Supplementary Appendix for "Optimal Automatic Stabilizers"

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This appendix contains four sections, which in turn: (i) provide auxiliary steps to some results stated in section 3; (ii) prove the propositions in section 4; (iii) prove the propositions in section 5; and (iv) describe the methods used to solve the model in section 6.

A Additional steps for deriving the results in section 3

A.1 The value of employment

In equilibrium, $a_{i,t} = 0$, and search effort is determined by comparing the value of working and not working according to equation (15). This section of the appendix derives the two key steps that make this difference independent of the household's skill, so that all households choose the same search effort.

Lemma 1. *The household's value function has the form*

$$V(\alpha, n, \mathcal{S}) = V^{\alpha}(\alpha, \mathcal{S}) + nV^{n}(\mathcal{S}) \tag{A.1}$$

for some functions V^{α} and V^{n} where $\mathcal{S}$ is the aggregate state. The choice of search effort is then the same for all searching households regardless of α.

Proof: Suppose that the value function is of the form given in equation (A.1). We will establish that the Bellman equation maps functions in this class into itself, which implies that the fixed point of the Bellman equation is in this class by the contraction mapping theorem. The household's search problem is

$$V^s(\alpha, \mathcal{S}) = \max_q \left\{ MqV(\alpha, 1, \mathcal{S}) + (1 - Mq)V(\alpha, 0, \mathcal{S}) - \frac{q^{1+\kappa}}{1+\kappa} \right\}.$$

Substitute for the value functions to arrive at

$$V^s(\alpha, \mathcal{S}) = \max_q \left\{ Mq \left[V^\alpha(\alpha, \mathcal{S}) + V^n(\mathcal{S})\right] + (1 - Mq)\left[V^\alpha(\alpha, \mathcal{S})\right] - \frac{q^{1+\kappa}}{1+\kappa} \right\} \tag{A.2}$$

$$V^s(\alpha, \mathcal{S}) = \max_q \left\{ MqV^n(\mathcal{S}) - \frac{q^{1+\kappa}}{1+\kappa} \right\} + V^\alpha(\alpha, \mathcal{S}) \tag{A.3}$$

where we have brought $V^\alpha(\alpha, \mathcal{S})$ outside the max operator as it appears in an additively separable manner. As there is no α inside the max operator, the optimal q is independent of α. Note that we can write V^s as $V^s(\alpha, \mathcal{S}) = g(\mathcal{S}) + V^\alpha(\alpha, \mathcal{S})$ where g is the solution to the maximization problem above.

The Bellman equation for employed and unemployed are

$$V^\alpha(\alpha, \mathcal{S}) + V^n(\mathcal{S}) = \log\left[\lambda\left(\alpha(wh + d)\right)^{1-\tau}\right] - \frac{h^{1+\gamma}}{1+\gamma} + \beta\mathbb{E}\left[(1 - \upsilon)\left(V^\alpha(\alpha', \mathcal{S}') + V^n(\mathcal{S}')\right) + \upsilon V^s(\alpha', \mathcal{S}')\right]$$

$$V^\alpha(\alpha, \mathcal{S}) = \log\left[\lambda b\left(\alpha(wh + d)\right)^{1-\tau}\right] - \xi + \beta\mathbb{E}\left[V^s(\alpha', \mathcal{S}')\right], \tag{A.4}$$

where we have used the budget constraint to substitute for consumption and the result that h is independent of α. Taking the difference yields

$$V^n(\mathcal{S}) = -\log(b) - \frac{h^{1+\gamma}}{1+\gamma} + \xi + \beta(1 - \upsilon)\mathbb{E}\left[V^n(\mathcal{S}') - g(\mathcal{S}')\right] \tag{A.5}$$

and plugging $V^s(\alpha, \mathcal{S}) = g(\mathcal{S}) + V^\alpha(\alpha, \mathcal{S})$ into the continuation value of the unemployed in (A.4) gives

$$V^\alpha(\alpha, \mathcal{S}) = \log\left[\lambda b\left(\alpha(wh + d)\right)^{1-\tau}\right] - \xi + \beta\mathbb{E}\left[g(\mathcal{S}') + V^\alpha(\alpha', \mathcal{S}')\right].$$

□

A.2 Optimal search effort

To derive optimal search effort in equation (20) we use the results of Lemma 1, specifically equations (A.3) and (A.5), using $\upsilon = 1$. This leads to

$$\max_q \left\{ Mq \left[\log \frac{1}{b} - \frac{h^{1+\gamma}}{1+\gamma} + \xi \right] - \frac{q^{1+\kappa}}{1+\kappa} \right\}.$$

The first order condition yields equation (20).

A.3 Proof of lemma 2

First, the Euler equation for a household is

$$\frac{1}{c_{i,t}} \geq \beta R_t \mathbb{E}\left[\frac{1}{c_{i,t+1}}\right]$$

as usual. Using (16) we have

$$\left[\alpha_{i,t}^{1-\tau}(n_{i,t} + (1-n_{i,t})b)\tilde{c}_t\right]^{-1} \geq \beta R_t \mathbb{E}\left[\left[\alpha_{i,t+1}^{1-\tau}(n_{i,t+1} + (1-n_{i,t+1})b)\tilde{c}_{t+1}\right]^{-1}\right].$$

Notice that $\mathbb{E}\left[\frac{\alpha_{i,t}^{1-\tau}}{\alpha_{i,t+1}^{1-\tau}}\right] = \mathbb{E}\left[\epsilon_{i,t+1}^{\tau-1}\right]$ is common across households and is known at date t. Now consider the two cases for $n_{i,t}$ and use the EU and UU transition probabilities to arrive at

$$\tilde{c}_t^{-1} \geq \beta R_t \mathbb{E}\left[\left[1 + \upsilon\left(1 - q_{t+1}M_{t+1}\right)\left(b^{-1} - 1\right)\right]\tilde{c}_{t+1}^{-1}\right]\mathbb{E}\left[\epsilon_{i,t+1}^{\tau-1}\right] \tag{A.6}$$

$$\tilde{c}_t^{-1} \geq \beta R_t \mathbb{E}\left[b\left[1 + \left(1 - q_{t+1}M_{t+1}\right)\left(b^{-1} - 1\right)\right]\tilde{c}_{t+1}^{-1}\right]\mathbb{E}\left[\epsilon_{i,t+1}^{\tau-1}\right]. \tag{A.7}$$

The right-hand side of these inequalities is larger for the employed (we establish this formally below), so there are two possibilities: the Euler equation of the employed holds with equality or both inequalities are strict.

Here we follow Krusell et al. (2011), Ravn and Sterk (2017), and Werning (2015) in assuming that the Euler equation of the employed/high-income household holds with equality. This household is up against its constraint $a' = 0$ so there could be other equilibria in which the Euler equation does not hold with equality. The equilibrium we focus on is the limit of the unique equilibrium as the borrowing limit approaches zero from below. See Krusell et al. (2011) for further discussion of this point. Equation (A.6) holding with equality yields the desired result.

The right-hand side of equation (A.6) weakly exceeds that of (A.7) if

$$q_{t+1}M_{t+1} + \frac{\upsilon}{b}\left(1 - q_{t+1}M_{t+1}\right) \geq 0.$$

Notice that qM is a job finding rate $\in [0,1]$ and $\upsilon \in [0,1]$ and $b \in [0,1]$.

A.4 Equilibrium definition

We first state the intermediate firm's problem and some additional equilibrium conditions and then state a definition of an equilibrium.

Firm's problem and inflation with Calvo pricing. The intermediate firm's problem is

$$\max_{p_t^*, \{y_{j,s}, n_{j,s}, v_{j,s}\}_{s=t}^{\infty}} \mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t}\left[\frac{p_t^*}{P_s} y_{j,s} - n_{j,s}h_{j,s}w_s - \psi_1 M_s^{\psi_2} v_{j,s}\right]$$

subject to

$$\begin{aligned} y_{j,s} &= \left(p_t^*/P_s\right)^{\mu/(1-\mu)} Y_s \\ y_{j,s} &= \eta_s^A h_s n_{j,s} \\ n_{j,s} &= (1-\upsilon)n_{j,s-1} + v_{j,s}. \end{aligned}$$

The solution to this problem satisfies

$$\frac{p_t^*}{p_t} = \frac{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t}\left(\frac{p_t^*}{p_s}\right)^{\mu/(1-\mu)} Y_s \mu \left[\frac{w_s h_s + \psi_1 M_s^{\psi_2} - R_{s+1}^{-1}(1-\upsilon)\psi_1 M_{s+1}^{\psi_2}}{\eta_s^A h_s}\right]}{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t}\left(\frac{p_t^*}{p_s}\right)^{1/(1-\mu)} Y_s}. \qquad \text{(A.8)}$$

The term in square brackets is real marginal cost at date s. As is standard, inflation and price dispersion evolve according to:

$$\pi_t = \left[(1-\theta) / \left[1 - \theta \left(\frac{p_t^*}{p_t} \right)^{1/(1-\mu)} \right] \right]^{1-\mu} \tag{A.9}$$

$$S_t = (1-\theta)\, S_{t-1} \pi_t^{-\mu/(1-\mu)} + \theta \left(\frac{p_t^*}{p_t} \right)^{\mu/(1-\mu)}. \tag{A.10}$$

where p_t^*/p_t is the relative price chosen by firms that adjust their price in period t.

