Inflating Away the Public Debt? An Empirical Assessment

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Goal

Quantify likelihood of inflation eroding real value of U.S. debt.

1. Looking forward, today.
2. With the actual privately-held debt distribution.
3. With plausible scenarios for inflation paths, disciplined by data.
4. With reaction of markets and bond prices.

Existing answers

- Historical.
- Assume one period debt and pick arbitrary Δ:

$$\frac{B_t}{P_t(1+\Delta)}$$

- Exponential debt, fixed real rate.

$$\sum_{t=0}^{\infty} \frac{H^t B^t}{P} \approx \int \frac{B e^{rt - t/m}}{P e^{\pi t + \Delta t}} \approx \frac{B}{P}\left(\frac{1}{r + \pi + 1/m + \Delta}\right)$$

ad hoc shock: permanent increase counterfactual.

We would like to have: ex ante but data-disciplined approach, effect of future paths of inflation on fiscal burden, using actual maturity of privately-held government debt.

Steps

1. Derive formula that isolates effect of inflation. It shows interactions of inflation process and maturity of debt.

2. Build data on private holdings of U.S. debt by maturity.

3. Extract risk-neutral marginal densities for inflation at different horizons from inflation options. Employ a new method of moments copula based technique to recover a joint distribution for inflation.

4. Deliver value-at-risk measures of the likelihood of debt debasement due to inflation.

5. Counterfactuals (stress tests) show interaction between debt maturity and expected inflation.

Theory and data

Theory: the public debt

- The real market value of government debt:

$$W_t = \sum_{j=0}^{\infty} \frac{H_t^j B_t^j}{P_t} + \sum_{j=0}^{\infty} Q_t^j K_t^j.$$

Problem: H^j depend on inflation.

- Law of motion for debt:

$$W_{t-1} = s_{t-1} + \sum_{j=0}^{\infty} \frac{H_{t-1}^{j+1} B_t^j}{P_{t-1}} + \sum_{j=0}^{\infty} Q_{t-1}^{j+1} K_t^j.$$

Problem: looks backwards.

Looking forward

Working assumption: there is a SDF $m_{t,t+j}$, such that:

$$1 = \mathbb{E}_t \left(\frac{m_{t,t+j}}{Q_t^j} \right) = \mathbb{E}_t \left(\frac{m_{t,t+j} P_t}{H_t^j P_{t+j}} \right).$$

The intertemporal budget constraint (iterating the law of motion of debt, multiplying by the SDF, and taking expectations):

$$W_0 = \mathbb{E} \left[\sum_{t=0}^{\infty} m_{0,t} \left(\frac{B_0^t}{P_t} \right) \right] + \mathbb{E} \left[\sum_{t=0}^{\infty} m_{0,t} K_0^t \right] = \mathbb{E} \left[\sum_{t=0}^{\infty} m_{0,t} s_t \right]$$

A formula

Proposition

The debt burden is a weighted average of the nominal payments that the government must make at all present and future dates:

$$\sum_{t=0}^{\infty} \omega_t B_0^t$$

with weights given by:

$$\omega_t = R_t^{-1} \int \left(\frac{f(\pi_{0,t})}{\pi_{0,t}} \right) d\pi_{0,t}$$

where $f(.)$ is the risk-neutral distribution for inflation and R_t is the risk-free real rate.

Our approach

$$\sum_{t=0}^{\infty}\left(R_t^{-1}\int\left(\frac{f(\pi_{0,t})}{\pi_{0,t}}\right)d\pi_{0,t}\right)B_0^t$$

- Data on $f(\pi_{0,t})$ and B_0^t.
- Foreseen future paths for inflation $\{\pi_{0,t}\}$, not arbitrary.
- Include direct and indirect effect of inflation, not fixed.
- Probabilities include: (i) shocks to fundamentals, (ii) policy regimes weighted by probabilities, not ad hoc shocks.
- Likelihood of debt reduction value at risk, not stress test.

