HOW LIKELY IS AN INFLATION DISASTER?

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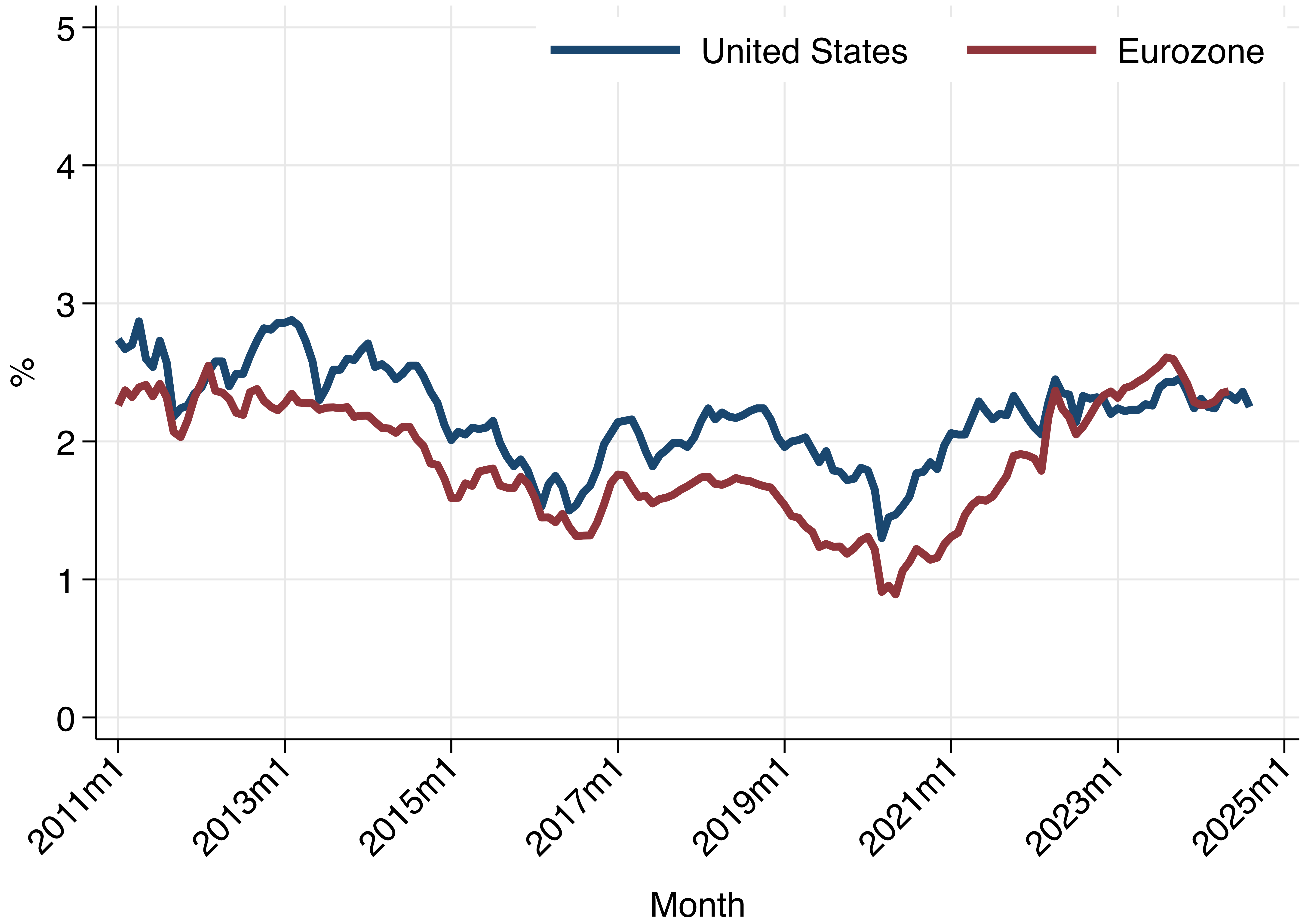
Market expected 5y5y inflation



- Easy to build, carefully monitored, influences policy
- But expected value.
- Robust decision making (and costs of inflation) cares about distribution and especially about tails.
- This paper: disasters
  $Prob[\pi_{T,T+H}/H > \bar{\pi} + d]$
  $Prob[\pi_{T,T+H}/H < \bar{\pi} - d]$

What is the current date t market perceived probability that inflation will be persistently above or below the annual target between T and T + H?

Outline / contributions

- **Methodological**: from option prices to probabilities
  (i) Start with standard methods to get N-probabilities
  (ii) Adjust for erosion in real value in payoff state to get Q-probabilities
  (iii) Adjust for horizon to get forward probabilities
  (iv) Adjust for risk to get P-probabilities

- **History of expectations**: how well anchored have they been
  (v) Likelihood of deflation trap in US in 2011-14 was overstated. But European deflation risk has been persistent, in spite of policies
  (vi) Expectations unanchored din 2021-22, re-anchored with policy, left scars
  (vii) US seems more solidly conditional anchored than EZ

Connection to the literature

- **Uses of inflation options data** (Kitsul Wright, 2013, Fleckenstein, Lopngstaff, Lustig (2017), Mertens, Williams (2021).
  (i) Start where they stopped
  (ii) Inflation and horizon adjustments to discuss anchoring of expectations
  (iii) Risk adjustment focussed on tails
  (iv) Revisit deflation episode, consider inflation disaster, EZ vs US, policy

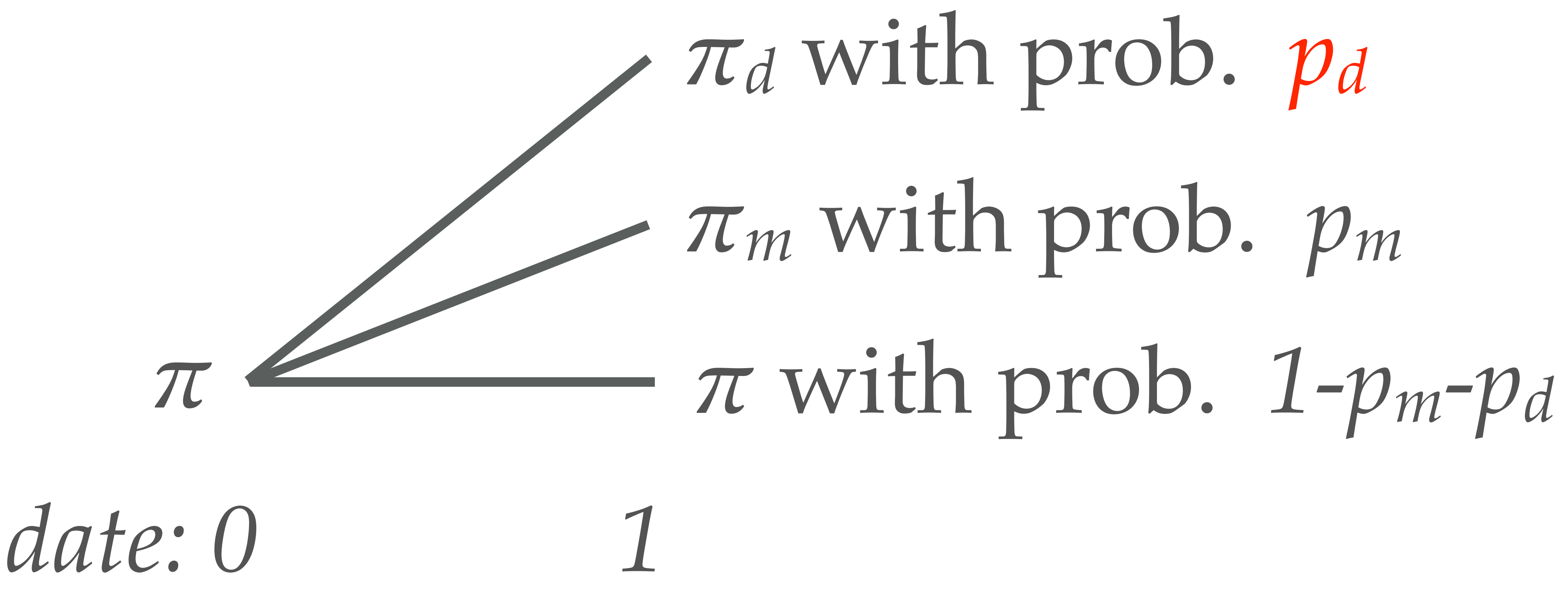
- **Measure inflation risk** (Christensen, Lopez Rudebusch (2015), Haubrich, Pennachi, Ritchekn (2012), Hordahl Tristani (2012)).
  (v) Follow Barro, Gabaix, Barro Liao to measure inflation disasters

- **Focus tails** (Kilian Manganelli (2007) Banerjee et al (2020) Andrade, Ghysels, Idre (2012), Lopez-Salido Loria (2020), Reis (2022), Ryngaert (2022)
  (vi) Use market prices instead of observed frequencies or surveys

The method

The standard reported probabilities

Panel A. Inflation event-tree



- An option that pays \$1 if disaster sells for price $a_d(1) = p_d\, m_d\, exp(-\pi_d)$
- Standard approach: $n_d(1) = a_d(1)\ exp(i(1))$. Probabilities since >0, add to 1
- But this has <u>no match in economic theory</u>!