Firm's problem and inflation with sticky information. Under the assumption of a unit separation rate, the real marginal cost of the firm is:

$$\frac{w_t + \psi_1 M_t^{\psi_2}/h_t}{\eta_t^A}.$$

Marginal costs are the sum of the wage paid per effective unit of labor and the hiring costs that had to be paid, divided by productivity. The price-setting first order condition is

$$\frac{p_t^*}{P_t} = \mu \left(\frac{w_t h_t + \psi_1 M_t^{\psi_2}}{\eta_t^A h_t} \right) \tag{A.11}$$

for firms with full information and $\mathbb{E}_{t-1}(p_t^*)$ for others. The price level satisfies

$$P_t^{\frac{1}{1-\mu}} = \theta p_t^{*(\frac{1}{1-\mu})} + (1-\theta)\, \mathbb{E}_{t-1}(p_t^*)^{\frac{1}{1-\mu}} \tag{A.12}$$

and price dispersion is given by

$$S_t = \left(\frac{p_t^*}{P_t} \right)^{\mu/(1-\mu)} \left[\theta + (1-\theta) \left[\frac{1}{1-\theta} \left(\frac{p_t^*}{P_t} \right)^{\frac{1}{\mu-1}} - \frac{\theta}{1-\theta} \right]^{\mu} \right]. \tag{A.13}$$

Equilibrium The aggregate resource constraint is:

$$Y_t - J_t = C_t + G_t. \tag{A.14}$$

The Fisher equation is:

$$R_t = I_t / \mathbb{E}_t \left[\pi_{t+1}\right]. \tag{A.15}$$

The link between $\tilde{c}_t$ and C_t depends on $\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau}\right]$. This evolves according to:

$$\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau}\right] = (1-\delta)\mathbb{E}_i \left[\alpha_{i,t-1}^{1-\tau}\right] \mathbb{E}_i \left[\epsilon_{i,t}^{1-\tau}\right] + \delta.. \tag{A.16}$$

The net revenues of the firm are paid out to the employed workers in the form of wages and dividends so we have

$$Y_t - J_t = (w_t h_t + d_t)(1 - u_t) \tag{A.17}$$

using the aggregate production function

$$Y_t = A_t h_t (1 - u_t) \tag{A.18}$$

and substituting into equation (14) we arrive at

$$h_t^{\gamma} = \frac{(1-\tau) w_t}{A_t h_t \frac{Y_t - J_t}{Y_t}}. \tag{A.19}$$

An equilibrium of the economy can be calculated from a system equations in 17 variables and three exogenous processes. The variables are

$$C_t, \tilde{c}_t, u_t, \mathbb{E}_i \left[\alpha_{i,t}^{1-\tau}\right], Q_t, R_t, I_t, \pi_t, Y_t, G_t, h_t, w_t, S_t, \frac{p_t^*}{p_t}, J_t, q_t, M_t.$$

And the equations are: (3), (4), (5), (6), (7), (17) (18), (19), (20), (A.11), (A.12), (A.13), (A.14), (A.15), (A.16), (A.18), and (A.19). The exogenous processes are η_t^A, η_t^G, and η_t^I.

In the quantitative model with Calvo pricing, we replace (A.11) with (A.8), (A.12) with (A.9), and (A.13) with (A.10). Moreover, with persistent employment we replace (20) with (15) and we must keep track of the value of employment in excess of unemployment, which is forward-looking, independent of α and can be calculated from (A.5).

B Proofs for section 4

Proof of Lemma 3. Given b, τ, M_t, η_t^A, the variables h_t, q_t, u_t, Y_t, A_t, w_t, J_t, S_t, $\frac{p_t^*}{P_t}$ satisfy a system of 9 equations in these 9 variables. The equation that determines h_t is (A.19). In what follows we establish by guess and verify that if h_t satisfies the form $\mathcal{H}_h(b, \tau, M_t, \eta_t^A)$, then the solution to equation (A.19) satisfies the same form. In order to do so, we first establish that other variables in the system have equilibrium mappings of the same form.

Start with q_t, which is given by equation (20). Substitute $h_t = \mathcal{H}_h(b, \tau, M_t, \eta_t^A)$ into (20) to obtain a mapping $q_t = q(b, \tau, M_t, \eta_t^A)$. Next, note that by $u_t = 1 - q_t M_t$ (i.e. (3) with $\upsilon = 1$) . Substitute in $q_t = q(b, \tau, M_t, \eta_t^A)$ to obtain a mapping $u_t = u(b, \tau, M_t, \eta_t^A)$. In turn, w_t is given by the wage rule in equation (5) and we substitute for u_t. Using equation (A.11) we substitute for w_t and h_t to establish a mapping for p_t^*/P_t of the same form and equation (A.13) then gives the mapping for $S_t = S(b, \tau, M_t, \eta_t^A)$. Using the aggregate production function $Y_t = \frac{\eta_t^A}{S_t} h_t(1 - u_t)$ we substitute to obtain a similar mapping and so too with the resources spent on recruiting $J_t = \psi_1 M_t^{\psi_2}(1 - u_t)$. Finally, we get to equation (A.19) rearranged as

$$h_t = \left[\frac{(1-\tau)w_t}{\frac{\eta_t^A}{S_t}\left(1 - \frac{J_t}{Y_t}\right)} \right]^{1/(1+\gamma)} . \tag{A.20}$$

As all the variables on the right-hand side are functions of b, τ, M_t, and η_t^A, the solution has the form $h_t = \mathcal{H}_h(b, \tau, M_t, \eta_t^A)$ for some function $\mathcal{H}_h(.)$. □

Proof of Lemma 4: When there is no mortality, $\delta = 0$, we can compute the cumulative welfare effect of a change in $F(\epsilon_{i,t}, u_t)$ including the effects on current and future skill dispersion. In particular

$$\begin{aligned}
\mathbb{E}_i \log\left(\alpha_{i,t}^{1-\tau}\right) &= \mathbb{E}_i \log\left(\alpha_{i,t-1}^{1-\tau}\epsilon_{i,t}^{1-\tau}\right) \\
&= \mathbb{E}_i \log\left(\alpha_{i,0}^{1-\tau}\epsilon_{i,1}^{1-\tau}\cdots\epsilon_{i,t}^{1-\tau}\right) \\
&= \mathbb{E}_i \log\left(\alpha_{i,0}^{1-\tau}\right) + \mathbb{E}_i \log\left(\epsilon_{i,1}^{1-\tau}\right) + \cdots + \mathbb{E}_i \log\left(\epsilon_{i,t}^{1-\tau}\right).
\end{aligned}$$

Similarly

$$\begin{aligned}\log\left(\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]\right) &= \log\left(\mathbb{E}_i\left[\alpha_{i,t-1}^{1-\tau}\right]\mathbb{E}_i\left[\epsilon_{i,t}^{1-\tau}\right]\right)\\ &= \log\left(\mathbb{E}_i\left[\alpha_{i,0}^{1-\tau}\right]\mathbb{E}_i\left[\alpha_{i,1}^{1-\tau}\right]\cdots\mathbb{E}_i\left[\epsilon_{i,t}^{1-\tau}\right]\right)\\ &= \log\left(\mathbb{E}_i\left[\alpha_{i,0}^{1-\tau}\right]\right)+\log\left(\mathbb{E}_i\left[\alpha_{i,1}^{1-\tau}\right]\right)+\cdots+\log\left(\mathbb{E}_i\left[\epsilon_{i,t}^{1-\tau}\right]\right)\end{aligned}$$

Notice that in this no-mortality case, the date-t loss from skill dispersion can be written as:

$$\mathbb{E}_i\log\left(\alpha_{i,t}^{1-\tau}\right)-\log\left(\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]\right)=\mathbb{E}_i\log\left(\alpha_{i,0}^{1-\tau}\right)-\log\left(\mathbb{E}_i\left[\alpha_{i,0}^{1-\tau}\right]\right)+\sum_{s=1}^{t}\left[\mathbb{E}_i\log\left(\epsilon_{i,s}^{1-\tau}\right)-\log\left(\mathbb{E}_i\left[\epsilon_{i,s}^{1-\tau}\right]\right)\right].$$

