exchange rate volatility. Although, based on their default risk and macroeconomic consequences, dollar-denominated contracts seem to be inefficient, they result from the free choice of borrowers and lenders, who presumably understand the risks involved in their credit contracts. Therefore, to judge the efficiency of these credit arrangements, it is important to identify potential market imperfections behind the choices of borrowers and lenders.

The main contribution of this paper is to reassess the motives behind domestic dollarization. This paper proposes a new interpretation of this apparent puzzle based on the demand for insurance against real shocks. Because devaluations occur more frequently during recessions, dollar assets provide insurance in economies with incomplete financial markets, and the devaluation response to aggregate shocks increases the contingent value of dollar assets.

Having analyzed the motive underlying the denomination of credit con-

[1] The level of dollarization is found to increase the likelihood of crisis and the vulnerability of the economy toward real exchange rate perturbations (Hausmann et al. (2001); Calvo et al., (2004)). For a theoretical approach, see, among others, Jeanne and Zettelmeyer (2003), Chang and Velasco (1999a), and Aghion et al. (2001).

tracts, this paper examines the interaction between the currency composition of the credit market and the optimal devaluation response of the Central Bank (CB) to aggregate shocks. This interaction may result in multiple equilibria: i) an equilibrium with a high degree of dollarization in which the CB minimizes exchange rate fluctuations, and ii) another equilibrium in which contracts are mainly denominated in domestic currency and the exchange rate is very volatile.[2] Based on this complementarity, the model explains persistence in the share of dollar liabilities in economies with low price and exchange rate risk.

The framework of this model is based on a risk-sharing problem between risk-averse consumers and risk-neutral domestic firms. Firms face a real productivity shock, which is the only source of aggregate risk in this economy. They are protected by limited liability and default in this model is socially costly. This paper finds that dollar and peso contracts enable consumers to trade off between insurance and default risk. Because devaluations are more likely to occur in bad states of the world, dollar assets provide insurance against the risk of recession, though they face a larger default risk. On the other hand, because devaluation and inflation are positively correlated, real payments of peso debt are lower in bad states. Thus, peso assets involve lower default risk at the expense of a more uneven consumption schedule. The ability of these assets to approximate complete financial markets depends on the magnitude of the devaluation response to aggregate shocks.

[2] In a different context, the interaction between monetary policy and the degree of domestic dollarization is also present in Caballero and Krishnamurthy (2001, 2003), Chamon and Hausmann (2002), Cowan and Do (2003), and Ize and Powell (2004).

If exchange rate fluctuations are small, in order to get a particular extent of consumption insurance, individuals must hold a larger fraction of their wealth denominated in dollars.

The interpretation of multi-currency denominated debt as an approximation for contingent contracts is also present in Ize and Levy-Yeyati (2003), Neumeyer (1998), and Holmstrom and Tirole (2002). In those models, the insurance capacity of foreign currency denominated assets increases with the exchange rate volatility. The crucial difference between these and that in this paper is that dollar instruments are issued by the domestic corporate sector, which has a limited capacity of providing insurance. Then, a highly volatile exchange rate increases the default risk of firms with a high share of dollar liabilities and the insurance capacity of existing financial assets is reduced. This paper identifies the failure in the credit market that can explain the excessive default risk associated with a high share of dollar liabilities, namely, the fact that credit contracts are nonexclusive. When firms have multiple creditors, default risk depends on the representative contract and not on the contract with an individual lender. As a result, individual contracts set in domestic currency are not characterized by lower default risk, since such risk depends on the entire liability composition of the firm. The resulting currency composition of liabilities is inefficient and typically results in excessive risk taking.

The exchange rate response to the shock is the relevant variable determining the efficiency of the financial market. Under full commitment, the CB's optimal exchange rate response enables dollar and peso assets to replicate a

complete financial market. However, under discretion, a welfare-maximizing CB limits exchange rate volatility when the firms have large share of dollar denominated liabilities. Consistent with empirical findings, the CB in this model finds it optimal to reduce exchange rate volatility and prevent the negative balance sheet effect associated with high shares of dollar liabilities.[3] A complementary arises between public demand for dollar denominated assets and central bank response to shocks arises, which can lead to multiple equilibria. As a result, low exchange rate volatility is expected to persist in economies with high share of dollar liabilities.[4] This is typically the case in economies with history of high and volatile inflation, where dollar denominated assets served as storage of value during inflationary episodes. Based on the interplay between the CB and the public, this model predicts contracts to remain denominated in dollars even if the risk of inflation is passed.

The rest of this paper is organized as follows. Section 2 describes the economic environment. Section 3 solves the credit market equilibrium for a given anticipated devaluation response. Section 4 endogenizes the optimal devaluation response and closes the model. Section 5 presents this paper's conclusions.

[3] The predictions of this model are largely supported by available empirical evidence. Calvo and Reinhart (2000, 2002), Eichengreen et al. (2002), and Alesina and Wagner (2005) find that dollarization of liabilities in emerging economies is positively correlated with managed floating or fixed exchange rate regimes.

[4] Figure 1 in the appendix illustrates this phenomenon. It shows predicted exchange rate volatility and share of dollar deposits in domestic banking sector for a selected group of emerging economies.

2 The model

The model describes a small economy open to trade. There are three goods, a nontradable final good and two intermediates, one tradable and one nontradable. The economy is populated by risk-averse consumers and a unit measure of risk-neutral entrepreneurs. There is a single source of uncertainty in the economy, the realization of the productivity shock, which affects the economy's overall production.

The capital account is assumed to be closed. This is to capture the fact that in emerging economies a large share of small firms and atomistic consumers do not have access to foreign capital and are unable to diversify country risk. The model then describes the risk-sharing problem between domestic risk-averse consumers and risk-neutral entrepreneurs. Consumers are endowed with an investment good, which is lent to domestic entrepreneurs before the realization of the shock. There is an imperfect set of contracts. Credit contracts cannot be set contingent to the realization of the aggregate shock; they can, instead, only be expressed in terms of a fixed amount of foreign or domestic currency.

The timing of events is as follows: Date 1 is a fully flexible period in which all contracts are set. At date 2, the productivity shock is realized, firms decide whether to repay their debts, and consumption takes place.

2.1 Goods market

This section derives the contingent real return on domestic and foreign currency denominated contracts for a given co-movement of the exchange rate to the shock, which is endogenized in Section 4.

2.1.1 Technology

Entrepreneurs have access to two production alternatives: they either home-produce an amount K of the final good or undertake a risky project. The project requires a unit of initial investment and results in the joint output of tradable and nontradable intermediate goods on date 2:

$$y_{is}^{T} = \widetilde{\tau} A_{is} \qquad\qquad y_{is}^{N} = (1 - \widetilde{\tau}) A_{is}$$

where $\widetilde{\tau}$ is a fixed proportion $\widetilde{\tau} \in (0, 1)$. The firm productivity is affected by an aggregate shock z_s and the firm's nonobservable idiosyncratic sensitivity toward it, a_i :

$$A_{is} \equiv A (1 + a_i z_s) \tag{1}$$

The state of nature $s \in \{B, G\}$ with $\Pr(s) = 0.5$ is defined by the realization of the aggregate productivity shock $z_s \in \{-z, z\}$. The idiosyncratic elasticity toward the shock, also realized in period 2, is uniformly distributed over the unitary interval $a_i : U[0, 1]$.

Consumers combine tradable and nontradable intermediate goods to produce the final consumption good, y_s^F. The structure of production can ac-

commodate for foreseen relative prices but cannot instantaneously adjust to unexpected changes in the relative price of inputs. This feature is modeled with a "putty-clay" technology for the production of the final good. In the underlying technology inputs are perfect substitutes: $y_s^F = y_s^N + y_s^T$.At time 1, before the productivity shock is realized, consumers freely choose the share of tradable and nontradable inputs, η. At date 2, however, after the state of nature is revealed, the production possibilities take on a Leontieff form and there is no ex post substitutability between tradable and nontradable inputs. This technology specification implies that the marginal cost of producing the final good increases with the price of both tradable and nontradable inputs. Although this feature is common to most technology specifications, under this extreme form of a putty-clay technology the marginal cost of the final good is a linear combination of the input prices, which simplifies considerably the analysis:

$$MC_s = \eta p_s^T + (1-\eta)\, p_s^N \tag{2}$$

where η is the input requirement chosen at time 1, and p_s^T and p_s^N are the prices of tradable and non-tradable intermediates in state $s \in \{B, G\}$.

