The Four R-stars:
From Interest Rates to Inflation and Back*

Ricardo Reis

LSE

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Abstract

R-star is a useful benchmark for the real interest rate. Sometimes, it refers to the steady-state equilibrium rate where savings equal investment (m), other times to the long-run value for the yield on safe government bonds (y), other times to the counterfactual return earned by inputs when the level of output is at potential (ρ), and some other times to the neutral monetary policy rate at which inflation is at its target value (i). This paper first documents the differing trends of these four R-stars in the quarter-century before the pandemic. Then, it proposes a general framework to make sense of their joint evolution, together with how they affect inflation. Looking ahead, if the steep rise in y and the slight fall in m observed since the pandemic until 2023 become new trends, the framework points to what will be the new challenges facing fiscal and monetary policy.

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1 Introduction

Where are interest rates going? The answer to this question can be stated in terms of what is *R-star* (or r^*): the long-run common component of all relevant interest rates in the economy.

By definition, we cannot directly observe this variable. It is a latent trend driving the co-movement of different variables, a model counterfactual, or a policy benchmark. Measuring it is a daunting task. And yet, economists have done it, in many and intricate ways, since the stakes are high.

Why does it matter? At the end of 2024, monetary authorities across the OECD are looking at the inflation disaster of 2021-24 through the rearview mirror and asking whether to cut policy rates any further. Their main doubt is how far away they are from the normal steady state for the stance of monetary policy. That is, what is *R-star* for the policy rate?

In turn, fiscal policymakers in the G-7 countries spent the first twenty years of this century running large public deficits while paying in interest every year roughly a constant fraction of GDP. This was possible because the interest rate they paid on the public debt steadily fell, and was often below the growth rate of the economy. Following the pandemic spending, public debt at the end of 2024 is even higher. Fiscal authorities are asking whether interest rates will stay low, fall further, or (god forbid!) rise, which would force tax rises or spending cuts to pay for it. What is *R-star* for the public debt?

Macroeconomists build their study of growth, business cycles, and much else on the neoclassical model they learned from Ramsey and Solow. At its heart, this is a model where savings equals investment to pin down the real return to investing in capital. What moves savings and investment drives economic growth and consumption. Understanding the frictionless steady-state level of the real interest rate in that model tells us what lies ahead for human welfare. At what price will the market for savings clear, *R-star*?

Finally, all economists, when confronting problems that involve a trade-off between dates, need a measure of the relative price of resources today versus in the future. They need an interest rate with which to discount payoffs in a distant future into present terms. What is the right discount rate *R-star* to do so?

All combined, measuring today (and at all times) what is *R-star* may be hard. But it would be foolish not to try. The answers to too many important questions depend on the estimates of *R-star*.

The current approaches to measuring it. In the literature, there are, at least, four approaches to measuring *R-star*.

The first of these focuses on extracting the long-run component of realized returns. The natural approach is to include long samples of data and to use time-series models that separate trends from cycles and that tease out common from idiosyncratic components. The confidence bands around the resulting estimates tend to be wide, which is an inevitable consequence of the data for interest rates both showing jumps up and down every week, while moving smoothly and steadily when averaged over many years.[1]

The second approach uses equilibrium models of savings and investment, with parameters that are calibrated or estimated. Within a model, researchers can define counterfactual equilibria for the interest rate, either as time goes to infinity, or as multiple frictions go to zero. These estimates of *R-star* can change substantially with the assumptions in the model, or with different definitions of the counterfactual.[2]

A third approach uses other variables that could indicate whether current interest rates are far from *R-star*. For instance, in models of monetary policy in the Wicksellian tradition (like new Keynesian models), if the policy rate is above (below) it, then inflation will fall (rise). Combining aggregate time series with models of inflation and measures of expected rates leads to estimates that are as imprecise as our forecasts of inflation and as limited as our understanding of the channels of monetary policy.[3]

Finally, a fourth approach draws inspiration from the study of financial markets. If the returns on any asset are equal to the common *R-star* plus a premium that is linked to duration, risk, or illiquidity, then being able to extract these premia would reveal the desired object. Yet, while some progress can be made down this path, the asset which has the smallest premium because it is overnight, the safest, and the most liquid is... a deposit at the central bank. Concluding that the best measure of *R-star* is the current policy rate is not terribly useful, especially since one of the key motivations of the exercise was to figure out where the policy rate was going to.[4]

This paper's approach. Part of the difficulty with measuring *R-star* is that each of the above methods was measuring a different object. It was not just the data or the econometric technique that differed between estimates, but also the underlying economic concepts.

[1]See Jordà et al. (2019) or Rogoff, Rossi and Schmelzing (2024).
[2]See Goodhart and Pradhan (2020) or Rachel and Summers (2019).
[3]See Holston, Laubach and Williams (2017), Del Negro et al. (2017), or Cúrdia et al. (2015).
[4]See Del Negro et al. (2019) or Kiley (2020).

Thinking about each of them for long enough reveals four *R-stars* that are related to each other, but are not the same.

This provides a perspective on why it has been so hard to pin down *R-star*. Each econometrician is using one rifle to hunt one hare, without realizing that there are four hares running in the field. Even as she changes rifles with different ranges, each of them more appropriate to a different hare, she continues shooting at all four and, as a result, is hitting none.

This paper tries to make progress by characterizing each of the hares separately. Namely, I conceptually distinguish between four *R-stars*, characterizing which market each clears, what fundamentals separately or jointly drive them, and how they relate to each other in general equilibrium.[5] I present rough estimates of each of them since 1995 that strongly suggest that they have had different trends. I lay out a model of the four equilibrium interest rates that, while not micro-founded, relies heavily on existing canonical models.[6] I then look at the data between 2021 and 2024 through the lenses of the model, which suggests a new trend for two of the *R-stars*. For the other two, I speculate on three different scenarios matching three different choices made by fiscal and monetary policy, to highlight the challenges that may lie ahead.

2 The trends in the R-stars between 1995 and 2019

This section conceptually defines the four distinct *R-stars*: (m, y, ρ, i), and measures their trends in the data.

The expected return on productive investment (m). The key interest rate at the heart of the Ramsey-Solow model of savings and investment is the marginal product of capital. This is the expected return from investing in the private economy to produce output.[7] I call it m.

Measuring a marginal return is hard, but measuring its average counterpart is relatively straightforward. Simply divide the payments to capital in the economy by the size of the

[5] There are other *R-stars* that I do not discuss, such as those linked to financial instability (Akinci et al., 2020), those in different regions with imperfect capital mobility (Carvalho et al., 2025), or those associated with misallocation of capital across sectors and firms (Reis, 2013).

[6] Among others, see Rachel (2023) for the model of investment, Reis (2020) for the model of unproductive and productive savings, Acharya and Dogra (2021) for the model of actual and potential output, and Galí (2008) for the model of monetary policy.

[7] It is also the concept closest to that discussed by Wicksell (1936) in its classic analysis of *R-star*.

private capital stock. In practice, one must be careful with what to put in the numerator and in the denominator because of differing trends in relative prices, in depreciation rates, in types of capital, and many other dimensions. Yet, after going through many of them, Reis (2022*b*) concludes that the trends over the 1995-2019 period are similar.

Panel (a) in figure 1 shows two straightforward measures of m for the US. The first divides net private operating surplus from the national income and product accounts, by the private capital stock from the fixed assets table, and subtracts 10-year expected inflation from the mean answer in the Survey of Professional Forecasters. The second calculates a weighted-average cost of capital, by combining the first measure with the yield on Baa-rated bonds, and adjusting for the effective tax treatment of leverage. This is a leveraged return to private capital.

Both series give the same conclusion: m was essentially unchanged for the 25 years displayed. This is consistent with the Ramsey-Solow theoretical prior that the US economy has been near its steady state throughout its period. The marginal product of capital does move from year to year, and studies of business cycles with a neoclassical core dissect those movements carefully. From the perspective of stars, or trends, though, the appropriate inference is of a constant *R-star*.

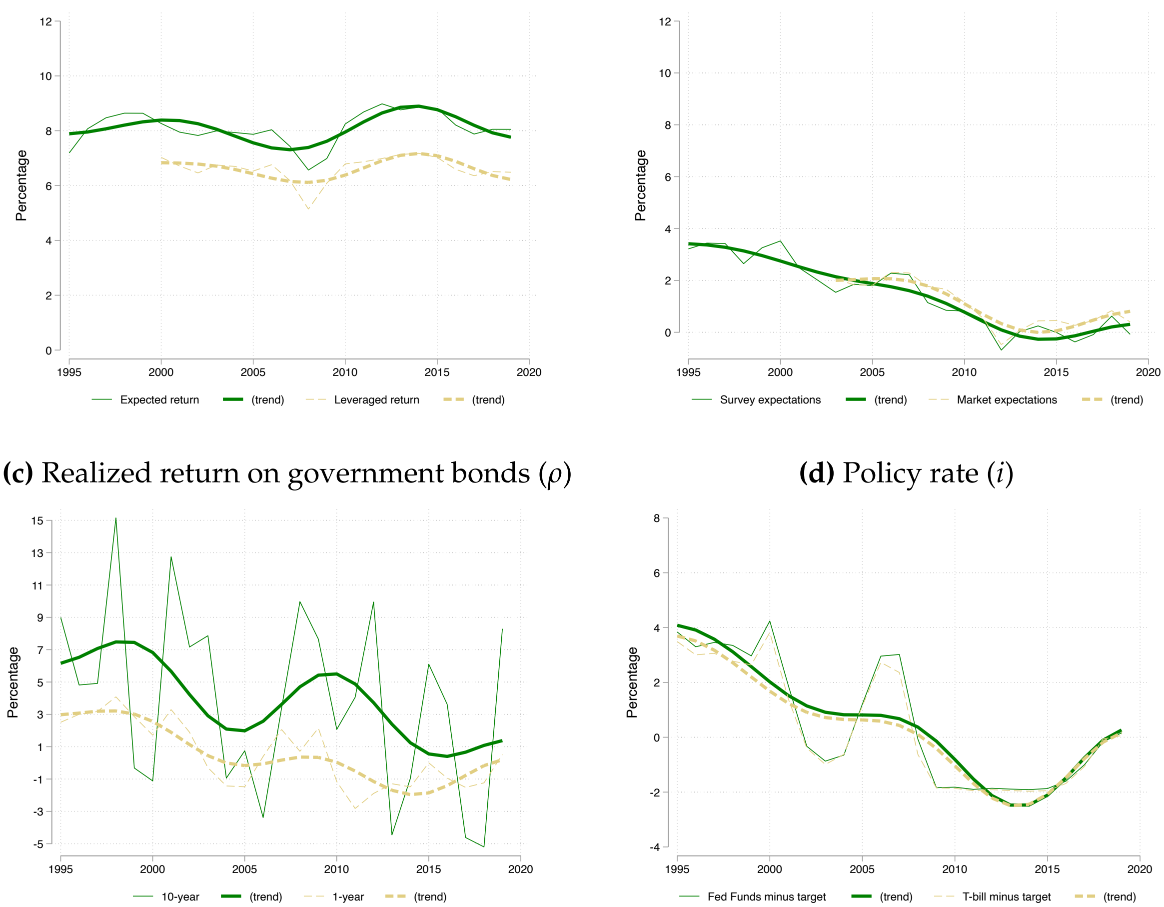
Appendix A expands on this conclusion in two directions. First, I look further back in time, for the US until the 1920s, and then all the way until 1856 by shifting to UK data. Second, I look further wide in the world to include the other countries in the G-7, as well as Chile. While there are larger fluctuations, partly because the data is measured less precisely, it is hard to reject the hypothesis that the returns to private capital have been roughly constant. Neither the last 25 years, nor the United States, stand out as exceptional.