Finally, take the expected discounted sum of this expression and rearrange to prove the result. □

Lemma 2. *For a random variable X,*

$$\frac{d}{d\tau}\left\{\mathbb{E}\left[\log\left(X^{1-\tau}\right)\right]-\log\left(\mathbb{E}\left[X^{1-\tau}\right]\right)\right\}=\frac{\mathrm{Cov}\left(X^{1-\tau},\log X\right)}{\mathbb{E}\left[X^{1-\tau}\right]}$$

Proof:

$$\begin{aligned}\frac{d}{d\tau}\left\{\mathbb{E}\left[\log\left(X^{1-\tau}\right)\right]-\log\left(\mathbb{E}\left[X^{1-\tau}\right]\right)\right\} &= -\mathbb{E}\left[\log\left(X\right)\right]+\frac{\mathbb{E}\left[X^{1-\tau}\log X\right]}{\mathbb{E}\left[X^{1-\tau}\right]}\\ &= -\mathbb{E}\left[\log\left(X\right)\right]+\frac{\mathbb{E}\left[X^{1-\tau}\right]\mathbb{E}\left[\log X\right]+\mathrm{Cov}\left(X^{1-\tau},\log X\right)}{\mathbb{E}\left[X^{1-\tau}\right]}\\ &= \frac{\mathrm{Cov}\left(X^{1-\tau},\log X\right)}{\mathbb{E}\left[X^{1-\tau}\right]}\end{aligned}$$

□

Proof of Proposition 1. For this proof, in addition to the social welfare function, (22), the relevant equations of the model are (3), (A.18), (4), (A.14), (20), and (A.19). Conceptually we can write the period t contribution to the objective function as $W_t = W(b,\tau,q,h,M,\eta_t^A)$ where h and q are functions of (b,τ,M_t,η_t^A) by Lemma 3. Specifically

$$\begin{aligned}W_t = &\left[\mathbb{E}_i\log\left(\alpha_{i,0}^{1-\tau}\right)-\log\left(\mathbb{E}_i\left[\alpha_{i,0}^{1-\tau}\right]\right)\right]+\mathcal{R}_t+u_t\log b-\log(1-u_t+u_tb) \\ &+(1+\chi)\log\left(\frac{\eta_t^A}{S(b,\tau,M_t,\eta_t^A)}h_t(1-u_t)-\psi_1M_t^{\psi_2}(1-u_t)\right)-(1-u_t)\frac{h_t^{1+\gamma}}{1+\gamma}-\frac{q_t^{1+\kappa}}{1+\kappa}-\xi u_t\end{aligned} \quad \text{(A.21)}$$

where one should interpret u_t as $1 - q_t M_t$.

The first order condition of the objective function with respect to b is then

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{dW}{db} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left.\frac{\partial W_t}{\partial b}\right|_{M,q,h} + \left.\frac{\partial W_t}{\partial q_t}\right|_M \left.\frac{\partial q_t}{\partial b}\right|_M + \left.\frac{\partial W_t}{\partial h_t}\right|_M \left.\frac{\partial h_t}{\partial b}\right|_M + \frac{dW_t}{dM_t}\frac{dM_t}{db} \right\} = 0.$$

The first term on the right-hand side corresponds to the insurance term, the next two terms give the effect of incentives, and finally we have the macro-stabilization term. The rest of the proof expresses these derivatives as the terms shown in the proposition.

Begin with the insurance term

$$\begin{aligned}\left.\frac{\partial W_t}{\partial b}\right|_{M,q,h} &= \frac{u_t}{b} - \frac{u_t}{1-u_t+u_t b} \\ &= u_t \left(\frac{1}{b} - 1 + 1 - \frac{1}{1-u_t+u_t b} \right) \\ &= u_t \left(\frac{1}{b} - 1 \right) \left(1 - \frac{u_t b}{1-u_t+u_t b} \right).\end{aligned}$$

Here we have made use of the rigid price assumption to treat $S_t = 1$. Note that

$$\begin{aligned}\left.\frac{\partial \log(b\tilde{c}_t)}{\partial \log b}\right|_{u,h} &= \frac{\partial}{\partial \log b} \log\left(\frac{bC_t}{\Omega_t(1-u_t+u_t b)} \right) \\ &= 1 - \frac{u_t b}{1-u_t+u_t b}\end{aligned} \tag{A.22}$$

where the partial derivative on the right hand side of (A.22) is with respect to b alone.[1] So, we have:

$$\left.\frac{\partial W_t}{\partial b}\right|_{M,q,h} = u_t \left(\frac{1}{b} - 1 \right) \left.\frac{\partial \log(b\tilde{c}_t)}{\partial \log b}\right|_{u,h} \tag{A.23}$$

Turning to the incentives terms

$$\begin{aligned}\left.\frac{\partial W_t}{\partial q_t}\right|_M &= \frac{\partial W_t}{\partial u_t} \left.\frac{\partial u_t}{\partial q_t}\right| - q_t^{\kappa} \\ \frac{\partial W_t}{\partial u_t} &= \frac{-1}{C_t}\left(A_t h_t - \psi_1 M_t^{\psi_2} \right) + \frac{1-b}{1-u_t+u_t b} + \log b + \frac{h_t^{1+\gamma}}{1+\gamma} - \xi + \frac{d\mathcal{R}_t}{du_t}\end{aligned} \tag{A.24}$$

[1] As $\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]$ is an endogenous state that depends on the history of u_t, we are taking the partial derivative holding fixed this history.

and combining these terms along with (20) we have

$$\left.\frac{\partial W_t}{\partial q_t}\right|_M = \left(\frac{-1}{C_t}\left(A_t h_t - \psi_1 M_t^{\psi_2}\right) + \frac{1-b}{1-u_t+u_t b} + \frac{d\mathcal{R}_t}{du_t}\right)\left.\frac{\partial u_t}{\partial q_t}\right|_M .$$

Now note that

$$\tilde{c}_t \equiv \frac{C_t}{\Omega_t(1-u_t+u_t b)} = \frac{A_t h_t(1-u_t) - \psi_1 M_t^{\psi_2}(1-u_t) - G_t}{\Omega_t(1-u_t+u_t b)}$$
$$\left.\frac{\partial \log \tilde{c}_t}{\partial u_t}\right|_{M,\Omega,G} = \frac{-1}{C_t}\left(A_t h_t - \psi_1 M_t^{\psi_2}\right) + \frac{1-b}{1-u_t+u_t b}$$

so

$$\left.\frac{\partial W_t}{\partial q_t}\right|_M = \left(\left.\frac{\partial \log \tilde{c}_t}{\partial u_t}\right|_{M,\Omega,G} + \frac{d\mathcal{R}_t}{du_t}\right)\left.\frac{\partial u_t}{\partial q_t}\right|_M$$

and

$$\left.\frac{\partial W_t}{\partial q_t}\right|_M \left.\frac{\partial q_t}{\partial b}\right|_M = \left(\left.\frac{\partial \log \tilde{c}_t}{\partial u_t}\right|_{M,\Omega,G} + \frac{d\mathcal{R}_t}{du_t}\right)\left.\frac{\partial u_t}{\partial b}\right|_M .$$

Now for the incentives for the intensive margin of labor supply

$$\left.\frac{\partial W_t}{\partial h_t}\right|_M \left.\frac{\partial h_t}{\partial b}\right|_M = (1-u_t)\left(\frac{A_t}{C_t} - h_t^{\gamma}\right)\left.\frac{\partial h_t}{\partial b}\right|_M .$$

Finally, note that if $\theta \in (0,1)$ then we also need to differentiate $S(b, \tau, M_t, \eta_t^A)$ with respect to b, which adds a term to $\frac{\partial W_t}{\partial b}\big|_{M,q,h}$. The additional term is

$$-\frac{1+\chi}{Y_t - J_t}\frac{\eta_t^A h_t(1-u_t)}{S_t^2}\left.\frac{\partial S_t}{\partial b}\right|_M = -\frac{Y_t}{C_t S_t}\left.\frac{\partial S_t}{\partial b}\right|_M .$$

□

Proof of Proposition 2. Conceptually, the proof follows the same steps as Proposition 1. The first order condition with respect to τ is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{dW_t}{d\tau} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left.\frac{\partial W_t}{\partial \tau}\right|_{M,q,h} + \left.\frac{\partial W_t}{\partial q_t}\right|_M \left.\frac{\partial q_t}{\partial \tau}\right|_M + \left.\frac{\partial W_t}{\partial h_t}\right|_M \left.\frac{\partial h_t}{\partial \tau}\right|_M + \frac{dW_t}{dM_t}\frac{dM_t}{d\tau} \right\} = 0.$$