First adjustment: risk neutral probabilities

- Arrow-Debreu security pays 1 unit of consumption, price $p_d\, m_d$, probability:

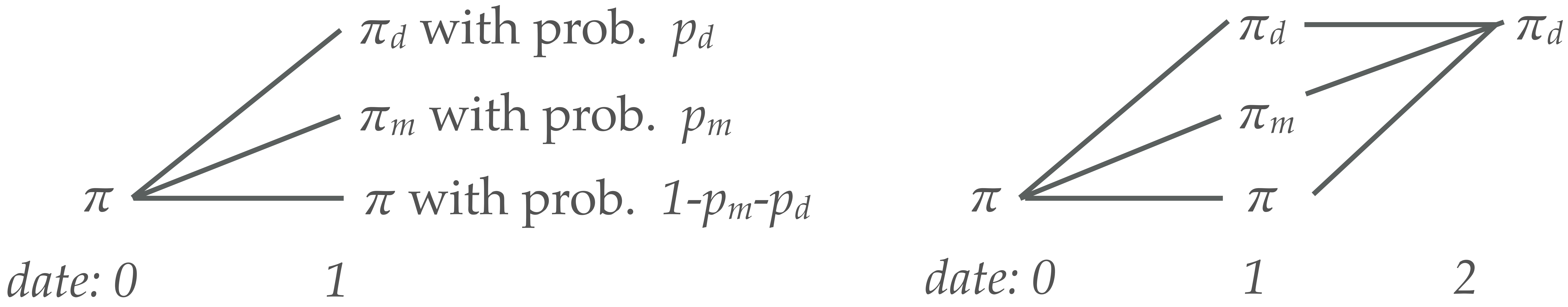
$$q_d(1) = p_d\, m_d\, exp(r(1))$$

$$= n_d(1)\, exp(r(1) + \pi_d - i(1)\,) \approx n_d(1)\, exp(d)$$

- This is desired p_d if risk-neutral, so Q-probability. Standard right only for $d=0$
- <u>Intuition</u>: when option pays, real payoffs are smaller, so option is cheaper.
- If horizon is short, or calculating near probabilities, adjustment is 1. But if 10-year ahead, 3% disaster, then adjustment is: exp(10x1.03) = 1.35 (or 0.67)

Second adjustment: forward probabilities

Panel A. Inflation event-tree Panel B. Distant inflation disaster



$$p_d(2) = p_m\, p_{md} + p_d\, p_{dd} + (1 - p_m - p_d\,)p_d$$

- Have first period probability: $p_d < p_d(2)$ ($p_m p_{md}/p_d$ large enough)
- And have cumulative probability: $p_d(1\&2) = p_d\, p_{dd} < p_d(2)$
- <u>Answer</u>: get $p_d(2)$ from a forward-dated option and model of persistence

Third adjustment: risk

- Familiar one, well $\tilde{m}$ is ratio of marginal utility in disaster and $\tilde{p}$ is probability of consumption disaster conditional on inflation disaster. Then:

$$q_d(1) \approx \left[(\tilde{m} - 1)\tilde{p} + 1\right] p_d$$

- Disaster are bad times, $\tilde{m} > 1$ so probability of disasters overstated.
- But do not need full model of risk!
- Only need conditional distribution of output and inflation disaster for $\tilde{p}$ and average output drop in inflation disaster for $\tilde{m}$
- Both are much smaller than in the rare events equity literature

Proposition: three adjustments to data

$$p(\pi_{T,T+H}) = \underbrace{n(\pi_{T,T+H})}_{\textit{Options Data}}$$

$$\times \underbrace{\left(e^{(\pi_{T,T+H}-\pi^{e}_{T,T+H})H}\right)}_{\textit{Real Factor}}$$

$$\times \underbrace{\left(e^{-r_{T,T+H}H} m(\pi_{T,T+H})\right)}_{\textit{Risk Factor}}$$

$$\times \underbrace{\sum_{\pi_{0,T}} \left[\left(\sum_{\ldots=\pi_{T,T+H}} q(\pi_{T,T+1},\ldots,\pi_{T+H-1,H} | \pi_{0,T}) \right) \frac{q(\pi_{0,T})}{q(\pi_{0,T+H})} \right]}_{\textit{Horizon Factor}}$$

Data and adjustment factors

Options

- Data on options to get these distributions:
  (i) Bloomberg, November 2009 to April 2024, will be updating monthly
  (ii) Horizon 5 and 10 years, as well as 1-years forward from 5y to 9y.
  (iii) Eight bins: $\pi(i) = \{ \leq -1 , (-1,0] , (0,1] , (1,2] , (2,3] , (3,4] , (4,5] , >5 \}$
  (iv) Concerns with liquidity: enforced no arbitrage, used all quotes, monthly only, focus on trends.

- Standard method, give price *a(k)* of option with strike price *k*, the N-density follows from (and matches previous estimates):

$$N(k) = 1 + e^i a'(k)$$

The adjustments: high inflation

Panel A: High inflation disaster (>4%) probabilities, 9/21 - 8/23						
	N_5y	Q_5y	N_10y	Q_10y	Q_5y5y	P_5y5y
US, 9/21-8/23	20.7%	22.8%	14.0%	17.2%	6.3%	4.2%
US	6.0%	6.7%	8.9%	11.0%	5.2%	3.5%
EZ	1.4%	1.7%	2.8%	3.6%	4.9%	3.2%

Panel B: High inflation disaster probability adjustment factors				
	N to Q, 5y	N to Q, 10y	Q, 10y to 5y5y	Q to P, 5y5y
US, 9/21-8/23	1.09	1.24	0.38	0.66
US	1.12	1.23	0.41	0.66
EZ	1.17	1.33	0.93	0.66

- Inflation adjustment, *N* to Q: higher recently (high inflation) and higher with longer horizon
- Horizon adjustment to 5y5y: strong for US, transitory market view of inflation
- Risk adjustment: significant, consistent with inflation risk premia of 23bp

The adjustments: deflation

Panel C: Deflation (<0%) probabilities, 1/11 - 12/12

	N_5y	Q_5y	N_10y	Q_10y	Q_5y5y	P_5y5y
US, 1/11-12/12	6.7%	5.6%	6.9%	4.8%	6.4%	6.2%
US	2.7%	2.3%	2.1%	1.5%	2.5%	2.4%
EZ	5.0%	4.6%	5.0%	4.2%	6.6%	6.3%

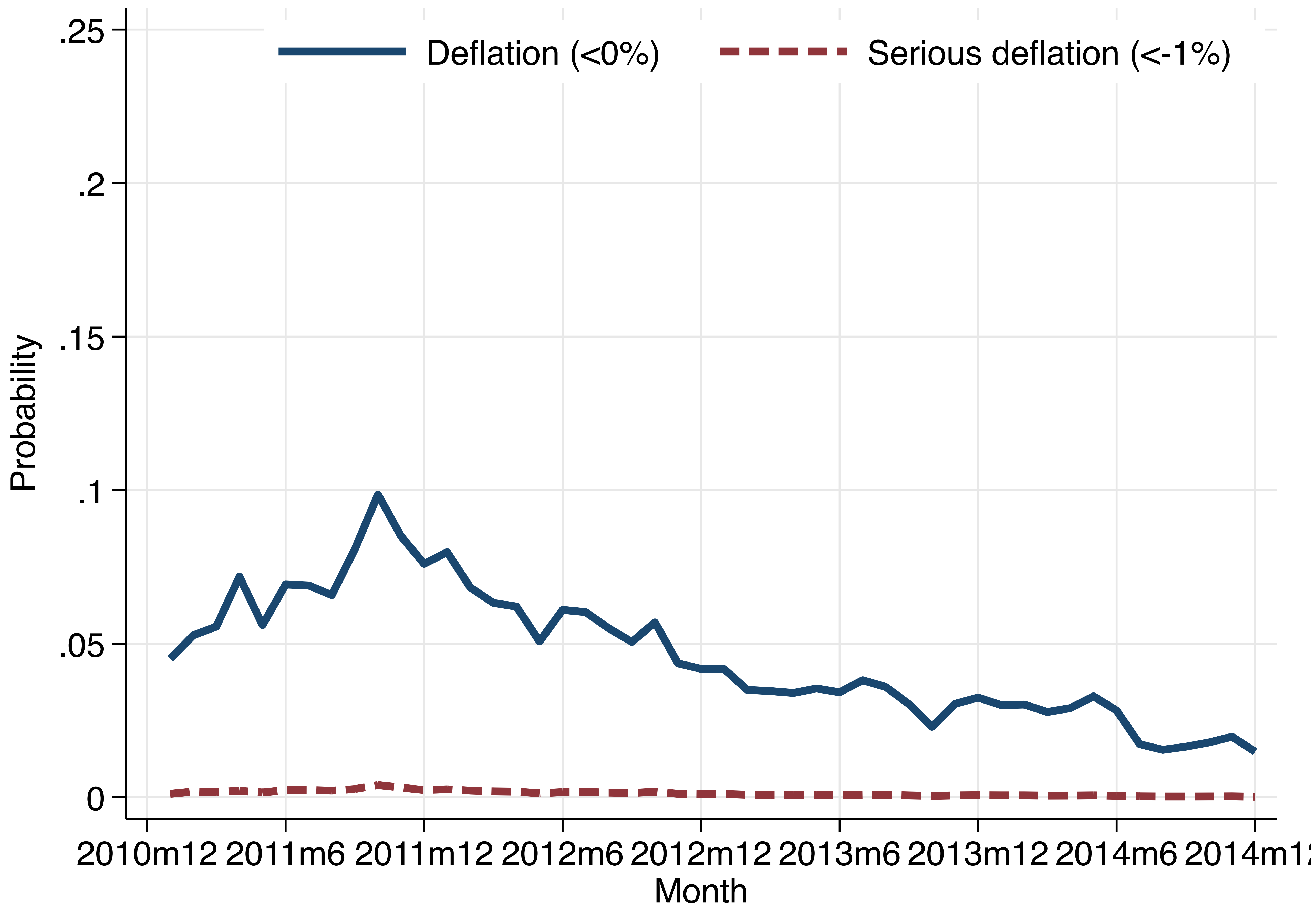
Panel D: Deflation probability adjustment factors

	N_5y to Q_5y	N_10y to Q_10y	Q_10y to Q_5y5y	Q_5y5y to P_5y5y
US, 1/11-12/12	0.84	0.69	1.41	0.96
US	0.85	0.72	1.31	0.96
EZ	0.90	0.80	2.26	0.96

- Inflation adjustment, *N* to *Q*: opposite direction, *N* leads to overstatement
- Horizon adjustment to 5y5y: Large for EZ, market perceives a deflation trap
- Risk adjustment: small because most deflations came with small output loss