The expected return on unproductive savings (y). Not all savings go into the productive capital stock. Many of them, instead, go to stores of value, like land or government bonds.

One measure of the returns to these savings is the yield from holding a long-term government bond. As in Diamond (1965)'s classic article, saving in government bonds is a way to store resources that is an alternative to using them for production. This return is easier to measure, widely accessible to most households, and closer to interpret as free from risk and liquidity premia. Naturally, this is the preferred measure for studies of *R-star* that take a finance approach, going back to the description of the literature in the introduction.

Panel (b) in figure 1 shows two measures of this real return, y, in the US data. The

Figure 1: The four R-stars and their trends between 1995 and 2019



(a) Return on productive capital (m)

(b) Return on unproductive savings (y)

(c) Realized return on government bonds (ρ)

(d) Policy rate (i)

Notes: Panel (a): Expected return is the US nominal return on private capital minus the average of expected 10-year-ahead inflation in the SPF. Leveraged return is a weighted average cost of capital using Baa bonds and effective tax rates. Panel (b): "Survey expectations" subtracts from the 10-year yield on US government bonds the average expected 10-year-ahead inflation in the SPF, while market expectations is the yield on a 10-year Treasury inflation-protected security. Panel (c): Holding period real return on 10-year and 1-year US government bonds. Panel (d): Federal Funds rate or 3-month government bond rate minus 2%. Trends calculated using a Mueller-Watson filter with a 10-year window.

first measure subtracts expected inflation (again the mean answer across the SPF survey respondents) from the 10-year yield, while the second measures directly the yield on bonds whose returns are indexed to the rate of inflation. Appendix A plots similar measures for the G-7 countries.

The data tells a consistent story across measures: y had a smooth downward trend during these 25 years, falling by between 2 and 3 percentage points. This is strikingly different from what we saw for m.

The realized return on savings (ρ). If y captures the expected return, then ρ captures the actual return. From the perspective of time-series approaches that measure average returns common across different assets, this is often the preferred measure. This is also the measure that enters the government budget constraint and is more relevant for fiscal authorities and for assessing the sustainability of the public debt.

Panel (c) in figure 1 plots the holding-period return for one year on a 10-year government bond minus the realized inflation during that year. As an alternative, it also plots the return on a 1-period bond, which is just its yield minus actual inflation. The 10-year returns were significantly above the 1-year ones through the sample: this is the consequence of the fall in yields in panel (b), since these correspond to a rise in bond prices, and so successive capital gains with holding these bonds.

The trend in this figure is less visible because of the large fluctuations in realized returns. Approximately though, it seems that between 1995 and 2015, ρ fell just as much, or maybe slightly more, than the fall in y. However, in the last 5 or so years of the sample, there is a clear increase in both measures of realized returns, ρ, even as expected returns, y, kept on falling.

The monetary policy rate (i). Finally, when discussing the link between interest rates, inflation, and other business-cycle variables, the typical and adequate focus is on an interest rate that is controlled by monetary policy. In the US, the natural measure of this is the overnight Federal Funds rate, although the return on a 3-month Treasury bill is just as adequate and captures both the current policy rate as well as what it is expected to be in the next 3 months. Panel (d) of figure 1 plots both, subtracting 2% from either, so that they are in pseudo-real units using as reference the Federal Reserve's inflation target, and so are comparable with the other panels.

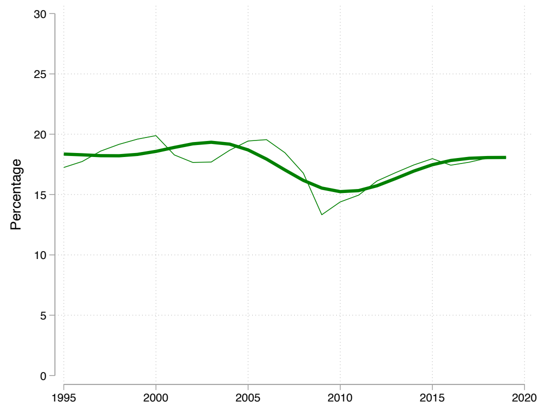
The path of the policy rate had two stages in the sample. Between 1995 and 2015, it fell decisively and more than the other rates, by approximately 6 percentage points. Between 2015 and 2019, it recovered by about 2 percentage points. This i looks quite different from either m or y.

Other indicators. Figure 2 shows four other macro-financial indicators. An account of the trends in *R-star* during this period should be able to fit these as well.

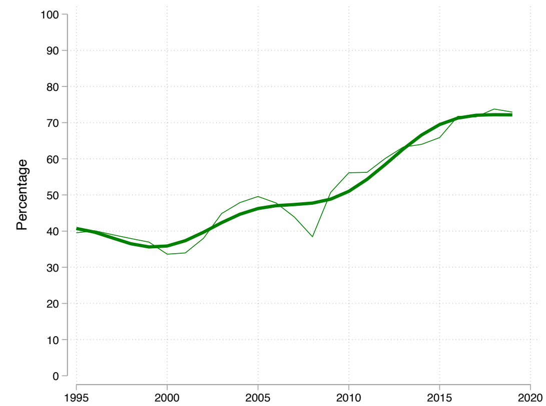
First, and since a measure of the returns to productive investment should be connected to the levels of that investment, panel (a) shows private investment as a ratio of GDP. I

Figure 2: Other relevant indicators, 1995-2019

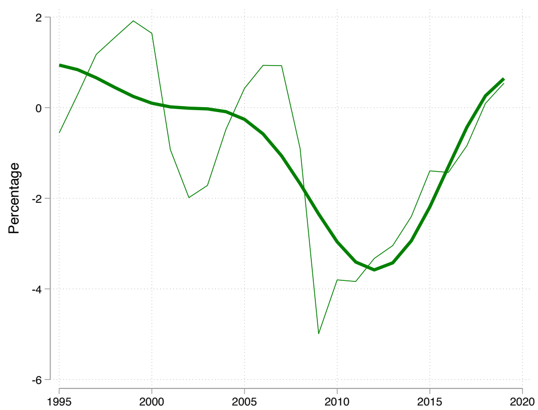
(a) k: investment over output



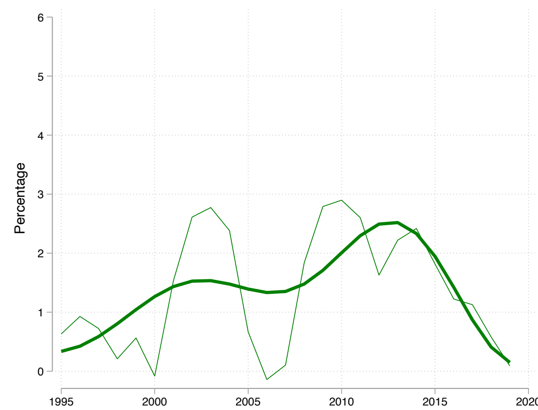
(b) b/k: ratio of unproductive to productive assets



(c) x/x^p: output relative to potential



(d) t: term premium



Notes: Panel (a): ratio of private investment to GDP. Panel (b): ratio of government debt plus the capital stock on dwellings to the private capital stock excluding dwellings. Panel (c): GDP relative to potential GDP according to the Congressional Budget Office. Panel (d): Difference between the yield on 10-year and 1-year government securities. Trends calculated using a Mueller-Watson filter with a 10-year window.

will refer to this in the model as k, since in a steady state, the flow of investment and the stock of capital are linked one to one. During most of this period, it was slightly below its usual value, although not much so. This constancy matches the constancy of the return to capital.

Second, panel (b) shows the ratio of the stock of unproductive investment to the productive stock. I denote this by b/k, and measure it as the ratio of government bonds plus the residential capital stock to the non-residential capital stock. This measure has a clear and significant upward slope during this period, partly driven by an increase in the stock of government bonds. Even as their yield was falling, the stock of bonds rose throughout, keeping government interest expenses roughly constant.

Third, realized returns and inflation are linked in many theories of aggregate demand to the level of unused productive capacity or slack. If output is denoted by x and potential output by x^p, the output gap is the ratio: x/x^p. Panel (c) shows that during almost a decade, between 2005 and 2015, output was below potential in the US.

Finally, panel (d) shows a measure of the link between interest rates at different maturities. This is the term premium, or the difference between the yield on 10-year and 1-year government bonds, denoted by t. It rose between 1995 and 2010 and then sharply fell for the last ten years. This is associated with unconventional monetary policies, like forward guidance and quantitative easing, which were explicitly designed to lower the term premium, in an attempt to raise inflation towards its target.

3 A framework for the four r-stars

In many models, m, y, ρ and i are either constant or move together in steady state. In the Ramsey-Solow neoclassical model, the return to private capital in the steady state is constant. In most models that build on it, the return on other forms of savings must likewise be constant, although they may differ from each other in levels to account for risk or other premia. In steady state, expected and realized returns are the same. Finally, most models of monetary policy have at their core a Fisher equation that equates the policy rate minus the inflation target to the return on capital or bonds.