2.1.2 Equilibrium

Prices of intermediate goods are set at date 1. The price of nontradables is set in domestic currency while the prices of tradable goods are given by their international price, assumed constant and equal to one. Thus, in the

local currency denomination, the price of tradables in each state of nature is given by the realization of the exchange rate, $p_s^T = e_s$, while the price of nontradables, p^N, is constant across states of nature.

The goods market is in equilibrium if for any state of nature $s \in \{B, G\}$ the nontradable goods market clears and the trade balance condition is satisfied. This requires the share of tradable intermediate goods produced to be equal to the share demanded: $\eta = \widetilde{\tau}$. Since at the time η is chosen, tradable and nontradable inputs are perfect substitutes, η is interior — that is, $\eta = \widetilde{\tau} \in (0, 1) -$ only if consumers are ex ante indifferent to using either of the two intermediate goods. Consumer indifference pins down the relative expected price between tradables and nontradables. Finally, the price of the final good is set competitively such that p_s^F is equal to the marginal cost of production, given by equation (2), which can be expressed in equilibrium as

$$p_s^F = \widetilde{\tau} p_s^T + (1 - \widetilde{\tau})\, p^N. \tag{3}$$

2.2 Credit market

Consumers are endowed with a unit of final good, which is sold to the entrepreneurs at the market price p_1^F. Consumer resources are lent to the entrepreneurs until date 2, when consumption takes place. Entrepreneurs are risk neutral, have no initial resources, and are protected by limited liability.

2.2.1 Credit contracts

Credit contracts can be denominated in domestic currency or dollars and stipulate an amount $\widetilde{r}_p$ and $\widetilde{r}_d$ to be paid in the respective currencies. Then, in real terms, claims on dollar and peso contracts are given by the realization of the exchange rate and consumption prices:

$$R_{ds} = \widetilde{r}_d \frac{p_1^F}{p_s^F} p_s^T \qquad\qquad R_{ps} = \widetilde{r}_p \frac{p_1^F}{p_s^F}$$

Importantly, in this context, expected changes in the exchange rate and prices have no real implications, since they are accounted for in the equilibrium interest rates. Deviations from expected depreciations and inflation, however, which are necessarily symmetric, impact on relative prices and consumption allocations. To emphasize this point, the credit market equilibrium is characterized in terms of deviations from the expected exchange rate of the form

$$\delta_s \equiv \frac{p_s^T - E\left(p_s^T\right)}{E\left(p_s^T\right)} \in \{\delta, -\delta\} \tag{4}$$

where δ is understood as the CB's devaluation response to the aggregate shock, which is considered exogenous for consumers and entrepreneurs. Along the same lines, it is useful to characterize the realization of the price of the final good in (3) as deviations from expectations:

$$\pi_s \equiv \frac{p_s^F - E\left(p_s^F\right)}{E\left(p_s^F\right)} = \tau \delta_s \tag{5}$$

where τ is a renormalization of the shares of tradable and nontradable intermediates in production $\tau \equiv \widetilde{\tau}\frac{E\left(p_s^T\right)}{E(p_s^F)}$. Then, the real claims on dollar and peso contracts can be rewritten as follows:

$$R_{ds} = r_d \frac{1+\delta_s}{1+\pi_s} \qquad\qquad R_{ps} = r_p \frac{1}{1+\pi_s}$$

where the interest rates for dollar and peso contracts, r_d and r_p, already incorporate the expected evolution of the inflation and exchange rate: $r_d \equiv \widetilde{r}_d \frac{p_1^F}{E(p_s^F)} E\left(p_s^T\right)$ and $r_p \equiv \widetilde{r}_p \frac{p_1^F}{E(p_s^F)}$. Replacing for price and exchange rate in equations (4) and (5), the real claims on peso and dollar contracts are approximated by the following linear expressions:

$$R_{(i)ps} = \begin{cases} r_{(i)p} - \tau\delta & s = B \\ r_{(i)p} + \tau\delta & s = G \end{cases} \tag{6}$$

$$R_{(i)ds} = \begin{cases} r_{(i)d} + (1-\tau)\,\delta & s = B \\ r_{(i)d} - (1-\tau)\,\delta & s = G \end{cases} \tag{7}$$

where variables with subscript i are firm specific and otherwise refer to market equilibrium. Note from equations (7) and (6) that the devaluation response to the aggregate shock differentiates dollar from peso contracts. If devaluations happen in the B state, that is, $\delta \geq 0$, dollar contracts involve larger real payments in the negative realization of the shock. From expression (5), inflation and devaluation are positively correlated; thus the real return on peso assets is lower in the B state. The contingent value of assets is given by the size of the devaluation response to the productivity shock.

2.2.2 Credit market equilibrium

The economic structure described above can now be summarized as a risk-sharing problem between risk-averse consumers and risk-neutral entrepreneurs, who are protected by limited liability.

Firms' idiosyncratic risk can be perfectly diversified such that the return on assets only follows the realization of the aggregate shock $s \in \{B, G\}$. Consumers choose the optimal portfolio composition subject to short-selling constraints $\mu \in [0, 1]$ to maximize the expected utility $Eu\left(c_s^c\right)$ subject to the following budget constraint:

$$c_s^c = \mu q_{ps} R_{ps} + (1 - \mu)\, q_{ds} R_{ds}$$

where $u' > 0$ and $u'' < 0$, R_{ps} and R_{ds} are, respectively, the real return on peso and dollar assets in state s, given by equations (7) and (6), and q_{ps} and q_{ds} are, respectively, the proportions of peso and dollar contracts repaid.

Entrepreneurs choose whether or not to undertake the project. If they do, they borrow from consumers to finance investment. Note that it is optimal for entrepreneurs to default on their debt whenever their profits are negative, in which case consumption is zero. Then, each entrepreneur $i \in [0, 1]$ chooses a strategy $\{\imath_i, \upsilon_i\} \in \{0, 1\} \times [0, 1]$ to maximize expected consumption, $E\left(c_{is}^e\right)$, subject to the following budget constraint:

$$E\left(c_{is}^e\right) = (1 - \imath_i)\, K + \imath_i \sum_{s \in \{B,G\}} 0.5 \Pr\left[A_{is} > R_{is}\right] E\left[A_{is} - R_{is} | A_{is} > R_{is}\right] \tag{8}$$

where $\imath_i \in \{0, 1\}$ is an indicator function that takes on value 1 if the entrepreneur undertakes the project and the real value of the firm's total liabilities in state s, R_{is}, is determined by the currency composition of debt, which is subject to short-selling constraints, $v_i \in [0, 1]$:

$$R_{is} = v_i R_{ips} + (1 - v_i) R_{ids}$$

Liquidation is assumed to be socially costly. For simplicity, in the case of default, it is assumed that the firm makes zero profits and consumers get no liquidation value.[5]

Parametric restrictions are made to assure that defaults only happen in the B state — that is, $2K \geq Az \geq K -$. From equation (1), the probability of repayment in the B state has a cutoff form: All firms with sensitivity to the shock higher than a cutoff point $\overline{a}$ will default on their debt, where $\overline{a}$ corresponds to the sensitivity of the firm with $A_{iB} = R_{iB}$. Given that a_i follows a uniform distribution over the unitary interval, the probability of repayment is given by

$$\Pr\left(A_{iB} > R_{iB}\right) = \frac{A - v_i R_{ipB} - (1 - v_i) R_{idB}}{Az} \tag{9}$$

Finally, firms compete in the credit market and the zero profit condition holds: Expected entrepreneurs' profits are equal to their opportunity cost, that is, the home production of K units of final good. In equilibrium, the free entry condition (10) pins down the interest rates r_d and r_p, so that

[5] The results are qualitatively the same if a defaulting firm has a fixed liquidation value.

investors retain the expected net present value of production:

$$E\left(c_{is}^{e}\right)=K \tag{10}$$

The credit market is in equilibrium if, at the equilibrium interest rate given by $r_p = r_{ip}$ and $r_d = r_{id}$ for all $i \in [0,1]$, consumers' optimal shares of dollar and peso assets are equal to the optimal shares of dollar and peso liabilities in the corporate sector, and the proportion of honored dollar and peso contracts is consistent with the default probabilities of the firms.