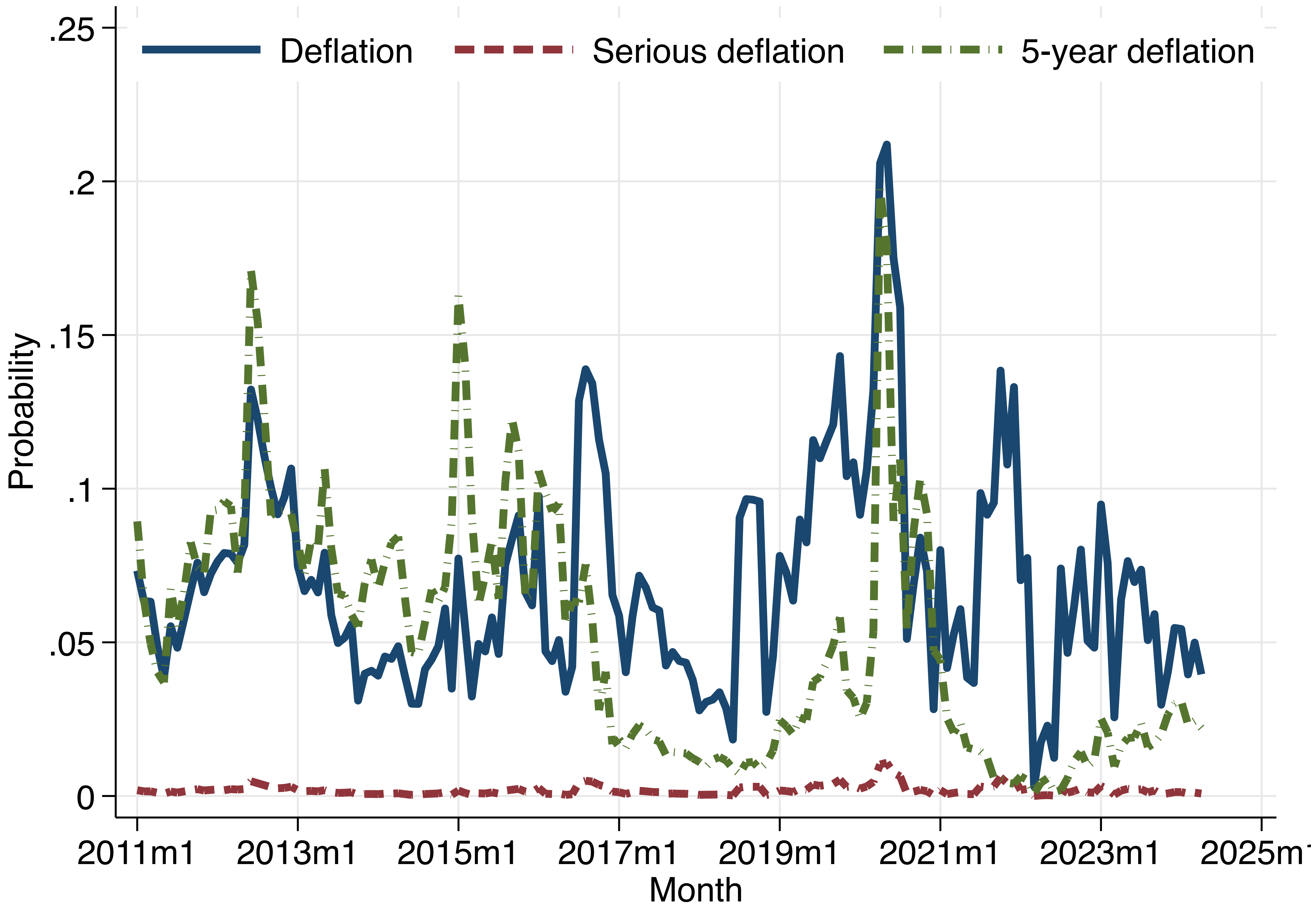
History of anchoring

The US unjustified fear of deflation



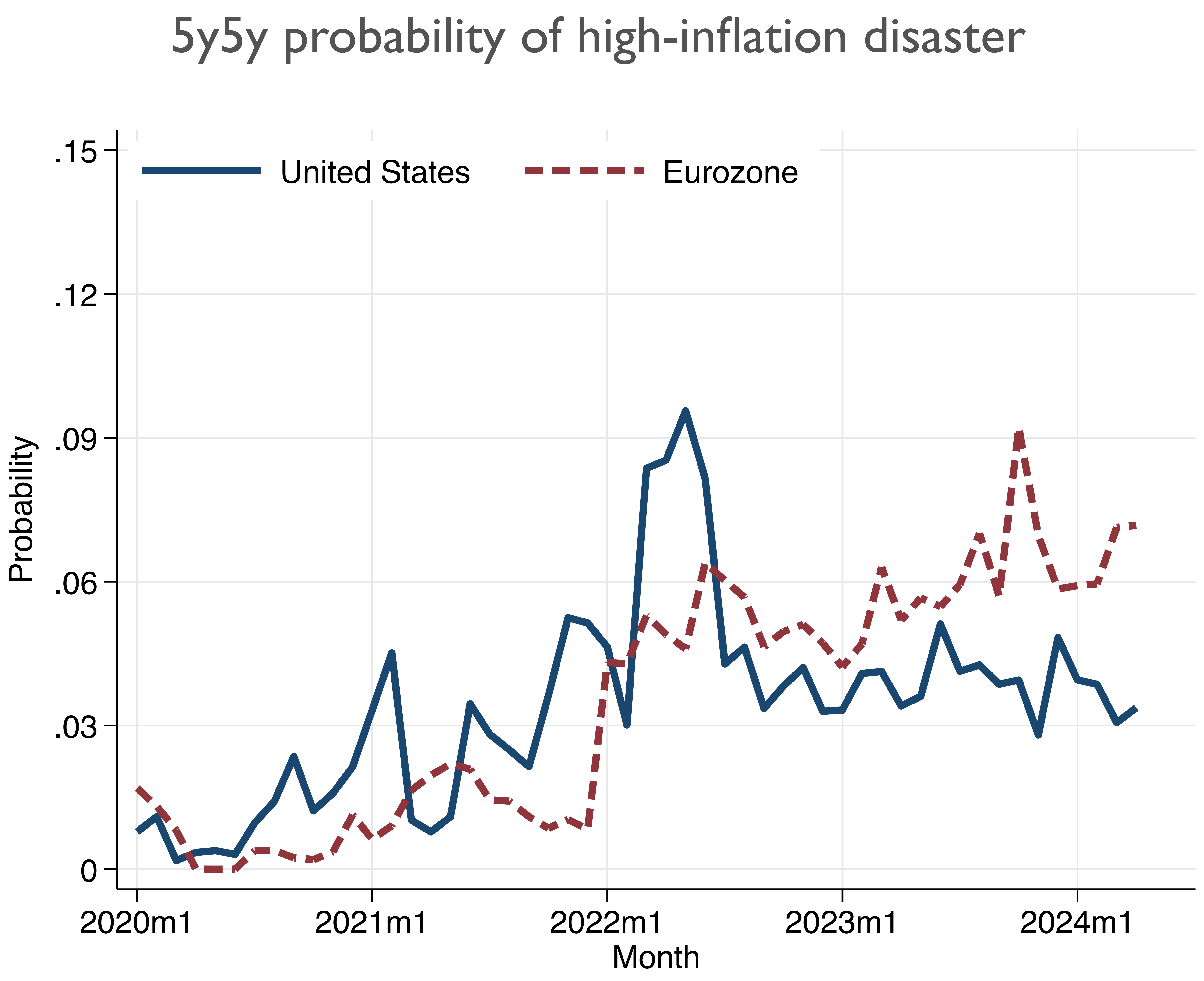
- Justified unconventional money and fiscal to avoid ZLB and liquidity trap
- But, gone after 2011.
- Literature had overstated it because of: (i) inflation adjustment factor, (ii) near horizon risk, not deflation trap
- Revision of history

The EZ deflation trap lingering concerns



- Still high in 2014, justifying QE, and other unconventional policies
- Started rising again in 2018 and peaked with pandemic.
- Since fallen, but stabilize at 5%, persistently higher than in US
- Switch after 2015 on 5y vs 5y5y: deflation trap is more likely than deflation in short horizon.

The inflation disaster of 2021-24



- 2021-24 is an inflation disaster. Three stages in inflation anchor

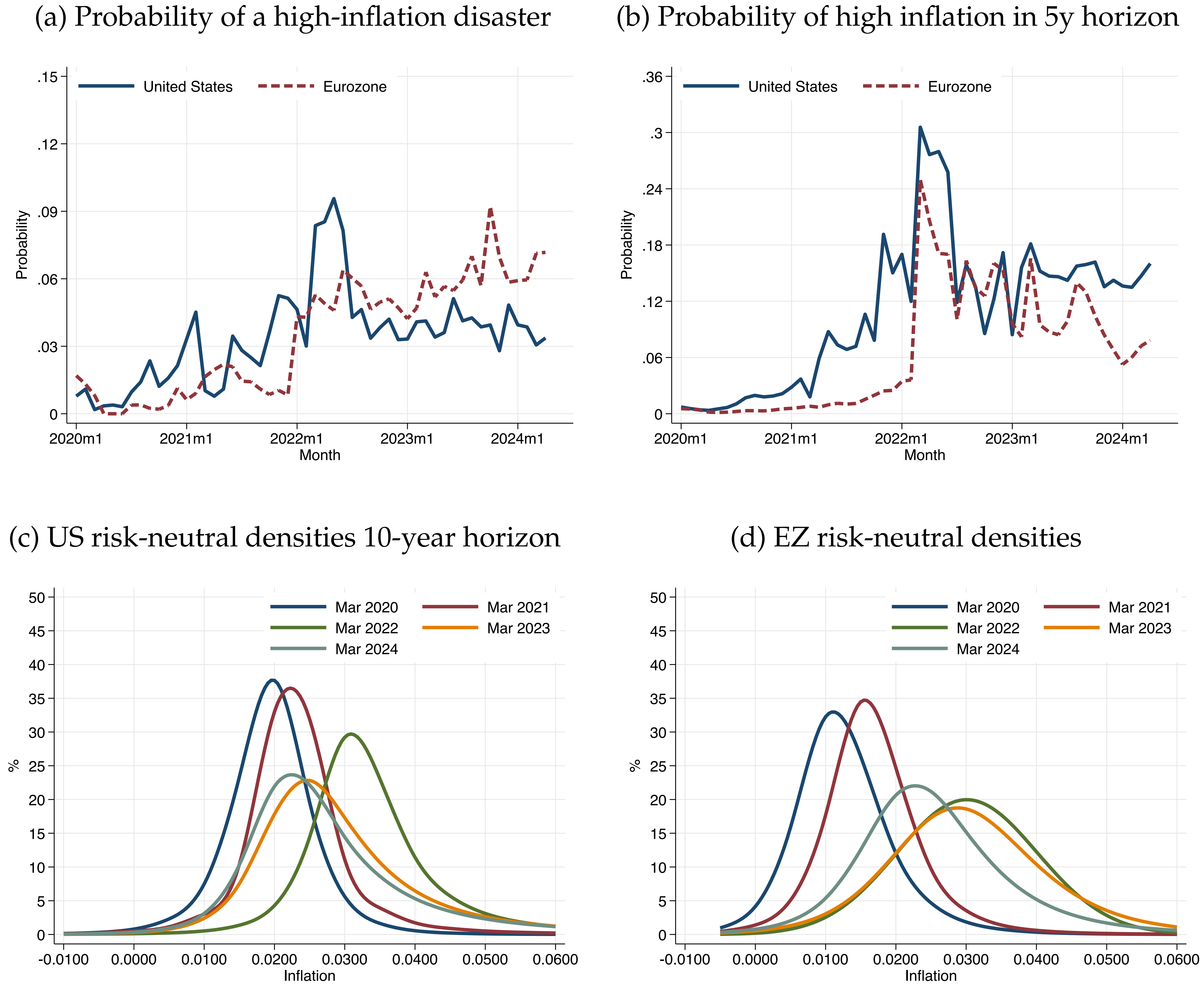
1. Deanchoring in 2021: response to events or to loose policy?