Yet, in the US data of the last 25 years that we just looked at, only m is constant, and the trends in the other three do not match with each other. This section sketches a simple model where the four *R-stars* can differ. I present their steady-state relations, in some generality, but without specifying the micro-foundations or the dynamics that lead to these steady states. For readers who prefer to see the equations behind each curve, they are listed in appendix B.

Investment and savings. The demand for savings to invest productively is lower the higher is the required return on those savings. This is because of the usual assumption that the marginal return to capital diminishes as capital rises. There is a negative relation between k and m, plotted in blue in panel (a) of figure 3 and labelled as *Investment*.

The investment curve is steeper the faster diminishing returns set in. In the standard case of a Cobb-Douglas production function, this slope is higher the smaller is the capital share of income.

The curve shifts left-down, when the same level of private investment gives a lower marginal return. This could be because: (i) productivity is lower, so each unit of capital gives less output; (ii) depreciation of capital is higher; or (iii) the price of capital goods is expected to fall. Moreover, (iv) less competition, (v) more regulation, or (vi) higher income taxes all reduce the returns to investment, and shift the relation left-down. Finally, insofar as k is total capital, then a fall in public investment is also a shift in this direction.

As for the supply of savings, in the Ramsey model, it would be a horizontal line in (k, m) space. The vertical intercept would be proportional to the subjective rate at which households discount the future minus the growth rate of the economy at this steady state.

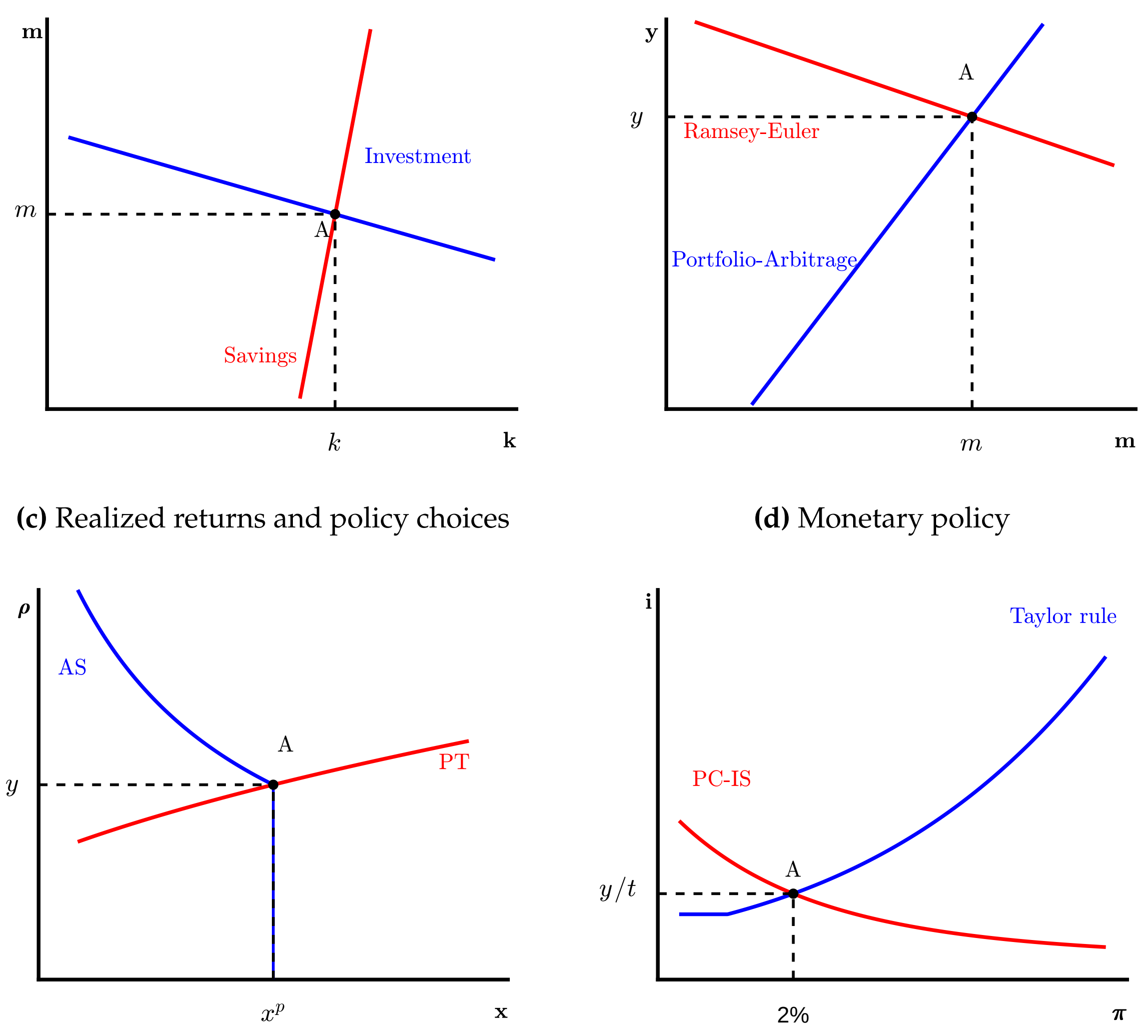
In panel (a) of figure 3, I plot instead a red *Savings* schedule that is upward sloping, so that, the higher is investment, the higher must be the returns offered to savers. The traditional way to justify the upward slope is incomplete markets, so that capital provides a service as self-insurance. Unpacking this service and why markets fail to equalize returns leads to the next part of the model.

Productive and unproductive savings. Savers can opt between putting their resources into productive uses, like buying equity or lending to firms, or into unproductive savings, like government bonds or land. Their choices determine in equilibrium a relation between the returns on the two alternatives: m and y. This equilibrium depends on two conditions.

The first is an arbitrage condition between the two returns that arises from the optimal portfolio choice of savers. The higher is the return on productive savings, the higher must be the return on government bonds, so that savers continue to be indifferent at the margin between saving an extra unit in either of these two uses. If they only cared about the expected financial return, this condition would be a 45^o line in the (m, y) space. However, unproductive savings provide liquidity or safety services. This makes the relation be always below the 45^o line, and the vertical distance to it measure what is often called the convenience yield of government bonds.

This *Portfolio-Arbitrage* relation is plotted in blue in panel (b) of figure 3. It shifts right-down when productive investments are perceived as riskier or less liquid, thus raising the premium they must pay relative to the expected return on government bonds. More broadly, it shifts right-down if the financial frictions preventing the direction of funds towards productive uses are more intense.

Figure 3: A model of four returns



(a) Productive investment and savings

(b) Unproductive or productive savings

(c) Realized returns and policy choices

(d) Monetary policy

The second relation in the figure is the familiar optimal savings conditions from the Ramsey model that comes from the Euler equation: the average return on savings is equal to the expected growth rate of consumption (scaled by intertemporal substitution) plus the subjective discount rate. Because of the incompleteness of markets, there is not a single return driving savings behavior, but rather a weighted average of the return on the two forms of savings.

For a fixed growth rate, the average return must be constant. Therefore, the higher is the return on productive capital m, the lower must be the return on unproductive storage y. The *Ramsey-Euler* schedule in red in panel (b) of figure 3 therefore slopes down. It shifts left-down when either: (i) expected growth is lower; (ii) the discounting of the future is lower, for instance because of an increase in longevity; or (iii) inequality is higher and the rich have a higher propensity to save so that expected aggregate consumption growth is lower.

The equilibrium for the two returns is given by point A in panel (b) of figure 3. How that equilibrium moves with k traces out the *Savings* curve in panel (a) of figure 3. When productive investment k rises, this lowers the ratio of unproductive to productive assets b/k. With a lower b/k, there is higher demand for safety and liquidity, so the *Portfolio-Arbitrage* curve shifts to the right-down. The equilibrium m rises and so the *Savings* curve in panel (a) slopes up. A simple intuition is that the higher is k then the higher must be the return offered to savers by productive savings, otherwise they would rather channel their savings to government bonds.

Panel (b) of figure 3 also unveils what is behind shifts in the *Savings* curve in panel (a). It will shift right-down when: (i) growth is lower, (ii) population ages, or (iii) inequality falls, since all of these shifts the *Ramsey-Euler* schedule left-down. Likewise, the *Savings* curve will shift left-up in panel (a) when financial frictions are more intense and productive investments are perceived as riskier or less liquid than storage, so the *Portfolio-Arbitrage* equation shifts right-down in panel (b).

Realized returns and the output gap. Production combines capital with variable inputs, like labor. The suppliers of those inputs write contracts to supply them taking into account an expected real reward. Those expectations have slow-moving norms for what is an adequate nominal payment. If inflation is in line with this norm, then inputs are hired according to their marginal product, given the installed capital. If so, output x is equal to potential x^p.

However, if inflation comes down below the norm, then the real compensation of the variable inputs turns out to be higher. Firms are willing to hire fewer of them, and output falls below potential. This mechanism, made famous by Friedman (1968), is elaborated and explored in many models of nominal rigidities in wages.

It leads to an *AS*—for Aggregate Supply—blue curve in panel (c) of figure 3. When inflation is below the expected norm, realized returns are higher than expected returns ($\rho > y$) and variable input suppliers and bondholders receive more than they expected. The other side of the coin is that firms hire fewer of them so output is lower than potential. The relation is downward-sloping in the (x, ρ) space.

When inflation is instead higher than the norm, so returns ρ are lower, then the economy is operating at potential. All the installed capital is fully employed, and the *AS* relation becomes vertical. The kink point is when inflation is just equal to the norm, so that realized and expected returns are the same: $\rho = y$.

The *AS* curve shifts left-down if there is less capital k and so lower potential output x^p. From panel (a), this may be the result of lower productivity or higher markups, which shift the *Investment* curve left-down. From panel (b) instead, if the equilibrium expected return on bonds is lower, then the kink point for the *AS* in panel (c) is lower. These are the direct general equilibrium forces from the top panels in figure 3 to panel (c).