For the insurance term we use Lemmas 4 and 2 to arrive at

$$\left.\frac{\partial W_t}{\partial \tau}\right|_{M,q,h} = \frac{\mathrm{Cov}\left(\alpha_{i,0}^{1-\tau}, \log \alpha_{i,0}\right)}{\mathbb{E}\left[\alpha_{i,0}^{1-\tau}\right]} + \left.\frac{\partial \mathcal{R}_t}{\partial \tau}\right|_u .$$

For the insurance term we proceed as in the proof of Proposition 1 to write

$$\left.\frac{\partial W_t}{\partial q_t}\right|_M \left.\frac{\partial q_t}{\partial \tau}\right|_M = \left(\left.\frac{\partial \log \tilde{c}_t}{\partial u_t}\right|_{M,\Omega,G} + \frac{d\mathcal{R}_t}{du_t}\right) \left.\frac{\partial u_t}{\partial \tau}\right|_M$$

and

$$\left.\frac{\partial W_t}{\partial h_t}\right|_M \left.\frac{\partial h_t}{\partial \tau}\right|_M = (1-u_t)\left(\frac{A_t}{C_t} - h_t^\gamma\right) \left.\frac{\partial h_t}{\partial \tau}\right|_M .$$

□

C Proofs for section 5

Proof of Proposition 3. Differentiating (A.21) with respect to M_t yields

$$\frac{dW_t}{dM_t} = \frac{\partial W_t}{\partial u_t}\frac{du_t}{dM_t} + \frac{1}{C_t}\left[-\frac{Y_t}{S_t}\frac{dS_t}{dM_t} + A_t(1-u_t)\frac{dh_t}{dM_t} - \psi_1\psi_2 M_t^{\psi_2-1}(1-u_t)\right] - (1-u_t)\frac{dh_t}{dM_t} - q_t^\kappa \frac{dq_t}{dM_t}.$$

Now use $\frac{du_t}{dM_t} = \left.\frac{\partial u_t}{\partial M_t}\right|_q - M_t\frac{dq_t}{dM_t}$, and equations (A.24) and (20) to get:

$$\begin{aligned}\frac{dW_t}{dM_t} =& -\left[\xi - \log b - \frac{h_t^{1+\gamma}}{1+\gamma}\right]\left.\frac{\partial u_t}{\partial M_t}\right|_q \\ &+ \left[\frac{-1}{C_t}\left(A_t h_t - \psi_1 M_t^{\psi_2}\right) + \frac{1-b}{1-u_t+u_t b} + \frac{d\mathcal{R}_t}{du_t}\right]\frac{du_t}{dM_t} \\ &+ \frac{1}{C_t}\left[-\frac{Y_t}{S_t}\frac{dS_t}{dM_t} + A_t(1-u_t)\frac{dh_t}{dM_t} - \psi_1\psi_2 M_t^{\psi_2-1}(1-u_t)\right] - (1-u_t)h_t^\gamma \frac{dh_t}{dM_t}.\end{aligned} \tag{A.25}$$

Using the resource constraint $C_t = A_t h_t(1-u_t) - \psi_1 M_t^{\psi_2}(1-u_t) - G_t$ and the definition $J_t = \psi_1 M_t^{\psi_2}(1-u_t)$ we have

$$\left.\frac{\partial C_t}{\partial u_t}\right|_{M,G} = -A_t h_t - \psi_1 M_t^{\psi_2} \tag{A.26}$$

$$\left.\frac{\partial J_t}{\partial M_t}\right|_{u} = \psi_1 \psi_2 M_t^{\psi_2 - 1}(1-u_t). \tag{A.27}$$

Rearranging (A.25)-(A.27) yields the result. □

D Description of methods for section 6

D.1 Estimated income process

The material in this appendix describes the estimation of the time-varying skill risk process following McKay (2017). The income process is as follows: $\alpha_{i,t}$ evolves as in (2). Earnings are given by $\alpha_{i,t} w_t$ when employed and zero when unemployed. Notice that here we normalize $h_t = 1$ and subsume all movements in h_t into w_t. While this gives a different interpretation to w_t it does not affect the distribution of earnings growth rates apart from a constant term. The innovation distribution is given by

$$\epsilon_{i,t+1} \sim F(\epsilon; x_t) = \begin{cases} N(\mu_{1,t}, \sigma_1) & \text{with prob. } P_1, \\ N(\mu_{2,t}, \sigma_2) & \text{with prob. } P_2, \\ N(\mu_{3,t}, \sigma_3) & \text{with prob. } P_3 \\ N(\mu_{4,t}, \sigma_4) & \text{with prob. } P_4 \end{cases}$$

The tails of F move over time as driven by the latent variable x_t such that

$$\mu_{1,t} = \bar{\mu}_t,$$

$$\mu_{2,t} = \bar{\mu}_t + \mu_2 - x_t,$$

$$\mu_{3,t} = \bar{\mu}_t + \mu_3 - x_t,$$

$$\mu_{4,t} = \bar{\mu}_t.$$

where $\bar{\mu}_t$ is a normalization such that $\mathbb{E}_i[exp\{\epsilon_{i,t+1}\}] = 1$ in all periods.

The model period is one quarter. The parameters are selected to match the median earnings growth, the dispersion in the right tail (P90 - P50), and the dispersion in the left-tail (P50-P10) for one, three, and five year earnings growth rates computed each year using data from 1978 to 2011. In addition we target the kurtosis of one-year and five year earnings growth rates and the increase in cross-sectional variance over the life-cycle. The moments are computed from the Social Security Administration earnings data as reported by Guvenen et al. (2014) and Guvenen et al. (2015). Our objective function is a weighted sum of the squared difference between the model-implied and empirical moments.

The estimation procedure simulates quarterly data using the observed job-finding and -separation rates and then aggregates to annual income and computes these moments. To simulate the income process, we require time series for x_t and w_t. We assume that these series are linearly related to observable labor market indicators (for details see McKay, 2017). Call the weights in these linear relationships β. We then search over the parameters P, μ, σ, and β subject to the restrictions $P_2 = P_3$ and $\sigma_2 = \sigma_3$.

Guvenen et al. (2014) emphasize the pro-cyclicality in the skewness of earnings growth rates. The estimated income process does an excellent job capturing this as shown in the top panel of figure 1. The estimated β implies a time-series for x_t which shifts the tails of the earnings distribution and gives rise the pro-cyclical skewness shown in figure 1. We regress this time-series on the unemployment rate and find a coefficient of 16.7. The fourth component of the mixture distribution occurs with very low probability, and in our baseline specification we set it to zero. This choice is not innocuous, however, because the standard deviation σ_4 is estimated to be very large and this contributes to the high kurtosis of the earnings growth distribution. In particular, omitting this component leads to a substantially smaller τ as a result of having less risk in the economy. We prefer to omit this from our baseline calibration because the interpretation of these high-kurtosis terms is unclear and we are not entirely satisfied with modeling them as permanent shocks to skill.

The resulting income process that we use in our computations is as follows: The innovation

