How Likely Is an Inflation Disaster?*

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Abstract

Long-dated inflation swap contracts provide widely used estimates of expected inflation. We develop methods to estimate complementary tail probabilities for persistently very high or low inflation using inflation options prices. We show that three new adjustments to conventional methods are crucial: inflation, horizon, and risk. We find that: (a) U.S. deflation risk in 2011–2014 has been overstated, (b) ECB unconventional policies lowered deflation disaster probabilities, (c) inflation expectations deanchored in 2021–2022, (d) reanchored as policy tightened, (e) but the 2021–2024 disaster left scars, and (f) U.S. expectations are less sensitive to inflation realizations than in the eurozone.

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1 Introduction

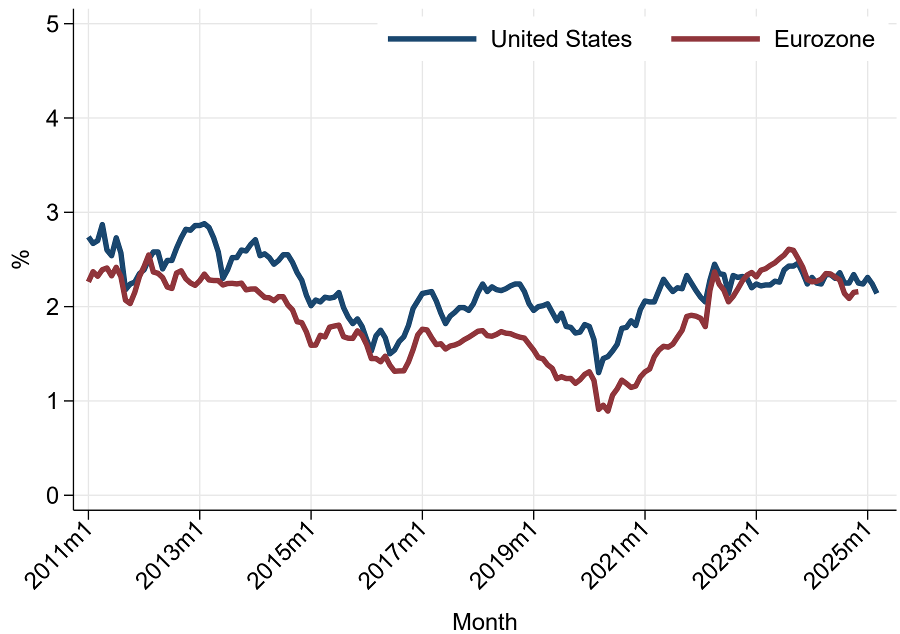
The 5-year-5-year (5y5y) forward expected inflation rate measures expected inflation in 5 years' time averaged over the following 5 years. It is a common indicator of whether long-run inflation expectations are well-anchored at the central bank's target (e.g., Gürkaynak, Levin and Swanson 2010). Policy makers find it useful because it both strips out current temporary fluctuations and averages over a long period of time, thereby providing focus on what monetary policy can achieve. In speeches, they often point to the approximate constancy of the 5y5y over the past 25 years (see Figure 1) to claim success at anchoring expectations, and even small changes in this measure can trigger large shifts in policy: a decline in the eurozone in 2011–2014 justified the start of quantitative easing.[1]

However, the 5y5y rate is a point estimate of an average. The distribution of its values could be extremely tight or dispersed. Making decisions under uncertainty typically requires knowing the whole distribution of future inflation rates, not just their expected value. Especially important for risk management are probabilities of extreme inflation realizations, which we will refer to as inflation disasters. It is these tail events that are associated with large costs of inflation, both in models of monetary policy and in opinion polls, as happened over 2021–2024, or during the German hyperinflation of the 1920s, or the stagflation of the 1970s.

This paper develops the methods to provide counterparts to Figure 1 in the form of tail probabilities of inflation disasters using traded option prices and minimal assumptions about preferences for pricing risk or inflation dynamics. Our objects of interest are two

[1]Reporting from the August 2014 Jackson Hole meeting where the ECB justified its use of quantitative easing, the *Financial Times* (2014) noted: "Mr Draghi had highlighted the inflation swap rate...never before August's Jackson Hole speech had a president of the ECB made such a clear link between its behavior and policy action."

Figure 1: Expected 5y5y inflation



This figure plots the 5-year-5-year forward inflation rates for the United States and eurozone. The figure shows that the rates for both regions fluctuate but remain generally stable. *Sources*: FRED for the United States and Bloomberg for the eurozone.

probabilities:

$$Prob_t[\pi_{T,T+H}/H > \bar{\pi} + d] \quad \text{and} \quad Prob[\pi_{T,T+H}/H < \bar{\pi} - d]. \tag{1}$$

Starting from the present t, a distant future is T years away, and a long horizon is denoted as H further years. Future long-term inflation is $\pi_{T,T+H}$, defined as the change in the log of the price level between the two dates in the subscript, while $\bar{\pi}$ is the inflation target, and d is the size of the disaster. These probabilities answer the question: What is the current market-perceived probability that inflation will be persistently above or below the $\bar{\pi}$% annual target between T and $T + H$? For example, what is the current probability that average inflation will be above 4% (2pp above the 2% target on average) between 5 and 10 years from now?

In our empirical implementation, we provide 5y5y estimates of these probabilities for the United States and the eurozone starting in October of 2009 and January of 2011, respectively, until October 2024. For disasters, we consider both high inflation and deflation, $d = 0.02$, or severe high inflation and deflation, $d = 0.03$.[2] We use these estimates to measure the success of monetary policy at anchoring market inflation expectations, and to judge the impact of different frameworks and policies on the stability of these anchors.

This paper makes a methodological contribution, which in turn leads to a revision of the recent history of inflation expectations given new empirical estimates.

We provide steps to translate the prices of traded inflation derivatives into risk-neutral and physical-measure probabilities of inflation. We show that using standard methods on options data to measure the goal in Equation (1) results in inaccurate estimates that can grossly over- or understate the desired probabilities. Three adjustments are required, which are conceptually understood when measuring expected average inflation with yields data, but have not been appreciated when measuring expected tail probabilities of inflation with options data.

First, units have to be adjusted to match Arrow-Debreu probabilities. When inflation is more likely, this raises the nominal payoff of a call option but it also lowers its real payoff. The conventionally used nominal state prices are therefore too low for high inflation states since a nominal payoff of \$1 in a future high-inflation state is worth less. For low inflation, the opposite is true.[3]

Second, traded options pay out based on realizations of inflation at $\pi_{0,T}$ and $\pi_{0,T+H}$,

[2]Given the 5-year horizon, high inflation is a cumulative 10 log-point deviation of inflation from target, and severely high is equal to 15 log-points, justifying the use of the word disaster. Higher choices of d are difficult to implement due to little trade of options further in the tails.

[3]This point applies to other derivatives as well, so our method can be used to adjust other financial-market-based probabilities. However, for noninflation options, this would require knowing the distribution of inflation conditional on the fundamental that the option is written on. For inflation options, that conditional distribution is a trivial point mass, making the adjustment simple.

but not for the desired forward horizon $\pi_{T,T+H}$.[4] If inflation expectations become unanchored gradually, this sluggishness implies that both the 5-year (5y) and the 10-year (10y) probabilities can understate the probability of a 5y5y inflation disaster. Given our focus on expectations, we innovate by estimating the perceived sluggishness revealed by the pricing of options at different horizons, as opposed to using past sluggishness in inflation realizations as has been done before.

Third, option prices imply probabilities adjusted for risk (or risk-neutral probabilities), but since marginal utility is likely high during disasters, their prices will over-state the actual, or physical-measure, tail probabilities. Building on recent work on rare output disasters, we propose an adjustment that does not require specifying the full dynamics of the stochastic discount factor that prices inflation risk, but instead uses prices of out-of-the-money options.

Empirically, we find that the three adjustments can be large. For instance, during the 2021–23 period of rising U.S. inflation, the median adjustment factors for inflation, horizon and risk were 1.24, 0.38, and 0.66, respectively. As a result, while a simple reading of the 10y option prices would suggest a median 14.0% probability of a 5y5y inflation disaster, the 5y5y actual probability was 4.2%.

As always, what adjustments should be made depends on the application. Only the inflation adjustment is needed if the goal is to have risk-neutral probabilities of tails for inflation disasters between the present and a distant future. Adding the risk adjustment gives corresponding physical probabilities, and adding the horizon adjustment gives forward physical probabilities between two distant futures. Alternatively, making only the inflation and horizon (but not risk) adjustments provides estimates of risk-neutral forward probabilities.[5]

[4] There are forward-starting options for 1-year horizons ($H = 1$) (which we will use later) as opposed to the longer horizon $H = 5$ that aligns with policy makers' interest in the 5y5y horizon.

[5] The website https://r2rsquaredlse.github.io/web-inflationdisasters/ provides time series of the infla-

Of independent interest, we provide estimates of the dynamic properties of inflation as perceived by market participants. They show a fall in stochastic volatility in the last decade, and a perception that disasters are short-lived. Likewise, we provide tail-focused estimates of the inflation risk premium. We find that periods of high inflation carry a large risk adjustment. In contrast, periods of deflation are associated with a smaller drop in output, thus resulting in a correspondingly lower risk adjustment.

Our second contribution is to the history of expectations. We use our estimates to reassess whether or not inflation expectations have been anchored and how monetary policy has affected anchoring. Our sample includes periods of elevated probabilities for both future deflation and high inflation. We reach six conclusions.

First, we reexamine the market-perceived probability of the United States falling into a deflation trap in 2011–2014. At the time, it was judged to be very high and justified expansionary monetary policy to fight the liquidity trap. Estimates based on our new methodology show that this probability was significantly lower than previously appreciated using conventional measures. We find that the risk of short-term deflation was at times elevated, but not the risk of a deflation trap at the 5y5y horizon.

Second, we find that the risk of an eurozone deflation trap persisted throughout the sample and is significantly higher than in the United States. The unconventional monetary policies since 2014 and the ECB's mission review of 2022 succeeded in lowering the probability of deflation in the near future, but not completely at lowering the perceived risk of a deflation trap over the long run.

Third, we find a large increase in the probability of a high-inflation disaster in the United States between the third quarter of 2021 and the second quarter of 2022. The same is true in the eurozone, but starting later and more concentrated in the first half of 2022. The mean of the distribution of expected inflation moved little, leading policy makers at

tion disaster probabilities.

the time to conclude that expectations were anchored and the observed increase in inflation would be temporary (Lagarde 2021, Powell 2021). But the tails showed deanchoring. The probability of an inflation disaster peaked in the United States at 10% in May 2022 and jumped from 1% to 6% in the eurozone between December of 2021 and June of 2022.

Fourth, we find that this deanchoring had a U-turn in 2022 that coincided almost to the month with the U-turn in monetary policy and the hiking of policy rates. By the end of 2022, the probability of a disaster had stabilized below 4% in the United States, while it has on average been 8.0% in the eurozone in the last twenty-four months of the sample. Since the deanchoring coincided with unusually loose monetary policy, and the reanchoring with tightening policy, this provides support for a tight link between monetary policy and the inflation anchor.