3 Benchmark: Optimal risk sharing with limited liability

This section characterizes a benchmark economy with complete financial markets. With limited liability, the second best allocation can be attained with a contingent contract $\{R_B, R_G\}$, where R_B and R_G denote consumers' claims on the corporate sector in the B and G states, respectively, that maximizes consumer expected utility $Eu\left(c_s^c\right)$ subject to

$$\begin{aligned} c_B^c &= \Pr\left(A_{iB} > R_B\right) R_B \\ c_G^c &= R_G \end{aligned}$$

and the free entry condition of the firm is satisfied:

$$K = 0.5 \Pr\left(A_{iB} > R_B\right) E\left(A_{iB} - R_B | A_{iB} > R_B\right) + 0.5 E\left(A_{iG} - R_G\right).$$

The solution to this program is independent of the exchange rate volatility, δ, as can be seen in the first-order condition characterizing the second best allocation:

$$foc_{SB}\left(R_B^{SB}\right) : \left[u'\left(c_B^c\right) - u'\left(c_G^c\right)\right] \Pr\left(A_{iB} > R_B^{SB}\right) - u'\left(c_B^c\right) \frac{R_B^{SB}}{Az} = 0 \quad (11)$$

The first term in (11) corresponds to the marginal benefit of improving insurance, while the second term is its cost in terms of default risk. Improving smoothness requires greater payments from the corporate sector to consumers in the adverse realization of the shock, precisely when firms' revenues are lower. Consequently, improving consumers insurance results in higher probability of default. Optimal allocation is always interior: Neither perfect consumption smoothness nor zero default risk.

This paper describes an economy with an incomplete set of financial assets two assets: Dollar- and peso-denominated debt. In principle, if these two assets can span the space of the shocks, such an economy could attain efficient allocation. Indeed, it is shown in the online appendix that in the absence of limited liabilities, an economy with only dollar and peso debt contracts attains the first best allocation.[6] The economy described in the present paper, however, presents different degrees of market imperfections that prevent an economy with only dollar and peso contracts from generally attaining the constrained efficient allocation. These imperfections are the following: i) firms are protected by limited liability and default is

[6] The same result applies if default risk is independent of the devaluation risk. This case is analyzed in Ize and Levy-Yeyati (2003).

socially costly; ii) there are short-selling constraints; and iii) contractual relationships are nonexclusive, in the sense that consumers (creditors) cannot effectively monitor firms' financial arrangements with other lenders.[7]

The following section analyzes how these different market imperfections affect the ability of dollar- and peso-denominated contracts to replicate complete financial markets. In particular, this paper emphasizes that nonexclusive contractual relationships can effectively explain the existence of excessive dollar debt in the domestic corporate sector and, consequently, their overexposure to devaluation risk.

4 Characterization of the credit market equilibrium

If lenders and creditors have exclusive contractual relationships, the probability of default of a given firm is governed by a single credit contract.[8] In this case, ex ante identical firms are induced to hold a currency composition ν such that

$$R_B^{SB} = \nu \left(r_p - \tau\delta\right) + \left(1 - \nu\right)\left(r_d + \left(1 - \tau\right)\delta\right) \tag{12}$$

where R_B^{SB} corresponds to the optimal contingent contract, characterized by the first-order condition for the second best program in equation (11). Notice that there is a minimum devaluation response to the shock, $\delta_{SB} > 0$,

[7] See Arnott and Stiglitz (1993) for an analysis of nonexclusive contractual relationships.

[8] The credit market equilibrium under exclusive contracts is fully characterized in the online appendix.

such that this contract is feasible under the short-selling constraint $\nu \in [0,1]$. Since this paper is interested in characterizing emerging markets, the following analysis is restricted to the case in which, for low devaluation response, consumers are constrained in their demand for insurance. The case under analysis satisfies the following condition:

Condition 1 (underinsurance) *For $\delta = 0$, the first order condition (11) evaluated at r is positive:*

$$u'\left(\frac{r(A-r)}{Az}\right)\frac{(A-2r)}{(A-r)} - u'(r) > 0 \tag{13}$$

where for $s = \{B, G\} : r = R_{ps} = R_{ds}$ is given by the free entry condition (10).[9]

The currency composition of debt under exclusive contractual relationship is plotted in Figure 1. Under assumption (13), for all $\delta < \delta_{SB}$, the optimal contract is fully dollarized and $r_d + (1-\tau)\delta < R_B^{SB}$. For all $\delta > \delta_{SB}$, the second best allocation is implemented and the share of dollar debt optimally decreases in the exchange rate volatility δ, such that the default probability of the firm is kept constant at the second best level.

In other words, if contracts are exclusive, entrepreneurs are induced to efficiently diversify the currency of their liabilities. The only difference between this program and that characterizing a market with perfectly contingent debt in equation (11) results from the existence of short-selling con-

[9] The interest rate that satisfies the free entry condition for $\delta = 0$ is $r = A + Az - 2\sqrt{AzK}$.

straints, which are only binding for low devaluation responses to the stock.

When a firm has nonexclusive contractual relationships, the currency denomination of one credit contract does not characterize that firm's overall currency composition of debt. As a result, firms cannot be not induced to hold well-diversified liabilities. The resulting equilibrium is not efficient: Firms are overly exposed to currency risk and fail to provide insurance to risk-averse consumers.

4.1 Currency composition of liabilities with nonexclusive contracts

When entrepreneurs choose the currency composition of debt $v_i \in [0,1]$ to maximize their expected consumption $E\left(c_{is}^{e}\right)$ — given by equation (8) – they face a convex objective function. Entrepreneurs prefer to maximize profits in that state of nature in which the probability of being active is larger, even at the expense of a higher default risk. Thus, entrepreneurs choose extreme currency compositions of liabilities, entirely denominated in either pesos ($v_i = 1$) or dollars ($v_i = 0$).[10] The proportions of firms with dollar- and peso-denominated debt are determined by consumers' portfolio choice μ:

$$\{v_i\}_0^{\mu} = 0, \{v_i\}_{\mu}^{1} = 1$$

As long as consumers diversify their portfolio composition $\mu \in (0,1)$, firms with dollar and peso debt must coexist. Thus, the free entry condition is

[10] Chamon (2003) and Broda and Levy-Yeyati (2006) also find that *currency-blind* credit contracts motivate entrepreneurs (or banks) toward excessive currency risk.

satisfied for both types of firms:

$$E\left(c_{ps}^{e}\right) = E\left(c_{ds}^{e}\right) = K \tag{14}$$

where $E\left(c_{ps}^{e}\right) = E\left(c_{ps}^{e}|\nu_i = 1\right)$ and $E\left(c_{ds}^{e}\right) = E\left(c_{ps}^{e}|\nu_i = 0\right)$ refer to the expected consumption of entrepreneurs with only peso and dollar debt, respectively.

The credit contracts $\{R_{pB}, R_{pG}\}$ and $\{R_{dB}, R_{dG}\}$ are determined by the free entry condition and the devaluation response to the shock:

$$R_{pG} - R_{pB} = 2\tau\delta \tag{15}$$

$$R_{dG} - R_{dB} = -2\left(1-\tau\right)\delta \tag{16}$$

The probabilities of default of firms with peso and dollar debt are different and so are the probabilities of default associated with the respective credit contracts. The proportion of performing peso-denominated (dollar-denominated) contracts in the B state is equal to the probability that a firm with only peso debt (dollar debt) remains active.