2. Reanchoring coincided with rise in rates (but less in EZ)

3. Lingering scars from episode as probabilities persistently higher than before

Digging deeper

Figure 4: Perceptions of a future inflation disaster during the 2021-24 inflation disaster



Note: Top panels: 5y5y (forward) and 5y (near term) inflation disaster (> 4%) probability. Bottom panels: 10y horizon risk neutral (Q) densities.

(b) Higher in near horizon, tight link between actual data, forecasts in near horizon, long-run anchor

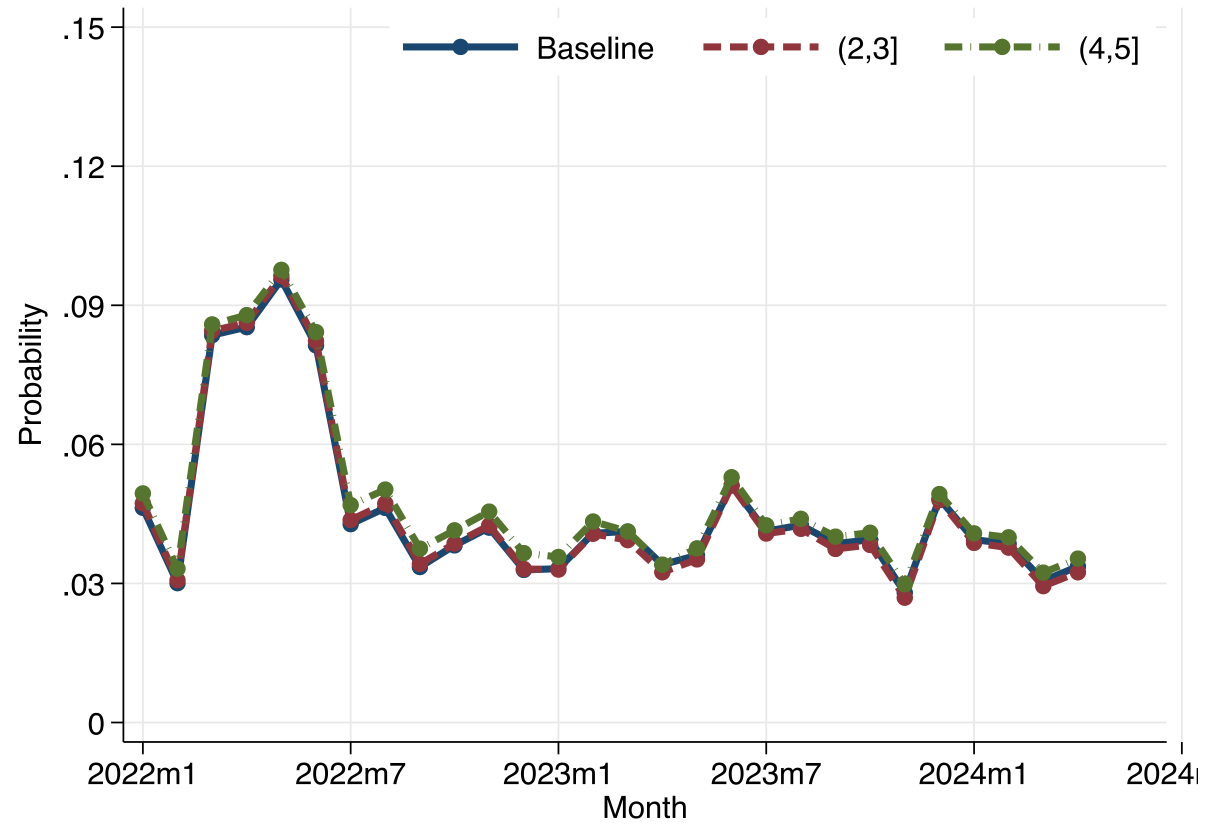
(c) Movement in the tails before median moves, leading indicators

(d) Skewness as a useful measure of drifts

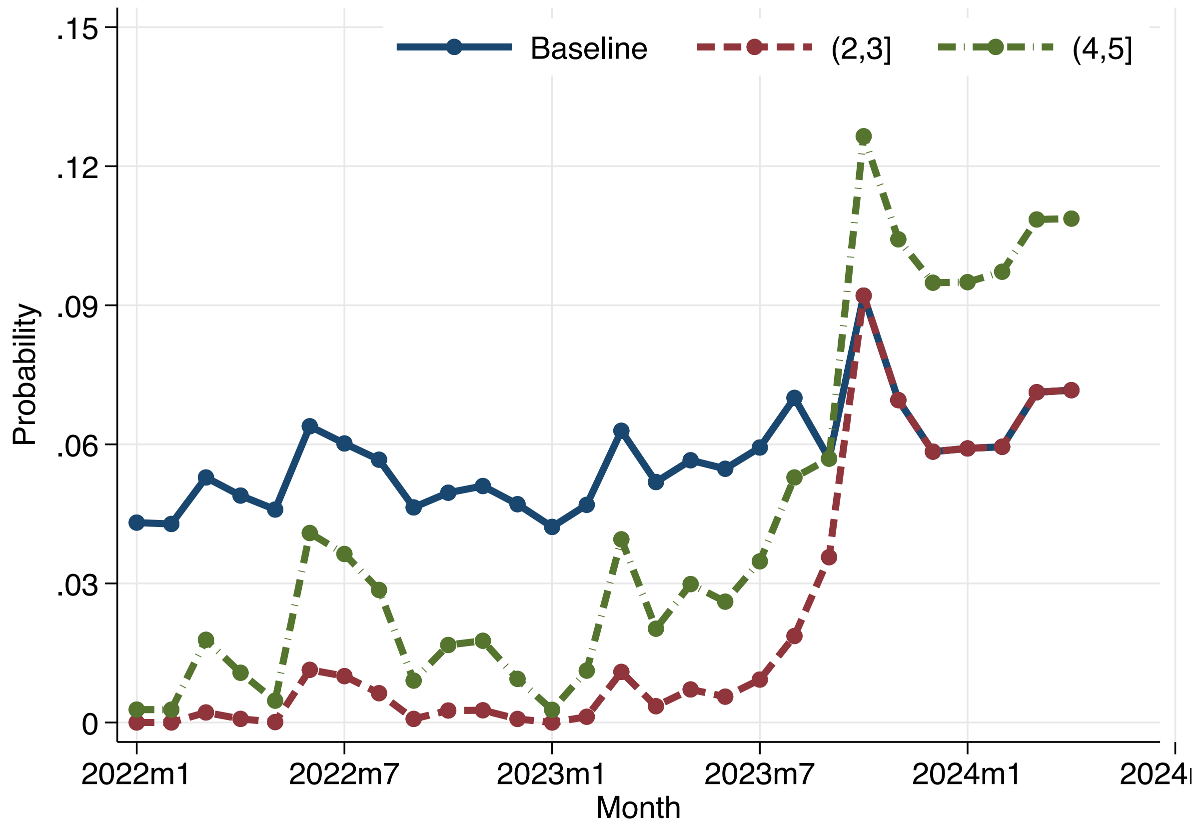
Conditional anchor

Figure 5: The conditional anchoring of expectations

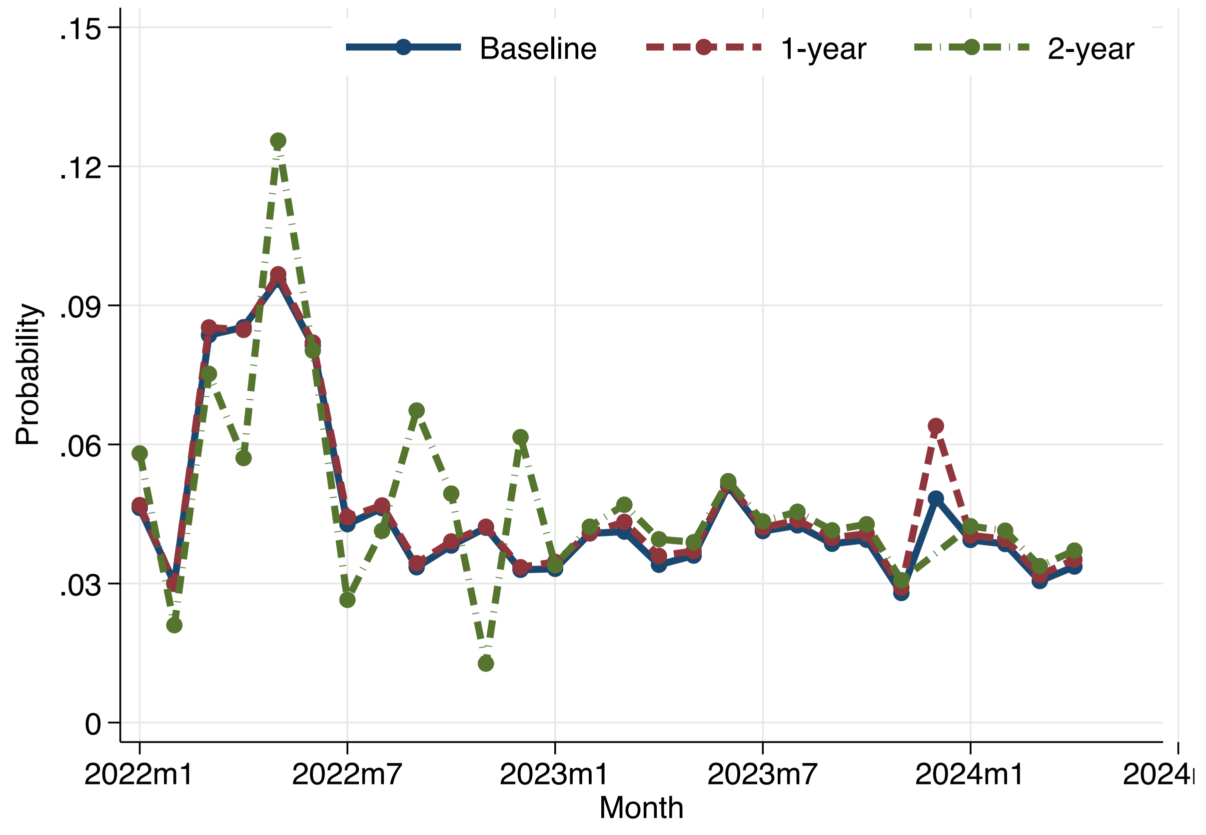
(a) The influence of initial conditions in the US