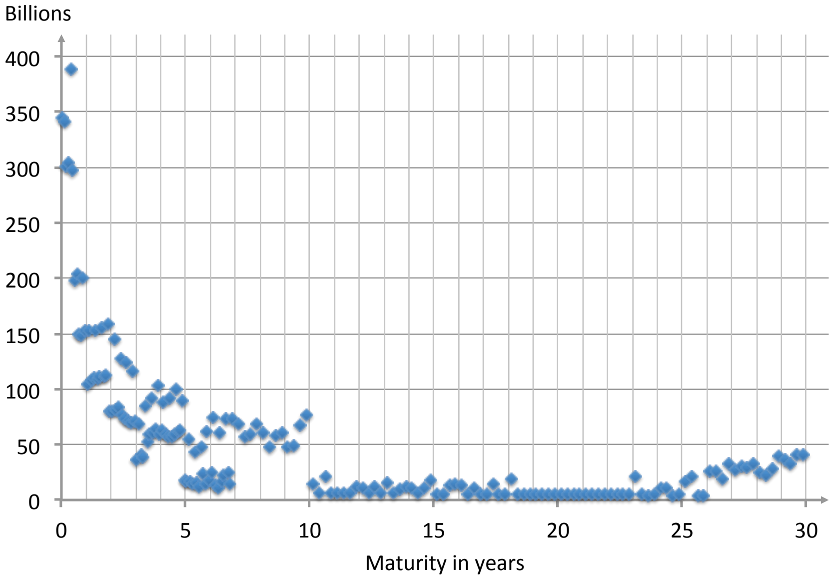
Data on public debt

- Problem with official numbers:
  1. Include TIPS ($0.9 trillion)
  2. Non-marketable debt ($5.4 tr.) and Social security ($4.8 tr.)
  3. State and local government holdings ($0.7 tr.)
  4. Fed holdings SOMA account ($1.9 tr.)

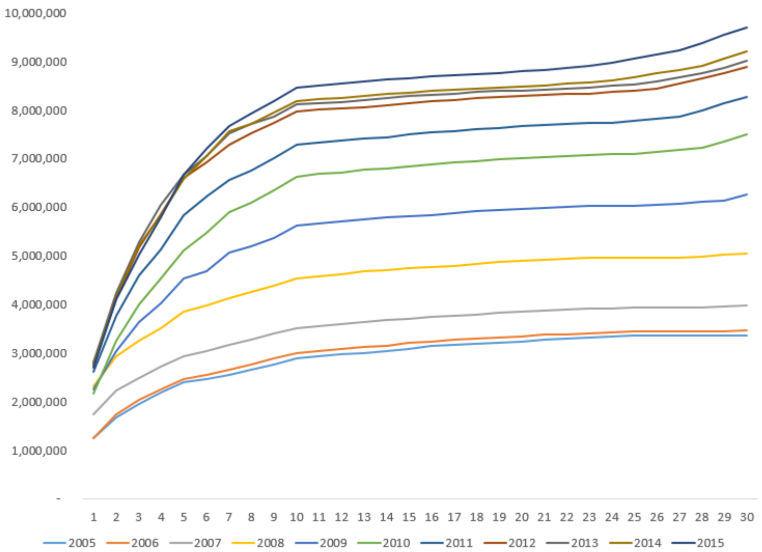
- Actual value of debt is 51% of GDP (not 101%).

- Breakdown by maturity and holder: data from CRSP, Treasury statements, SOMA balances.