Fifth, the probabilities of an inflation disaster by the end of the sample are two to three times higher than they were between 2011 and 2019. The inflation disaster of 2021–2024 has left scars in market perceptions. This supports theories where credibility depends on realized outcomes.

Sixth and finally, we calculate how sensitive are the probabilities of disaster to temporarily high inflation, either in the present or in the near future. We find that in the United States, expected inflation is well anchored in the sense of being insensitive to inflation realizations, but this was less so until recently in the eurozone.

2 Connection to the Literature

This paper is related to three strands of the literature.

2.1 Other uses of inflation options data

Closest to our paper, a small literature has constructed inflation probability measures from the market prices of inflation options. Kitsul and Wright (2013), hereafter KW, the first paper in this literature, uses U.S. data over a three-and-a-half year period starting in October 2009 and estimates probabilities of deflation at 1-, 3-, 5-, and 10-year horizons (starting from today).

Methodologically, our paper begins where KW ends: we apply our three adjustments to the state prices that they produce. While their measures could be used for the valuation of nominal payoffs, to assess physical probabilities of inflation requires following our method. In the applications, we use a much larger sample that includes the recent inflation disaster and the other largest currency in the world; we revisit the probability of deflation during the common sample (reaching a different conclusion from theirs); and we study the connection between anchoring of expectations and monetary policy.

There are five important differences between KW and our paper. First, KW uses conventional methods to extract state prices from options prices. We adjust for the effect of inflation on the real payoffs, so that the state prices can match Arrow-Debreu probabilities (our first adjustment).

Second, a main focus of our paper is the construction of forward probabilities (our second adjustment). KW only present probabilities from today onwards. The overwhelming focus of policy makers on the 5y5y measure testifies to the relevance of this forward approach, as do common debates on whether inflation is transitory or persistent.[6] In our applications, we find that this distinction is quite important for the key macroeconomic debates: in 2011–2020, the probability of a short-lived deflation was very different from

[6]In their working paper, Kitsul and Wright (2012) present distributions of forward inflation for four dates in 2011 and 2012 based on the very restrictive assumptions of risk neutrality and inflation innovations being independent over time. These assumptions are not consistent with market participants' expectations or data on rare disasters.

the probability of a deflation trap, and when an inflation disaster actually happens, as in 2021–2024, assessing whether long-run expectations are anchored requires excluding the near future.

Third, KW adjusted the probabilities for risk (as in our third adjustment) but by estimating and using a statistical model for the stochastic discount factor with respect to inflation risk that depends on realizations of actual U.S. inflation. Instead, we let the options prices at different horizons tell us what the market believes are the dynamics of inflation. We only use historical data from eighteen countries on inflation and GDP growth to measure marginal utilities explicitly during inflation disasters. Moreover, KW study the relation of their stochastic discount with inflation, but do not use it to construct physical probabilities of disasters from the options data, which is our focus.

Fourth, our sample period is approximately four times as long. KW use data from an options market that had just become active, while our sample includes many more years when this market was mature. The longer sample also allows us to apply the estimates to important debates surrounding inflation and monetary policy in the last 10 years, and especially to the recent period of dramatically elevated inflation where the question of inflation expectations becoming unanchored was central. Almost none of our applications could have been studied using the KW sample period.

Fifth, and finally, we construct eurozone inflation disaster probabilities and discuss how eurozone policy affected them. The challenge the ECB has faced in fighting deflation, as well as the comparison between the United States and the eurozone are an important part of our paper.

Related, Fleckenstein, Longstaff and Lustig (2017) use U.S. data on inflation swaps and options through October 2015 to estimate a stochastic volatility model of inflation dynamics that allows for time-varying risk premiums. They also calculate probabilities of inflation being low or high at various horizons. The differences relative to this paper are the same as for KW: the three adjustments, as well as the longer sample period and the inclusion of the eurozone allowing us to discuss a rich set of applications. Moreover, when adjusting for risk, we allow for disaster-specific prices of risk. Fleckenstein, Longstaff and Lustig (2017) instead have one inflation risk premium that is not state specific.

Mertens and Williams (2021) fit a New Keynesian model to U.S. data on interest rates and option prices. It calculates forward probabilities assuming that inflation follows a Gaussian random walk, an assumption which we show to be inconsistent with the market's expectations. Also, it does not make the inflation adjustment to nominal payoffs, nor adjust for risk. As we show, each of these adjustments matters significantly.[7]

2.2 Term structure models

A large literature extracts information about inflation from market prices by fitting term structure models to real and nominal yield data, sometimes including inflation swap prices (Christensen, Lopez and Rudebusch 2010, Christensen, Lopez and Rudebusch 2015, Haubrich, Pennacchi and Ritchken 2012, Hördahl and Tristani 2012). These papers focus on estimating the expected average inflation rate, with adjustments for inflation, horizon, and risk that exploit the linearity of the expectations operators. We, instead, focus on the tails of the distribution for inflation, and on using data from option prices, which requires different methods altogether to deal with inflation, horizon, and risk.

Our adjustment for inflation is exactly 1 for expected average inflation, so it is not relevant for that literature. In turn, adjusting for horizon is easy for expected inflation: the 5y5y expected inflation is just the 10y expected inflation minus the 5y expected inflation. This is not so for the probability of an inflation disaster (or any percentile for that matter), where the horizon adjustment requires the full distribution of outcomes, which we show

[7] Gimeno and Ibanez (2018) are closer to us in goal but impose assumptions that are more restrictive than ours. Hilscher, Raviv and Reis (2022) make the inflation adjustment, but do not explain it or quantify its effect, nor do they adjust for horizon and risk. They also discuss a very different set of issues.

how to estimate using the options data alone. Finally, because of our focus on the tails, we only estimate a risk adjustment that is specific to inflation-output disasters, as opposed to risk for any realisation of inflation as in typical term structure models.

Of course, any fully specified time-series model of inflation contains predictions for the distribution of future outcomes. Many term structure models do so, even if they are estimated using only the expectations of the average in yields. Because our options data is for the tails, we provide estimates for the disasters directly without making all the assumptions that come with specifying a full probability model for inflation dynamics.

2.3 Other measures of the inflation risk premium

Our model of inflation risk draws on the literature on equity disasters (Barro 2006, Gabaix 2012, Barro and Liao 2021), while we focus on inflation disasters. Our method can also be used to construct probabilities for the S&P 500 or for currency changes, subject to knowing the probability that large changes in those prices coincide with high or low inflation. But, options on equities or currencies almost always have short horizons, between 1 week and 1 year, for which the adjustments are less quantitatively important.

2.4 Tail macroeconomic outcomes

A small literature focuses on tail outcomes for inflation disasters, specifically inflation at risk (Kilian and Manganelli 2007, Banerjee et al. 2024, Andrade, Ghysels and Idier 2012, Lopez-Salido and Loria 2020). Estimates based on empirical distributions have to pool across many countries and long periods of time with different inflation regimes. Instead of outcomes of realized inflation, we measure market perceptions of this risk. Because the possibility of extreme and persistent inflation events is constantly traded, they provide many more observations on the likelihood of inflation disasters that are region specific.

2.5 Surveys

A few papers look at expectations of disasters in surveys (Reis 2021, Ryngaert 2022). We instead take the perspective of financial markets. Very few surveys ask respondents about tail probabilities of distant-horizon inflation, and the few that do (the Survey of Professional Forecasters for the United States) move little over time. Time series of dispersion in surveys about long-horizon inflation are more useful, but disagreement, which many surveys capture, and uncertainty, which we measure, are not the same (Reis 2020, Coibion et al. 2021).

3 Constructing Probabilities of Inflation Disasters: Theory

We explain the intuition behind our method in a stylized setting with only high-inflation disasters before deriving its applicability in a general setup.

3.1 The intuition of the method in a simple setup

There are three periods, $0, 1, 2$, and four possible realizations of inflation: low, target, moderately high, and disaster. Let $\bar{\pi}$ denote the inflation target and assume that inflation is on target in period 0. After that, the probability that inflation is on target is p_π, and we assume that expected inflation for periods 1 or 2 is equal to the target. Disaster inflation is $\bar{\pi} + d$.

From today's perspective, the probability of an inflation disaster in period 1 is p_d. The conditional probability that there is an inflation disaster in period 2 is p_d if inflation was on target, p_{md} if inflation was moderate in period 1, and p_{dd} if inflation was a disaster. Note that we assume that the economy does not enter a high-inflation disaster from the low-inflation state. These are stylized assumptions that we make only for the purposes of exposition in this section.

Finally, $i(1)$ is the nominal interest rate between dates 0 and 1, $r(1)$ its real counterpart, and m_d is the (real) stochastic discount factor when there is an inflation disaster.

3.1.1 Conventional measures

Imagine that we have data on options that pay one nominal unit if π_d is realized in period 1, and zero otherwise. Assuming no arbitrage, the price of that option is equal to $a_d(1) = p_d \exp(-\bar{\pi} - d) m_d$: when the event with probability p_d is realized, it pays \$1, which in real terms requires an inflation adjustment $\exp(-\bar{\pi} - d)$, that is discounted by the real stochastic discount factor m_d, reflecting the marginal utility of the future payoff.

The conventional approach is to construct the price for the disaster state as: $n_d(1) = a_d(1) \exp(i(1))$. This can be thought of as a probability in the sense that it is nonnegative and summing it with the state prices for the other three states gives one. But what does it measure?

3.1.2 First adjustment: Risk-neutral (inflation-adjusted) probabilities

Arrow-Debreu securities pay one unit of *consumption*, not \$1, in each future state. Therefore, the price of the disaster A-D security is $p_d m_d$. The associated A-D probability is then $q_d(1) = p_d m_d \exp(r(1))$. This is the real risk-neutral probability.

It follows right away that:

$$q_d(1) = n_d(1) \exp(r(1) + \bar{\pi} + d - i(1)) \approx n_d(1) \exp(d). \tag{2}$$

The first equality comes from the definition of $n_d(1)$, while the approximation comes from the assumption that expected inflation—the gap between the nominal and the real interest rates—is equal to the target inflation level. The conventionally measured probabilities from options $n_d(1)$ must therefore be adjusted by the disaster size d. This is our first adjustment.

Intuitively, when the disaster happens, and the option pays, its $1 is now worth less in real terms. Economic agents therefore pay less for this option than if they were suffering from money illusion. Researchers in turn need to adjust for this effect as well.

If we are calculating probabilities near the inflation target (so d is close to 0), as sometimes is done by central banks, then this adjustment factor is negligible. Likewise, even for $d = 0.03$, if the horizon is short, then the adjustment factor is quantitatively not that significant. However, if we are looking at disasters over long horizon, say 10 years, then the adjustment factor is $\exp(10 \times 0.03) = 1.35$. Reporting $n_d(1)$ based on the price of a well out-of-the-money long-dated high inflation option thus significantly underestimates the risk-neutral probability $q_d(1)$.

3.1.3 Second adjustment: Forward (horizon-adjusted) probabilities

Imagine that our goal is to measure the probability of having a disaster in period 2. From the perspective of the present, this is $p_m p_{md} + p_d p_{dd} + p_\pi p_d$.

The price of an Arrow-Debreu security that paid one unit of consumption in period 2, if there is an inflation disaster in that period, would provide an estimate of the risk-neutral probability, $q_d(2)$.