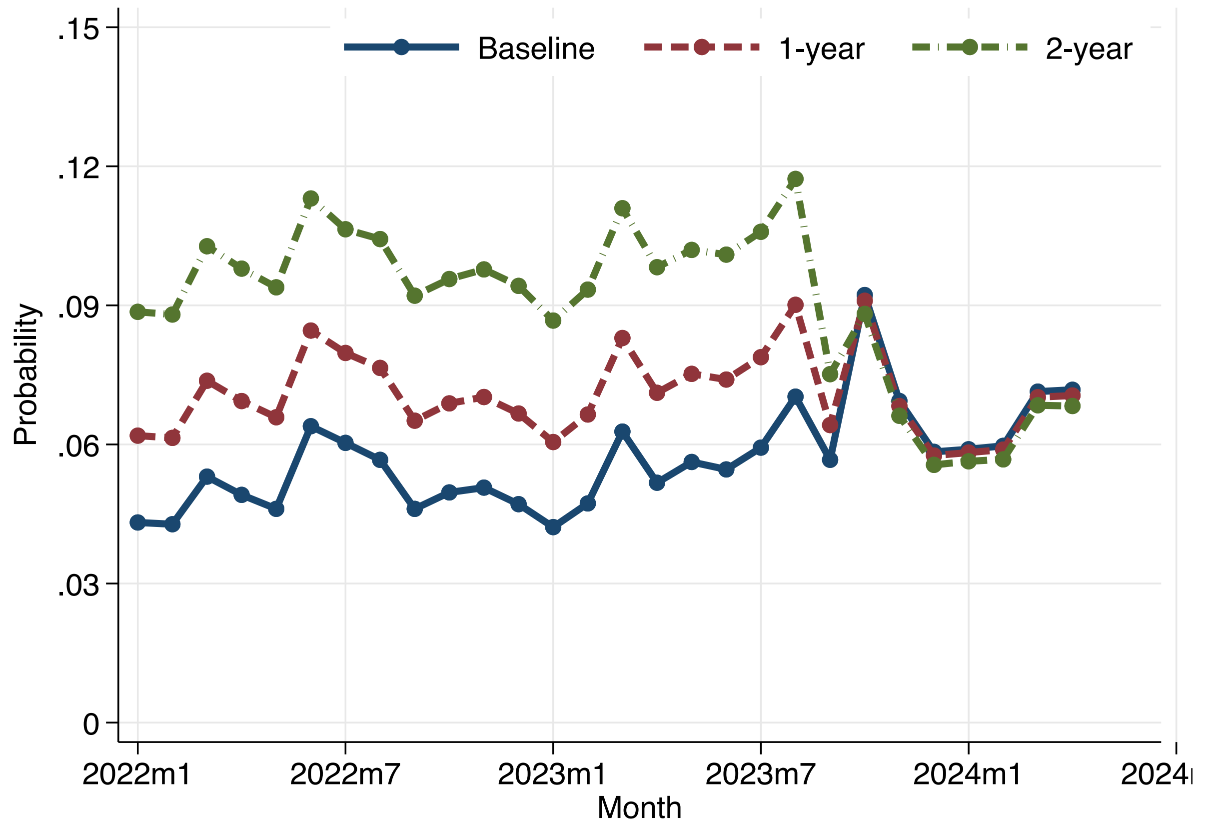
(b) The influence of initial conditions in the EZ



(c) Conditioning on the near future in the US



(d) Conditioning on the near future in the EZ



Note: The figure reports various conditional 5y5y inflation disaster (> 4%) probabilities. Top row: Baseline, based on actual current inflation, and varying current inflation (over the previous year) to lying in different ranges, either 2%-3% or 3%-4%; bottom row: Changing, in addition, inflation over the next two years.

- Are expectations insensitive to realizations of inflation (in recent past or near future)?
- Initial conditions: very much so in US, less so in EZ.
- Conditional probabilities: inflation perceived as being transitory in US, less so in the EZ.

Empirical implementation of the three adjustments

Q probabilities, inflation adjustment

- Almost same steps as Breeden Litzenberger's seminal paper:

$$Q(k) = e^r k a''(k) = N(k) e^{k-i+r}$$

- Q(.) coincides with N(.) if every realization of inflation is equal to expected inflation… Silly, only if no uncertainty about inflation.

- Q coincides with P if people are risk neutral. As is standard.

Dataset with Probabilities of Inflation Disasters from Options Prices

Download the dataset and view figures that use it

View on GitHub

Summary

The prices of inflation options give the cost of insuring against extreme events. They reveal the probability of these events as perceived by market participants. However, to construct probabilities of inflation disasters at the conventional 5-year-5-year horizon, the standard option pricing formus have to be modified in three ways: to account for the erosion of the real value of the options' payoff, to account for the forward 5-year ahead starting horizon, and to account for the compensation for inflation risk. Below are data and figures for the probabilities of inflation disasters, making these adjustments. The data starts in January of 2011 and refers to US and EA inflation. This dataset can be **freely used** by other researchers.

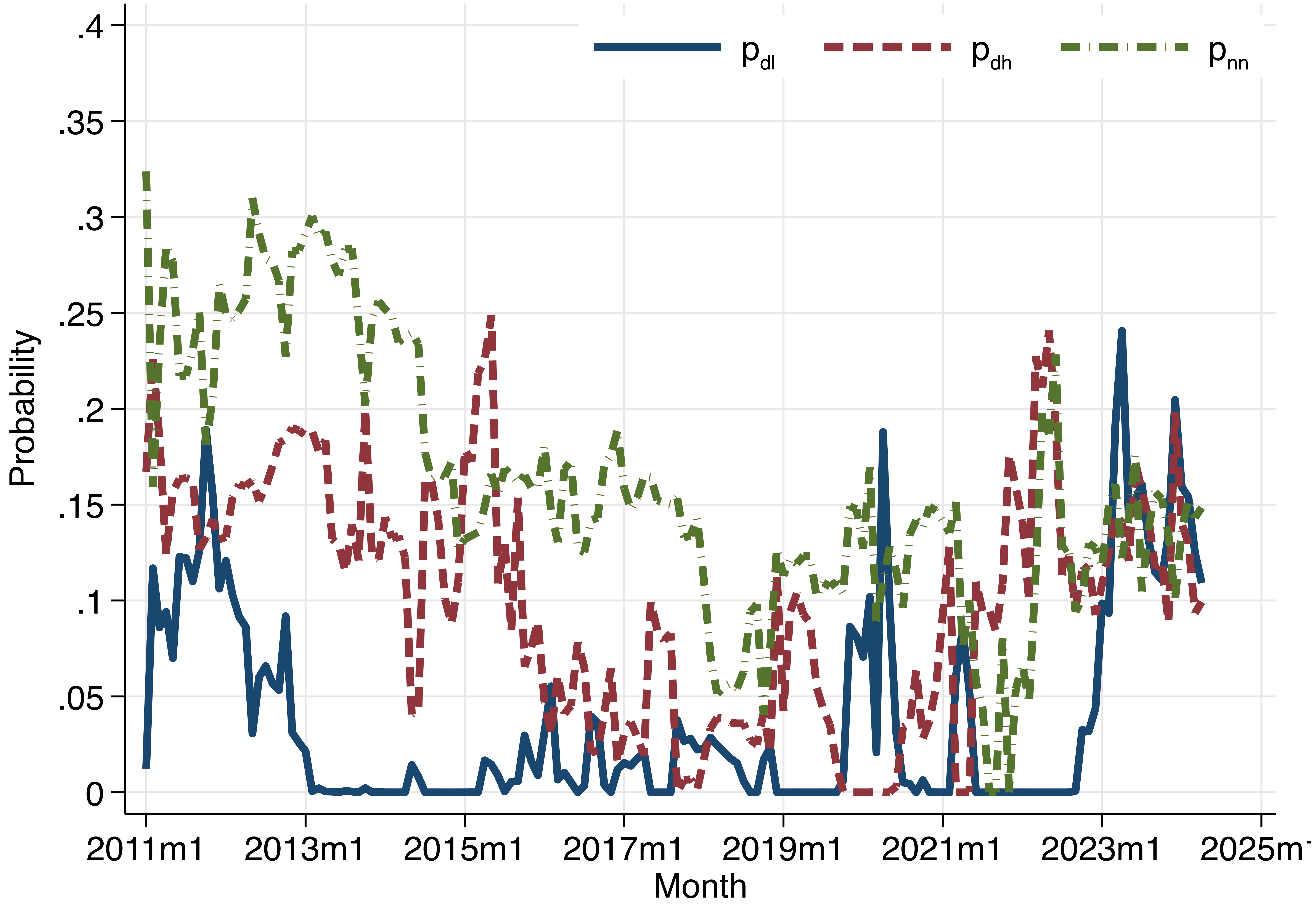
Horizon adjustment: model of dynamics

$$\pi_{t+\Delta} = \underbrace{\bar{\pi}}_{\text{target}} + \underbrace{\varepsilon_{t+\Delta}}_{\text{smooth shocks}} + \underbrace{d^h_{t+\Delta} - d^l_{t+\Delta}}_{\text{disasters}}$$

- Assumptions: (i) jump large relative to smooth shocks, (ii) mean reverting Ito

$$\mathbf{P} = \begin{bmatrix} 1-5p_l & p_l & p_l & p_l & p_l & p_l & 0 & 0 \\ p_{dl}+p_{nn} & p_{ml} & p_{mr} & 0 & 0 & 0 & 0 & 0 \\ p_{dl} & p_{nn} & p_m & p_{mr} & 0 & 0 & 0 & p_{dh} \\ p_{dl} & 0 & p_{nn} & p_n & p_{nn} & 0 & 0 & p_{dh} \\ p_{dl} & 0 & 0 & p_{nn} & p_n & p_{nn} & 0 & p_{dh} \\ p_{dl} & 0 & 0 & 0 & p_{mr} & p_m & p_{nn} & p_{dh} \\ 0 & 0 & 0 & 0 & 0 & p_{mr} & p_{mh} & p_{dh}+p_{nn} \\ 0 & 0 & p_h & p_h & p_h & p_h & p_h & 1-5p_h \end{bmatrix}.$$

US model parameter estimates



Constant parameters:

- If rise, 50% chance will revert
- Probability of leaving disaster after one year is 99% for US, same for EZ high, but 69% for deflation

Time varying parameters:

- Fall in stochastic volatility
- Recent rise in H jumps.