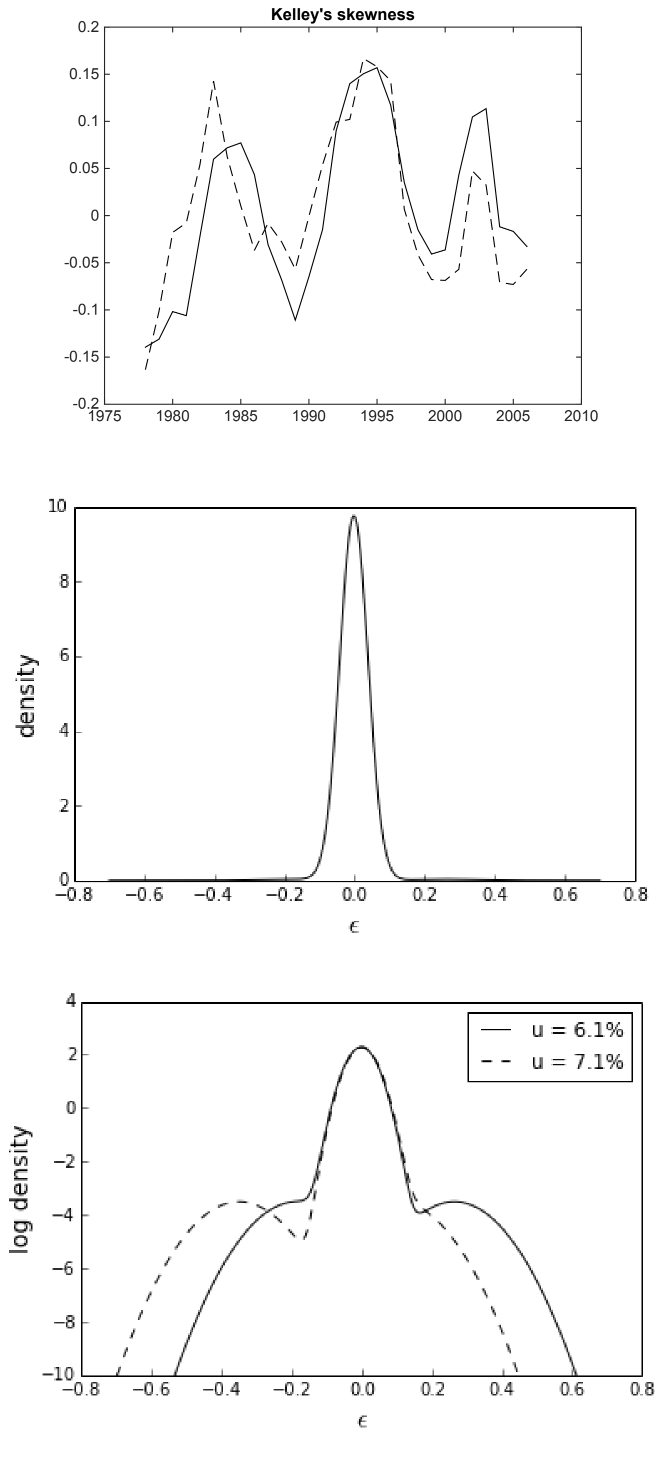

Figure 1: Properties of $F(\epsilon)$.

distribution is given by

$$\epsilon_{i,t+1} \sim F(\epsilon; u_t) = \begin{cases} N(\mu_{1,t}, 0.0403) & \text{with prob. } 0.9855, \\ N(\mu_{2,t}, 0.0966) & \text{with prob. } 0.00727, \\ N(\mu_{3,t}, 0.0966) & \text{with prob. } 0.00727 \end{cases}$$

with

$$\begin{aligned} \mu_{1,t} &= \bar{\mu}_t, \\ \mu_{2,t} &= \bar{\mu}_t + 0.266 - 16.73(u_t - u^*), \\ \mu_{3,t} &= \bar{\mu}_t - 0.184 - 16.73(u_t - u^*), \end{aligned}$$

where u^* is the steady state is unemployment rate in our baseline calibration. The bottom panels of figure 1 show the density of ϵ and how it changes with an increase in the unemployment rate.

D.2 Global solution method

As a first step, we need to rewrite the Calvo-pricing first-order condition recursively:

$$\frac{p_t^*}{p_t} = \frac{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{\mu/(1-\mu)} Y_s \mu \ell_s}{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{1/(1-\mu)} Y_s},$$

where

$$\ell_s \equiv \frac{h_s w_s + \psi_1 M_s^{\psi_2} - R_{s+1}^{-1}(1-\theta)(1-\upsilon)\mathbb{E}_s \psi_1 M_{s+1}^{\psi_2}}{\eta_s^A h_s}$$

is a measure of real marginal cost. Define p_t^A as

$$p_t^A = \mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{\mu/(1-\mu)} Y_s \mu \ell_s$$

and p_t^B as

$$p_t^B = \mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{1/(1-\mu)} Y_s$$

such that

$$\frac{p_t^*}{p_t} = \frac{p_t^A}{p_t^B}.$$

p_t^A and p_t^B can be rewritten as

$$p_t^A = \mu Y_t \ell_t + (1-\theta)\mathbb{E}_t\left[\left(\frac{I_t}{\pi_{t+1}}\right)^{-1} \pi_{t+1}^{-\mu/(1-\mu)} p_{t+1}^A\right] \tag{A.28}$$

$$p_t^B = Y_t + (1-\theta)\mathbb{E}_t\left[\left(\frac{I_t}{\pi_{t+1}}\right)^{-1} \pi_{t+1}^{-1/(1-\mu)} p_{t+1}^B\right]. \tag{A.29}$$

The procedure we use builds on the method proposed by Maliar and Maliar (2015) and their application to solving a New Keynesian model. We first describe how we solve the model for a given grid of aggregate state variables and then describe how we construct the grid.

There are seven state variables that evolve according to

$$\begin{aligned}
\mathbb{E}_i\left[\alpha_{i,t+1}^{1-\tau}\right] &= (1-\delta)\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]\mathbb{E}_i\left[\epsilon_{i,t+1}^{1-\tau} u_t\right] + \delta \\
\mathbb{E}_i\left[\log \alpha_{i,t+1}\right] &= (1-\delta)\left[\mathbb{E}_i\left[\log \alpha_{i,t}\right] + \mathbb{E}_i\left[\log \epsilon i, t+1 | u_t\right]\right] \\
S_{t+1}^A &= S_t \\
u_{t+1}^{\text{Lag}} &= u_t \\
\log \eta_{t+1}^A &= \rho^A \log \eta_t^A + \varepsilon_{t+1}^A \\
\log \eta_{t+1}^G &= \rho^G \log \eta_t^G + \varepsilon_{t+1}^G \\
\log \eta_{t+1}^I &= \rho^I \log \eta_t^I + \varepsilon_{t+1}^I,
\end{aligned}$$

where S^A is the level of price dispersion in the previous period and the ε terms are i.i.d. normal innovations.

There are six variables that we approximate with complete second-order polynomials in the state: $(1/C_t)$, p_t^A, p_t^B, J_t, V^n and V_t, where V_t^n is the value of being employed and V_t is the value of the social welfare function. We use (17) and (18) to write the Euler equation in terms of C_t and this equation pins down $1/C_t$. p_t^A and p_t^B satisfy (A.28) and (A.29). V_t^n satisfies (A.5). V_t satisfies

$$V_t = W_t + \beta\,\mathbb{E}_t\left[V_{t+1}\right].$$

J_t satisfies $J_t = \psi_1 M_t^{\psi_2}(\upsilon - u_t)$. Abusing language slightly, we will refer to these variables that we approximate with polynomials as "forward-looking variables."

The remaining variables in the equilibrium definition can be calculated from the remaining equations and all of which only involve variables dated t. We call these the "static" variables.

To summarize, let $\mathcal{S}_t$ be the state variables, $\mathcal{X}_t$ be the forward-looking variables, and $\mathcal{Y}_t$ be the static variables. The three blocks of equations are

$$\begin{aligned}
\mathcal{S}' &= \mathcal{G}^{\mathcal{S}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \varepsilon') \\
\mathcal{X} &= \mathbb{E}\,\mathcal{G}^{\mathcal{X}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \mathcal{S}', \mathcal{X}', \mathcal{Y}') \\
\mathcal{Y} &= \mathcal{G}^{\mathcal{Y}}(\mathcal{S}, \mathcal{X})
\end{aligned}$$

where $\mathcal{G}^{\mathcal{S}}$ are the state-transition equations, $\mathcal{G}^{\mathcal{X}}$ are the forward-looking equations and $\mathcal{G}^{\mathcal{Y}}$ are the state equations. Let $\mathcal{X} \approx \mathcal{F}(\mathcal{S}, \Omega)$ be the approximated solution for the forward-looking equations for which we use a complete second-order polynomial with coefficients given by Ω. We then operationalize the equations as follows: given a value for $\mathcal{S}$, we calculate $\mathcal{X} = \mathcal{F}(\mathcal{S}, \Omega)$ and $\mathcal{Y} = \mathcal{G}^{\mathcal{Y}}(\mathcal{S}, \mathcal{X})$. We then take an expectation over ε' using Gaussian quadrature. For each value of ε' in the quadrature grid, we compute $\mathcal{S}' = G^{\mathcal{S}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \varepsilon')$, $\mathcal{X}' = \mathcal{F}(\mathcal{S}', \Omega)$ and $\mathcal{Y}' = \mathcal{G}^{\mathcal{Y}}(\mathcal{S}', \mathcal{X}')$. We can now evaluate $\mathcal{G}^{\mathcal{X}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \mathcal{S}', \mathcal{X}', \mathcal{Y}')$ for this value of ε' and looping over all the values in the quadrature grid we can compute $\hat{\mathcal{X}} = \mathbb{E}\,\mathcal{G}^{\mathcal{X}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \mathcal{S}', \mathcal{X}', \mathcal{Y}')$. $\hat{\mathcal{X}}$ will differ from the value of $\mathcal{X}$ that was obtained initially from $\mathcal{F}(\mathcal{S}, \Omega)$. We repeat these steps for all the values of $\mathcal{S}$ in our grid for the aggregate state space. We then adjust the coefficients Ω part of the way towards those implied by the solutions $\hat{\mathcal{X}}$. We then iterate this procedure to convergence of Ω.