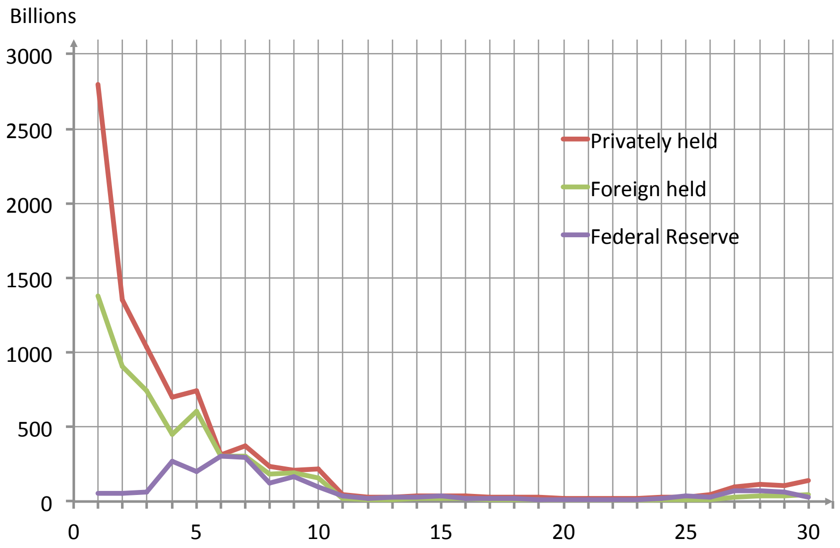
Payments to private sector by maturity



Change over time



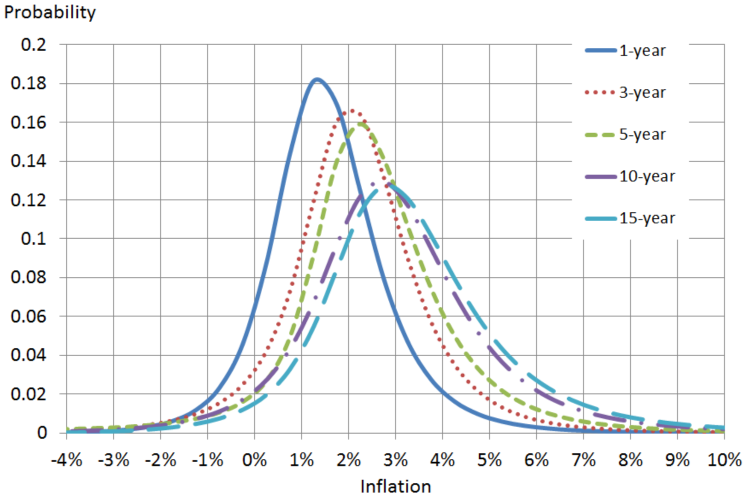
Payments by maturity and by holder



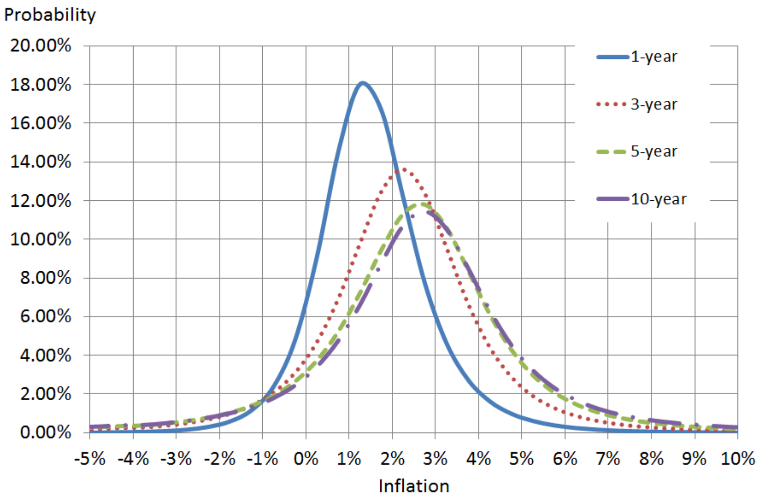
Data on inflation contracts

- CPI options, caps and floors, daily quotes from Bloomberg. 31st of December 2012.
- For cumulative and yoy inflation: in increments of 0.5%, horizons 1-10 years.
- Non-parametric estimate: $f(\pi_{0,t}) = R_t \pi_t \left(\frac{\partial^2 a_0}{\partial k^2} \right)$.
- Unlike surveys (subjective), break-even (point), term structure models (parametric).

Risk-neutral densities cumul. inflation



Risk-neutral densities for yoy inflation



Estimating joint distributions

- Sklar's theorem: there exists a copula function $C(.) : [0,1]^2 \to [0,1]$ such that:

$$f(\ln \pi_{t,t+1}, \ln \pi_{t+1,t+2}) = C\left(f(\ln \pi_{t,t+1}), f(\ln \pi_{t+1,t+2})\right).$$

- Parametric copula: $\hat{C}\left(f(\ln \pi_{t,t+1}), f(\ln \pi_{t+1,t+2}), \rho\right)$.

- N moments:

$$f(\ln \pi_{t,t+2}) = f(\ln \pi_{t,t+1} + \ln \pi_{t+1,t+2})$$
$$= \int_{\Pi} \hat{C}\left(f(\ln \pi_{t,t+1}), f(\ln \pi_{t+1,t+2}), \rho\right) d\pi_{t,t+1} d\pi_{t+1,t+2}$$

where $\Pi = \{\pi_{t,t+2} : \ln \pi_{t,t+1} + \ln \pi_{t+1,t+2} = \ln \pi_{t,t+2}\}$.
Restricted model: $\ln \pi_{t,t+1} = p_{t+1} + s_{t+1}$.