Risk adjustment: model of rare disasters

$$\pi_{t+\Delta} = \bar{\pi} + \overbrace{u^{\pi}_{t+\Delta} + e_{t+\Delta}}^{\varepsilon_{t+\Delta}} + d^h_{t+\Delta} - d^l_{t+\Delta}$$

$$\log(c_{t+\Delta}) = \log(c_t) + g + u^c_{t+\Delta} + \beta_0 e_{t+\Delta} - \beta^h d^h_{t+\Delta} - \beta^l d^l_{t+\Delta}$$

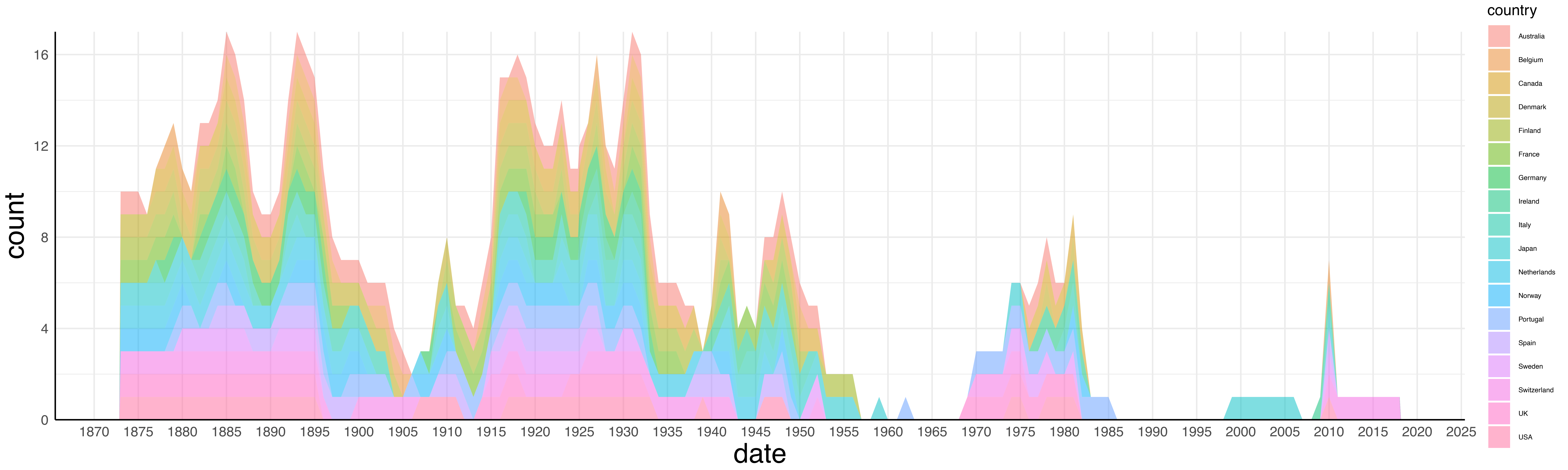
- Crucial parameters are β^h and β^l

- With probability p, disaster $\beta^h d = 1\text{-}1/z$, where z has a Pareto distribution:

$$F(z^h) = 1 - \left(\frac{z^h}{z^h_0}\right)^{-\alpha^h} \quad \text{with} \quad z^h \geq z^h_0 > 1, \alpha^h > 0.$$

- Key parameters are minimum size of jumps z and thickness of tails α. Then risk aversion 3 (E-Z utility).

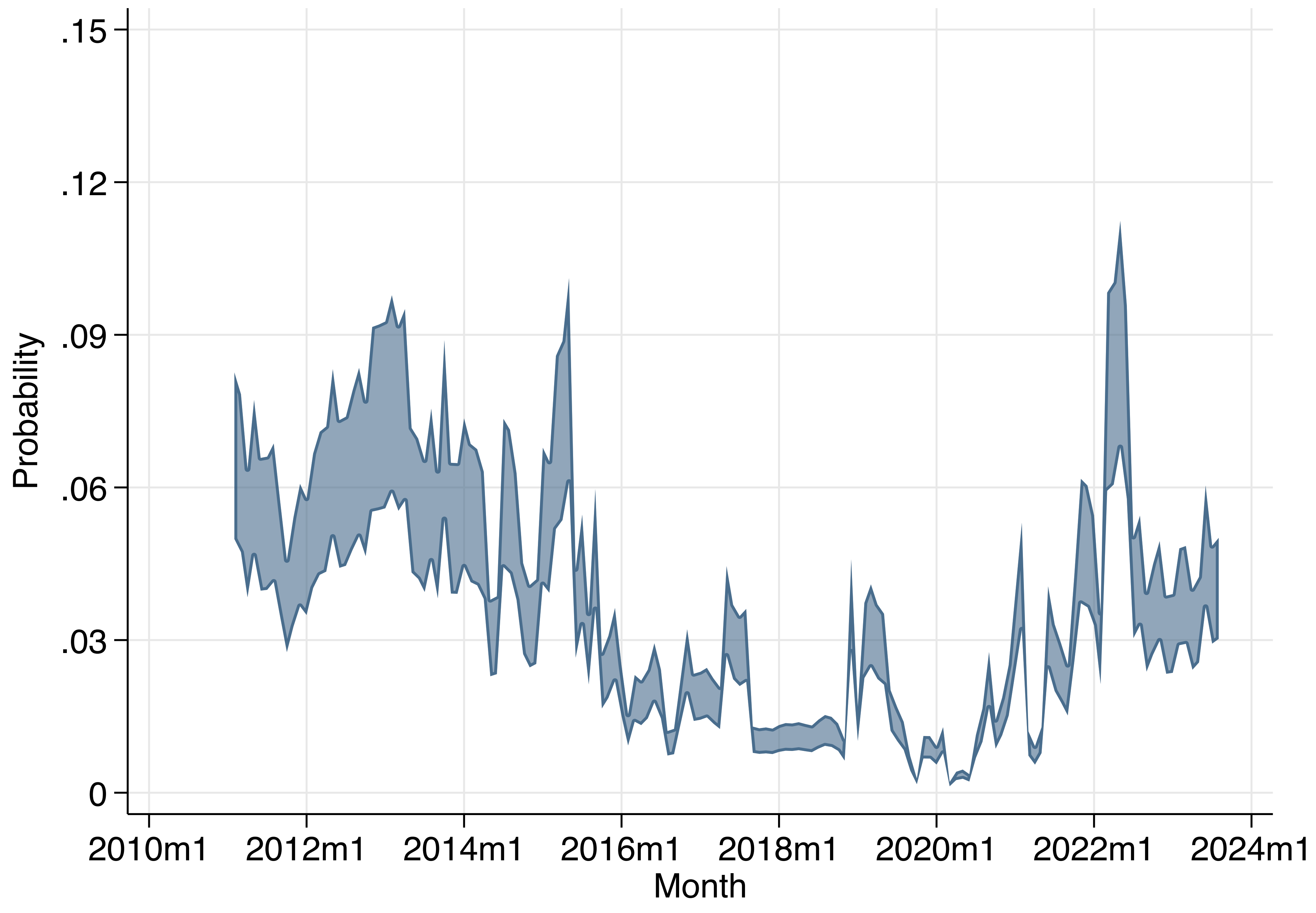
Pareto distribution



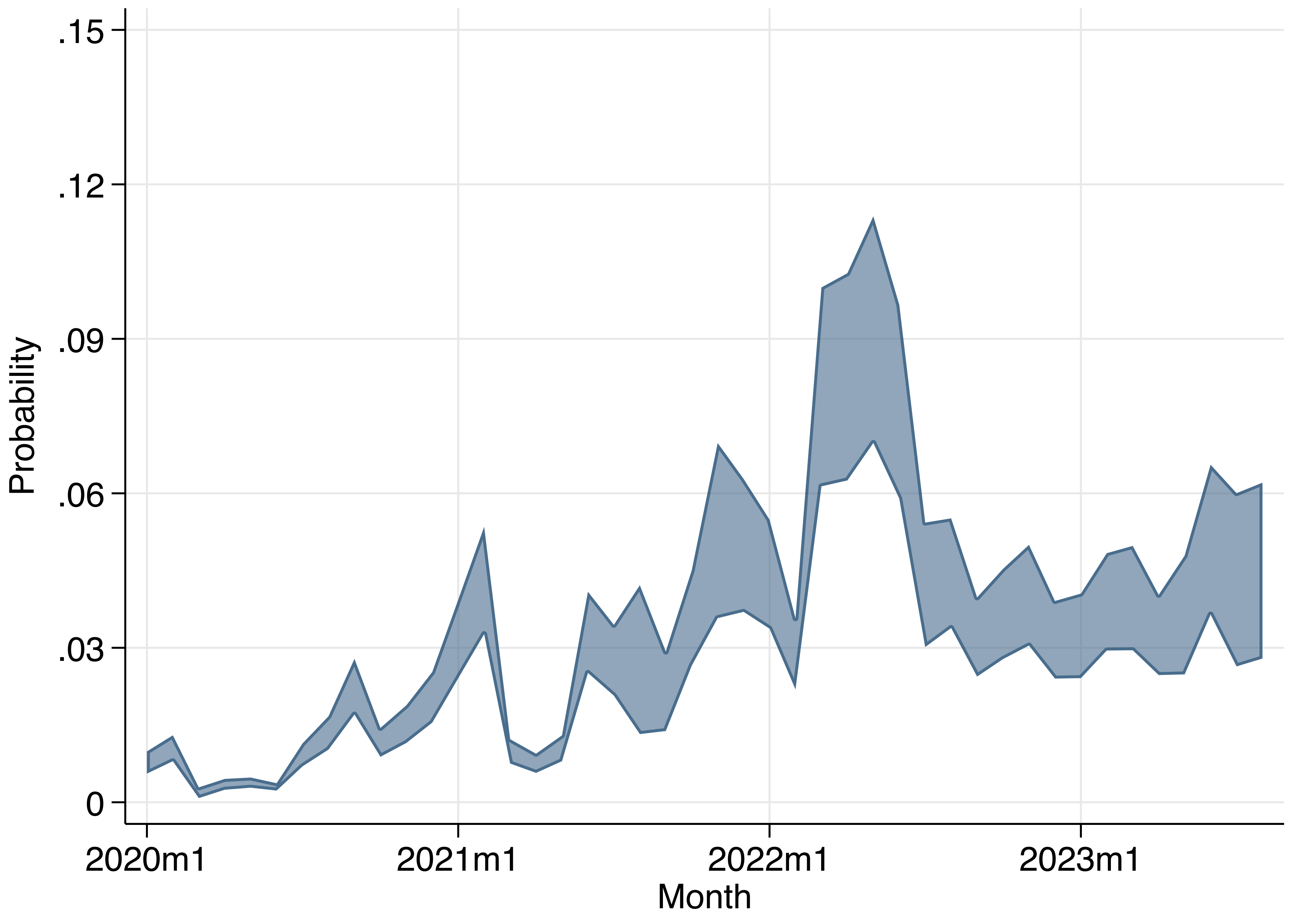
- Data: Barro (2006) consumption, Jorda Schularick Taylor (2019) inflation. Probability of output disaster conditional inflation disaster: 20.0%
- Estimates: $\alpha^h = 5.45,\ z^h{}_0 = 1.03$ and $\alpha^l = 15.18,\ z^d{}_0 = 1.06$

Confidence bands for adjustments

Risk adjustment only



Both risk and horizon adjustments



Note: The left panel shows the 90% confidence band for the US 5y5y inflation disaster (> 4%) probability when standard errors take into account the uncertainty in the risk adjustment estimate; the right panel adds uncertainty in the horizon adjustment.