However, we do not have the option prices that match this security. Looking at either short-dated or long-dated cumulative inflation options does not capture the desired probability. The short-dated option from the previous section that pays if there is a disaster in the first period provides an estimate of p_d via $q_d(1)$, while a long-dated option that pays if there has been a disaster that lasts for two periods would give an estimate of $p_d p_{dd}$. Since inflation moves sluggishly, $p_m p_{md} / p_d$ is likely above one, so that both short and long-dated options understate the desired forward probability.

To calculate the forward probability, we need information on the extent to which inflation is sluggish. Fortunately, there are traded forward-starting options, but for subperiods of our hypothetical period 2. Namely, in the data, there are forward contracts for annual inflation within our 5-year desired periods. They provide the missing data.

3.1.4 Third adjustment: Actual or physical measure (risk-adjusted) probabilities

Finally, with an estimate of m_d, we can go from the risk-neutral to the physical probabilities. Importantly, to answer the question in this paper, one does not need a full model of risk that gives the stochastic discount factor at all states. Only the risk that is correlated with inflation in disaster times is relevant. Moreover, it is likely that the disaster state adjustment will be the largest adjustment of the three states.

Imagine then that the main source of variation in the stochastic discount factor is whether or not there is a consumption disaster. So, m_d is a function of consumption, which can either be normal or be in a disaster. Conditional on an inflation disaster, let $\tilde{p}$ be the conditional probability that there is a consumption disaster as well. Because a consumption disaster is a time of elevated marginal utility, then the ratio of m_d when there is a consumption disaster to the marginal utility when there is none—call this ratio $\tilde{m}$—is well above 1.

Continuing with the approximation that disasters are small-probability events, so that the marginal utility without a disaster is approximately equal to the expected one, the risk-neutral probability is:

$$q(1) \approx [(\tilde{m} - 1)\, \tilde{p} + 1]\, p_d. \tag{3}$$

Since $\tilde{m} > 1$, the risk-neutral probability will over-state the probability of an inflation disaster.

The rare disasters literature has argued that $\tilde{m}$ can be quite large. However, for inflation, the picture is a bit different. First, the relevant probability is $\tilde{p}$: that conditional on an inflation disaster, there is a consumption disaster. This is well below one. There are many times, especially outside the United States, where inflation has been reasonably high or low without any sharp fall in economic activity. Second, $\tilde{m}$, which measures the marginal utility of both an inflation and a consumption disaster relative to normal times is on average lower than in a "standard" consumption disaster because, historically, there are several episodes where an inflation disaster came with only a mild (or no) recession. Therefore, the adjustment for risk is not as dramatic as the one in the literature on the equity premium.

What the formula shows is that to calculate the necessary adjustment factor the two relevant quantities to measure are $\tilde{m}$ and $\tilde{p}$. Since there is already a well-established literature measuring them for consumption disasters, and since we have corresponding data on inflation, combining them provides a path forward to identify the two parameters.

3.2 The theoretical result

This section sets out a general framework and derives the key theoretical result on the three adjustment factors to go from option prices to the probability of inflation disasters.

3.2.1 Uncertainty about inflation

Inflation is a random variable and has an associated probability distribution $p(\pi)$.[8]

[8] In the Internet Appendix we use a setup that allows for different states of the world s that have the same level of inflation but differ in other dimensions, for example, consumption.

3.2.2 Inflation securities and inflation risk

The nonnegative price in consumption units of an Arrow-Debreu inflation security that pays one unit of the consumption good if inflation is π at the future date is equal to:

$$b(\pi) = p(\pi)m(\pi), \tag{4}$$

where $m(\pi)$ measures the average marginal utility across states of the world where inflation is the same. Since $m(\pi)$ varies *only* with inflation, it has all the information relevant to assess inflation risk.[9]

3.2.3 Risk-neutral Q-probabilities

The real risk-free security pays a constant one unit of consumption. The inverse of its price is e^r, where r is the real interest rate. Since this security has an identical payoff as buying one inflation security for each possible value of inflation, it follows that by no-arbitrage: $e^{-r} = \sum_\pi b(\pi) = \sum_\pi p(\pi)m(\pi)$.[10] Defining $q(\pi) = b(\pi)e^r$, it is nonnegative and adds to 1 across inflation rates. This is the risk-neutral probability of this inflation rate.

3.2.4 N-probabilities

In match what is traded in the market, consider a security that pays one nominal unit at the future state-date. Its price is $a(\pi) = b(\pi)e^{-\pi}$. Importantly, if inflation is high, this is lower than that of $b(\pi)$, because the nominal unit delivered by this security is worth less in real terms than that of the Arrow-Debreu inflation security. The nominal interest

[9] With other sources of risk in the economy, $m(\pi)$ would average across them. The Internet Appendix generalizes this. This pattern is present in the data: between 2000 and 2020, the U.S. economy went through booms and busts, but inflation was approximately unchanged.

[10] Also, as is standard, e^{-r} is the expected SDF or marginal utility of consumption growth.

rate i is likewise defined as the inverse of the price of a security that pays one nominal unit for sure next period $e^{-i} = \sum_{\pi} b(\pi)e^{-\pi}$. Combining these two, one can define an N-probability (for "nominal risk-neutral probability") as $n(\pi) = b(\pi)e^{i-\pi}$, which is itself nonnegative and adds up to 1.

3.2.5 Linking $Q-$ and $N-$ probabilities

Let $\pi^e = i - r$, be the break-even expected inflation. It immediately follows that risk-neutral and nominal probabilities are related according to:

$$q(\pi) = n(\pi)e^{\pi - \pi^e}. \tag{5}$$

The Q-probability of average expected inflation coincides with the N-probability. But as we go towards the tails, away from that average they are increasingly apart.

3.2.6 Time and horizons

Starting from the present, the joint risk-neutral probability density of inflation over the following T periods, and over the remaining H periods, is $q(\pi_{0,T}, \pi_{T,T+H})$. From the definition of marginal and conditional distributions: $q(\pi_{T,T+H}) = \sum_{\pi_{0,T}} q(\pi_{0,T}, \pi_{T,T+H})$ and $q(\pi_{T,T+H}|\pi_{0,T}) = q(\pi_{0,T}, \pi_{T,T+H})/q(\pi_{0,T})$. Finally, because of the definition of inflation, $\pi_{T,T+H}H = \pi_{T,T+1} + \pi_{T+1,T+2} + ... + \pi_{T+H-1,T+H}$, and there is a joint distribution of $q(\pi_{T,T+1}, \pi_{T+1,T+2}, ..., \pi_{T+H-1,T+H})$. Combining all of these probabilities:

$$\begin{aligned} q(\pi_{T,T+H}) = q(\pi_{0,T+H}) \sum_{\pi_{0,T}} \Bigg[& \frac{q(\pi_{0,T})}{q(\pi_{0,T+H})} \times \\ & \sum_{\pi_{T,T+1},...,\pi_{T+H-1,T+H}} q\left(\pi_{T,T+1}, ..., \pi_{T+H-1,T+H} | \pi_{0,T}, \sum_{j=1}^{H} \pi_{T+j-1,T+j} = \pi_{T,T+H} \right) \Bigg] \end{aligned} \tag{6}$$

The expression in the bottom line takes into account the persistence of inflation across successive periods within the interval of time $(T, T+H)$. On the top line is the adjustment for the sluggishness of inflation over the long horizons.

3.2.7 Final result

Combining all the steps, we get the result as a formula to obtain the desired disaster probabilities:

Proposition 1. *The probabilities of high and low inflation disasters are, respectively:* $\sum_{\pi_{T,T+H} > H(\bar{\pi}+d)} p(\pi_{T,T+H})$ *and* $\sum_{\pi_{T,T+H} < H(\bar{\pi}-d)} p(\pi_{T,T+H})$*, where*

$$\begin{aligned} p(\pi_{T,T+H}) = & \underbrace{n(\pi_{T,T+H})}_{\textit{Options Data}} \\ & \times \underbrace{\left(e^{(\pi_{T,T+H} - \pi^e_{T,T+H})H}\right)}_{\textit{Real Factor}} \\ & \times \underbrace{\left(e^{-r_{T,T+H}H} m(\pi_{T,T+H})\right)}_{\textit{Risk Factor}} \\ & \times \underbrace{\sum_{\pi_{0,T}} \left[\left(\sum_{\ldots = \pi_{T,T+H}} q(\pi_{T,T+1}, \ldots, \pi_{T+H-1,H} | \pi_{0,T}) \right) \frac{q(\pi_{0,T})}{q(\pi_{0,T+H})} \right]}_{\textit{Horizon Factor}}. \end{aligned} \tag{7}$$

4 Data and Empirical Implementation

The empirical implementation of the adjustment factors in proposition 1 requires data from option prices, a statistical model of the incidence of economic disasters over horizons, and an economic model of the stochastic discount factor when there is an inflation disaster.

4.1 Data on inflation options

There is an active market for U.S. and eurozone inflation options. The same players that buy and sell nominal and inflation-indexed government bonds, or that trade in the inflation swap markets, are often present in these option markets to hedge some of their positions. Therefore, even though trading volumes will differ, these data are as good as those behind Figure 1, which are used frequently.[11]

Price data exist for both call and put options for average inflation between the present and up to 15 years in the future for strike prices between -2% to 6% with 0.5% jumps. The typical call security with a strike price k pays at the future date the difference between the gross inflation rate (e^{π}) until that date and the strike price k, if the difference is positive, or zero otherwise. The price of that option today is $a(k)$.

We use U.S. options data from October 2009 to October 2024 from Bloomberg; for the eurozone, the sample starts in January 2011. While option prices are available daily, sometimes the data quality is low. To be conservative, we construct data at the monthly frequency. We focus on horizons of 5 and 10 years, which are two of the more commonly traded markets for these securities. We also compare put-call-parity real rates with those implied by the inflation swap contracts to confirm that prices are not only consistent within the options market but also across inflation derivative markets. The Internet Appendix describes how we construct the data.[12]

[11] Baumann et al. (2021) and Feldman et al. (2015) describe the use of these options data at the ECB and the Fed, respectively.

[12] These options are traded over the counter, so a valid concern is whether inflation disasters are also times when there is a higher likelihood that the sellers of the options default on their contracts. If so, this would show up in the price of other options sold by the same intermediaries. While this might have been a concern at the start of our sample, there is no indication that it is significant for most of the period that we cover.

4.2 Inflation adjustment: Risk-neutral probabilities

Recovering Q-probabilities is just as easy as recovering N-probabilities, as it relies on the same methods from options pricing, and it should always be done.[13]

The no-arbitrage pricing condition for traded securities is:[14]

$$a(k) = \sum_{\pi} \left(p(\pi) m(\pi) \max \left\{ \frac{e^{\pi} - k}{e^{\pi}}, 0 \right\} \right) \approx \int_{k}^{\infty} \left(\frac{e^{\pi} - k}{e^{\pi}} \right) b(\pi) d\pi, \tag{8}$$

where the approximation comes from assuming a continuum of inflation states and using the definition of the Arrow-Debreu prices in Equation (4).