Evaluating some of the equations of the model involves taking integrals against the distribution of idiosyncratic skill risk $\epsilon_{i,t+1} \sim F(\epsilon_{i,t+1}, u_t)$. We do this using Gaussian quadrature within each of the components of the mixture distribution.

We use a two-step procedure to construct the grid on the aggregate state space. We have seven aggregate states so we choose the grid to lie in the region of the aggregate state space that is visited by simulations of the solution. We create a box of policy parameters $[b_L, b_H] \times [\tau_L, \tau_H]$. We then create a grid of twelve Sobol points on this box and for each pair (b, τ) we use the procedure of Maliar and Maliar (2015) to construct a grid on the aggregate state space and solve the model.

This procedure iterates between solving the model and simulating the solution and constructing a grid in the part of the state space visited by the simulation. This gives us twelve grids, which we then merge and eliminate nearby points using the techniques of Maliar and Maliar (2015). This leaves us with one grid that we use to solve the model when we evaluate policies. Each of the grids that we construct have 100 points.

D.3 The policy trade-offs in the quantitative model

We now explain how the policy trade-offs documented in Propositions 1, 2, and 3 can be calculated in the richer quantitative model.

The social welfare function is

$$\begin{aligned}
&V\left(\mathbb{E}_i\left[\alpha_i^{1-\tau}\right], S_{-1}, \eta, u_{-1}, \mathbb{E}_i \log\left(\alpha_{i,t}\right)\right) \\
&= (1-\tau)\mathbb{E}_i \log\left(\alpha_i\right) - \log\left(\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]\right) + u\log b - \log(1-u+ub) \\
&+ (1+\chi)\log\left(\frac{\eta^A}{S}h(1-u) - J\right) - (1-u)\frac{h^{1+\gamma}}{1+\gamma} - \left(u_{-1} + \upsilon\left(1-u_{-1}\right)\right)\frac{q^{1+\kappa}}{1+\kappa} - u\xi \\
&+ \beta\mathbb{E}\left[V\left(\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]', S, \eta', u, \mathbb{E}_i\log\left(\alpha_i\right)'\right)\right].
\end{aligned}$$

In addition we will use the following equations of the model

$$\begin{aligned}
u &= \left[u_{-1} + \upsilon\left(1-u_{-1}\right)\right](1-qM) \\
h^{1+\gamma} &= (1-\tau)\bar{w}\left(1-\frac{J}{Y}\right)^{-1} S\left(\frac{1-u}{1-\bar{u}}\right)^{\zeta} \\
q^{\kappa} &= MV^n\left(\mathbb{E}_i\left[\alpha_i^{1-\tau}\right], A, S_{-1}, \eta^I, \eta^G, u_{-1}, \mathbb{E}_i\log\left(\alpha_{i,t}\right)\right) \\
J &= \psi_1 M^{\psi_2}\left[1-u-(1-\upsilon)(1-u_{-1})\right] \\
V^n(\cdots) &= \left[-\log(b) - \frac{h^{1+\gamma_1}}{1+\gamma_1} + \xi\right] \\
&\quad + \beta(1-\upsilon)\mathbb{E}\left[\left(1-\frac{\kappa}{1+\kappa}q'M'\right)V^n\left(\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]', A', S, \eta^{I\prime}, \eta^{G\prime}, u, \mathbb{E}_i\log\left(\alpha_i\right)'\right)\right]
\end{aligned}$$

Insurance term (b) Take the derivative of V with respect to b taking q, h, and M as given

$$V_{\text{Insur}} = \frac{u}{b} - \frac{u}{1-u+ub} + \beta\mathbb{E}\left[V_{\text{Insur}}'\right].$$

Incentives term (b) First, take the derivative of V with respect to q and multiply it by dq/db taking M as given:

$$\begin{aligned}V_{\text{Incen-q}} &= \frac{\partial W}{\partial u}\frac{\partial u}{\partial q}\frac{\partial q}{\partial b} \\ &\quad - \left(u_{-1}+\upsilon\left(1-u_{-1}\right)\right)q^{\kappa}\frac{\partial q}{\partial b}+\beta\mathbb{E}\left[V'_{\text{Incen-q}}\right]+\beta\mathbb{E}\left[V'_{u_{-1}}\right]\frac{\partial u}{\partial q}\frac{\partial q}{\partial b}\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial W}{\partial u} &= \left(\log b+\frac{1-b}{1-u+ub}\right)-\frac{1}{C}\frac{1+\chi}{1+\chi\eta^{G}}\left(\frac{\eta^{A}}{S}h+\frac{\partial J}{\partial u}\right)+\frac{h^{1+\gamma}}{1+\gamma}-\xi+\frac{d\mathcal{R}}{du} \\
\frac{\partial u}{\partial q} &= -M\left[u_{-1}+\upsilon\left(1-u_{-1}\right)\right] \\
\frac{\partial q}{\partial b} &= \frac{1}{\kappa}\left(MV^{n}\right)^{\frac{1}{\kappa}-1}M\frac{\partial V^{n}}{\partial b}=\frac{1}{\kappa}\frac{q}{V^{n}}\frac{\partial V^{n}}{\partial b} \\
\frac{\partial h}{\partial b} &\approx \frac{\partial h}{\partial u}\frac{\partial u}{\partial q}\frac{\partial q}{\partial b}+\frac{\partial h}{\partial \bar{u}}\frac{\partial \bar{u}}{\partial q}\frac{\partial \bar{q}}{\partial b}=\frac{\partial h}{\partial q}\frac{1}{\kappa}\frac{q}{V^{n}}\frac{\partial V^{n}}{\partial b}+\frac{\partial h}{\partial \bar{u}}\frac{\partial \bar{u}}{\partial b} \\
\frac{\partial V^{n}}{\partial b} &= -\frac{1}{b}-h^{\gamma}\frac{\partial h}{\partial b}+\beta(1-\upsilon)\mathbb{E}\left[\left(1-\frac{\kappa}{1+\kappa}q'M'\right)\frac{\partial V^{n}}{\partial b}'-\frac{\kappa}{1+\kappa}M'\frac{\partial q}{\partial b}'V^{n\prime}\right] \\
&= -\frac{1}{b}-h^{\gamma}\frac{\partial h}{\partial b}+\beta(1-\upsilon)\mathbb{E}\left[\left(1-\frac{\kappa}{1+\kappa}q'M'\right)\frac{\partial V^{n}}{\partial b}'-\frac{1}{1+\kappa}M'^{1/\kappa+1}\left(V^{n\prime}\right)^{\frac{1}{\kappa}}\frac{\partial V^{n\prime}}{\partial b}\right] \\
&= -\frac{1}{b}-h^{\gamma}\frac{\partial h}{\partial b}+\beta(1-\upsilon)\mathbb{E}\left[\left(1-\frac{\kappa}{1+\kappa}q'M'\right)\frac{\partial V^{n\prime}}{\partial b}-\frac{1}{1+\kappa}M'q'\frac{\partial V^{n\prime}}{\partial b}\right] \\
&= -\frac{1}{b}-h^{\gamma}\frac{\partial h}{\partial b}+\beta(1-\upsilon)\mathbb{E}\left[\left(1-q'M'\right)\frac{\partial V^{n\prime}}{\partial b}\right] \\
&= -\frac{1}{b}-h^{\gamma}\left(\frac{\partial h}{\partial q}\frac{1}{\kappa}\frac{q}{V^{n}}\frac{\partial V^{n}}{\partial b}+\frac{\partial h}{\partial \bar{u}}\frac{\partial \bar{u}}{\partial b}\right)+\beta(1-\upsilon)\mathbb{E}\left[\left(1-q'M'\right)\frac{\partial V^{n\prime}}{\partial b}\right] \\
&= \left(1+h^{\gamma}\frac{\partial h}{\partial q}\frac{1}{\kappa}\frac{q}{V^{n}}\right)^{-1}\left[-\frac{1}{b}-h^{\gamma}\frac{\partial h}{\partial \bar{u}}\frac{\partial \bar{u}}{\partial b}+\beta(1-\upsilon)\mathbb{E}\left[\left(1-q'M'\right)\frac{\partial V^{n\prime}}{\partial b}\right]\right] \\
\frac{\partial h}{\partial u} &= -\frac{\zeta}{1+\gamma}\frac{h}{1-u} \\
\frac{\partial h}{\partial \bar{u}} &= \frac{\zeta}{1+\gamma}\frac{h}{1-\bar{u}}
\end{aligned}$$