Conclusion

Conclusion

- How to calculate counterpart to 5y5y market-bases expected inflation that focusses on tails of distribution to judge inflation disasters?

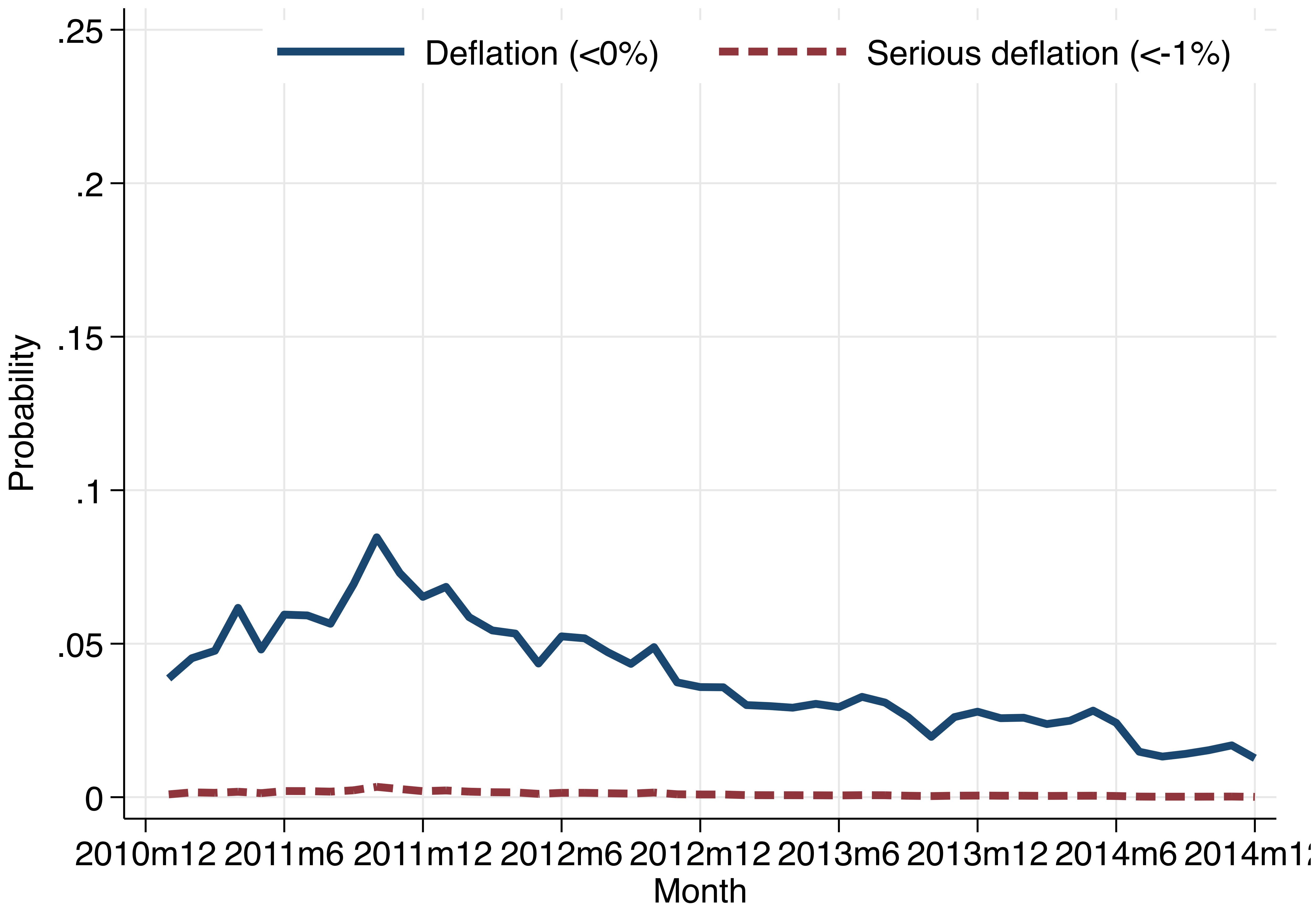
- Natural to use options, but needed to develop machinery to use the data

- Applications results (noting that these are market perceptions):
  1. Fed deflation fears 2011-14 were exaggerated, but persist in EZ and unconventional policies as well as mission reviews only offered temporary respite
  2. Deanchoring of expected inflation between mid 2021 and mid 2022, coinciding his high realized inflation and loose policy
  3. Reanchoring of expectations quite sharply once monetary policy tightness, but still lingering scars for the future
  4. Temporary inflation shocks have larger influence on markets in EZ than in the US