4.2 Credit market equilibrium

Given that firms are not induced to hold a diversified currency composition of debt, equilibrium cannot generally replicate the second best allocation. Credit market equilibrium in the case of nonexclusive contracts replicates the following program: $\mu \in [0,1]$ is the share of firms with only peso debt

that maximizes consumers' expected utility $Eu\left(c_s^c\right)$ subject to the free entry condition for both types of firms (14), the contingent value of dollar and peso debt in (15)-(16), and the consumers' budget constraints; that is,

$$\begin{aligned} c_B^c &= \mu R_{pB} \Pr\left(A_{iB} > R_{pB}\right) + (1-\mu) R_{dB} \Pr\left(A_{iB} > R_{dB}\right) \\ c_G^c &= \mu R_{pG} + (1-\mu) R_{dG} \end{aligned}$$

For low devaluation volatility, $\delta < \delta_{SB}$, this problem is equivalent to that under exclusive contractual relationships. In both contractual cases, the optimal currency composition of consumers' portfolios and firms' liabilities does not require diversification. Both are entirely denominated in dollars. The contingent value of dollar assets is insufficient to provide the optimal level of insurance under short-selling constraints and consumers are underinsured to the realization of the shock. Augmenting the exchange rate volatility increases the contingent value of assets and reduces the value of the multipliers on the financial restrictions.

For $\delta > \delta_{SB}$, however, the implementation of the second best allocation requires firms to diversify their composition of liabilities according to expression (12). This is not an equilibrium outcome under nonexclusive contractual relationships; instead, firms do not diversify the currency composition of debt. The default risk of firms with dollar debt increases with devaluation volatility and the insurance capacity of dollar assets is therefore reduced. For a large enough devaluation response, the default rate on dollar contracts is so high that they fail to provide any insurance and the market

is fully composed of peso contracts. These results are plotted in Figure 1 and summarized in the following proposition.

[FIGURE 1 AROUND HERE]

Proposition 1 *The credit market equilibrium is characterized by non-currency diversification of firm debt. Moreover,* $\exists \delta_1, \delta_2 : \delta_2 \geq \delta_1 \geq \delta_{SB}$ *such that i)* $\forall \delta < \delta_1 : \mu(\delta) = 0$*, ii)* $\forall \delta > \delta_2 : \mu(\delta) = 1$*, and iii)* $\forall \delta > \delta_{SB} : \frac{\partial Eu(c_s^c)}{\partial \delta} < 0$ *and* $\forall \delta < \delta_{SB} : \frac{\partial Eu(c_s^c)}{\partial \delta} > 0$*.*

Proof. See Appendix ■

5 Policy equilibrium

The size of the exchange rate response to the shock is the relevant variable determining the efficiency of the financial market. This paper does not analyze the sign of the covariance between the real shock and the exchange rate, which would require a more complete description of the real side of the economy.[11] Instead, this paper starts by recognizing that depreciations tend to happen in adverse states of nature and then analyzes the optimal size of the response based only on its implications for the domestic financial market. The analysis is then restricted to devaluation responses of the form $\{\delta_B, \delta_G\} = \{\delta, -\delta\}$, where $\delta \geq 0$.

[11] This is a reduced form of a model where devaluation has, additionally, an expansionary effect on the economy. An extension that endogenizes the lower bound for the optimal devaluation in the B state is available upon request.

With a commitment technology, a credible CB that maximizes consumer welfare can push the economy toward an equilibrium that replicates one under complete financial markets. The optimal monetary policy under full commitment results from maximizing consumers' expected utility $Eu\left(c_s^c\right)$ before the credit contracts are set, internalizing that the policy impacts on the currency composition of contracts. One can verify that such equilibrium corresponds to $\delta = \delta_{SB}$ and the credit market is fully dollarized, $\forall i \in [0,1]$: $v_i = \mu = 0$.

A time-inconsistent monetary authority, however, will not implement this ex ante optimal devaluation response. The following assumes that the exchange rate intervention occurs after the credit contracts have been set. Taking the portfolio currency composition and the market interest rates as given, the CB chooses a devaluation response that maximizes consumer expected utility. The mechanism presented here is in line with the common agency problem developed in Tirole (2003). Government is a common agent of all consumers and its incentives depend on a representative investor's portfolio, and not on a single investor's choice. Then, consumers exert externalities on each other through their impact on the CB's incentives.

Definition 1 (Policy equilibrium) *The policy (subgame perfect) equilibrium is a set*
$\{\{r_{ip}\}_{i=0}^{1}, \{r_{id}\}_{i=0}^{1}, \{v_i\}_{i=0}^{1}, \mu, \delta\}$ *such that i)* $\{\{r_{ip}\}_{i=0}^{1}, \{r_{id}\}_{i=0}^{1}, \{v_i\}_{i=0}^{1}, \mu\}$ *is a credit market equilibrium given a devaluation schedule* $\{\delta_B, \delta_G\} = \{\delta, -\delta\}$ *and ii) the devaluation schedule* $\{\delta_B, \delta_G\} = \{\delta, -\delta\}$*, with* $\delta \geq 0$*, maximizes consumers' utility* $Eu\left(c_s^c\right)$ *subject to their budget constraint, for a given*

credit market equilibrium $\{\{r_{ip}\}_{i=0}^{1}, \{r_{id}\}_{i=0}^{1}, \{v_i\}_{i=0}^{1}, \mu\}$.

5.1 Optimal policy

Given a credit market equilibrium $\{r_p, r_d, \{v_i\}_{i=0}^{1}, \mu\}$, the CB chooses $\delta \geq 0$ to maximize consumers' expected utility $Eu(c_s^c)$ subject to their budget constraint:

$$\begin{aligned} c_B^c &= \mu R_{pB} \Pr(A_{iB} > R_{pB}) + (1-\mu) R_{dB} \Pr(A_{iB} > R_{dB}) \\ c_G^c &= \mu R_{pG} + (1-\mu) R_{dG} \end{aligned}$$

where $R_{ps} = r_p - \tau\delta_s$ and $R_{ds} = r_d + (1-\tau)\delta_s$ for $\{\delta_B, \delta_G\} = \{\delta, -\delta\}$.

From an ex ante perspective, a time-inconsistent CB is biased against insurance. To grasp this intuition, consider the case of a fully dollarized credit market, $\mu = 0$. Since at the time of the CB's intervention contracts are already set, the first-order condition only considers the effect of the devaluation response, δ, on the ex post return on contracts. It does not, however, internalize how the CB's policy feeds back into currency composition of contracts and the equilibrium interest rate r_d, agreed at time 1, which decreases with the probability of repayment, according to (14). This phenomenon can be observed when comparing the first-order condition for the CB with that characterizing the second best allocation in (11):

$$\begin{aligned} foc_{CB}(R_{dB}) &: u'(c_B^c) \Pr(A_{iB} > R_{dB}) - u'(c_G^c) + u'(c_B^c) \frac{R_{dB}}{Az} \\ foc_{SB}(R_{dB}) &: \left[u'(c_B^c) - u'(c_G^c)\right] \Pr(A_{iB} > R_{dB}) + u'(c_B^c) \frac{R_{dB}}{Az} \end{aligned}$$

When the market is fully dollarized, $\mu = 0$, the CB unconstrained optimal devaluation response is such that the real claim on dollar assets in the B state, R_{dB}, makes the CB's first-order condition zero: $foc_{CB}(R_{dB}) = 0$. Notice that such a condition implies that the first-order condition characterizing the second best allocation is positive, $foc_{SB}(R_{dB}) > 0$, which indicates that consumers are underinsured. In this case, the volatility of the exchange rate is insufficient from an ex ante perspective, since the CB tries to avoid the potential negative balance sheet effect of a devaluation on dollarized liabilities. The exact opposite characterizes the CB's policy when credit contracts are mainly denominated in domestic currency: The CB's policy involves an excessive devaluation response to lower the real value of peso claims in the bad realization of the shock, so default risk is reduced beyond socially optimum.