The other relation in panel (c) of figure 3 comes from the management of aggregate demand by policy, both fiscal and monetary. Given a target for output and for inflation, optimal policy will tolerate some inflation above target only if output is below potential. If the shocks hitting the economy satisfy a "divine coincidence" then both output and inflation will be on target at all times. But, if there is an inevitable trade-off between the two—sometimes called a "supply shock"—then output may be below (above) potential, and inflation will be above (below) target. The preferences of the policymaker determine how it sets fiscal and monetary policy to manage aggregate demand and lower (raise) inflation to raise (lower) output to trade off the dual deviations from target.

Since realized returns depend negatively on realized inflation, this *PT* relation, for Policy Target, is an upward-sloping relation in (x, ρ) space. It is steeper if: (i) policymakers put a higher weight on keeping output near its target, so they are willing to tolerate wider fluctuations in ρ relative to deviations in x, and (ii) the Phillips curve trade-off between output and inflation is flatter, so the management of aggregate demand requires a larger movement in inflation for smaller changes in output.

From general equilibrium, this *PT* curve shifts right-down when the expected return on bonds y falls. More interesting, it also shifts right-down when policymakers' output target is higher relative to potential output, so they are willing to pursue high inflation and deliver low returns. The same applies if the inflation target is higher than the norm, perhaps because policymakers are willing to use unexpected inflation to lower the public debt burden. Fiscal and monetary policy can be understood as shifts in those targets, whether intentionally or because of the imprecision in controlling aggregate demand.

The natural goal of policy is to have the two schedules intersect precisely at the kink, as portrayed in panel (c) of figure 3. At this point, realized and expected inflation are the same, and output is at potential. This is achieved if the policymakers are able to calibrate their output target precisely at potential, and their target for inflation is achieved repeatedly, so that the inflation norm for private negotiations is just equal to that target.

But, if policy repeatedly fails, say by delivering a *PT* curve that intersects below the kink making inflation be persistently high, then the negotiation norm for nominal payments will adjust upwards. This will gradually shift the *PT* curve left-up. Eventually, the economy will move to a new steady state where the *PT* curve still intersects the *AS* at the same point, but behind the curves is now a higher level of the inflation target and of the inflation norm.

This is the equilibrating force from the rationality of expectations. Policy can deliver some unexpected inflation for some time, but not forever, unless it is willing to keep on providing more and more stimulus and having inflation spiral upwards.

The policy rate and inflation. An important component of aggregate demand management is monetary policy. In our context, it sets a key interest rate, i, that has its own *R-star* equilibrium. The final piece of the puzzle is how this is set and how it affects inflation π.

A lower policy rate i raises aggregate demand by lowering the returns to saving through a conventional Keynesian IS equation. This higher demand then pushes for high inflation given sticky prices by firms, through a conventional Keynesian Phillips curve equation. In the end, a lower i comes with higher π. This downward-sloping relation is portrayed in panel (d) of figure 3 as the blue *PC-IS* relation. It is flatter, the flatter is the Phillips curve and the more insensitive is spending to interest rates.

The stimulus to aggregate demand comes from the policy rate being temporarily below a natural or Wicksellian rate. This is equal to the expected rate on government bonds discounted by the term premium: y/t. Therefore, a fall in y from the top panels in the figure will shift left-down the *PC-IS* relation. In the other direction, an increase in the term premium t also shifts it left-down. Finally, this curve shifts left-down when firms' expected inflation falls relative to the inflation target.

The final component of the model is the *Taylor rule*. Higher inflation π makes policymakers raise the policy rate, as long as that rate is above its effective lower bound. The slope of this relation is higher the more aggressive are policymakers in controlling inflation. The curve shifts right-down when the inflation target of monetary policy is higher.

A policymaker committed to its inflation target has to target a Wicksellian, or neutral rate, which works as a shifter of the Taylor rule. Only if this is set at the right level, will inflation be equal to its target. That right level, in our model, corresponds to y/t that is the expected return on government bonds discounted by the term premium t. When the policy calibrates the Taylor rule to exactly intersect the *PC-IS* at that point, the inflation is equal to its 2% inflation target, as shown in the figure.

All together. Figure 3 has the four diagrams that pin down the four *R-stars* as the variables in the vertical axes of each diagram. The figure also links them to three important macroeconomic outcomes in the horizontal axes: investment, slack, and inflation. The equilibrium is depicted by the points A.

Clearly, nothing imposes that all four interest rates should be the same in this steady state of an economy. There are equilibrium forces within the model that draw them close to each other as well as exogenous drivers that will tend to move them together, like adjustments of norms, expectations, and policies. But over many years, or decades, these endogenous variables can move in different directions. Some of the forces driving them apart are structural features of the economy, like growth, demographics, productivity, competition, inequality, and the desire for safety or liquidity. Others are characteristics of contracts and expectations, like norms on compensation, expected inflation, and term premia between shorter and long-term bonds. And others depend on aggregate demand policy, through the targets it pursues for output, inflation, and policy rates. As they change, the framework in this model provides a way to analyze their consequences, as I will do next.

4 Using the framework to account for the 1995-2019 trends

Confident in a set of facts about the four *R-stars*, and armed with a model where they are jointly determined, I now turn to matching the two.

The movements in m and y. Rachel and Smith (2017) provides a thorough account of some of the major economic shifts in the early XXIst century driving changes in *R-star* (updated in Rachel, 2023).

Three of them are relevant for the *Investment* schedule. First, US economic growth was slightly lower than before, both in terms of productivity and population growth. Second, the price of investment relative to consumption goods had been falling for decades. Third, public investment was low relative to the past, partly because of prioritizing social spending, partly because of austerity policies following the great financial crisis.

Two other factors, highlighted by research since then go in the same direction in the model. Depreciation rates have risen, partly as more of the capital stock is intangible, and markups have risen, especially in the US economy as industries became more concentrated.

All combined, these five changes shift the *Investment* curve left-down, as shown in panel (a) in figure 4.

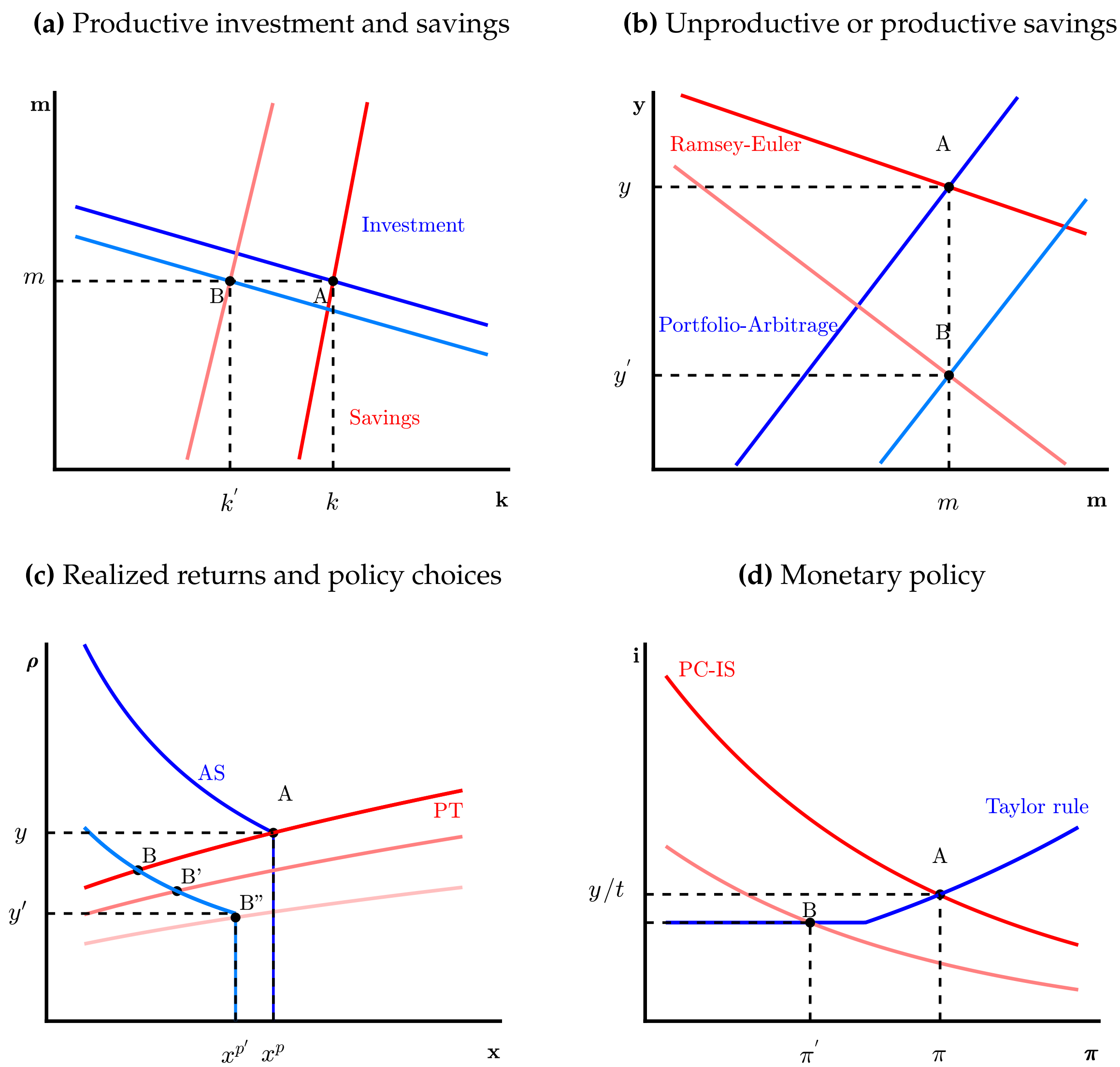
Were these the only changes, then we should have seen a fall in the return on capital m. Yet, we know from the data that the latter was constant. Therefore, the *Savings* curve must have shifted left-up in the 1995-2019 period, so as to keep this *R-star* roughly unchanged in spite of the headwinds facing investment. The economy moved from point A to B in panel (a) of the figure, which is also consistent with the depressed private investment during this period.

To understand what caused the decline in savings channeled to productive investment, we have to turn to panel (b). Going back to the literature, as distilled by Rachel and Smith (2017), the decline in the rate of growth in the economy during this period shifted the *Ramsey-Euler* curve left-down. In the same direction, population aging translates into people wanting to save more, or subjective patience rising in Ramsey's formula. Finally, the rise in inequality, together with a higher propensity to save by the rich, both raise the desire to save by households and so also shift the *Ramsey-Euler* curve left-down. By themselves, these would lower both y and m, not matching the evidence.