Following Breeden and Litzenberger (1978), take derivatives with respect to k recalling that $q(\pi) = b(\pi)e^{r}$:

$$e^{r} a'(k) = - \int_{k}^{\infty} e^{-\pi} q(\pi) d\pi. \tag{9}$$

Taking another round of derivatives with respect to k, and using the definition of a distribution function $Q(\pi)$ gives a simple formula to build this distribution:

$$Q(k) = e^{r} k a''(k). \tag{10}$$

Using this formula provides a way to build the Arrow-Debreu prices directly from the option prices. The right-hand side can be measured for different strike prices: it is how sensitive the price of the option is to the strike price. Since these strike prices are themselves inflation measures, one can easily build the whole distribution for different gross inflation k.

How does this connect to the $N(.)$ probabilities conventionally measured? Since $n(\pi) =$

[13] In the Internet Appendix, we compare our daily N-probabilities with those from KW in the overlap sample for the U.S. data (October 2009 to April 2013). They are almost identical.

[14] Note that the payoff of these securities only depends on inflation, not on the entire set of states (see the Internet Appendix).

$b(\pi)e^{i-\pi}$, differentiating Equation (8) with respect to k gives another simple formula to build this distribution:

$$N(k) = 1 + e^{i}a'(k). \tag{11}$$

These probabilities are sometimes reported in the financial media, and one could alternatively start from here to get $Q(.)$ by multiplying by $e^{\pi-\pi^e}$.

This adjustment should always be made. Otherwise, from Equation (4) and the definition of $n(.)$, then $n(\pi) = p(\pi)$ only if $m(\pi)e^{i-\pi} = 1$. That is, for the conventionally-calculated probabilities to match the actual physical probabilities, it must be not only that there is risk neutrality ($m(\pi)e^{r} = 1$), but also that $\pi = \pi^e$ for every realization of π. But this is only the case if there is no uncertainty about inflation, in which case the exercise of building a distribution of inflation is not interesting. Instead, from the definition of $q(\pi) = b(\pi)e^{r}$ and Equation (4), we have that $q(\pi) = p(\pi)$ as long as $m(\pi)e^{r} = 1$. This is the case if people are neutral with respect to inflation risk, or if the classical dichotomy holds, so inflation is uncorrelated with marginal utility. The adjustment is needed because, even if investors are risk neutral, they still care about receiving a payoff in a high-inflation state that has lower real value.

4.3 Horizon adjustment: Forward probabilities

Obtaining forward expectations of inflation is straightforward (Figure 1). Starting with measures of expected inflation between the present and a faraway date, T, and between the present and a farther-away date $T + H$, the joint linearity of the expectations operator and of inflation (as a difference in logs) implies that $\mathbb{E}^{q}(\pi_{T,T+H}) = \mathbb{E}^{q}(\pi_{0,T+H}) - \mathbb{E}^{q}(\pi_{0,T})$. Going from probabilities on cumulative inflation to probabilities over a forward period is harder. Proposition 1 shows that more data beyond the two distributions for cumulative inflation, as well as a model for the time-series properties of inflation as perceived by markets, are required.

4.3.1 Data on forward starting options

There exist markets for forward-dated options at every date that will pay out depending on the realizations of inflation in $\pi_{T,T+1}$. These options are for inflation in one given year, not on the average over a longer period $H > 1$, which is our focus.[15]

The markets in which these trade are less liquid, so we want to be conservative in using them. In the data, the five distributions covering the 1-year-ahead inflation starting in 5 to 9 years are quite similar almost always. This indicates that a low-order Markov process with not too much persistence is an adequate model since, after 5 years, the marginal risk-adjusted distribution of inflation seems to have settled at its ergodic state. Therefore, and to allow for possible data concerns, we take the average of these five annual distributions and use that alone for estimation, making our approach more robust to the presence of measurement noise. Using the adjustments discussed in Section 4.2 provides an estimate of $q(\pi_{5,6}) \approx ... \approx q(\pi_{9,10})$.

4.3.2 A model of inflation persistence

Since the data are for risk-neutral inflation, the model of dynamics is for risk-neutral inflation as well. We assume that inflation is the sum of three parts: a deterministic part, which has been constant at the inflation target during our sample $\bar{\pi}$; a stochastic one capturing the ups and downs during normal times ε; and a stochastic one capturing sharp jumps during disasters, which may be positive d^h or negative d^l. Altogether, Δ represents a time period:

$$\pi_{t+\Delta} = \bar{\pi} + \varepsilon_{t+\Delta} + d^h_{t+\Delta} - d^l_{t+\Delta}. \tag{12}$$

[15] These data were used to estimate general stochastic processes for inflation in Hilscher, Raviv and Reis (2022), and are described there in detail, as well as in the Internet Appendix.

We assume that d_t^h and d_t^l are two independent common disasters that arrive as Poisson processes. We then make two major assumptions on ε_t. First, that the variance of ε_t is small relative to the size of the disaster jumps, so that inflation enters the disaster range only as a result of a disaster, or if inflation in the previous year was just below disaster levels. Second, assume that if Δ were infinitesimally small, then ε_t would approximately follow a mean-reverting Ito process with continuous sample paths in time. The result of these two assumptions is that inflation follows a first-order Markov process with a particular set of restrictions on the transition matrix.[16]

Because strike prices for inflation options come in jumps of 0.5%, our data comes in 8 bins: $\pi(i) = \{\leq -1, (-1,0], (0,1], (1,2], (2,3], (3,4], (4,5], > 5\}$. A discrete approximation of this process is a Markov chain over 8 states corresponding to these bins with an 8×8 Markov transition matrix $\mathbf{P}$:

$$\mathbf{P} = \begin{bmatrix} 1-5p_l & p_l & p_l & p_l & p_l & p_l & 0 & 0 \\ p_{dl}+p_{nn} & p_{ml} & p_{mr} & 0 & 0 & 0 & 0 & 0 \\ p_{dl} & p_{nn} & p_m & p_{mr} & 0 & 0 & 0 & p_{dh} \\ p_{dl} & 0 & p_{nn} & p_n & p_{nn} & 0 & 0 & p_{dh} \\ p_{dl} & 0 & 0 & p_{nn} & p_n & p_{nn} & 0 & p_{dh} \\ p_{dl} & 0 & 0 & 0 & p_{mr} & p_m & p_{nn} & p_{dh} \\ 0 & 0 & 0 & 0 & 0 & p_{mr} & p_{mh} & p_{dh}+p_{nn} \\ 0 & 0 & p_h & p_h & p_h & p_h & p_h & 1-5p_h \end{bmatrix}. \tag{13}$$

Starting with the low-inflation disaster state in the first row, the economy exits it with probability $5p_l$, which should be close to 1 to match the Poisson assumption on disasters. When the disaster disappears, the economy will return to any one of the normal

[16] Mertens and Williams (2021) compute forward distributions under the much stronger assumption that inflation follows a Gaussian random walk.

(nondisaster) values, though not to the state opposite and closest to the other disaster. We assume that they are equally likely reflecting the first-order Markov assumption that where it was before the disaster would not affect where it ends up now. Symmetrically, the same arguments explain the 8th row referring to the high-inflation disaster.

Turning to when inflation is close to 2%, in the third and fourth row, it may move up or down according to its normal process symmetrically with probability p_{nn}. This captures the normal inflation dynamics. Inflation may be hit by the high-inflation disaster with probability p_{dh}, or with the low-inflation disaster with probability p_{dl}.

Finally, in the 2nd and 3rd (and 6th and 7th) rows, a final ingredient appears, as there is mean reversion in the normal inflation component. The probability of staying close to the target is p_n, and the probability of staying above (or below) the target is p_m.[17] The probability of reverting towards target is p_{mr}, which in the data we find to be much higher than the probability of staying at that level.[18]

4.3.3 Estimating the model

There are six parameters to estimate: the probabilities of entering a high and low disaster p_{dh} and p_{dl}; the probabilities of exiting the disaster p_d, p_l; the probability of normal inflation moving, p_{nn}, which captures the local volatility of inflation; and the probability of elevated or low normal inflation moving back to the target, capturing mean-reversion in normal inflation p_{mr}. Given a set of parameters, we simulate many paths to calculate the probability distributions for inflation at the different horizons.

The data consist of 21 numbers per month, 7 for each of the 3 distributions: the cumulative distributions $q(\pi_{0,5})$ and $q(\pi_{0,10})$, and the average forward distribution $q(\pi_{5,6})$.

[17]Note that p_n and p_m are equal to combinations of the other parameters: $p_n = 1 - 2p_{nn} - p_{dl} - p_{dh}$. Similarly, $p_m = 1 - p_{nH} - p_{nn} - p_{mr} - p_{nL}$.

[18]For completeness, and again because probabilities have to add up to 1 within rows: $p_{ml} = 1 - p_{dl} - p_{nn} - p_{mr}$ and $p_{mh} = 1 - p_{dh} - p_{nn} - p_{mr}$.

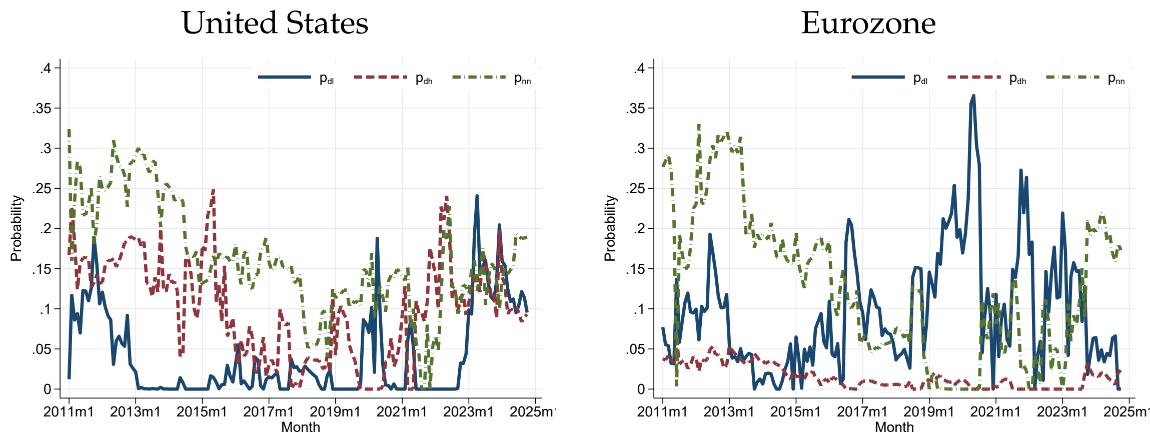
These are the moments that the model must hit, in a GMM procedure that assigns them equal weight. The overall fit, which we report in the Internet Appendix, is quite good.

In principle, we could estimate the model separately at each month, and recover parameters that are specific to each date. For parsimony, instead, we kept three of the parameters fixed over the whole sample, while letting the other three vary across months. We estimated several other candidate models, including models where four of the parameters vary over time, and where all six did so, as well as one where some parameters move at an annual frequency while others move at a monthly frequency. The Internet Appendix discusses these models, why our results are robust to this choice, and why the parsimonious setup is preferable.