Although socially inefficient, a time-inconsistent CB can find it optimal to implement a fixed exchange rate when the credit market is highly dollarized. Consider the interest rate r that satisfies the free entry condition (14) under a fixed exchange rate, that is, $\delta = 0$.[12] Replacing the probability of repayment with expression (9), the corresponding CB's first-order condition is given by

$$foc_{CB} : [(1-\mu)(1-\tau) - \mu\tau] \left\{ u'\left(\frac{r(A-r)}{Az}\right)\frac{A-2r}{Az} - u'(r) \right\} = -\lambda$$

where $\lambda \geq 0$ is the multiplier on the constraint $\delta \geq 0$. For parameters such

[12] For $\delta = 0$, the interest rate that satisfies the free entry condition of the firms is $r_d = r_d = r = A + Az - 2\sqrt{AzK}$.

that $u'\left(\frac{r(A-r)}{Az}\right)\frac{A-2r}{Az} - u'(r) < 0$, the constraint binds for high levels of dollarization $(\mu < 1-\tau)$. That is, the CB implements a fixed exchange rate when the credit market is heavily dollarized.

More generally, the CB's optimal policy is characterized by the following proposition.

Proposition 2 *For a given credit market equilibrium* $\{\{r_{ip}\}_{i=0}^{1}, \{r_{id}\}_{i=0}^{1}, \{v_i\}_{i=0}^{1}, \mu\}$, *the optimal devaluation schedule of the form* $\{\delta_B, \delta_G\} = \{\delta, -\delta\}$, *with* $\delta \geq 0$, *satisfies i) if* $u'\left(\frac{r_d(A-r_d)}{Az}\right)\frac{A-2r_d}{Az} - u'(r_d) < 0 : \exists \mu^* \in (1-\tau, 1)$, *such that* $\forall \mu < \mu^* : \delta = 0$, *and* $\forall \mu > \mu^* : \frac{\partial \delta}{\partial \mu} > 0$ *and ii) if* $u'\left(\frac{r_d(A-r_d)}{Az}\right)\frac{A-2r_d}{Az} - u'(r_d) > 0 : \exists \mu^* \in (0, 1-\tau)$, *such that* $\forall \mu > \mu^* : \delta = 0$, *and* $\forall \mu < \mu^* : \frac{\partial \delta}{\partial \mu} < 0$.

Proof. See Appendix ■

5.2 Policy equilibrium

The degree of dollarization and the market interest rates determine the CB's optimal policy. The ex post optimal policy feeds back into the credit market expectations and maps into a credit market equilibrium. Along the lines of Kydland and Prescott (1977), the policy equilibrium is the set of fixed points for which the market's foreseen devaluation response coincides with the ex post optimum.

If expression $u'\left(\frac{r(A-r)}{Az}\right)\frac{A-2r}{Az} - u'(r)$ is positive, the CB improves consumption smoothness by increasing the contingent value of dollar assets.

Because of its bias against insurance, however, the exchange rate intervention is insufficient to replicate the equilibrium under complete markets ($\delta < \delta_{SB}$). In this case, there is a unique equilibrium characterized by full dollarization of the credit market and a positive but suboptimal devaluation response.

More interesting is the case in which expression $u'\left(\frac{r(A-r)}{Az}\right)\frac{A-2r}{Az} - u'(r)$ is negative. In this case, the CB's monetary policy is aimed at lowering the number of defaulting firms at the expense of a reduction in insurance to creditors (consumers). A complementarity arises between the credit market currency composition, described in Proposition 1, and the optimal monetary policy, characterized by Proposition 2.i). When the credit market is mainly composed of peso assets, the CB chooses an excessive devaluation response relative to the optimum under full commitment. Because dollarized firms cannot bear great exchange rate volatility, the default risk on dollar assets is excessive. From Proposition 1, the market's reaction is to increase the share of peso-denominated assets, reinforcing the motive for a large devaluation response. Correspondingly, when the credit market is heavy dollarized, the CB minimizes the exchange rate volatility, departing from the optimal policy under full commitment. From Proposition 1, the credit market intensifies its degree of dollarization when the devaluation response is low, which exacerbates the monetary lack of response.

As a consequence, there are potentially two stable equilibria: one with low dollarization and excessive exchange rate volatility and another with full dollarization and fixed exchange rate (see Figure 2.a.). Consumers are

underinsured in both equilibria but are unambiguously worst off in the equilibrium with a high share of peso contracts (see Figure 2.b.). On the other hand, consistent with the empirical literature on "dollarization," in the equilibrium with a high share of peso debt, the number of defaulting firms is lower and total output is more stable across states.

[FIGURE 2 AROUND HERE]

These findings are summarized in the following proposition.

Proposition 3 *If* $1 > \frac{u'\left(\frac{r(A-r)}{Az}\right)}{u'(r)}\frac{(A-2r)}{Az} > \frac{(A-r)}{Az}$, *there are potentially two stable equilibria: i) full dollarization equilibrium with* $\forall i \in [0,1] : v_i = \mu = 0$ *and* $\delta_1^* = 0$ *and ii) low dollarization equilibrium with* $\forall i \in [0,1] : \{v_i\}_{i=0}^{\mu} = 1, \{v_i\}_{i=0}^{\mu} = 0, \mu \in (1-\tau, 1]$, *and* $\delta_2^* > \delta_{SB}$. *The equilibrium with full dollarization always exists. If* $\frac{u'\left(\frac{r(A-r)}{Az}\right)}{u'(r)}\frac{(A-2r)}{Az} < \frac{(A-r)}{Az}$, *there is a unique equilibrium that satisfies:* $\forall i \in [0,1] : v_i = \mu = 0$ *and* $\delta^* < \delta_{SB}$.

Proof. See Appendix ■

5.3 Persistence of dollarization after price stabilization

The easiest way to analyze the implications of changes in the inflationary risk is by introducing a mean-preserving spread over the inflation rate in (5): $\pi_s = \tau\delta_s + \varepsilon$, where $\varepsilon : N\left(0, \sigma_\varepsilon^2\right)$. To focus on pure monetary disturbances, let one assume that the relative price of tradables and nontradables is unaffected by the inflationary risk: $\delta_s - \pi_s = (1-\tau)\,\delta_s$.[13] Then, inflationary risk

[13] Normalizing the mean of the noise to zero is not a crucial assumption, since the interest rate in pesos (r_p) collects any expected inflation bias.

does not affect real returns on dollar assets. These returns are still given by the real claims on contracts, $R_{d,s}$ in (7), and the probability of repayment in the B state $\Pr(A_{iB} > R_{dB})$.

The equilibrium for a high peso share disappears for high enough inflation variance whereas the full dollar equilibrium is robust. An episode of high inflation volatility can trigger a jump into the complete dollarized credit market equilibrium (see Figure 3). Once the credit contracts are fully denominated in foreign currency, the CB's optimal policy is to reduce the devaluation response to aggregate shocks, which perpetuates consumers' preferences toward dollar assets. This equilibrium is stable even if the mean-preserving spread over the inflation rate disappears. Moreover, the equilibrium with full dollarization may be unique if market participants assign a positive probability that the economy returns to monetary instability with high inflation volatility.

[FIGURE 3 AROUND HERE]

6 Conclusion

After years of high inflationary risk, many developing countries have succeeded in stabilizing their monetary variables. Today, these economies' fears center on underinsurance against aggregate shocks. The main contribution of this paper is to illustrate this topic from the perspective of risk-averse residents. In this framework, dollar assets are demanded as insurance against real aggregate risks.

This paper identifies the failure in the credit market to explain the excessive default risk associated with high shares of dollar liabilities, namely, the fact that credit contracts are nonexclusive. When firms have multiple creditors (consumers), default risk depends on the representative contract but not on contracts with individual consumers. The currency composition of liabilities is inefficient and typically results in excessive risk taking.