In forming $\frac{\partial h}{\partial b}$ we ignore changes in J/Y as this ratio is always small. Second, take the derivative of V with respect to h and multiply it by dh/db taking M as given:

$$V_{\text{Incen-h}}=(1-u)\left[\frac{A}{C}\frac{1+\chi}{1+\chi\eta^{G}}-h^{\gamma}\right]\frac{\partial h}{\partial b}.$$

Note

$$\frac{d\mathcal{R}}{du}=\beta\frac{d\mathbb{E}\left[V'\right]}{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]}\frac{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]}{du}+\beta\frac{d\mathbb{E}\left[V'\right]}{d\mathbb{E}_i\log\left(\alpha_i\right)}\frac{d\mathbb{E}_i\log\left(\alpha_i\right)}{du}$$

Macro-stabilization term (b) Take the derivative of V with respect to M and multiply by the derivative of M with respect to b

$$\begin{aligned}
V_M = & \left[\left(\log b+\frac{1-b}{1-u+ub}\right)+\frac{h^{1+\gamma}}{1+\gamma}-\xi\right]\frac{du}{dM}\frac{dM}{db}-\left(u_{-1}+\upsilon\left(1-u_{-1}\right)\right)q^{\kappa}\frac{dq}{dM}\frac{dM}{db} \\
& +\left[-\frac{1}{C}\frac{1+\chi}{1+\chi\eta^G}\left(\frac{A}{S}h+\frac{\partial J}{\partial u}|_M\right)\right]\frac{du}{dM}\frac{dM}{db} \\
& -\frac{1}{C}\frac{1+\chi}{1+\chi\eta^G}\psi_1\psi_2M^{\psi_2-1}\left[1-u-(1-\upsilon)(1-u_{-1})\right]\frac{dM}{db} \\
& +\beta\mathbb{E}\left[V'_{u_{-1}}\right]\frac{du}{dM}\frac{dM}{db} \\
& +\left[\frac{1}{C}\frac{1+\chi}{1+\chi\eta^G}\frac{A}{S}(1-u)-(1-u)h^{\gamma}\right]\frac{dh}{dM}\frac{dM}{db} \\
& -\frac{1}{C}\frac{1+\chi}{1+\chi\eta^G}\frac{A}{S^2}h(1-u)\frac{dS}{dM}\frac{dM}{db}+\beta\mathbb{E}\left[V'_{S_{-1}}\right]\frac{dS}{dM}\frac{dM}{db} \\
& +\frac{d\mathcal{R}}{du}\frac{du}{dM}\frac{dM}{db}+\beta\mathbb{E}\left[V'_M\right]
\end{aligned}$$

The first line is unemployment risk; the second, third, and fourth lines are the Hosios terms as they reflect the gain in resources from reducing unemployment and the loss from tightening the labor market; the fifth line is the labor wedge; the sixth line is price dispersion; the seventh line is skill risk.

We can compute $\frac{du}{dM}$, $\frac{dq}{dM}$, $\frac{dh}{dM}$, and $\frac{dS}{dM}$ numerically from a monetary shock. We compute $\frac{dM}{db}$ from the finite difference across b.

The summary statistics of skill dispersion evolve according to:

$$\begin{aligned}
\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right] &= (1-\delta)\mathbb{E}_i\left[\alpha_{i,t-1}^{1-\tau}\right]\mathbb{E}_i\left[\epsilon_{i,t}^{1-\tau}\right]+\delta \\
\frac{d}{dM}\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right] &= (1-\delta)\mathbb{E}_i\left[\alpha_{i,t-1}^{1-\tau}\right]\frac{d}{dM}\mathbb{E}_i\left[\epsilon_{i,t}^{1-\tau}\right] \\
\mathbb{E}_i\left[\log\left(\alpha_{i,t}\right)\right] &= (1-\delta)\mathbb{E}_i\left[\log\left(\alpha_{i,t-1}\right)\right]+(1-\delta)\mathbb{E}_i\left[\log\left(\epsilon_{i,t}\right)\right] \\
\frac{d}{dM}\mathbb{E}_i\left[\log\left(\alpha_{i,t}\right)\right] &= (1-\delta)\frac{d}{dM}\mathbb{E}_i\left[\log\left(\epsilon_{i,t}\right)\right].
\end{aligned}$$

V_M reflects in part the change in value from how M affects future state variables, which can be calculated from the envelope conditions:

$$\begin{aligned}
V_{u_{-1}} &= -(1-\upsilon)\frac{q^{1+\kappa}}{1+\kappa} \\
&\left\{\left(\log b + \frac{1-b}{1-u+ub}\right) - \frac{1+\chi}{C}\frac{A}{S}h + \frac{h^{1+\gamma}}{1+\gamma} - \xi\right\}\frac{du}{du_{-1}} - \frac{1+\chi}{C}\frac{dJ}{du_{-1}} \\
&+ \left(\frac{1+\chi}{C}\frac{A}{S} - h^{\gamma}\right)(1-u)\frac{dh}{du_{-1}} - (u_{-1}+\upsilon(1-u_{-1}))\, q^{\kappa}\frac{dq}{du_{-1}} \\
&\beta\,\mathbb{E}\left[V'_{\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]}\frac{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]'}{du_{-1}} + V'_S\frac{dS}{du_{-1}} + V'_{u_{-1}}\frac{du}{du_{-1}}\right]
\end{aligned}$$

$$\begin{aligned}
V_{\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]} &= -\frac{1}{\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]} \\
&+ \left\{\left(\log b + \frac{1-b}{1-u+ub}\right) - \frac{1+\chi}{C}\frac{A}{S}h + \frac{h^{1+\gamma}}{1+\gamma} - \xi\right\}\frac{du}{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]} - \frac{1+\chi}{C}\frac{dJ}{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]} \\
&+ \left(\frac{1+\chi}{C}\frac{A}{S} - h^{\gamma}\right)(1-u)\frac{dh}{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]} - (u_{-1}+\upsilon(1-u_{-1}))\, q^{\kappa}\frac{dq}{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]} \\
&\beta\,\mathbb{E}\left[V'_{\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]}\frac{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]'}{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]} + V'_S\frac{dS}{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]} + V'_{u_{-1}}\frac{du}{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]}\right]
\end{aligned}$$

$$\begin{aligned}
V_{\mathbb{E}_i \log(\alpha_{i,t})} &= 1-\tau \\
&+ \left\{\left(\log b + \frac{1-b}{1-u+ub}\right) - \frac{1+\chi}{C}\frac{A}{S}h + \frac{h^{1+\gamma}}{1+\gamma} - \xi\right\}\frac{du}{d\mathbb{E}_i\log(\alpha_{i,t})} - \frac{1+\chi}{C}\frac{dJ}{d\mathbb{E}_i\log(\alpha_{i,t})} \\
&+ \left(\frac{1+\chi}{C}\frac{A}{S} - h^{\gamma}\right)(1-u)\frac{dh}{d\mathbb{E}_i\log(\alpha_{i,t})} - (u_{-1}+\upsilon(1-u_{-1}))\, q^{\kappa}\frac{dq}{d\mathbb{E}_i\log(\alpha_{i,t})} \\
&\beta\,\mathbb{E}\left[V'_{\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]}\frac{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]'}{d\mathbb{E}_i\log(\alpha_{i,t})} + V'_S\frac{dS}{d\mathbb{E}_i\log(\alpha_{i,t})} + V'_{u_{-1}}\frac{du}{d\mathbb{E}_i\log(\alpha_{i,t})}\right]
\end{aligned}$$

$$\begin{aligned}
V_{S_{-1}} &= -\frac{1+\chi}{C}\frac{Y}{S}\frac{dS}{S_{t-1}} \\
&+ \left\{\left(\log b + \frac{1-b}{1-u+ub}\right) - \frac{1+\chi}{C}\frac{A}{S}h + \frac{h^{1+\gamma}}{1+\gamma} - \xi\right\}\frac{du}{dS_{-1}} - \frac{1+\chi}{C}\frac{dJ}{dS_{-1}} \\
&+ \left(\frac{1+\chi}{C}\frac{A}{S} - h^{\gamma}\right)(1-u)\frac{dh}{dS_{-1}} - \left(u_{-1} + \upsilon(1-u_{-1})\right) q^{\kappa}\frac{dq}{dS_{-1}} \\
&\beta\,\mathbb{E}\left[V'_{\mathbb{E}_i[\alpha_i^{1-\tau}]}\frac{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]'}{dS_{-1}} + V'_S\frac{dS}{dS_{-1}} + V'_{u_{-1}}\frac{du}{dS_{-1}}\right].
\end{aligned}$$