The first constant parameter is p_{mr}, which captures the extent of mean reversion. For the United States, the estimate is 0.50, while for the eurozone it is 0.47, capturing the strong sluggishness of inflation. The other two are the exit probabilities for disasters, p_l, p_h. The estimates are almost exactly the same for the United States (0.1990 and 0.1998), implying that, as soon as the United States enters a disaster state, it leaves it with a probability of $5 \times 0.199 \approx 1$ shortly after. For the eurozone, instead they are 0.1999 and 0.0617, so that a high-inflation disaster is perceived to persist for more than 1 year in the eurozone with a probability of 69%.

The three time-varying parameters are the probabilities of entering a disaster, the main focus of our interest, and p_{nn}, which captures local volatility and captures the changes in inflation volatility over the sample. Figure 2 shows their estimates over time. The decline in p_{nn} for the United States since the start of the decade captures a fall in the perceived volatility of inflation, although since the pandemic that trend has reverted. Independently of this, the probability of jumping to a low-inflation disaster was high at the start, but became quite low after mid-2012, although with a significant jump in 2020. More erratic is the pattern in the probability of jumping to a high-inflation disaster. It significantly

Figure 2: Inflation dynamics: Model parameter estimates



The figure plots time-varying Markov model transition probabilities estimates for the United States and eurozone. We plot the probabilities of moving to a high or low inflation disaster and normal inflation moving.

declines after 2015, but, since the start of the pandemic, it has risen significantly.

For the eurozone, there is a similar decline in the stochastic volatility of inflation throughout the decade, and a similar uptick since the pandemic. However, the probability of a deflation disaster hitting the economy is higher than in the United States throughout the sample, and varies significantly, including a significant rise during the pandemic. The probability of a high inflation disaster stays small throughout, but rises at the very end of the sample.

4.4 Risk adjustment: Physical probabilities

If the Phillips curve was vertical at the long horizons that we consider, inflation would be uncorrelated with marginal utility. Therefore, $m(\pi)$ would be a constant, equal to the inverse of the real interest rate, and the risk-neutral probabilities would be equal to the actual probabilities.[19] However, it seems likely that inflation disasters are times where

[19] Note that people may still be arbitrarily risk averse: the stochastic discount factor may still be volatile over all the states and there is plenty of risk in the economy. But, all of it would be orthogonal to inflation:

marginal utility is high. Deflation and high inflation sometimes, even if not always, come at the same time as economic recessions. If so, at the tail of the distribution, $m(\pi)$ is high, in which case risk-neutral probabilities will over-state the actual physical-measure probabilities of disasters, because these events are particularly costly to investors. Relative to a full model of risk, however, we only need a model to price inflation risk at the tails.

4.4.1 A model of risk in inflation disasters

To model risk, we supplement our model of inflation dynamics with a model of how it comoves with consumption c_t:

$$\pi_{t+\Delta} = \bar{\pi} + \overbrace{u^{\pi}_{t+\Delta} + e_{t+\Delta}}^{\varepsilon_{t+\Delta}} + d^h_{t+\Delta} - d^l_{t+\Delta}, \tag{14}$$

$$\log(c_{t+\Delta}) = \log(c_t) + g + u^c_{t+\Delta} + \beta_0 e_{t+\Delta} - \beta^h d^h_{t+\Delta} - \beta^l d^l_{t+\Delta}. \tag{15}$$

Consumption is expected to grow at rate g subject to some shocks $u^c_{t+\Delta}$ that are independent of inflation shocks $u^{\pi}_{t+\Delta}$, but comoving in normal times due to the common shock e_t. This may be driven by multiple shocks, and may be correlated over time, but the parameter determining inflation risk premiums during normal times is the comovement scalar β_0.

Our focus is on d^h_t, d^l_t, the disasters that strike inflation, and which are nonzero with probabilities p^h, p^l. (Consumption disasters that do not come with high or low inflation do not trigger the options, so they are included in $u^{\pi}_{t+\Delta}$ and $u^c_{t+\Delta}$, respectively.) The coefficient β^h measures the size of the consumption drop when there is a high inflation disaster, while the coefficient β^l measures the size of the drop following a deflation disaster.

We follow and modify the approaches of Gabaix (2012) and Barro and Liao (2021) to model the size of the disaster. Defining the inverse fall in consumption by $z^h = 1/(1 -$

$m(\pi)$ is constant.

$\beta^h d)$, we assume that if a disaster strikes, then z^h follows a Pareto distribution:

$$F(z^h) = 1 - \left(\frac{z^h}{z_0^h}\right)^{-\alpha^h} \quad \text{with } \; z^h \geq z_0^h > 1, \alpha^h > 0. \tag{16}$$

The Pareto distribution has two parameters. The first, z_0^h is the minimum size of the jumps. The higher it is, the more average consumption falls during inflation disasters. The second is the exponent α^h capturing how quickly the tail of the distribution thins out. The lower it is, the more likely is a very large consumption disaster. The same applies for deflations, (z^d, z_0^d, α^d).

4.4.2 Estimating the model

We combine data on annual output from Barro (2006) (using real GDP per capita, as it did) with data on inflation from Jordà, Schularick and Taylor (2016) between 1875 and 2015. The data set covers 18 advanced economies, listed in the Internet Appendix, and we use it to identity periods where both inflation and output had disasters and estimate the Pareto distribution as well as the comovement parameters.

Details of the estimation and alternative estimation approaches are reported in the Internet Appendix. In the sample, the unconditional probability of an inflation disaster (10 log points above or below target for 5 years) is 12.9%, and they overlap with output disasters in 20.0% of the cases. Therefore, $\tilde{p} = 0.20$. Separating high and low inflation disasters, then $\tilde{p}^h = 0.356$ and $\tilde{p}^d = 0.085$. For the Pareto distribution, we estimate that $\alpha^h = 5.45$ and $z_0^h = 1.03$ for high-inflation disasters and $\alpha^l = 15.18, z_d^0 = 1.06$ for deflation disasters. That is, deflation disasters more rarely come with output disasters, and when they do, the falls in output are on average higher, but with significantly thinner tails.

Given an estimated model of inflation-consumption disaster comovement, we follow the standards of the rare-disasters literature (Gabaix 2012, Barro and Liao 2021) by using an Epstein-Zin model for marginal utility with a relative risk aversion coefficient of 3.

4.5 Assessing the uncertainty around the adjustments

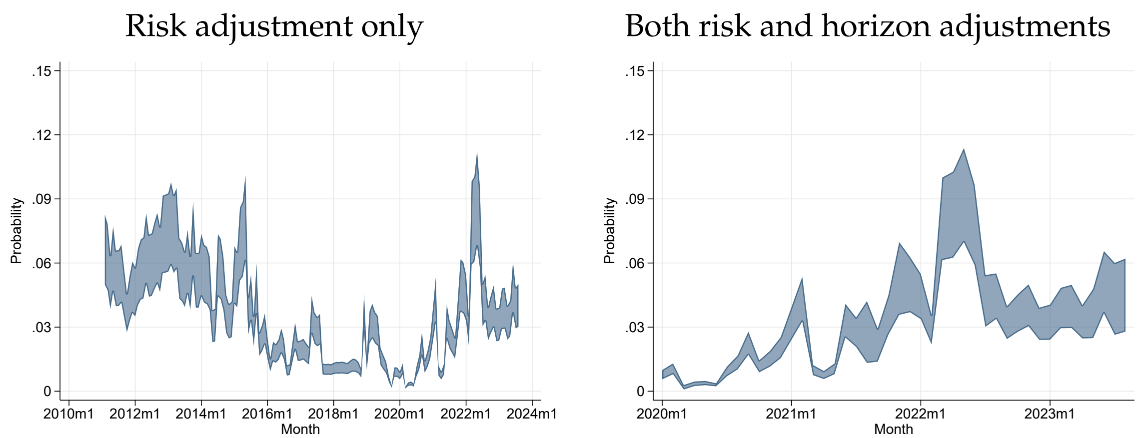
Two of our adjustments—horizon and risk—required statistical estimates, so they come with estimation uncertainty. For the risk adjustment, we estimated the parameters of the Pareto distribution. A bootstrap over the inflation-output data provides confidence bands. For the horizon adjustment, we estimated a statistical model using GMM. The co-variance matrix of the parameter estimates has the standard GMM asymptotic formula. Finally, the delta method translates these to estimates of the uncertainty around the probability of a disaster.

Figure 3 shows the 90% confidence band around the estimates of a high-inflation disaster for the United States. On the left panel are confidence bands treating the horizon adjustment as known, so only for the Pareto estimates for the risk adjustment. The right panel zooms in on the more recent period to be clearer, and considers the estimation uncertainty on horizon as well. The bands are relatively tight, between 1.5 and 2.5 percentage points in width in the more recent period. This shows that the estimation uncertainty around our two adjustment factors is relatively minor (there is no estimation needed when applying the inflation adjustment). More relevant is the time-series variation in the estimates, which is driven by changes in the market expectations reflected in the options price data.

4.6 Using market data and liquidity

Empirical results always depend on data quality. In the case of price data, the common concern is market liquidity. As always, this requires a brief discussion of the source of the data to have the right care in using our estimates and interpreting our results.

Figure 3: Confidence bands from the adjustments



The left panel shows the 90% confidence band for the U.S. 5y5y inflation disaster (> 4%) probability when standard errors take into account the uncertainty in the risk adjustment estimate; the right panel adds uncertainty in the horizon adjustment.

The derivatives market for inflation started in 2002, and grew very quickly. Conservatively, we follow several other studies (Kitsul and Wright 2013, Fleckenstein, Longstaff and Lustig 2017, Mertens and Williams 2021, Nagel 2016, Hilscher, Raviv and Reis 2022) and only use data from 2009 onward, when the market was quite liquid. Since then, Chipeniuk and Walker (2021) report that the volume of trading for inflation caps and floors quadrupled between 2009 and 2017. Since 2021, some claim that the U.S. interdealer market has virtually disappeared, while others report that since the pandemic, the market has been driven by clients' increased demand for inflation protection, predominantly through inflation caps in the dealer-to-client market.[20] The variation in our estimates during these times is reasonable and related to policy events.

Since the inflation options are actively used to hedge positions in inflation swaps, their liquidity concerns should be related. Bahaj et al. (2025) estimate liquidity premiums in the swaps market, and find that they are moderate at the long horizons that we focus

[20]See Williams (2023) and *Risk* (2023).

on in this paper, and that fundamentals drive more than 90% of the variation in prices.[21] Nevertheless, to be conservative, in our analysis of the recent history of U.S. and euro-zone inflation, we will focus on trends across many months, rather than month-to-month fluctuations, since these are likely to be more robust to liquidity changes.

Policy institutions use inflation options data regularly, to provide public data (the Federal Reserve Bank of Minneapolis produces a weekly series), research (Kozlowski 2024), and in speeches justifying policies (e.g., Schnabel 2022, Lane 2022). Policy interventions themselves can sustain liquidity in markets (Allen, Carletti and Gale 2009, Kargar et al. 2021, Falato, Goldstein and Hortaçsu 2021). An example of such proactive involvement is the creation of the FX options market by the Bank of Israel during the high inflation period of the 1980s (Fischer 2006).