Based on the interplay between the currency composition of the credit market and the CB's optimal policy, the model explains persistence in the share of dollar liabilities in economies with low inflationary risk. Indeed, this interplay may result in multiple stable equilibria: an equilibrium with a high degree of dollarization in which the CB minimizes exchange rate volatility and another in which contracts are mainly denominated in domestic currency and monetary policy is highly countercyclical. Under full dollarization, real aggregate shocks have a greater impact on output and the number of defaulting firms is larger but, as a counterpart, lenders have a smoother consumption schedule.

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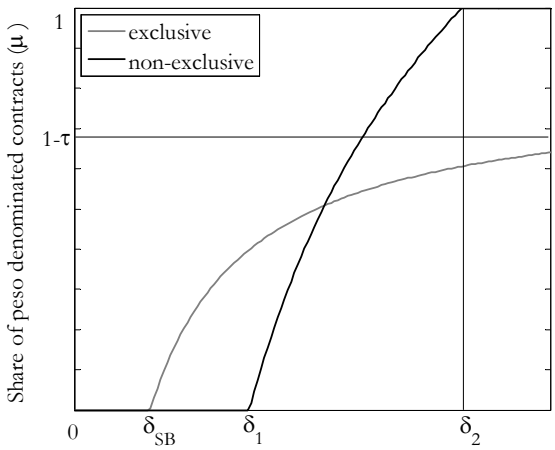

Figure 1.a.

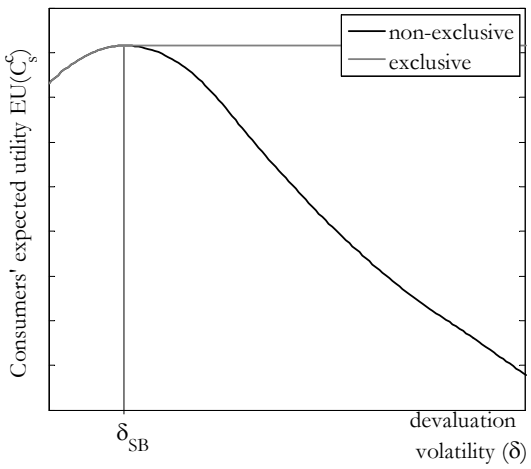

Figure 1.b.

Figure 1. Under exclusive contract, share of peso debt is interior and second best level of utility is attained for $\delta \geq \delta_{SB}$. Under non-exclusive contracts, $\mathrm{EU}\left(c_s^c\right)$ decreases for $\delta > \delta_{SB}$ and for $\delta > \delta_2$ consumers are constrained in their demand for peso assets $(\mu = 1)$.

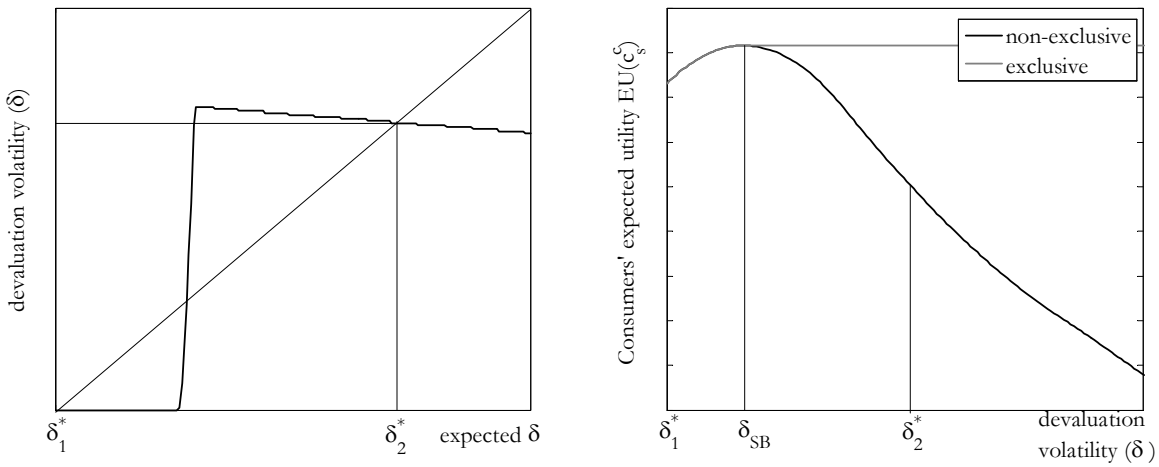

Figure 2.a.

Figure 2.b.

Figure 2. δ_1^* and δ_2^*, with $0 = \delta_1^* < \delta_{SB} < \delta_2^*$, are two stable policy equilibria. In both cases, expected utility is suboptimal.

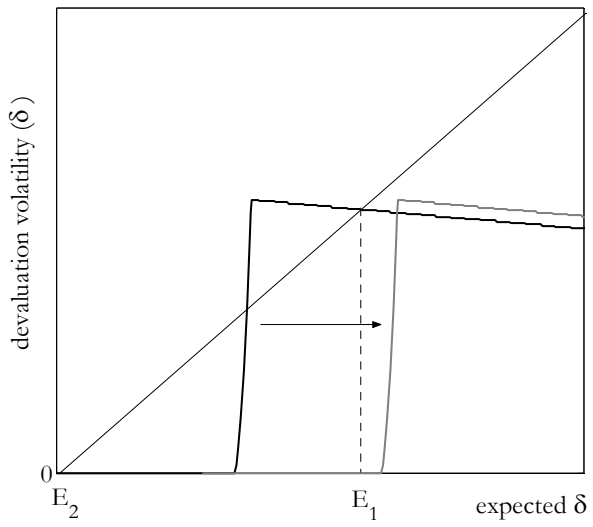

Figure 3. Increase in inflationary risk triggers jump from equilibrium E_1, with high devaluation volatility, to E_2, with fixed exchange rate.

7 Appendix

Proof. Proposition 1. To prove i), note that an allocation under nonexclusive contracts cannot be superior to the one under exclusive contracts. This is because in the case of nonexclusive contracts, interest rates cannot be firm-specific, while this possibility is allowed in the case of exclusive contractual relationships. Under assumption (13), for $\delta < \delta_{SB}$, the optimal exclusive contract implies full dollarization of liabilities and assets. This type of contract is also available in the case of nonexclusive contract. It follows that, under nonexclusive contracts, there is $\delta_1 \geq \delta_{SB}$ such that for all $\delta < \delta_1 : \mu = 0$.

To prove ii), it is sufficient to point out that $\frac{\partial \Pr(A_{iB} > R_{pB})}{\partial \delta} > 0$ and $\frac{\partial \Pr(A_{iB} > R_{dB})}{\partial \delta} < 0$. So there is a high enough $\overline{\delta}$ such that for all $\delta > \overline{\delta} : \Pr(A_{iB} > R_{pB}) > \Pr(A_{iB} > R_{dB}) = 0$. Law of large numbers holds so $q_p = \Pr(A_{iB} > R_{pB})$ and $q_d = \Pr(A_{iB} > R_{dB})$. Then, for all $\delta > \overline{\delta} : q_p R_{pB} > q_d R_{dB}$ and $R_{pG} > R_{dG}$ and the market is fully pesified. Thus, there is a $\delta_2 \leq \overline{\delta}$ such that for all $\delta > \delta_2 : \mu = 1$.