Insurance term (τ) Take the derivative of V with respect to τ taking q, h, and M as given

$$V_{\text{Insur}} = \frac{\text{Cov}(\alpha_i^{1-\tau}, \log\alpha_i)}{\mathbb{E}[\alpha_i^{1-\tau}]} + \beta\,\mathbb{E}\left[V_{\text{Insur}}\right]$$

Incentives term (τ) The structure of this term parallels that for b. In this case we need

$$\begin{aligned}
\frac{\partial q}{\partial\tau} &= \frac{1}{\kappa}\frac{q}{V^n}\frac{\partial V^n}{\partial\tau} \\
\frac{\partial h}{\partial\tau} &\approx -\frac{h}{1+\gamma}\frac{1}{1-\tau} + \frac{\partial h}{\partial q}\frac{\partial q}{\partial\tau} + \frac{\partial h}{\partial\bar{u}}\frac{\partial\bar{u}}{\partial\tau} \\
&= -\frac{h}{1+\gamma}\frac{1}{1-\tau} + \frac{\partial h}{\partial q}\frac{1}{\kappa}\frac{q}{V^n}\frac{\partial V^n}{\partial\tau} + \frac{\partial h}{\partial\bar{u}}\frac{\partial\bar{u}}{\partial\tau} \\
\frac{\partial V^n}{\partial\tau} &= -h^{\gamma}\frac{\partial h}{\partial\tau} + \beta(1-\upsilon)\,\mathbb{E}\left[-\frac{\kappa}{1+\kappa}M'V^{n\prime}\frac{\partial q'}{\partial\tau} + (1-\frac{\kappa}{1+\kappa}q'M')\frac{\partial V^{n\prime}}{\partial\tau}\right] \\
&= -h^{\gamma}\frac{\partial h}{\partial\tau} + \beta(1-\upsilon)\,\mathbb{E}\left[-\frac{1}{1+\kappa}M'q'\frac{\partial V^{n\prime}}{\partial\tau} + (1-\frac{\kappa}{1+\kappa}q'M')\frac{\partial V^{n\prime}}{\partial\tau}\right] \\
&= -h^{\gamma}\frac{\partial h}{\partial\tau} + \beta(1-\upsilon)\,\mathbb{E}\left[(1-q'M')\frac{\partial V^{n\prime}}{\partial\tau}\right] \\
&= \left(1 + h^{\gamma}\frac{\partial h}{\partial q}\frac{1}{\kappa}\frac{q}{V^n}\right)^{-1}\left\{\frac{h^{1+\gamma}}{1+\gamma}\frac{1}{1-\tau} - h^{\gamma}\frac{\partial h}{\partial\bar{u}}\frac{\partial\bar{u}}{\partial\tau} + \beta(1-\upsilon)\,\mathbb{E}\left[(1-q'M')\frac{\partial V^{n\prime}}{\partial\tau}\right]\right\}
\end{aligned}$$

Macro-stabilization term (τ) As above, but with $dM/d\tau$ in place of $dM/d\tau$.

D.4 Extended model with savings

Define $V(a, n, \epsilon)$ as the value of being employed ($n = 1$) or unemployed($n = 0$) with assets a and in group ϵ. We omit aggregate states for simplicity of notation. The value of searching, V^s, satisfies

$$V^s(a,\epsilon) = \max_q\left\{MqV(a,1,\epsilon) + (1-Mq)V(a,0,\epsilon) - \frac{q^{1+\kappa}}{1+\kappa}\right\}. \tag{A.30}$$

The decision problem of the employed household is

$$V(a,1,\epsilon) = \max_{c,h,a'} \left\{ \log(c) - \frac{h^{1+\gamma}}{1+\gamma} + \beta_\epsilon (1-\upsilon_\epsilon) V(a',1,\epsilon') + \beta_\epsilon \upsilon_\epsilon V^s(a',\epsilon') \right\} \tag{A.31}$$

subject to the budget constraint

$$a' + c = \lambda \left(wh + d\right)^{1-\tau} \alpha_\epsilon^{1-\tau} + Ra.$$

The decision problem of the unemployed household is

$$V(a,0,\epsilon) = \max_{c,a'} \left\{ \log(c) - \xi + \beta_\epsilon V^s(a',\epsilon') \right\} \tag{A.32}$$

subject to the budget constraint

$$a' + c = b\lambda \left(wh(a,\epsilon) + d\right)^{1-\tau} \alpha_\epsilon^{1-\tau} + Ra$$

where $h(a,\epsilon)$ is the hours the household would have worked had they been employed.

The optimal q solves

$$q(a,\epsilon)^\kappa = M\left(V(a,1,\epsilon) - V(a,0,\epsilon)\right).$$

And substituting this into the definition of V_t^s

$$V^s(a,\epsilon) = V(a,0,\epsilon) + \frac{\kappa}{1+\kappa} \left(M\left[V(a,1,\epsilon) - V(a,0,\epsilon)\right]\right)^{1+1/\kappa}.$$

The Euler equation for a household is

$$\frac{1}{c} \geq \beta R_t \mathbb{E}\left[\frac{1}{c'}\right].$$

The labor supply optimality condition is

$$h^\gamma = \frac{1}{c} \lambda (1-\tau) w \left(wh + d\right)^{-\tau} \alpha_\epsilon^{1-\tau}.$$

We track the distribution of wealth before the employment and group transitions occur at the start of the period, call the distribution $\Gamma_t(a,n,\epsilon)$. For convenience, let $\tilde{\Gamma}_t(a,n,\epsilon)$ be the distribution of wealth after transitions have occurred. The two are related according to

$$\tilde{\Gamma}_t(a,1,\epsilon') = \sum_{\epsilon} \Pr(\epsilon'|\epsilon) \left\{(1-\upsilon_\epsilon)\Gamma_t(a,1,\epsilon) + M_t q_t(a,\epsilon)\left[\Gamma_t(a,0,\epsilon) + \upsilon_\epsilon \Gamma_t(a,1,\epsilon)\right]\right\}.$$

We then have average labor supply among workers of

$$\mathcal{H}_t = \int h_t(a)\alpha_\epsilon d\tilde{\Gamma}_t(a,1,\epsilon) / \int d\tilde{\Gamma}_t(a,1,\epsilon).$$

and aggregate consumption is given by

$$\sum_n \int c_t(a,n,\epsilon) d\tilde{\Gamma}_t(a,n,\epsilon).$$

The government's receipts are from labor income taxes

$$\int \alpha_\epsilon \left[w_t h_t(a,\epsilon)\right] + d_t - \lambda_t \left(w_t h_t(a,\epsilon) + d_t\right)^{1-\tau} \alpha_\epsilon^{1-\tau} d\tilde{\Gamma}_t(a,1,\epsilon)$$

and it's outlays are G_t, interest $(R_t - 1)B$ where R_t is the ex post real return on bonds $R_t = (1 + i_{t-1})/\pi_t$, and UI payments

$$b\lambda_t \int \alpha_\epsilon^{1-\tau} \left(w_t h_t(a,\epsilon) + d_t\right)^{1-\tau} d\tilde{\Gamma}_t(a,0,\epsilon).$$

The firm's problem is the same as in the baseline economy with the exception that we replace h_t with the skill-weighted average work effort among employed households, denoted $\mathcal{H}_t$.

An equilibrium of the economy can be calculated from a system equations that is similar to the baseline economy, but with aggregate work effort and consumption replaced by the equations above. As those equations depend on the policy rules and distribution of wealth, we also require equations that dictate how the distribution of wealth evolves, and how the policy rules are determined. The Reiter (2009) method the Euler equation, labor supply condition, and search effort first-order condition, and Bellman equations, at many points for the individual states and interpolates the policy rules between them. The distribution of wealth is approximated as a histogram that evolves according to the idiosyncratic shocks and the policy rules.

To calibrate the model, we think of an age group as approximately 12 years of life and set the probability of stochastically aging to 1/48. We use the 2001 SCF and divide the sample into age groups 25 to 36, 37 to 48 and 49 to 60. For each group we compute the median liquid assets, median earnings, and unemployment rate. We then set the values of β_ϵ, α_ϵ, and υ_ϵ to target these moments. Liquid assets are defined as the sum of liquid accounts ("liq" in the SCF extracts sums checking, savings, and money market accounts), directly held mutual funds, stocks, and bonds less revolving debt. Following Kaplan et al. (2014), liquid account holdings are scaled by 1.05 to reflect cash holdings.

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