An alternative to market-based data is household and professional surveys (Armantier et al. 2022, Grishchenko and Wilcox 2024, Fofana, Patzelt and Reis 2024). However, the tails of surveys reveal the extent of disagreement, while the tails from market options reveal perceptions of rare disasters. The two are conceptually very different. In the same way that market prices may be affected by liquidity, surveys can be affected by biases in answering, so that combining them to extract as much information as possible is likely preferred (Nagel 2024).

[21] Ideally, future work would propose a fourth adjustment factor that captures potentially time-varying illiquidity of the option contracts. The literature is still far from delivering this, and the direction of its impact is not obvious: since all option prices move together with the real risk-free rate, only movements in the differential liquidity of options with strike prices that are nearer or more out of the money would affect the constructed probabilities.

5 Quantifying the Adjustment Factors in the United States and the Eurozone

We now summarize the impact of the three adjustments for inflation, horizon, and risk to build disaster probabilities.

Table 1 presents summary statistics of the various steps in finding the probabilities. Panels A and B focus on high-inflation disasters, both their final probabilities and the adjustment factors to get to them, respectively. We show the medians for the full sample, and for a period in the United States, between September 2021 and August 2023 when realized inflation reached 9.1%, and the high-inflation disaster probabilities were large. Panels C and D show the U.S. deflation probabilities between January 2011 until December 2012, a time when there were heightened deflation fears and the Fed undertook QE rounds two (in November 2010) and three (September 2012) as well as operation twist (September 2011).

5.1 Inflation adjustment

The first four (two) columns in panel A (B) show the inflation adjustment. During the recent period, the median 5-year N-probability was 20.7%, while the risk-neutral Q-probability was higher at 22.8%. The median adjustment factor to move from N to Q is 1.09.[22] As expected, the effect is larger for the 10y horizon, with a median adjustment factor of 1.24, as the median N-probability was 14%, but the Q-probability was 17.2%.

For deflation, in panels C and D, the adjustments work in the opposite direction. The median 5-year N deflation probability over the 24-month period when it was heightened was 6.7%, compared to 5.6% for the Q-probability, while for the 10y horizon, the differ-

[22]The adjustment factor depends on the probability density, which is not constant, so neither is the adjustment factor.

Table 1: Three inflation disaster probability adjustments

This table reports summary statistics for various inflation disaster probabilities. We focus attention on six measures. In columns 1–4 of panel A we report probabilities of average inflation lying above 4% over the next 5 or 10 years. N denotes nominal risk-neutral probabilities, and Q denotes risk-neutral probabilities, that is, probabilities after adjusting for the effect of inflation (inflation adjustment). Column 5 reports forward risk-neutral probabilities of 5-year forward probabilities, that is the probability of inflation lying above 4% in 5 years for 5 years (horizon adjustment). The final column adjusts that probability for risk, and is denoted by P (risk adjustment). Panel B reports adjustment factors: adjusting for inflation and therefore moving from N to Q probabilities (5y and 10y); adjusting for horizon, that is, moving from Q_10y to Q_5y5y probabilities; and moving from Q_5y5y to P_5y5y probabilities. Panels C and D report deflation probabilities and adjustment factors.

A: High inflation disaster (>4 %) probabilities, 9/21–8/23

	N_5y	Q_5y	N_10y	Q_10y	Q_5y5y	P_5y5y
U.S., 9/21–8/23, median	20.7%	22.8%	14.0%	17.2%	6.3%	4.2%
U.S., 90th percentile	21.3%	23.8%	23.3%	28.9%	10.7%	7.1%
Eurozone, 90th percentile	14.1%	16.4%	16.2%	20.6%	14.2%	9.4%

B: High inflation disaster probability adjustment factors

	N to Q, 5y	N to Q, 10y	Q, 10y to 5y5y	Q to P, 5y5y
U.S., 9/21–8/23, median	1.09	1.24	0.38	0.66
U.S., median	1.12	1.24	0.40	0.66
Eurozone, median	1.17	1.33	0.92	0.66

C: Deflation (<0%) probabilities, 1/11–12/12

	N_5y	Q_5y	N_10y	Q_10y	Q_5y5y	P_5y5y
U.S., 1/11–12/12, median	6.7%	5.6%	6.9%	4.8%	6.4%	6.2%
U.S., 90th percentile	6.8%	5.9%	6.2%	4.5%	7.5%	7.2%
Eurozone, 90th percentile	11.3%	9.9%	10.5%	7.7%	12.0%	11.6%

D: Deflation probability adjustment factors

	N_5y to Q_5y	N_10y to Q_10y	Q_10y to Q_5y5y	Q_5y5y to P_5y5y
U.S., 1/11–12/12, median	0.84	0.69	1.41	0.96
U.S., median	0.85	0.72	1.33	0.96
Eurozone, median	0.90	0.80	2.29	0.96

ence was larger; median adjustment factors were 0.84 and 0.69 for the two horizons.

In the full sample, the adjustment sizes are a little smaller because extreme inflation and therefore adjustments are less likely. Still, the adjustments are higher the longer is the horizon, and they also turn out to be larger for the United States than for the eurozone. Overall, N-probabilities overstate the risks of deflation and understate the risks of high inflation.

5.2 Horizon adjustment

The next adjustment is for the horizon; we use it to get the 5y5y forward probability. Panels A and C report the median 10y (column 4) and 5y5y (column 5) disaster probabilities, while panels B and D report median adjustment factors (column 3).

This adjustment is the largest of the three. In the 2021 to 2023 U.S. high-inflation subsample, the median forward $Q-$probability of an inflation disaster is 6.3% compared to the 10y probability of 17.2%. Similarly, in the full sample, the median forward $Q-$probability (5.2%) is smaller than the 10y probability, which is 11%. Reflecting this, the median adjustment factor is 0.41. According to the estimates, when U.S. inflation is high, market participants do not expect it to persist.

In contrast, in the eurozone, the adjustment factors are much higher, reflecting a higher market-perceived persistence of inflation. As a result, if one were to look at the 10y probabilities, one would think the United States is much more likely to have a disaster than the eurozone. In fact, at the forward horizon, the two are quite close to each other.[23]

In contrast, for deflation, forward probabilities are higher than 10y probabilities, both when deflation risk is elevated and for the full sample. When inflation is low, there is a worry that it may continue to be low in the future. The median eurozone forward defla-

[23]Note that 10y and 5y5y probabilities as well as adjustment factors are all time-varying and that we report medians for each measure separately; there is thus not a one-to-one mapping in adjustment factor and, for example, the relative size of the 10y and 5y5y probability.

tion $Q-$probability is 6.6% compared to a 10y deflation $Q-$probability of 4.2%, and the median adjustment factor for the eurozone is 2.26. These high numbers reflect persistent long run deflation fears by market participants.

5.3 Risk adjustment

We estimate separate risk adjustment factors (P/Q ratios) for high-inflation and deflation states using data for 18 advanced economies between 1875 and 2015. In the data, the output disasters associated with high and low inflation are of different sizes. Namely, episodes of deflation, like in the late nineteenth century, have not come with particularly severe depressions.[24] The adjustment factor is a mere 0.96. Instead, high inflation more often came with deep recessions, as in most countries during the 1970s, so the adjustment factor is substantial at 0.66.

If, following the literature, we assumed a common adjustment factor, then we estimate it to be 0.82. This is still large, so that not taking risk into account leads to an overstatement of the physical probability of an inflation disaster.

Of course, like any other empirical estimate, these factors depend on the sample. When we estimate our model with data post-1910, so that we do not include the frequent deflations of the late nineteenth century, then the adjustment factor for deflation falls to 0.91. Deflations are now associated with more serious recessions. At the same time, in this sample, the adjustment factor for high-inflation disasters is also smaller (0.62). Therefore, the difference between the two tails is almost the same, and high inflation still comes with significantly larger output disasters than deflations.

To compare these estimates to the literature, we calculate the corresponding risk premiums rp, defined as $q(\pi + rp) = p(\pi)$: the increase in inflation to equate risk-adjusted

[24] Atkeson and Kehoe (2004), using different historical data for 17 countries, write: "the only episode in which there is evidence of a link between deflation and depression is the Great Depression (1929–1934)." Bordo and Filardo (2014) reach the same conclusion.

and actual probabilities. Note that they are positive for high inflation and negative for deflation, but we will refer to their absolute value. We find only moderately high inflation risk premiums averaging 0.23%.

This is in line with literature that has used very different methods. Fleckenstein, Longstaff and Lustig (2017, 2016) estimate risk premiums in the range of 0.2%–0.25% by taking the difference between subjective expectations from analyst forecasts and market expectations from inflation swap rates. The FRB Cleveland estimates the affine term structure model of Haubrich, Pennacchi and Ritchken (2011) and during our common sample its average was 0.39%. However, because we separate high inflation and deflation episodes, we find a significant variability within this average. The average risk premium for high-inflation disasters is 0.61%, while the risk premium for deflation is close to zero.

5.4 Comparing disaster probabilities

Combining all the adjustments leads to the final inflation disaster physical probability in the last column of panels A and C. In the 2021–2023 U.S. sample, the median 5y5y inflation disaster physical probability was 4.2%, elevated relative to its median of 3.5% in the U.S. full sample and 3.2% in the eurozone full sample. These are all below 5%, indicating the success of the Fed and the ECB at convincing market participants that inflation will hover around its target. These probabilities are asymmetric, but in different directions for the two regions. In the United States, the probability of deflation in the full sample is lower, at 2.4%, but in the eurozone it is higher at 6.3%. Outside of the short period in 2011-2012, the probability of deflation in the United States is always small, but for the eurozone it is significantly higher. As a result, adding the two, the probability of a disaster is lower in the United States, 5.9%, relative to the eurozone, 9.5%, entirely driven by the higher probability of a deflation disaster for the latter. The next section decomposes these medians into the evolution of the probabilities over time.

6 A History of the Anchoring of Inflation Expectations

A priority in the pursuit of an inflation target is to anchor inflation expectations. Estimates of the market-perceived probability of inflation disasters give an objective measure of success. Ideally, the estimates should be always close to zero. This section shows that they are not, relates their variation over time to the major events in monetary policy since 2010, and compares the success of the Fed and the ECB at anchoring expectations.