To prove iii), note that, by construction, for $\delta < \delta_{SB} : R_{dB} = r_d + (1-\tau)\delta < R_B^{SB}$ and $\frac{\partial R_{dB}}{\partial \delta} > 0$. Therefore, for $\delta < \delta_{SB}$, the social planner's first order condition in (11), evaluated at R_{dB}, is positive and decreasing in δ. It follows that $\forall \delta < \delta_{SB} : \frac{\partial Eu(c_s^c)}{\partial \delta} > 0$. For $\forall \delta > \delta_{SB}$, the equilibrium credit contract under exclusive contract involves currency diversification of liabilities. That is: $R_B = R_B^{SB} = \mu [r_p - \tau\delta] + (1-\mu)[r_d + (1-\tau)\delta]$ requires $0 < \mu < 1$. This is because $\frac{\partial R_{dB}}{\partial \delta} > 0 > \frac{\partial R_{pB}}{\partial \delta}$ and therefore, for $\delta > \delta_{SB} : R_{pB} < R_B^{SB} < R_{dB}$ and $foc_{SB}(R_{pB}) > 0 > foc_{SB}(R_{dB})$. Combining firms' free entry condition with consumers' budget constraints, the effect of δ on welfare can be expressed as follows:

$$\frac{\partial Eu(c_s^c)}{\partial \delta} = -\frac{\mu\tau}{1 + \Pr(A_{iB} > R_{pB})} foc_{SB}(R_{pB}) + \frac{(1-\mu)(1-\tau)}{1 + \Pr(A_{iB} > R_{dB})} foc_{SB}(R_{dB})$$

which is negative for $\delta > \delta_{SB}$. ■

Proof. Proposition 2. Define $f(R_{pB})$ and $f(R_{dB})$ using the following functional form

$$f(R_B) : u'(c_B^c) \left[\Pr(A_{iB} > R_B) - \frac{R_B}{Az} \right] - u'(c_G^c)$$

where r_p and r_d satisfy the free entry condition of the firms (14) for a given expected devaluation schedule δ^e and are taken as given by the CB. The following result follows from the free entry condition (14)

Result 1: $f(R_{pB}) \geq f(R_{dB})$ for any expected policy $\delta^e \geq 0$ and any CB's policy $\delta \geq 0$ and equality only holds for $\delta^e = \delta = 0$. To derive this result, note that for any expected devaluation schedule $\delta^e \geq 0$, the free entry conditions of the firms are satisfied only if $r_p \leq r_d$. Therefore, $f(R_{pB}) - f(R_{dB}) \geq 0$ for $\delta = 0$. Moreover, for any given r_p and r_d, $f(R_{pB}) - f(R_{dB}) = \frac{2u'(c_B^c)}{Az}[R_{dB} - R_{pB}]$ increases in δ.

The first order condition of the Central Bank can be written as follows:

$$foc_{CB} : u'(c_B^c)\frac{\partial c_B^c}{\partial \delta} + u'(c_G^c)\frac{\partial c_G^c}{\partial \delta} = -\mu\tau \cdot f(R_{pB}) + (1-\mu)(1-\tau) \cdot f(R_{dB}) = -\lambda$$

where λ is the multiplier on the constraint $\delta \geq 0$. The interior optimum corresponds to $\lambda = 0$, which implies in the following result:

Result 2. The signs of $f(R_{pB})$ and $f(R_{dB})$ coincide for any interior optimum.

The characterization of the equilibrium follows from these two results:

Condition in $i)$ implies that $f(r_d) \leq f(r_p) < 0$ for $\delta = 0$. Then, $\delta = 0$ is an interior optimum if $\frac{\mu\tau}{(1-\mu)(1-\tau)} = \frac{f(r_d)}{f(r_p)} > 1$; that is, $\delta = 0$ is an interior optimum for a given $\mu^* > 1 - \tau$. Applying the implicit function theorem to the foc_{CB}, one can derive that $\frac{\partial \delta}{\partial \mu} = \frac{\tau f(R_{pB}) + \tau f(R_{dB})}{soc_{CB}}$, where soc_{CB} correspond to the second order condition of the CB's maximization problem ($\frac{\partial foc_{CB}}{\partial \delta} < 0$). Then, $\frac{\partial \delta}{\partial \mu} > 0$ as long as the following condition holds: $0 < f(R_{pB}) \leq f(R_{dB})$, which is necessarily the case given results 1 and 2.

Equivalently, the condition in $ii)$ corresponds to $0 < f(r_d) \leq f(r_p)$ for $\delta = 0$. Then $\delta = 0$ is an interior optimum if $\frac{\mu\tau}{(1-\mu)(1-\tau)} = \frac{f(r_d)}{f(r_p)} < 1$; that is, $\delta = 0$ is an interior optimum for a given $\mu^* < 1 - \tau$. Applying the implicit function theorem to the foc_{CB}, it follows that $\frac{\partial \delta}{\partial \mu} = \frac{\tau f(R_{pB}) + \tau f(R_{dB})}{soc_{CB}} < 0$ as long as the following condition holds: $0 > f(R_{dB}) \geq f(R_{pB})$, which is necessarily the case given results 1 and 2. ■

Proof. Proposition 3. If the interest rate $r = r_d = r_p$ consistent with $\delta = \delta^e = 0$ satisfies the following condition: $1 > \frac{u'\left(\frac{r(A-r)}{Az}\right)}{u'(r)}\frac{(A-2r)}{Az} > \frac{(A-r)}{Az}$, then assumption (13) and condition in proposition 2.i) are satisfied in equilibrium. From proposition 1, the credit market equilibrium is fully dollarized for $\delta <$

δ^{SB} and the CB's optimal policy, characterized in proposition 2.i) involves fixed exchange rate for any $\mu < 1-\tau$. Therefore, full dollarization with fixed exchange rate is a stable equilibrium and always exists. From proposition 1, there is $\delta_1 > \delta_{SB}$ such that $\frac{\partial \mu}{\partial \delta} > 0$ and, from proposition 2.i), there is a $\mu^* > 1-\tau$ such that $\frac{\partial \delta}{\partial \mu} > 0$. Thus, provided that a low dollarization equilibrium exists, it is characterized by $\mu > 1-\tau$ and $\delta > \delta^{SB}$.
If the interest rate $r = r_d = r_p$ consistent with $\delta = \delta^e = 0$ satisfies the following condition: $\frac{u'\left(\frac{r(A-r)}{Az}\right)}{u'(r)} \frac{(A-2r)}{Az} < \frac{(A-r)}{Az}$, then assumption (13) and condition in proposition 2.ii) are satisfied in equilibrium. Fixed exchange rate is never an equilibrium, as it is only a CB's optimal policy for $\mu > 1-\tau$, which is not an equilibrium outcome for $\delta = \delta^e = 0$. Therefore, the only policy equilibrium is characterized by $\delta = \delta^e > 0$ and $\frac{\partial \delta}{\partial \mu} < 0$. As a result, the maximum equilibrium exchange rate fluctuation corresponds to $\mu = 0$. Comparing the CB's first order condition with the one characterizing the second best (11), the optimal CB's policy under full dollarization implies underinsurance, which is attained by implementing a policy $\delta^* < \delta_{SB}$. The credit market equilibrium for $\delta^e = \delta^* < \delta_{SB}$ is characterized by $\mu = 0$. ■

On line appendix

First best: Absence of limited liabilities

In the case without limited liabilities, an economy with only peso and dollar contracts attains the First Best equilibrium. That is, if the corporate sector was strong enough not to default on its debt in bad states of nature, risk neutral entrepreneurs could freely insure risk averse consumers: $c_B^c = c_G^c$.

This allocation is attained in an economy with peso and dollar credit contracts. When there is no probability of default (or when the probabilities of default are independent of the state of nature): $\Pr(A_{iB} > R_{iB}) = \Pr(A_{iG} > R_{iG})$. Then, entrepreneurs' expected consumption is linear in the currency composition of debt v_i. Firms are indifferent between peso and dollar debt as long as the respective interest rates are equalized, $r_{ip} = r_{id}$. Moreover, all firms are ex-ante identical, so they all face the same interest rates, $r_{ip} = r_{id} = r$, which is pinned down from the free entry condition (10).