Yet, as important to savings as these changes in 1995-2019 was the so-called rise in global imbalances. Countries in South-East Asia saved large amounts in advanced economies abroad tied to the large surpluses in their trading relations. At the start of the century, a significant share of these savings were directed to build foreign reserves abroad, following the South-East Asia Crisis of the end of the XX^{th} century. These savings therefore had a strong relative preference for safe and liquid stores of value that could be easily accessed and used in a new crisis. This raised the desired supply of savings in the US, especially directed to unproductive stores of value, shifting the *Portfolio-Arbitrage* curve right-down.

At the same time, and for many years after, China grew to become one of the world's largest economies. In that process, China was also one of the largest exporters of savings as it had a persistent current account surplus. These savings did not flow freely, but were subject to capital controls through which authorities dictated investments abroad. Given the information asymmetries and other financial frictions limiting the desire to directly invest in US companies, there was a strong bias by these official authorities to value the safety and liquidity of government bonds. This increased preference for unproductive

Figure 4: Accounting for the 1995-2019 trends in light of the model



stores of value shifted the *Portfolio-Arbitrage* curve right-down (Reis, 2022*a*).

Two other factors following the global financial crisis plausibly shifted the *Portfolio-Arbitrage* schedule right-down as well. First, there was a decline in the supply of private assets deemed safe, as many asset-backed securities lost their safe ratings. Second, and related, there was an increase in risk aversion by investors, and an increase in regulation by authorities. Both led to a relative shift in the excess demand for safe and liquid unproductive stores of value over risky and illiquid productive investment.

All these factors combined led to the changes displayed in panel (b) of figure 4. The equilibrium shifted from point A to point B with a decline in the yield on government bonds y, while the marginal product of capital m stayed unchanged.

Back to panel (a), this translates to the savings schedule shifting left-up, not because overall savings fell, but because the supply of savings directed to risky illiquid productive investment contracted. The opening of the gap between the two *R-stars*, m and y, that we found in the data is consistent with the large shifts in the global market for savings and investment.

How policy responded to the challenge of high and rising $m - y$. The account that I just provided for the trends in m and y has implications for the potential output in the economy. The shift in desired savings away from productive investments, together with the higher markups and depreciation rates, all resulted in a lower steady-state level of investment in panel (a) of figure 4. In turn, the global imbalances and the stronger relative preference for safety and liquidity resulted in a fall in y in panel (b).

These macro-trends translate in panel (c) to a shift left-down of the AS schedule, as the level of productive capacity falls from x^p to $x^{p'}$. Moreover, as the yield on unproductive assets falls from y to y', the kink point in the AS is at a lower level. Point B'' in the figures is vertically below the old equilibrium point A. Both changes combined give the light-blue new schedule for the AS in figure 4.

How policy responded to these changes was different across regions and, especially, over time during this period. Arguably, during 2008-12, the combination of the global financial crisis and the European sovereign debt crisis caught policymakers by surprise, and constrained their ability to provide stimulus to aggregate demand right away. The PT curve stayed approximately in the same place (or only slightly shifted to the right-down). Therefore, the intersection with the new AS happened at its downward-sloping segment, at a point like B. Output was below its potential, and there was underemployment of variable inputs, like labor, during this period. Inflation was lower than expected, and realized returns ρ exceeded yields.

The policy challenge in the years after was to shift the PT schedule through monetary and fiscal stimulus. Fiscal policy engaged in large deficit programs, while conventional monetary policy set the main policy rate as low as possible for a prolonged period of time.

At the same time, as inflation stayed low, often below target (especially in the EZ), the norms for expected nominal increases slowly fell. The associated wage restraint was widely discussed during this time. It was linked to the decline in unionization rates, and with a perception that trends in offshoring meant that the bargaining power of workers had fallen, leading to "expected deflation forces".

In panel (c) of figure 4, this shows up as a gradual shift of the *PT* curve right-down, from an initial point B, to B′, and eventually reaching B″. As both norms and policy adjusted to the new lower value of *R-star* for government bond yields, and as the inflation target of 2% was reached, the economy achieved its potential level of activity.

Inflation and unconventional monetary policies. In this process of adjustment to the changes in macro-trends, monetary policy faced a particular challenge in regard to its *R-star* level for the policy rate i. All else equal, a lower y requires a proportionately lower i to deliver the inflation target. Yet, if i cannot fall because of an effective lower bound (ELB), then monetary policy will fall short in this adjustment.

In terms of the model, the fall in y leads to a shift left-down of the *PC-IS* curve. The new intersection, displayed in panel (d) of figure 4 is now in the horizontal segment of the Taylor rule, at point B. The resulting inflation is below target, at π'.

This is consistent with the B equilibrium in panel (c), as an inflation below target delivers high realized returns ρ that come with under-employment. The lack of a decisive shift in the *PT* curve in panel (c) is partly explained by monetary policy being constrained at its horizontal segment by the ELB in panel (d). Conventional policy, that for instance shifted the aggressiveness with which policy responds to inflation in the Taylor rule, would not have helped. This would make the *Taylor rule* steeper in its upward-sloping component, but would keep the horizontal component unchanged. The intersection would stay at the same point B.

How did the *PT* curve then shift in panel (c) to bring the economy back to capacity in point B''? Some of it happened through expansionary fiscal policy. Especially in the EZ, this came slowly, as there were constraints and scars left by the European sovereign-debt crisis of 2010. Both in the US and the EZ, monetary policy played a role, albeit through unconventional channels. Quantitative easing and forward guidance, among other policies, were deployed with the goal of lowering the term premium t: the wedge between the (long-term) government bond yield y and the policy rate i. These policies, in the model, shift the *PC-IS* back right-up.

In an extreme case, the shift is back to the old *PC-IS*. This would bring the economy back to equilibrium in point A, with inflation on target π. While the horizontal intersection point is the same, this now corresponds to a lower y' matched precisely by a lower t'. The lower *R-star* on government bonds comes with the same *R-star* for the policy rate, achieved by unconventional policies that lower the term premium between these two rates. This is approximately what happened in the US data.

In the EZ data, this process took longer. There was a mix of lower term premium achieved through unconventional policies and persistently lower conventional policy rates. This shift in conventional policy into a more ”dovish” stance extended the horizontal segment of the *Taylor rule*, and flattened it. In the end, it intersected a *PC-IS* that shifted right-up by the fall in the term premium, but not all the way back to its original position, at a new point. This delivered an inflation rate again close to the 2% target (or slightly below it) but now with a policy rate that was persistently lower.

5 The recent past: 2021-24

With a coherent account of what happened to the four *R-stars* in 1995-2019, I now turn to the harder challenge of interpreting recent movements in interest rates.

The recent data. The pandemic years of 2020-21 were unusual and since then, only three years have passed. Moreover, different advanced economies have faced different challenges and followed different policies. Therefore, inferring trends common to economies on the four *R-stars* come with great uncertainty.

With this caveat in mind, figure 5 shows the measures of the four returns from the previous figure 1 on the historical annual data, but now for US quarterly data between 2021 and 2024.

Starting with the return on productive capital m in the left panel, the data indicate that it may have slightly fallen but, roughly, it continues to be unchanged.

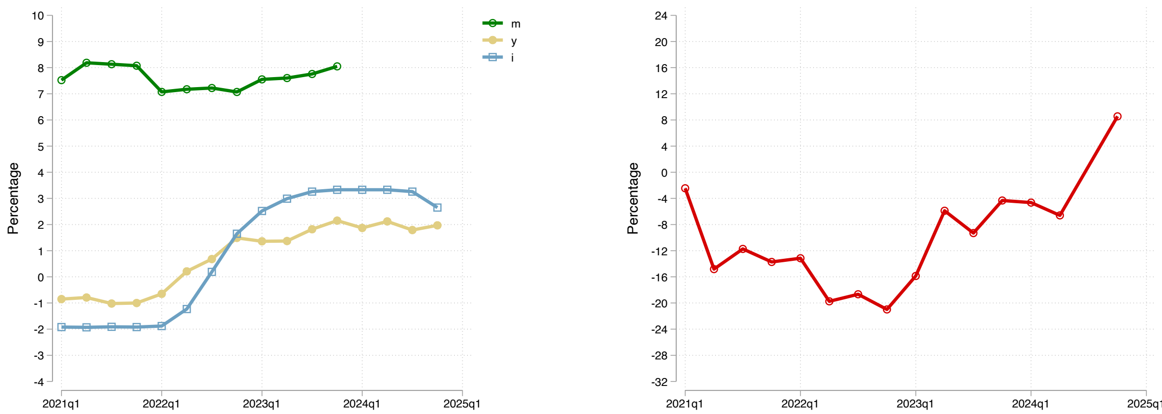
The yield on government bonds y, instead, has continuously and quite quickly risen since 2022. Inasmuch as the 1995-2019 period was characterized by the slow build up of a wedge between m and y, the gap between the return on productive capital and the yield on government bonds sharply fell in 2002-23.

The policy rate i is also in the left panel. It stayed low at first. But, since 2022 it has moved upwards, as monetary policy tightened.

In the right panel, with a scale blown up by a factor of four relative to the other three series is the realized return on government bonds ρ. It was sharply negative for most of this period, as a result of the sharp fall in bond prices and the large surprise inflation disaster.

Figure 5: The four returns since 2021

(a) m, y and i **(b)** ρ



Notes: see the notes in figure 1. For m this is the unleveraged return, for y it is the market yield, for ρ it is the annual holding return on a 10-year bond, and for i it is the Federal funds rate.

Interpreting the sharp fall in the $m - y$ wedge. The first place to look for why $m - y$ has fallen is the set of factors that had driven its rise in the decades before.

Start with the previous shift towards preferring safe, liquid, and unproductive stores of value. The large increase in US public debt during the pandemic, and the sustained and expected public deficits might well have reduced the perception of safety attached to government bonds. In the same direction, their liquidity might be questioned following the problems in the US Treasury market identified in 2020 (as well as parallelisms with the Truss event in the UK market) as well as with the halting of bond-buying programs by the central bank.