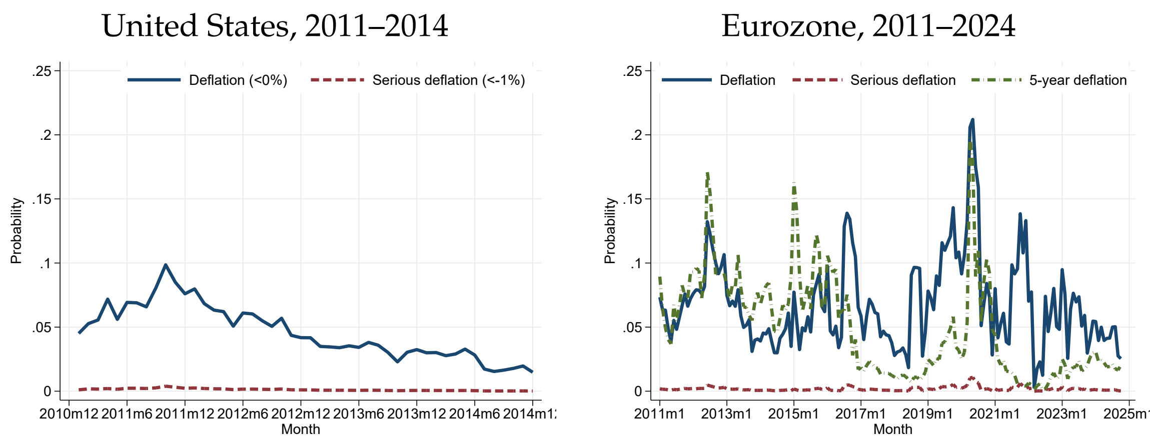
6.1 The fear of deflation

Between 2010 and 2020, both the Fed and the ECB feared that the binding zero lower bound for policy rates would give rise to inflation expectations being stuck persistently below target (Blanchard, Dell'Ariccia and Mauro 2010). Expectations might have been anchored below the 2% target as had likely happened in Japan (Borağan Aruoba, Cuba-Borda and Schorfheide 2017) and was predicted by some theory (Benhabib, Schmitt-Grohé and Uribe 2002). This situation justified the use of quantitative easing (Eggertsson and Woodford 2003) together with many other unconventional monetary and fiscal policies, all with the aim of moving the anchor back to 2% (Eggertsson 2020). The mission reviews of the Fed in 2020 and the ECB in 2021 were partly justified by the fear of deflation (Federal Reserve System 2020, European Central Bank 2021, Reichlin et al. 2021).

Figure 4 shows the evolution of the probability of deflation and serious deflation (less than -1%) at the 5y5y horizon over time. For the United States, on the left-hand side, we zoom in on the 2011–2014 period because the probabilities are very close to zero after that (with the exception of the pandemic, as will be discussed below).

At the start of our sample, the U.S. probabilities were high and rising. Investors were perhaps doubtful of the Fed's ability to steer inflation back on target after actual deflation in 2009. Yet, by the end of 2012, the probability of persistent deflation had fallen below

Figure 4: Probabilities of a deflation disaster



The figure plots 5y5y deflation and extreme deflation probabilities and risk-neutral (Q) 5y deflation probabilities.

5%, and the probability of serious deflation was close to 0, staying there afterward.

Christensen, Lopez and Rudebusch (2015), Kitsul and Wright (2013), and Fleckenstein, Longstaff and Lustig (2017), writing near this time, reported much higher probabilities of deflation. There are three reasons behind the discrepancy in this particular episode. First, those papers mostly focused on deflation in the near horizon, and over a single year, so without the large horizon adjustment factor. Our estimates instead are for the probability of a deflation trap, a persistent period of deflation over 5 years in the long run, the event that policy makers worry most about.[25] Second, and as already indicated in Table 1, the inflation adjustment is significant over this long horizon, and without it the probabilities are overstated. Third, as discussed in Section 5.3, the risk premium for deflation is smaller in our estimates than in the affine models used in previous work that impose a uniform risk premium. This last adjustment goes in the opposite direction of the other two, since a

[25]During this time, the 10-year actual probability (persistent long-term deflation) was even lower than the 5y5y deflation probability, while the forward risk-neutral probability of deflation in a single year was higher and more volatile. The latter is closer to those in the earlier work. Our estimated Markov model of inflation dynamics shows both strong mean reversion and a high probability of leaving the disaster state as soon as the economy has entered it. Therefore, while deflation was likely, a deflation trap was not.

larger deflation risk premium would make our estimates even smaller, and so even more distant from the previous literature.[26]

The right panel of Figure 4 shows the estimates for the eurozone. There are several noticeable differences compared to the United States. First, the probability of faraway deflation dropped by less in 2011–2014. The ECB justified the use of negative interest rates, quantitative easing, and other unconventional policies starting in 2014 with the aim of fighting deflation.[27] This justification is supported by our estimates, and the policy was initially successful in bringing the estimates persistently below 5% for a little more than 1 year.

However, second, after a short-lived spike at the end of 2016, the estimates started rising in the middle of 2018 and peaked with the pandemic, exceeding 20% in the second quarter of 2020. A very short-lived spike is also present in the U.S. data for 2020. Both central banks drastically increased quantitative easing and forward guidance in 2020, and the estimates suggest that the fear of a deflation trap caused by the pandemic was reflected in market expectations.

Third, the eurozone estimates have since fallen, and have again stabilized below 5% by the end of the sample. This is persistently higher than for the United States, where this probability has been very close to zero since the start of 2021. Arguably, the market continues to perceive a higher chance that the eurozone will fall into a deflation trap, in spite of the mission review.

Fourth, Figure 4 shows a third series, for a deflation disaster over the next 5 years to

[26]The Internet Appendix shows a version of Figure 4 using a pooled risk factor that is constrained to be the same for inflation and deflation. The qualitative conclusions in this section are the same, even if the quantitative estimates are different.

[27]In its 2021 mission review the ECB writes: "The deployment of unconventional monetary policy measures, especially since 2014, has made a significant contribution to countering disinflationary pressures, dispelling deflation concerns and averting a more pronounced downward drift in inflation expectations." https://www.ecb.europa.eu/press/economic-bulletin/articles/2021/html/ecb.ebart202105_01~d813529721.en.html

focus on the near term, as in Boninghausen, Kidd and de Vincent-Humphreys (2018). Until 2016, this tracked the 5y5y probability and was slightly above it. That is, the perception in markets of the probability of an inflation disaster was roughly the same over the next 5 years as over the following five. But, after that (with the exception of the spike in 2020) the probability of near deflation has been significantly lower than of long-run deflation. Therefore, the fear of deflation in the future is arguably driven not by the current levels of deflation, but by a perception by financial markets that there is something structural in the eurozone economy, or in the ECB's mandate and actions, that makes it different from the United States and prone to fall into a deflation trap.

6.2 The 2021–2024 inflation disaster

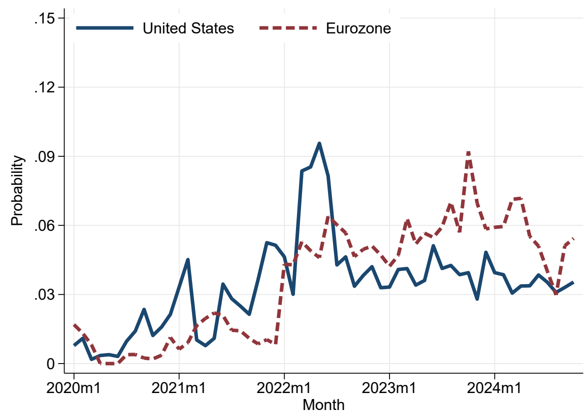
After three decades of inflation close to 2%, the 3-year period between 2021 and 2024 saw an explosion in the price level in the United States and the eurozone. Inflation was so high that the 2021–2026 five-year period will likely classify as an inflation disaster—average inflation above 4%—in both the United States and the eurozone. In December of 2016, the markets had put a probability of this happening at only 2.0% and 1.2%, for the United States and the eurozone, respectively.

The reasons behind the disaster were a combination of supply shocks (Bernanke and Blanchard 2025), drifting inflation expectations from loose monetary policy (Reis 2023) and fiscal stimulus (Bianchi and Melosi 2022), with their relative weights still being debated (Dao et al. 2024). Common to all of them, for the inflation spike to be transitory, it was key that long-run inflation expectations would stay anchored. Policy makers referred to the estimates in this paper in real time (first released in February of 2021, and updated regularly on our websites), and variants of them, to assess this risk (e.g., Schnabel 2022, Gopinath 2022).

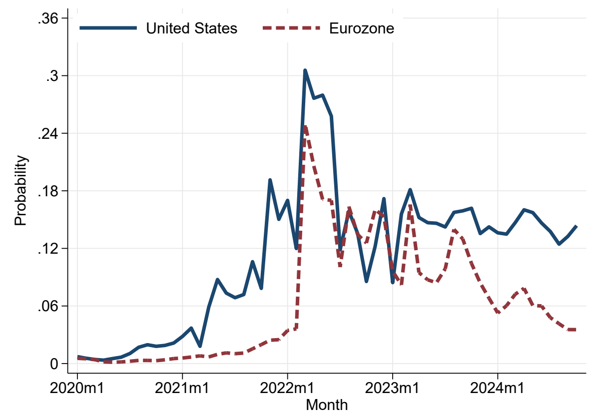
We now have the benefit of hindsight to revisit this turbulent period. The top-left plot

Figure 5: Perceptions of a future inflation disaster during the 2021–2024 inflation disaster

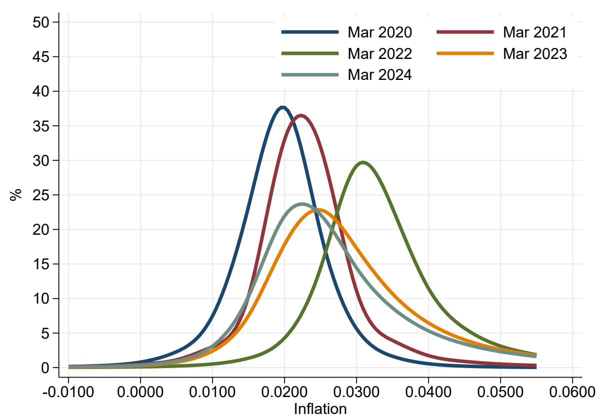
(a) Probability of a high-inflation disaster



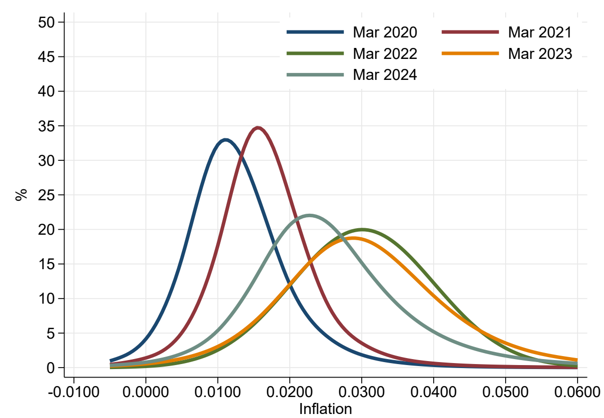
(b) Probability of high inflation at 5y horizon



(c) U.S. risk-neutral densities, 10-year horizon



(d) EZ risk-neutral densities, 10-year horizon



Top panels: 5y5y (forward) and 5y (near-term) inflation disaster ($> 4\%$) probabilities. Bottom panels: 10y horizon risk-neutral (Q) densities.

of Figure 5 shows the 5y5y probabilities of a high-inflation disaster, for the United States and the eurozone. The estimates tell a story in three stages.

First, throughout 2020, the probability of a high-inflation disaster was low and similar in the two regions, hovering between 0% and 3%. By 2021, the probability started rising for the United States alone, reaching 5% by the end of the year, while in the eurozone, it stayed low and constant. As both central banks kept monetary policy loose, the market-perceived probability of high inflation in the United States rose in tandem with the sharp increase in actual inflation at the time. From the perspective of economic theory, this evolution suggests that even long-horizon expectations are sensitive to extreme current realizations.