Estimating joint distributions

Proposition

Given data for the marginal distributions of cumulative inflation $f(\ln \pi_{t,t+j})$ *and yoy inflation* $f(\ln \pi_{t+j-1,t+j})$ *for* $j = 1, ..., J$*, one can estimate the joint distribution* $f(\ln \pi_{t,t+1}, \ln \pi_{t+1,t+2}, ... \ln \pi_{t+J-1,t+J})$ *by estimating the* M *parameters in the* ρ *vector that satisfy the* $(N-1)(J-1) \geq M$ *conditions:*

$$f(\ln \pi_{t,t+j}) = \int_{\Pi} \hat{C}\left(f(\ln \pi_{t,t+1}), ...f(\ln \pi_{t+J-1,t+J}), \rho\right) d \ln \pi_{t,t+1} ... d$$

The integration set Π *is such that:*
$\ln \pi_{t,t+1} + ... + \ln \pi_{t+j-1,t+j} = \ln \pi_{t,t+j}$*, for* $j = 1, ..., J$*.*

Results

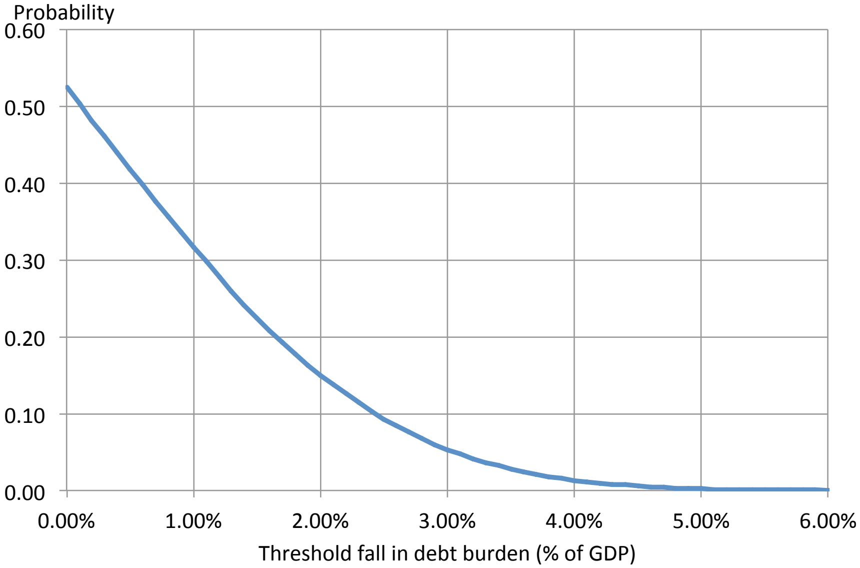
	Short term (1-3 years)	*Medium term (4-6 years)*	*Long term (7-10 years)*
Short term (1-3 years)	0.19	0.22	0.16
Medium term (4-6 years)		0.47	0.43
Long term (7-10 years)			0.54

Notes: Estimated average correlation coefficients for year-on-year inflation between date 2012+j and 2012+l, in column j, row l.

1. Non-stationary variance small
2. Little persistent over 3 years
3. But long memory

Inflating Away the Public Debt?

Value at Risk



Percentiles of the the distribution of losses for bondholders

	Holders of the debt				
Percentile	*Privately held (51%)*	*Domestic (19%)*	*Foreign (32%)*	*China (7%)*	*Central Bank (12%)*
90th	2.4%	0.8%	1.6%	0.4%	1.2%
95th	3.1%	1.1%	2.0%	0.5%	1.5%
99th	4.2%	1.4%	2.8%	0.7%	2.0%

Notes: Each cell shows the real losses of the debt held by the agent in the column that occur with the probability shown in the row.

Large numbers very unlikely, gain over foreigners, Fed loses the most.

Interpreting the probabilities

$$\mathbb{E}\left[\frac{m_{0,2}}{\pi_{0,2}}\right] = \mathbb{E}\left[\left(\frac{m_{0,1}}{\pi_{0,1}}\right) \times \left(\frac{m_{1,2}}{\pi_{1,2}}\right) \times \left(\frac{m_{0,2}}{m_{0,1}m_{1,2}}\right)\right]$$

- Conditional at date 2012: no new information arises over horizons (not time), last term is 1.
- Product of actual inflation and risk: (i) weighted by marginal utility, (ii) do not take a stand on Phillips curve.
- If want actual probabilities: if constant risk premium across distribution, adjustment to drift of inflation, but our estimates are about dispersion, no change.
- If risk premium different for different values of inflation: higher at extremes of distribution, then estimates of probabilities even lower.

Explaining the estimates

Stress tests

1. Higher average inflation: Shift marginal yoy distributions at every maturity so that the new median is at the old 90%.
2. Higher inflation for sure: Set to 0 the mass in the density below the 90% percentile, scale proportionately.
3. More variable inflation: Shift so median is old 90th percentile, but scale inflation proportionately.
4. Higher for sure: Same as 1 but with variance set to zero.
5. Partially expected: After initial unexpected jump of inflation upwards, adjust by conditional distributions.
6. Temporary: Shift to 90th first year, 80th percentile second year, and so on, so by 5 years no change till 5 years.
7. Gradual: One year unchanged, two-year to 60%, and so on, so by 5 years shift to 90th percentile.