The global imbalances that played a large role in the previous decades have also changed. The accumulation of reserves in the US by South East Asian countries has stagnated or reversed. Moreover, the desire by Chinese authorities to buy US government bonds has declined significantly. The fragmentation of world capital markets and geopolitical tensions may extend this for many years. All combined, the supply of savings from abroad to US markets, and especially its tilt towards unproductive stores of value, may well be persistently lower.

In the model, this change is depicted in panel (b) of figure 6. The *Portfolio-Arbitrage* schedule shifted left-up. Since, in the data, m has barely changed, then starting from the B equilibrium point in 2019, the economy shifted to a point like C, with the yield on government bonds rising from y' to y''.

This required either that the *Ramsey-Euler* schedule was very steep, or that it shifted right-up during this time, as plotted in the figure. Recalling what is behind the Ramsey formula, a shift of the schedule requires a consumption spree, or a savings glut. This is consistent with the post-pandemic data, at least for a few years. More persistent might be an increase in optimism about future economic growth, perhaps driven by artificial intelligence.

This leads to turning to panel (a) in figure 6, where the shifts in equilibrium in panel (b) translate into a shift right-down of the *Savings* schedule. As we see in the data, this led to a small rise in private investment in the US, which may persist, although it is too early to tell. At the same time, there was a rise in public investment with a turn towards industrial policy in the "Inflation Recovery Act". This might have led to a small shift of the *Investment* curve to the right-up. Either way, the return on private capital fell only slightly, from m' to m'', as displayed in the figure.

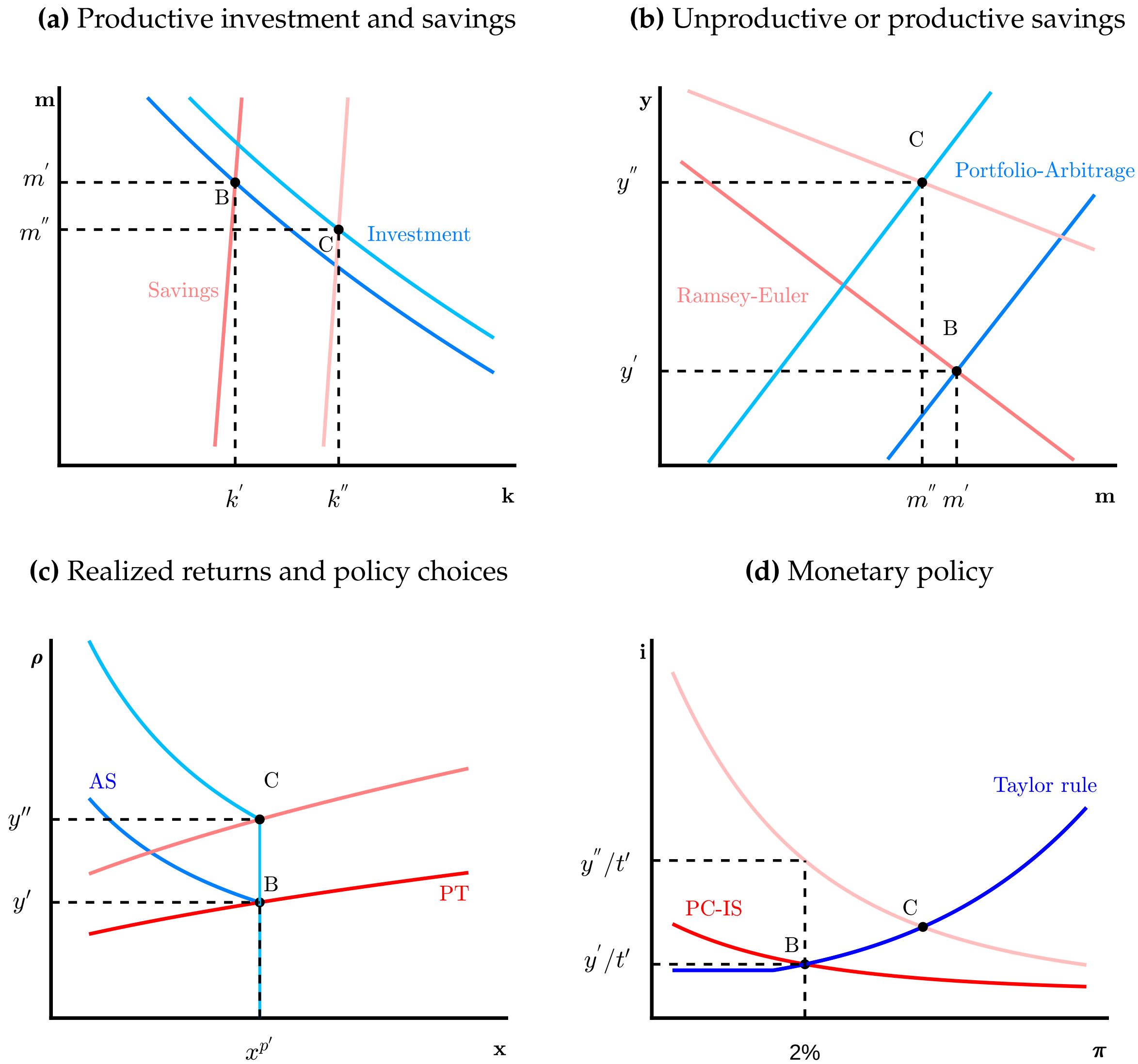
Inflation and realized returns. A sharp rise in the government bond yield to y'' would alter the policy tradeoffs facing the fiscal and monetary authorities.

Turning to panel (c) in figure 6, the AS curve would have shifted vertically up in the recent past. Eventually, the increase in investment will also shift this schedule to the right; how much of that shift has occurred does not change the arguments and challenges laid out here. With unchanged policy, the economy operates at full capacity but inflation spikes upwards. The economy goes to point B with very negative returns to bondholders, matching the data in 2021-22.

At first, aggregate demand policy might have tolerated the higher inflation, and even promoted it with large fiscal stimulus programs and loose monetary policy. The losses to bondholders from "inflating the debt away" partly paid for the pandemic increase in the public debt. Moreover, calibrating the tightening of fiscal or monetary policy was challenging and subject to the risk of over-doing it, leading to underemployment right after the pandemic shock. Eventually, and only gradually, policy did tighten. This shifted the *PT* curve left-up, as shown in figure 6. By the end of 2024, the economy was close to point C, where inflation was almost back on target and realized returns were equal to yields.

Panel (d) in figure 6 zooms in on the monetary policy rate. The higher y'' shifted the PC-IS curve to the right-up, reversing some of its slow shift in the opposite direction during the previous 25 years. The economy moved from the previous equilibrium at point

Figure 6: The four R-stars in 2021-24 according to the model



B with inflation on target to a new equilibrium at point C, with inflation at π'' well above the 2% target.

Keeping inflation on target would have required raising the intercept in the Taylor rule. As y''/t' is now higher, this means the neutral, or Wicksellian rate for policy was now higher. We turn to this, and other options next.

6 The future

If interpreting the recent past is challenging, speculating what will happen to the *R-stars* in the future is even more. This section offers some conjectures, without taking a stand on their relative likelihoods.

I start from the presumption that the state of affairs in the previous section is persistent. Namely, the reversal of the revealed preference for government bonds of the previous 25 years, together with some optimism surrounding future economic growth, has shifted the yield on government bonds persistently higher to y'', even as m'' is only slightly lower than it was in 1995-2019. Therefore, the equilibrium will stay as in the points C in the top two panels of figure 6

What happens to inflation and realized returns depends on how monetary and fiscal policy are set. This will affect the bottom two panels in the figure, starting from point C. I consider three scenarios.

Policy scenario 1: the benevolent benchmark The first scenario is where inflation stays on target and output is at capacity. Achieving this benevolent scenario requires two changes in policy relative to the past.

Starting with monetary policy, displayed in the right-panel of figure 7, policymakers would realize that with a higher *R-star* for government bond yields, then the neutral policy rate is persistently higher. The *Taylor rule* would shift up relative to the last 30 years, matching the shift up of the *PC-IS* because of the higher y''. The new equilibrium would be at a point like D, vertically above the pre-pandemic point B.

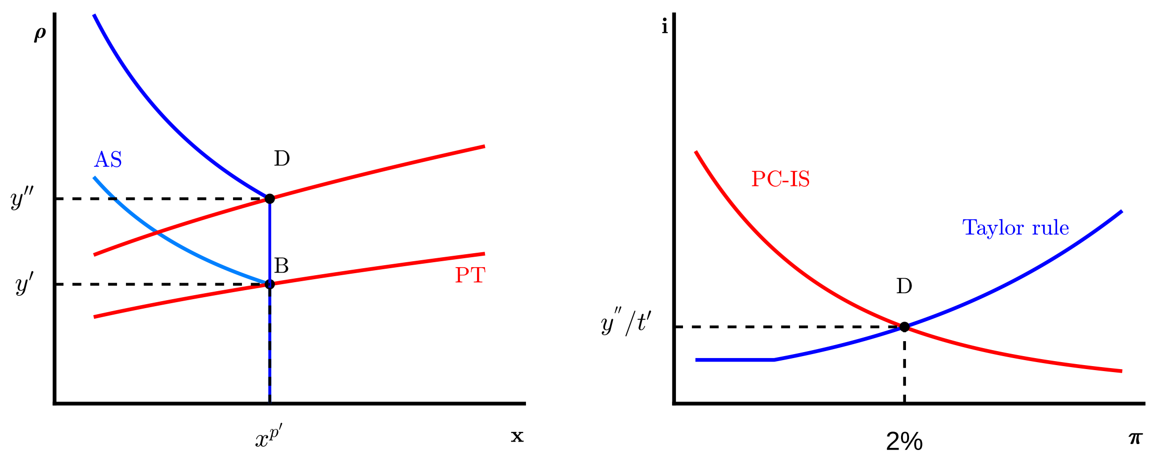
This would come with persistently higher policy rates. The central bank would still raise or lower policy rates as inflation goes up and down, but around a higher average level. The need for unconventional monetary policy or the concerns with the ELB would be left in the past.

Turning to fiscal policy, with steady inflation on target, $\rho = y$. Therefore, the *R-star* on the realized real return paid on government bonds would also be forever higher. Fiscal policy would adjust to the higher interest expenses on the debt by raising primary surpluses and keeping the public debt sustainable.