The aggregate productivity shock indirectly affects consumers through the realization of the devaluation response, $\delta_s \in \{\delta, -\delta\}$, which determines the real return on assets. The optimal portfolio composition is $\mu = 1 - \tau$. Consumers avoid the currency risk by holding a portfolio that replicates the share of tradables and nontradables in the consumption price index. Consumption in each state of nature $s \in \{B, G\}$ is simply given by the fixed real payment r. In this case, the size of the devaluation response δ is irrelevant and does not affect real allocations.

Risk sharing with exclusive contracts

When lenders and creditors have exclusive contractual relationships, the probability of default of a given firm is governed by a single credit contract. Then, credit contracts can optimally set firm-specific interest rates according to its probability of default. In this way, entrepreneurs internalize the effect of their default risk on the welfare of the representative consumer and hold a well diversified currency composition of debt.

Currency Composition of Liabilities with Exclusive Contracts

The interest rate faced by a firm i with share of peso debt ν_i is determined by the arbitrage conditions for the consumers: the optimal credit arrangement is such that consumers are indifferent between lending to a particular firm i and the market average:

$$\begin{aligned} u'\left(c_B^c\right)\Pr\left(A_{iB} > R_{iB}\right)R_{ipB} + u'\left(c_G^c\right)R_{ipG} &= u'\left(c_B^c\right)\Pr\left(A_{iB} > R_B\right)R_{pB} + u'\left(c_G^c\right)R_{pG} \\ u'\left(c_B^c\right)\Pr\left(A_{iB} > R_{iB}\right)R_{idB} + u'\left(c_G^c\right)R_{idG} &= u'\left(c_B^c\right)\Pr\left(A_{iB} > R_B\right)R_{dB} + u'\left(c_G^c\right)R_{dG} \end{aligned}$$

where R_{ids} and R_{ips} for $s \in \{B, G\}$ correspond to the real interest rate faced by the firm i according to equations (7) and (6), and R_{ds} and R_{ps} correspond to the market interest rates. Then, the interest rate that a representative consumer charges to a firm i decreases in its probability of repayment, which is itself given by the firm's currency composition of liabilities (from equation

(9)):

$$\frac{\partial R_{ips}}{\partial \nu_i} = -\psi(\nu_i)\frac{\partial \Pr(A_{iB} > R_{iB})}{\partial \nu_i} R_{ipB} < 0 \tag{17}$$

$$\frac{\partial R_{ids}}{\partial \nu_i} = -\psi(\nu_i)\frac{\partial \Pr(A_{iB} > R_{iB})}{\partial \nu_i} R_{idB} < 0 \tag{18}$$

where $\psi(\nu_i) = \frac{u'(c_B^C)}{u'(c_B^C)\Pr(A_{iB}>R_{iB})+u'(c_G^c)}$. Then, under exclusive contracts, consumers provide the right incentives to firms: entrepreneurs are "punished" with higher dollar and peso interest rates when they increase their share of dollar liabilities. As a result, firms are induced to diversify the currency composition of debt.[14] And, since firms are ex-ante identical, in equilibrium the composition of debt is equalized across firms and is equal to consumers' portfolio choice:

$$\forall i \in [0,1] : v_i = \mu \tag{19}$$

Since all firms have the same currency composition of liabilities ν_i, they also have the same default risk. Furthermore, since firms hold both dollar and peso debt, the default risk of both types of assets is identical; that is

$$q_{dB} = q_{pB} = \Pr(A_{iB} > R_{iB}) = \Pr(A_{iB} > R_B) \tag{20}$$

[14] When the interest rate charged to each entrepreneur adjusts to the firm currency composition of liabilities, the entrepreneurs's problem is concave in ν_i :

$$\frac{\partial^2 E(c_{is}^e)}{\partial \nu_i^2} = Az\left[\frac{\partial \Pr(A_{is} > R_{iB})}{\partial \nu_i}\right]^2 \left[1 - 2(1 + \Pr(A_{is} > R_{iB}))\frac{Az\psi(\nu_i)}{Az - \psi(\nu_i)R_B}\right]$$

where $\psi(\nu_i) = \frac{u'(c_B^c)}{u'(c_B^c)\Pr(A_{is}>R_{iB})+u'(c_G^c)}$ and $R_{iB} = \nu_i R_{ipB} + (1-\nu_i)R_{idB}$. A sufficient condition for the second order condition to be negative is $\psi(\nu_i) > \frac{1}{2}$, which is always true since $u'(c_B^c) > u'(c_G^c)$.

where R_B is the overall claim in the B state stipulated in an exclusive contract:

$$R_B = \mu\left(r_p - \tau\delta\right) + \left(1-\mu\right)\left(r_d + \left(1-\tau\right)\delta\right) \tag{21}$$

Credit Market Equilibrium

The credit market equilibrium in the case of exclusive contracts replicates the social planner allocation analyzed in section 3.1. with the addition of two restrictions representing the short selling constraints:

$$foc_{exc}: \left[u'\left(c_B^c\right) - u'\left(c_G^c\right)\right] \Pr\left(A_{iB} > R_B\right) + u'\left(c_B^c\right) R_B \frac{\partial \Pr\left(A_{iB} > R_B\right)}{\partial R_B} = \lambda_d - \lambda_p \tag{22}$$

where λ_d and λ_p are the multiplier for the short-selling constraints. The only difference between this program and the one characterizing a market with contingent debt in equation (11) results from the existence of these constraints, which can only bind for low devaluation response to the sock. Augmenting the exchange rate volatility increases the contingent value of assets and reduces the value of the multipliers on the financial restrictions. Once the binding short selling constraint is relaxed, the solution in (22) is invariant to the volatility of the exchange rate. In this case, the currency denomination of contracts, characterized in (21), adjusts to the size of the devaluation response so to keep real claims on the corporate sector (R_B) constant. Dollar and peso contracts can substitute for a full set of contingent debt in this case.

Proposition 4 (exclusive contracts) *The credit market equilibrium is char-*

acterized by i) $\exists \delta_{SB}$ *such that* $\forall \delta < \delta_{SB} : \frac{\partial Eu(c_s^c)}{\partial \delta} > 0$ *and* $\forall \delta > \delta_{SB} : \frac{\partial Eu(c_s^c)}{\partial \delta} = 0$, *ii)* $\forall \delta < \delta_{SB} : \mu(\delta) = 0$ *and* $\forall \delta > \delta_{SB} : \frac{\partial \mu(\delta)}{\partial \delta} > 0$, *and iii)* $\lim_{\delta \to \infty} \mu(\delta) = 1 - \tau$.

Proof. Let $\left\{R_B^{SB}, R_G^{SB}\right\}$ be the optimal contingent claim (second best with limited liabilities) defined by condition (11). Note that condition (11) is equivalent to the first order condition under exclusive contracts in (22) when $\lambda_d = \lambda_p = 0$. The short selling constraints can be rewritten in terms of the contingent value of nondiversified portfolios:

$$\begin{aligned} R_G - R_B &\leq 2\tau\delta \\ R_G - R_B &\geq -2(1-\tau)\delta \end{aligned}$$

Then, the multiplier of the binding constraint decreases with δ and there is always a high enough δ_{SB} such that the binding constraint is satisfied with equality. It follows that for all $\delta < \delta_{SB} : \frac{\partial Eu(c_s^c)}{\partial \delta} > 0$ and for all $\delta \geq \delta_{SB} : \lambda_d = \lambda_p = 0$ and $\frac{\partial Eu(c_s^c)}{\partial \delta} = 0$.
Under assumption (13), $R_B^{SB} = r_d + (1-\tau)\delta_{SB}$. Thus, for $\delta \geq \delta_{SB}$ the solution is equivalent to the optimal contingent contract $R_B = R_B^{SB} = \mu[r_p - \tau\delta] + (1-\mu)[r_d + (1-\tau)\delta]$, which is independent of δ. The shares of peso and dollar debt adjust to the devaluation rate so that contract is constant:

$$\frac{\partial \mu}{\partial \delta} = -\frac{\mu - (1-\tau)}{[r_d - r_p + \delta]^2} > 0$$

Note that $\lim_{\delta \to \infty} \mu = 1 - \tau$. ■