Why so small? Inflation

	Horizon			
Distribution for inflation	1-year	3-year	5-year	30-year
Baseline	1.5%	2.1%	2.4%	3.0%
Counterfactuals				
1. Permanently higher	3.2%	4.0%	4.5%	5.6%
2. Right tail only	4.4%	4.7%	5.2%	5.2%
3. Higher and more variable	3.1%	4.0%	4.5%	5.6%
4. Higher for sure	3.2%	4.0%	4.6%	5.7%
5. Partially anticipated	3.0%	2.9%	3.2%	3.3%
6. Temporary increase	3.2%	3.3%	3.2%	3.1%
7. Gradual increase	1.5%	2.5%	3.4%	5.5%

Notes: Each cell reports $1/E(1/\pi_{0,n})$, the harmonic mean of inflation at horizon n.

Never that high.

Stress tests

Inflation counterfactual	Holders of the debt				
	Privately held (51%)	*Domestic (19%)*	*Foreign (32%)*	*China (7%)*	*Central Bank (12%)*
1. Permanently higher	3.7%	1.3%	2.3%	0.6%	1.8%
2. Right tail only	4.4%	1.5%	2.9%	0.7%	2.1%
3. Higher and more variable	3.4%	1.2%	2.2%	0.5%	1.7%
4. Higher for sure	3.8%	1.4%	2.4%	0.6%	1.9%
5. Partially anticipated	1.3%	0.4%	0.9%	0.2%	0.6%
6. Temporary increase	1.2%	0.4%	0.8%	0.2%	0.4%
7. Gradual increase	2.3%	0.9%	1.4%	0.3%	1.4%

Notes: Each cell shows the fall in the real value of debt as a ratio of GDP.

Estimates always small

Why so small? Maturity

Inflation counterfactual	*Including only debt of maturity up to:* 1 year	4.5 years	All	With the maturity distribution of debt in 2000
1. Permanently higher	0.1%	1.1%	3.7%	5.1%
2. Right tail only	0.2%	1.5%	4.4%	5.8%
3. Higher and more variable	0.1%	1.1%	3.4%	4.7%
4. Higher for sure	0.1%	1.1%	3.8%	5.3%
5. Partially anticipated	0.1%	0.6%	1.3%	1.6%
6. Temporary increase	0.1%	0.7%	1.2%	1.2%
7. Gradual increase	0.0%	0.3%	2.3%	3.8%

Notes: Each cell shows the fall in the real value of debt as a ratio of GDP.

In 2012, 6% of the market value of debt was $>$ 10 years, in 2000 it was 17%.

Financial repression

Government delays payment of debt for N years, like required reserves at the central bank.

Duration of repression	*Repression*	*Higher inflation*	*Total*
1 year	0.7%	4.6%	5.3%
5 year	4.8%	8.1%	12.9%
10 year	12.2%	10.7%	22.9%

Notes: Each cell shows the fall in the real value of debt as a ratio of GDP as a result first of repression, and then inflation, under experiment 1. The last column is the sum of the two previous.

In first column of numbers, financial repression alone has large effect. The next column has experiment 1, so see effect of inflation is higher. The total effect is substantial.

Compare to rules of thumb

- $B/Y \times d \times \mathbb{E}(\Delta\pi) = 101\% \times 5.4 \times 1\% = 5.5\%$.
    1. Actual probability of that happening is 0.07%.
    2. Must use correct B/Y, which is 51%.
    3. Must use duration of privately-held debt, not average maturity of all debt: 3.7 years.
    4. Revised estimate: 1% more inflation 1.9% of GDP

- Gives local estimate in response to a marginal and permanent increase in inflation.
    1. It is local – right answer is 1.6% of GDP for 1% inflation.
    2. It is a sensitivity – effects of a parallel shift of the distributions. Compare to our stress tests.
    3. It is permanent – shifts all distributions the same. Actual correlation is 0.19.

- Interaction of both *maturity* and *inflation process* explains our results. Both debt and inflation options data.

Conclusion

Inflating away the debt?

- Steps in the process:
  - Method for value-at-risk measures.
  - Very left-skewed private holdings of government debt.
  - Option prices shows inflation has little persistence in 2-3 year horizon.
  - Small estimates throughout, unlikely can deflate debt.
- Numbers would be bigger if higher and more persistent inflation, higher private holdings of long-term debt, financial repression.
- If inflation will not pay for the public debt, then what will?