The joint goal of monetary and fiscal policy in managing aggregate demand would be to ensure that the *PT* curve intersects the *AS* at the point D, vertically above the pre-

Figure 7: Benevolent future policy scenarios

(a) Inflation and realized returns **(b)** Inflation and policy rates



pandemic point B, as shown in figure 7.[8] With the intersection at the kink, output would be at potential. Real payments of variable inputs would be in line with norms given productivity growth and the inflation target, so expectations of inflation would be steady and anchored.

While this scenario may seem optimistic, it comes with three important policy challenges.

First, the movement to a lower wedge between the return on private capital and the return on government debt might well not be smooth. Calibrating policy to achieve the outcomes in figure 7 with real-time imperfect data will be far from easy.

Second, taking as given that the $m - y$ wedge has permanently become smaller, it makes a difference what the source of that change is. On the other hand, if this happens because there is less demand for safety and liquidity, or a perception that government bonds are less likely to have these properties, then this calls for tighter monetary and fiscal policy to achieve the same growth rate or unemployment rate.

On the other hand, say the fall in the $m - y$ wedge happens because there is an increase in productivity, or a re-channeling of investment towards productive capital, which in turn may raise the productive capacity of the economy. With it come higher tax collections making it easier to pay for the higher interest. Monetary and fiscal policy can be looser relative to the previous case.

More generally, the model makes clear that a fall in $m - y$ through a rise in y calls for

[8]Point D in figure 7 is the same as point C in figure 6 .

shifting the PT curve left-up; while if it comes through a fall in m, then the PT has to shift right-down. Distinguishing the two is not easy, and detecting separate trends in m and y in real time is near-impossible.

Third, forever-higher returns on government bonds would put pressure on the Treasury to honor its commitment to bondholders. This pressure would be bound to spill over to the policy targets. Central bank independence would likely be tested, and the inflation target re-considered, as often happens during fiscal crises. In terms of the model, managing the debt and saving on interest expenses will call for the PT curve to move right-down. On average, this can only happen if inflation is higher.

Policy scenario 2: forever-higher inflation. The second scenario most likely arises if monetary and fiscal policy give in to the pressure from high interest payments on the public debt. Also pointing in the same direction might be an heightened fear of under-employment. The post-pandemic boom and its reduction in inequality and improvement in the well-being of the poor may have raised the appreciation for the relative benefits of "running the economy hot."

In the model, either the desire to inflate the public debt, or to run the economy hot, lead to a PT that is to the right-down. As shown in figure 8, it would then intersect the AS at a point like D'. This way, the returns paid to bondholders would be lower at ρ'', reducing the interest paid on the debt. By having the equilibrium below the kink, the economy would be running hot.

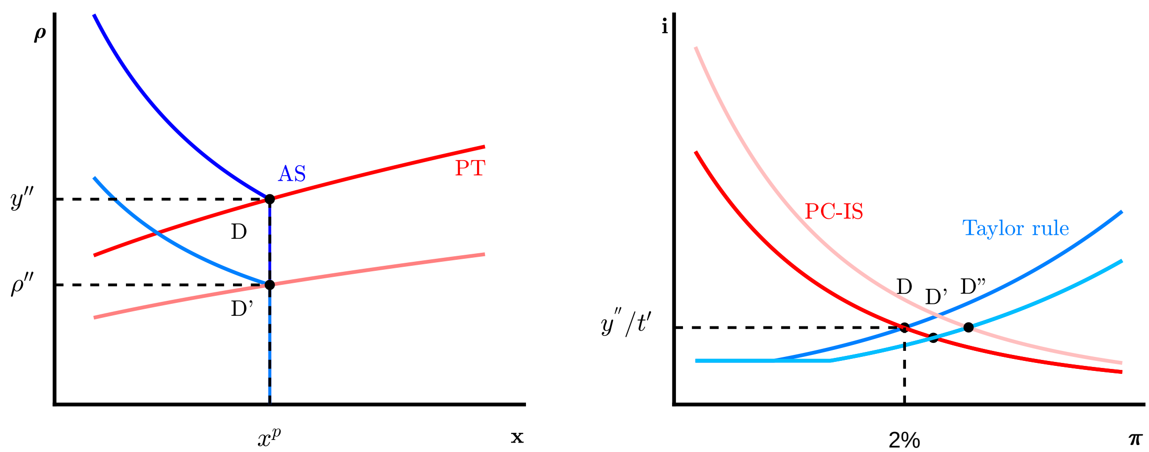
At first, these two consequences might make this a desirable scenario. But, they rely on the norms for remunerating bondholders, workers, and other suppliers of variable inputs holding steady in line with the inflation target. As realized inflation would be persistently above that, and all of these economic agents earn realized returns below what they expected, those norms would change. Expected inflation would rise and nominal wages would grow faster. Policymakers can only achieve the low-returns and hot-economy situation by aiming for even higher inflation. Eventually, inflation would run out of control.

In terms of the model, as norms about inflation rise, the PT curve endogenously shifts left-up. The economy naturally moves towards point D. Fiscal and monetary policy would therefore be continuously loosening to bring the economy back to D'. Keeping $\rho'' < y''$ to lower interest payments or run the economy hot can only be achieved with forever-higher inflation.

This sequence of events for monetary policy is displayed in the right-side plot in figure

Figure 8: Policy scenario with high and rising inflation

(a) Inflation and realized returns **(b)** Inflation and policy rates



8. Starting from point D with inflation on target, running the economy hot and having realized returns low requires shifting the *Taylor rule* right-up to achieve a point D'. The inevitable result is that inflation is above target.

With time, expected inflation adjusts upwards. This shifts the *PC-IS* right-up, and moves the economy to point D'', with even higher inflation. This process continues with the two curves shifting right and inflation spiraling upwards. Throughout, policy rates are kept too low.

This description of events is familiar. It matches what Friedman (1968) feared would happen, and in part did happen, in the United States in the 1970s. Friedman (1968) wrote in terms of the central bank trying to target a real interest rate away from its equilibrium value. From the perspective of this article, he was talking about policy trying to peg a value for ρ below the value for y. Understanding the distinction between these two *R-stars* provides some perspective on his arguments.

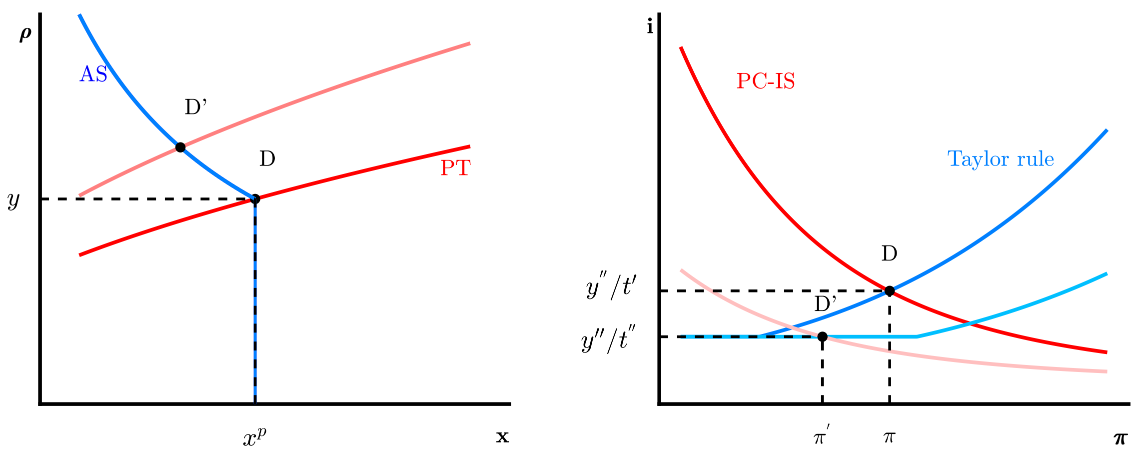
Policy scenarios: a double trap. A final scenario is less familiar, but also has a long intellectual tradition in the study of the Keynesian liquidity trap. It is displayed in figure 9.[9]

A feature of the US data in the second half of 2024 was a sharp rise in the term premium. There are a few reasons for why this might have happened and might persist in the future ahead. One is a rise in expected inflation and in inflation risk premia. This might be

[9] See Adrian, Gaspar and Gourinchas (2024) for related points with a different perspective.

Figure 9: Policy scenario with a double trap

(a) Inflation and realized returns **(b)** Inflation and policy rates



because of forward-looking bondholders perceiving the risk of the second scenario above, or backward-looking ones adjusting to the large losses they had in 2021-24. Another reason is a rise in the perception that the large public debt and political unwillingness to lower public deficits might lead to financial repression over the holders of long-term bonds, which investors are pricing in. A third, more direct, factor might be the unwinding of quantitative easing or, looking ahead, the lack of effectiveness of this unconventional policy after its over-use in the previous 15 years.

In the model, if the term premium rises from t' to t'', this moves the *PC-IS* curve to the left-down as shown in the right-hand side plot of figure 9. Just as was the case in 2010-15, monetary policy might find itself at the ELB. Even if it adjusts its inflation target, extending the horizontal segment of the *Taylor rule* to the right, this might not be enough. The higher yields on government bonds y'' are offset by the higher term premium t'' to leave policy rates against the ELB. The economy would find itself in a too-low inflation trap in point D'.

In turn, imagine that fiscal policy is forced into a path of austerity by the higher yields. In part this is because of the permanently-higher yield on government bonds y'', but it can also come from a heightened fear of sovereign default. Either way, the stance of fiscal policy may be contractionary.

The combination of tight fiscal and monetary policy then imply that the *PT* curve in the left-hand side plot of figure 9 moves left-up. The economy would find itself in a too-low activity trap in point D'.

In this scenario, there is a double trap. Both fiscal and monetary policy find themselves constrained, one by fiscal capacity, and the other by the lower bound on policy rates. The economy operates under capacity, inflation is too low, and policy finds itself unable to escape.

This liquidity trap occurs partly because of sins of the past. The inflation disaster of 2021-24 would have fed into permanently higher term premium, while the erosion of the perceived safety and liquidity of government bonds would have fed into higher yields. The latter would reduce fiscal capacity, while the two combined would reduce room for monetary policy. Aggregate demand would be insufficient and the economy would be in a persistent recession.