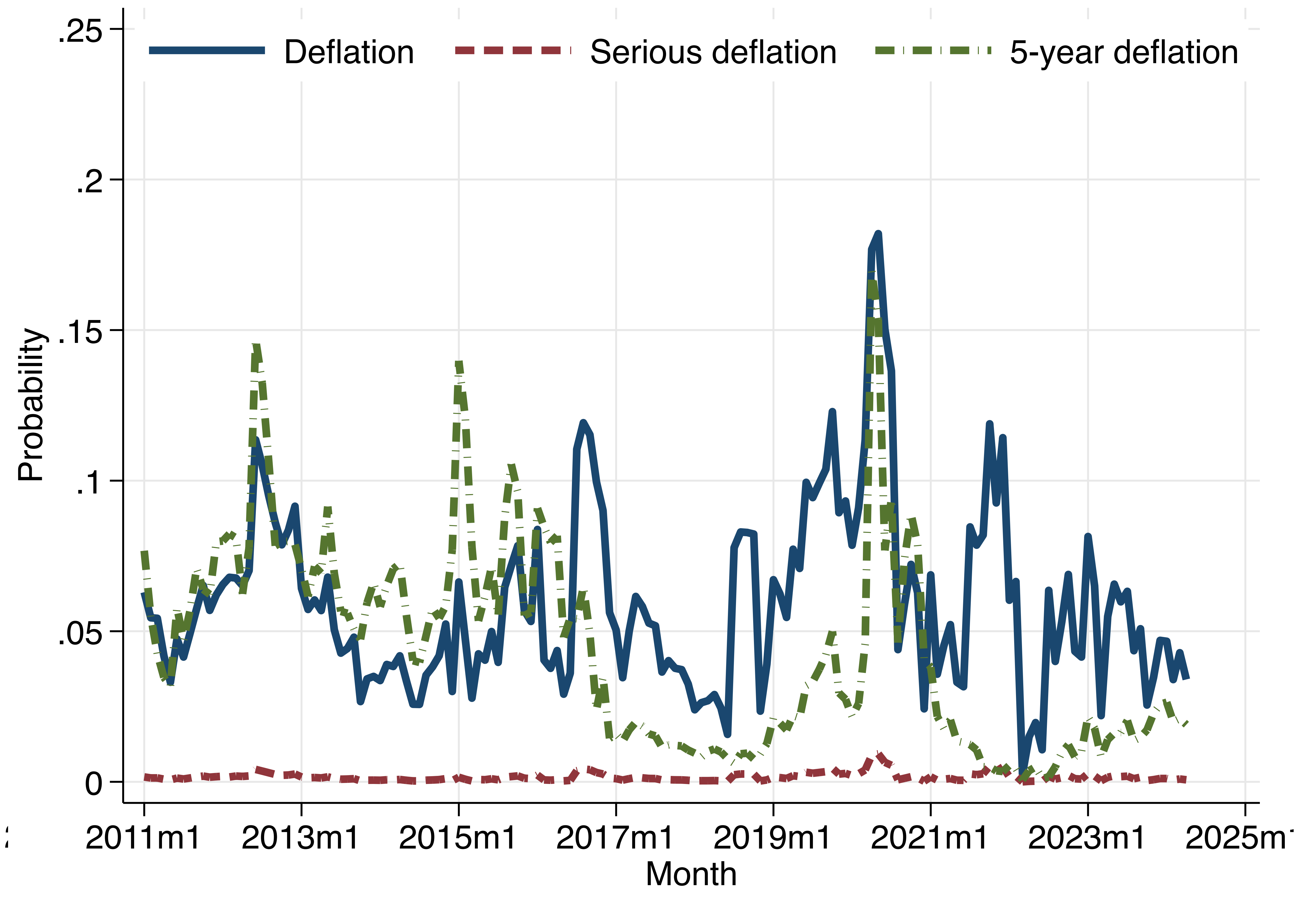
Appendix: extras

Deflation with pooled risk estimates

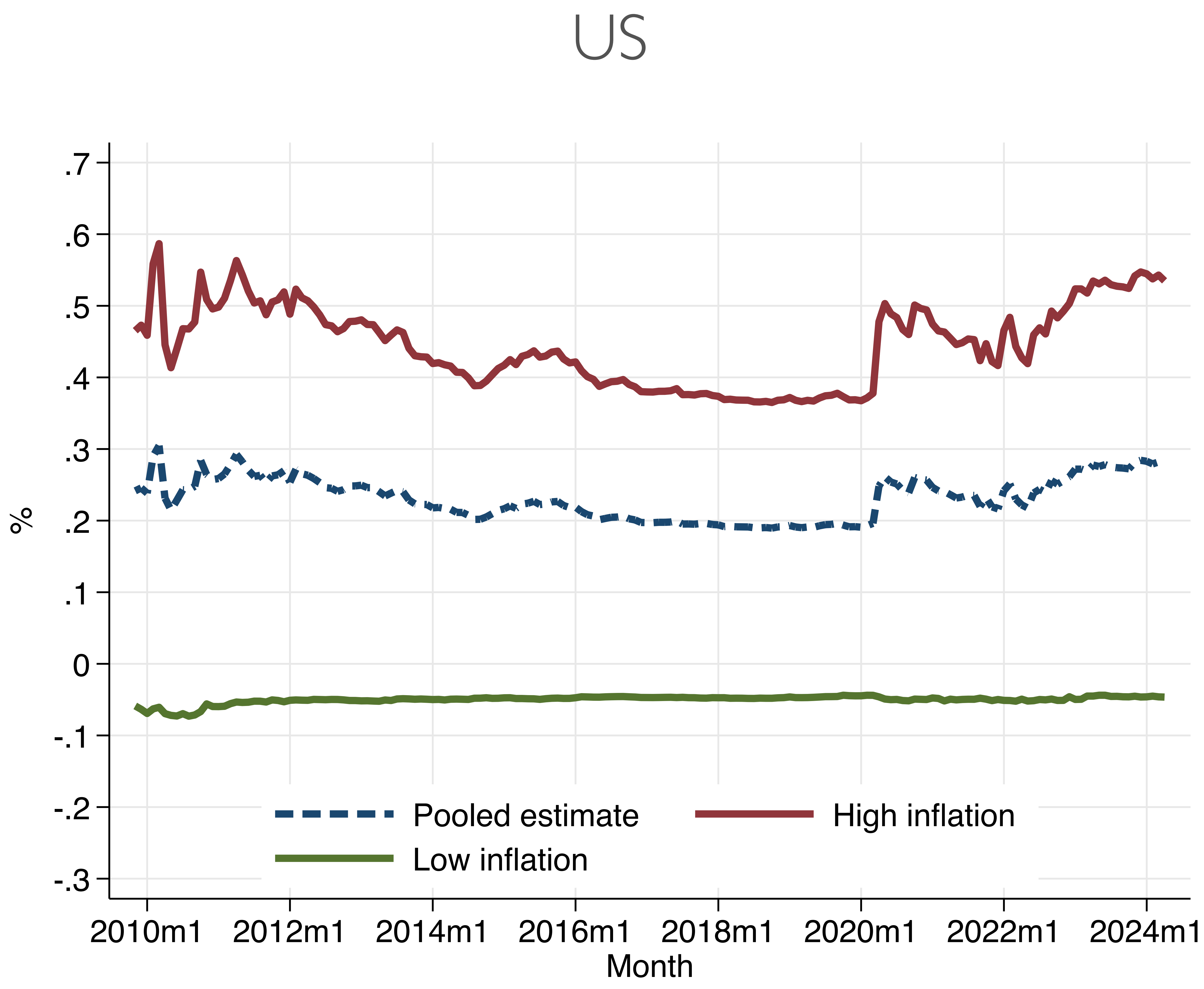
US



EZ



Risk premia


US
.7
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.4
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-.2
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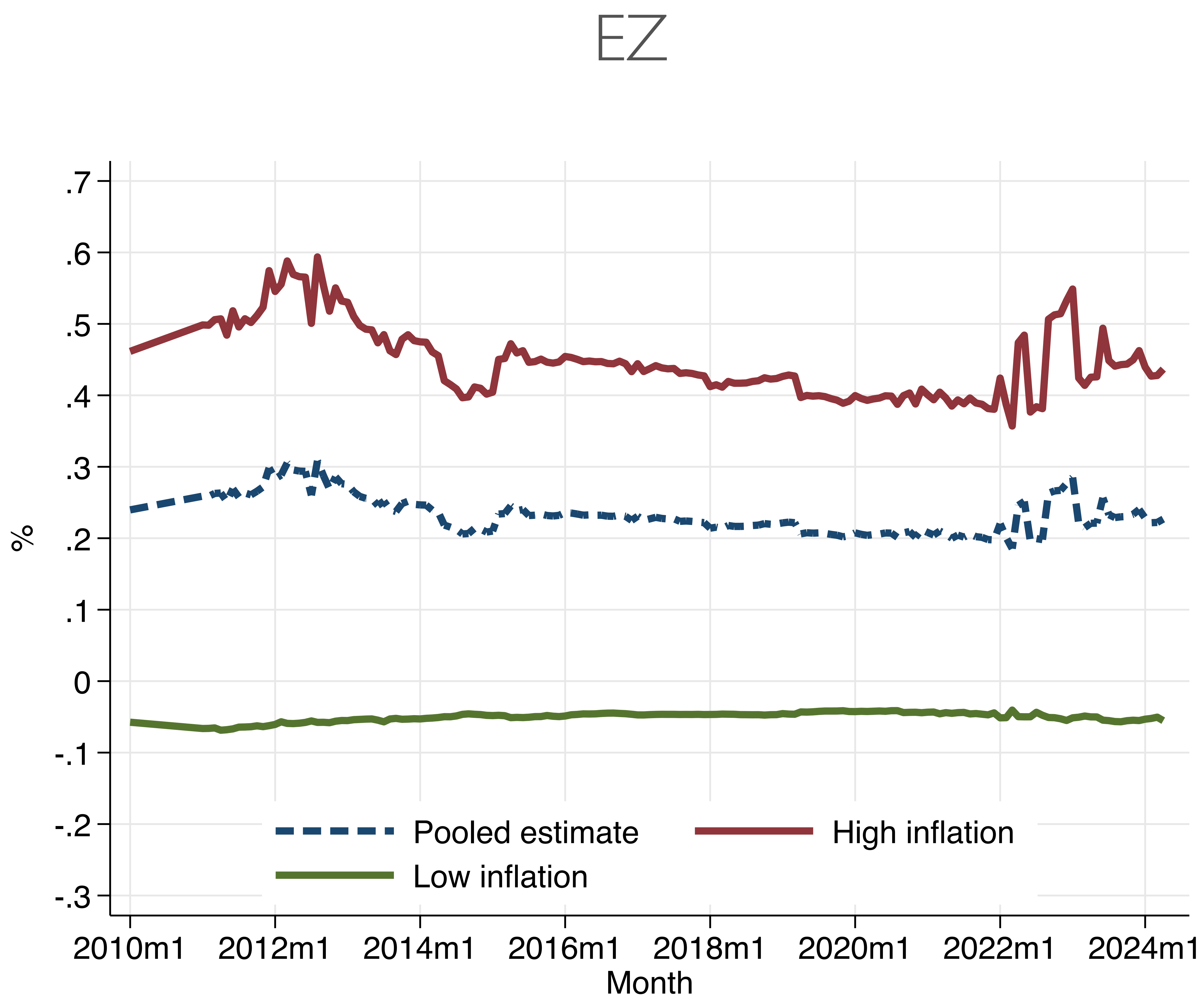
%
2010m1
2012m1
2014m1
2016m1
2018m1
2020m1
2022m1
2024m1
Month
Pooled estimate
High inflation
Low inflation


EZ
.7
.6
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.4
.3
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.1
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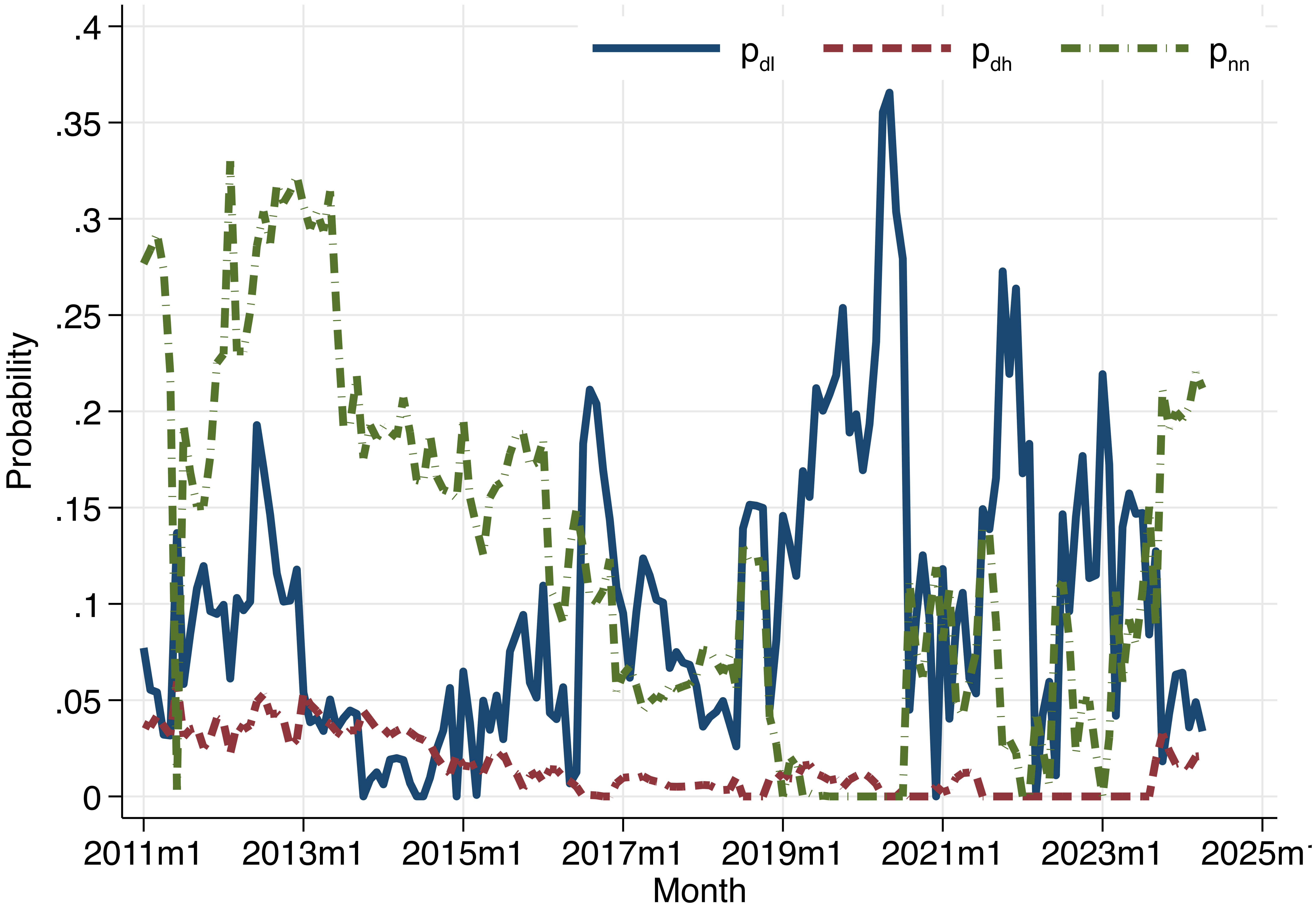
%
2010m1
2012m1
2014m1
2016m1
2018m1
2020m1
2022m1
2024m1
Month
Pooled estimate
High inflation
Low inflation

EZ dynamic model parameter estimates



Pareto distribution over pooled sample