Second, in the first half of 2022, the U.S. probability kept rising, peaking at 10% in May. The Fed reacted aggressively to the rise in inflation with a 50-bp hike on May 5th, that was followed by several more so that rates between the start of May and the end of December that year increased by 400 basis points. At the same time, likely in response, the U.S. probability of a disaster sharply fell, reaching 3% by the end of 2022. It has remained close to that value since. The sharp policy adjustment, from being highly accommodative to aggressively fighting inflation, came with a corresponding change in inflation expectations. Expectations were unanchored for more than 1 year, but reanchored once the policy priorities were reestablished.

Turning to the eurozone, as inflation took off, so did the eurozone disaster probabilities. Since July of 2022, they have been above those of the United States, averaging 7.6%. Similar to the United States, this increase coincided with the change in monetary policy, as the ECB raised its policy rates for the first time in 11 years at its July 27, 2022, meeting.

Third, at the end of our sample, at the start of 2024, the probability of an inflation disaster is hovering between 3% and 4% in the United States, but it is much higher in the eurozone. In the last 6 months of 2024, the 5y5y eurozone probability rose because the 5-year probability fell, while the 10-year probability was almost unchanged. Both for the United States and the eurozone, the probability of an inflation disaster is significantly larger at the end of of 2024 than it was before 2021, when these probabilities had been near 1% in both regions for many years. The inflation disaster has left scars for the future.

In summary, our estimates show that (a) inflation expectations deanchored in 2021–2022, (b) the tightening of policy had a noticeable effect in stopping or reversing that deanchoring, and (c) there is a scar from the episode into the future as the probability of an inflation disaster has been permanently higher than before.

The top-right panel of Figure 5 digs deeper by looking at the 5y probabilities, to assess the market perceptions of an immediate disaster. Interestingly, they increased in 2021 and 2022 together with actual inflation and with the baseline faraway disaster probabilities, but by much more (the axis scale is doubled). Probabilities peaked at 31% in the United States and 25% in the eurozone, both in March of 2022. This suggests a tight link between actual inflation, forecasts of inflation, and anchoring of expected inflation. It is consistent with models where the credibility of the central bank depends on its current performance.

The bottom row of Figure 5 shows instead the 10-year risk-neutral distributions (so without horizon or risk adjustment) at different points in time, to understand the shifts behind the movements in the tails. In 2021–2022, the increase in the tails of the distributions was much more pronounced than the increases in the median. Skewness rose significantly. Then, by 2024, distributions shifted back, but not to where they started in 2020–2021. Looking at the prices of the 10-year inflation swaps, which match the mean of these distributions, gave an impression of only a slight unanchoring during this time (recall Figure 1). Looking instead at the probabilities of a disaster provided in this paper shows a much more worrying drift, and a clear impact of monetary policy.[28]

Based on the experience of the 5y5y inflation swaps (Figure 1) one might assume that inflation expectations stayed anchored throughout. Indeed, Powell (2024) concludes that this contributed to inflation stabilizing quickly after monetary policy tightened, with little impact on unemployment. The estimates in this paper support a different interpretation: expectations deanchored, policy had to and partially succeeded in reanchoring them, but even now they have not gone back to their initial state.

[28]The distribution of survey expectations of inflation in the 1970s shows similar behavior (Reis 2021).

6.3 Measures of anchoring

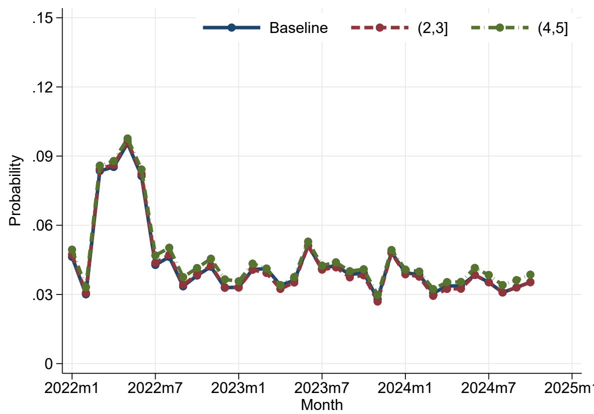
The simplest definition of anchored expectations is that, unconditionally, they do not move at all. The equivalent for our disaster probabilities is that these are close to zero almost always. A conditional statement of anchoring is instead whether expectations of future inflation are insensitive to realizations of inflation. The discussion of the 2021-24 episode suggests that they are not, and that this depends on the stance of monetary policy. We now explore this further by taking advantage of our methods that give probabilistic measures for the tails.

The baseline estimates of the probability of a high-inflation disaster in a month are conditional on what inflation was in that month (as well as the parameters of the model, some of which are time-varying). Given the estimated model of inflation dynamics, we calculated what that probability would have been if this initial inflation was a different value. The top row of Figure 6 plots the answers for two hypothetical initial inflation rates: between 2% and 3% and between 3% and 4%. It does so from 2022 onward, when inflation was well above the two hypothetical ranges: before that, for much of the sample, as inflation was in these ranges, baseline and one of the hypotheticals coincide. The hypotheticals ask to what extent was the heightened probability of an inflation disaster not the result of currently elevated inflation, but of the updated probabilities on future shocks and persistence of inflation embedded in the option prices.

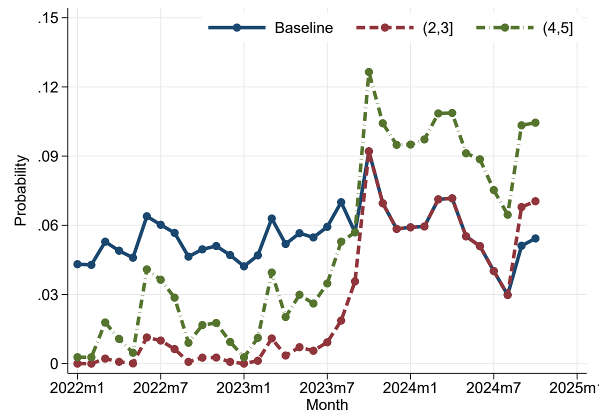
For the United States, on the left, the hypotheticals are practically indistinguishable from the baseline. This suggests that initial conditions have little impact on the market-perceived probability of an inflation disaster 5 years down the line. On the one hand, this implies that the higher perceived probability of a disaster in the United States was not just the result of inflation being high, but rather was driven by changes in perceived future shocks and dynamics. This explains why, even as inflation fell back near its target in 2024, the probability of an inflation disaster remained unchanged. On the other hand,

Figure 6: The conditional anchoring of expectations

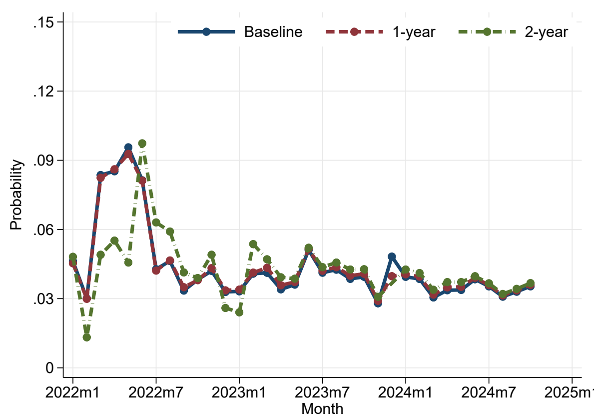
(a) The influence of initial conditions in the United States



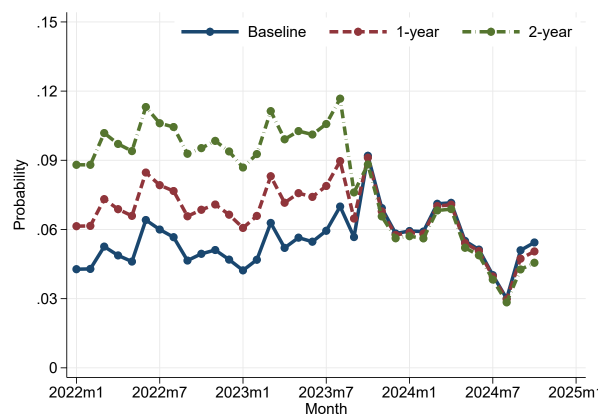
(b) The influence of initial conditions in the eurozone



(c) Conditioning on the near future in the United States



(d) Conditioning on the near future in the eurozone



The figure reports various conditional 5y5y inflation disaster (> 4%) probabilities. Top row: Baseline, based on actual current inflation, and varying current inflation (over the previous year) to being in different ranges, either 2%-3% or 3%-4%; bottom row: Changing, in addition, inflation over the next 2 years.

this shows that changes in disaster probabilities can be interpreted as reflecting changes in future policy credibility. Inflation expectations in 2021-2022 therefore were temporarily unanchored, since the increase in the disaster probability was not driven by high inflation realizations. Overall, one might say that expectations can be quick to unanchor in the United States, but also quickly reanchor.

For the eurozone, the opposite is true. Had inflation been lower, then the market-perceived probability of an inflation disaster would have been significantly lower. This only stops being the case from the second half of 2023 onward. Before that, disaster probabilities were high mainly because of high inflation realizations, not because of a change in expected future policy.

The bottom row of Figure 6 calculates two separate counterfactuals. Taking as given that inflation was high, it asks what would have been the market-perceived probability of a disaster conditional on knowing that inflation would stay high in the next year, as well as the year after. These conditional probabilities measure anchoring by showing whether the option prices at different horizons expect high inflation to persist.

Again, for the United States, the conditional and unconditional probabilities are quite similar. Even if markets were convinced in 2022 that inflation would stay high for the next 2 years, that would not change their trust that the Fed would prevent a disaster in 2027-2032. But this is not the case for the eurozone before the middle of 2023. Markets expect that if inflation persists for 1 or 2 years, the willingness or ability of the ECB to prevent a far-ahead inflation disaster is lower. From this perspective as well, inflation expectations were less anchored in the eurozone than in the United States.

7 Conclusion

This paper develops methods to use inflation options data to back out market-perceived probabilities for tail events in inflation at distant horizons. We show that producing accurate estimates requires taking into account: (a) inflation options' nominal payoffs need to be adjusted to get real Arrow-Debreu probabilities; (b) disaster probabilities for forward horizons can differ from short or long horizons because of the sluggishness of inflation; and (c) the risk premium for inflation is not the same at its two tails compared to the center of the distribution. We provide simple, but we hope robust, methods to make all of these adjustments. We show that the adjustments are quantitatively large relative to constructing probabilities using conventional methods.

We apply our methods to data from the United States and the eurozone between 2009 and October 2024. Starting with the market perceptions of a deflation trap, contrary to previous wisdom, we find that they were low and short-lived in the United States from 2011 to 2014, but have persisted in the eurozone and unconventional monetary policies only provided temporary respite. Turning to high inflation, we find a significant deanchoring of inflation expectations that peaked in mid-2022, and then reanchored as monetary policy tightened. By the end of the sample, we find scars of the high-inflation episode in persistently elevated probabilities of a future inflation disaster. Finally, temporary shocks to inflation, either in the recent past or in the near future, have a larger influence on the expectations anchor in the eurozone than in the United States.

In the future, we hope that our methods will allow researchers and policy makers to measure the risk of inflation disasters more accurately, and use them to assess how changes in policy, economic fundamentals, or temporary or permanent shocks affect inflation expectations and the inflation anchor.

Code Availability: The replication code and data are available in the Harvard Dataverse at https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/LSVW9M.

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