7 Conclusion

This lecture distinguished between four *R-stars* to make sense of why it has been so hard to measure this concept in the data. Both theory and the 1995-2019 experience support making this distinction. I offered a framework that explains why the four *R-stars* will differ given the macro trends that we observed. Each of the *R-stars* is important for different questions that are central to macroeconomics and finance, and the framework connected their evolution to the levels of investment, output, and inflation.

Looking at the recent 2021-24 past, there are signs in the data that the wedge between the marginal return to private investment and the yield on government bonds is closing. I laid out arguments in theory for why this may be a persistent trend, and suggested that this will come through a fall in the safety and liquidity premium of government bonds and perhaps due to an investment boom lowering the returns to private investment. Forecasting the future is very hard, so these arguments should be interpreted as having very wide confidence bands around them. But speculating on these trends is important because it sheds light on what challenges policymakers will face.

In particular, I laid out three possible policy scenarios in which the US and other advanced economies might find themselves. In a first benevolent scenario, monetary policy would set policy rates forever higher on average, and fiscal policy would raise primary surpluses to pay for the higher interest on the public debt. Inflation would stay on target. In a second scenario, policy would try to run the economy hot and deliver low returns to bondholders. This would lead to high and rising inflation. In a third scenario, both monetary and fiscal policy would find themselves constrained, by the effective lower bound and by fiscal capacity, respectively. The economy would be stuck with stagnation, operating below capacity and with too-low inflation.

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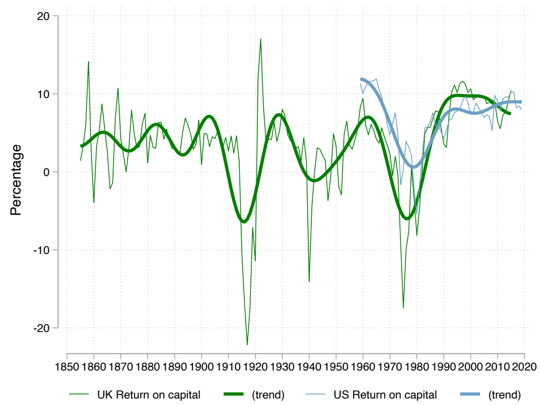
Appendix

A Other variables

Figure A1 shows three other measures of m, perhaps the hardest of the four *R-stars* to measure. They all tell the same story: a near constancy of the returns to private capital during this period. The figure also shows measures of y across countries, all of which share the decline in the return on unproductive savings.

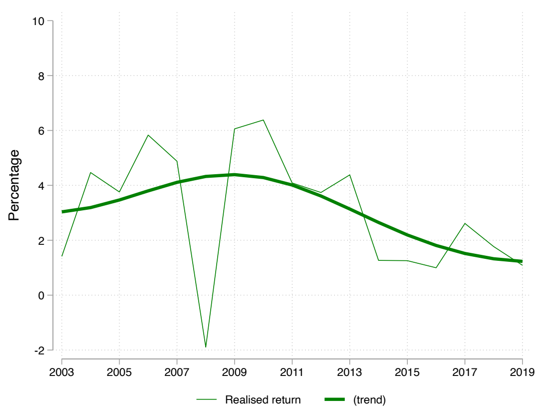
Figure A1: Alternative measurements

(a) Return on productive capital in US, UK (m)



(b) Return on productive capital in G7 (m)

(c) Return on productive capital in Chile (m)



(d) Return on unproductive capital in G7 (y)

Notes: Panel (a): Historical real returns on productive capital in the US and the UK. UK historical data is from "A Millennium of Macroeconomic Data for the UK" from Bank of England for capital stock of non-dwellings. US historical data partly comes from BEA, partly from Jorda-Schularik-Taylor Macrohistory Database using private capital. Panel (b): Real return on productive capital in G7 countries is nominal return on private capital minus realized inflation. Panel (c): Real return on productive capital in Chile is nominal return on private capital minus inflation. Panel (d): Yield on government bonds in G7 countries minus inflation. Trends calculated using a Mueller-Watson filter with a 20-year window for panel a, and 10-year window for panels b-d.

B The model in equations

Since I am discussing steady states, all endogenous variables (denoted with small letters) have no time subscript. Capital letters refer to exogenous variables that may shift, while greek letters refer to parameters. Note that all returns are in gross terms.

The *Investment* curve comes from the standard firm optimality condition that the payment from renting the capital stock equals the marginal product of capital:

$$m = 1 - \delta + Ak^{\theta - 1},$$

where δ is the depreciation rate, and θ is the capital share parameter in a Cobb-Douglas production function. As for A, it depends positively on the level of total factor productivity and negatively on markups.

The *Savings* curve comes from combining the *Portfolio-Arbitrage* and the *Ramsey-Euler* relations. Turning to those, the *Portfolio-Arbitrage* schedule is:

$$m = yB\left(\frac{k}{b}\right)^{\gamma}.$$

Starting from the frictionless framework where $m = y$, there is a convenience discount on government bonds that falls with the size of their holdings b relative to the holdings of capital k. The elasticity of the convenience yield to the supply of bonds is γ. As for B, it controls the relative safety and liquidity of government bonds: it is higher if they are perceived to satisfy the demand for those attributes more, and thus can pay a lower return relative to productive capital.

The *Ramsey-Euler* equation is standard from any macro textbook:

$$G = \alpha m + (1 - \alpha)y \quad \Rightarrow \quad y = \frac{G - \alpha m}{1 - \alpha}.$$

The expected growth rate of consumption, adjusted by the inverse of the intertemporal elasticity of substitution, plus the subjective discount rate is captured by the shifter G. It is equal to the average returns from savings, which are a weighted average of the returns from savings in productive capital, and the return from savings in bonds, with the weight on the first being α. Note that there is an approximation assumed here: the weights on the two forms of savings are kept constant even as b and k change. In general, a rise in b/k would lower α providing a countervailing force. The force I highlight in this model is the dominant one in a micro-founded version of the model (Reis, 2020).

Combining the *Ramsey-Euler* with the *Portfolio-Arbitrage* equations to replace out y gives the *Savings* curve:

$$m = \frac{G}{\alpha + \frac{1-\alpha}{B}\left(\frac{K}{b}\right)^{-\gamma}}.$$

This completes the first two panels, and determines the first two *R-stars*: m and y.

Turning to the management of aggregate demand, the *AS* or aggregate supply curve is:

$$\rho = y \max\left\{1\ ;\ \left(\frac{x}{x^p}\right)^{-\eta}\right\}.$$

Realized returns ρ are anchored by expected returns y. However, when output is below potential, then they are higher, with a slope given by η. This reflects the demand curve for variable inputs, since $\rho > y$ means that the realized returns from supplying those inputs is higher, and so the firm will want to demand less of them. The potential output depends on capital, which I write as $x^p = Dk^\theta$, where recall that θ was the capital share in the Cobb-Douglas production function, while D is a shifter linked to productivity, markups, or regulation that lowers the output produced with a given amount of capital.

As for aggregate demand, the standard first-order condition from minimizing quadratic deviations of output and inflation from target, subject to a positively sloped frontier that trades off the two is: $\pi/\bar{\pi} = (x/\bar{x})^{-\lambda}$, where $\bar{\pi}$ is the inflation target, $\bar{x}$ is the output target, and λ is the ratio of the relative weight on output in the policymakers' objective function to the slope of the trade-off between inflation and output (the slope of the Phillips curve). Now, by the definition of actual returns after realized inflation π, and expected returns according to the norm π^n, it must be that $\rho\pi = y\pi^n$.

Combining the two relations in the previous paragraph gives the *PT* curve:

$$\rho = yEx^\lambda \quad \text{where} \quad E = \left(\frac{\pi^n}{\bar{\pi}}\right)\bar{x}^{-\lambda}.$$

The *PT* schedule shifts up when the norm rises, and down when the inflation target falls or when policy starts targeting a higher level of output.

Finally, we turn to monetary policy. Recall that i stands for the nominal policy rate divided by the inflation target $\bar{\pi}$. Starting with the *Taylor rule*, it is:

$$i = \max\left\{\frac{1}{\bar{\pi}}\ ,\ F\left(\frac{\pi}{\bar{\pi}}\right)^{\phi}\right\}.$$

The first term on the right-hand side captures the zero lower bound on nominal rates. The second term is a standard Taylor rule. The coefficient ϕ is the responsiveness of policy to inflation, which is typically taken to be a number above 1 to satisfy Taylor's principle.

The shifter F plays an important role in monetary policy: it is sometimes called the Wicksellian natural rate. As long as $F = y/t$ then inflation will be on target $\bar{\pi}$, but to understand this result requires introducing the other equilibrium relation.

The *PC-IS* equilibrium relation starts with a Phillips curve that links unexpected inflation to deviations of output from a natural rate: $\pi/\pi^e = (x/x^n)^{1/\kappa_0}$. It combines this with a new Keynesian IS linking that output gap to the deviation of the real short-term rate to the natural rate of interest. Now, since $i\bar{\pi}$ is the nominal short-term rate, then $i\bar{\pi}/\pi^e$ is the real short-term rate, where π^e is expected inflation. The natural rate of interest is then y/t, the expected rate on government bonds adjusted for the term premium. Therefore, the IS is: $x/x^n = ((i\bar{\pi}/\pi^e)/(y/t))^{-1/\kappa_1}$.

Combining the two relations to eliminate the output gap delivers the *PC-IS*, which is:

$$i = \left(\frac{y}{t}\right) H\pi^{-\kappa} \quad \text{, where} \quad H = \left(\frac{\pi^e}{\bar{\pi}}\right)(\pi^e)^{\kappa}.$$

The new coefficient $\kappa = \kappa_0\kappa_1$ is positive. It is higher, and so the *PC-IS* is steeper, if the Phillips curve is very steep (κ_0 is high) or if the sensitivity of spending to interest rates is very high (κ_1). Since both of these seem to run against the evidence, we expect the *PC-IS* to be relatively flat. As for shifts, it will shift up when the expected return y rises, when the term premium t falls, and when expected inflation π^e rises.

We can now verify combining the two curves that, when expectations are anchored on target $\bar{\pi} = \pi^e$, then inflation is $\pi = \bar{\pi}((y/t)/F)^{1/(\phi+\kappa)}$. Therefore, it must be that $F = y/t$ for inflation to be on target $\pi = \bar{\pi}$.