Appendix (for Online Publication) to

How Do Central Banks Control Inflation?

A Guide for the Perplexed

by Laura Castillo-Martinez and Ricardo Reis

June 2025

A Model with uncertainty

This section of Online Appendix drops the assumption that all uncertainty is resolved in period 1, as we did in the main text. We also allow the information of the central bank to be limited not only by imperfect real-time estimates of the state of the economy, but also of the desired inflation target. In what follows, we only show the equations that differ from the ones in the text.

A.1 Steering inflation using the interest rate on reserves

Nominal pegs: Under a nominal peg, the Fisher equation becomes

$$r_t = x_t - \mathbb{E}_t(\pi_{t+1}) \,. \tag{1}$$

Now, with a peg, inflation from date 2 forwards is also undetermined.

Interest rate feedback rules: Now the terminal condition that we impose is that the difference between inflation and the inflation target does not explode at a rate higher than ϕ: $\lim_{T\to\infty} \phi^{-T}\, \mathbb{E}_t\left(\pi_{t+T} - \pi^*_{t+T}\right) = 0$. Then, the difference equation has a unique solution:

$$\pi_t = \pi^*_t + \sum_{j=0}^{\infty} \phi^{-j-1}\, \mathbb{E}_t\left(r_{t+j} + \pi^*_{t+1+j} - \phi\pi^*_{t+j} - x_{t+j}\right). \tag{2}$$

Note that this equation holds for all $t \geq 0$. Since p_{-1} is given, the price level is determinate at all dates, including 0.

The feedback rule that keeps expected inflation near its target sets the interest rate to respond to inflation as well as to the central bank's forecast of real interest rates and the inflation target: $x_t = \hat{r}_t + \hat{\pi}^*_{t+1} - \phi\hat{\pi}^*_t$. Its effectiveness is:

$$\pi_t - \hat{\pi}_t = \pi_{t-1} - \hat{\pi}_{t-1} + \sum_{j=0}^{\infty} \phi^{-j-1}\, \mathbb{E}_t \left[r_{t+j} - \hat{r}_{t+j} + \pi^*_{t+1+j} - \hat{\pi}^*_{t+1+j} - \phi(\pi^*_{t+j} - \hat{\pi}^*_{t+j}) \right] . \tag{3}$$

Escape clauses as anchors: Going back to the solution for inflation with a Taylor rule, by iterating the Fisher equation up until a finite date T, we reach:

$$\pi_t = \pi^*_t + \sum_{j=0}^{T-t} \phi^{-j-1}\, \mathbb{E}_t \left[r_{t+j} + \pi^*_{t+1+j} - \phi\pi^*_{t+j} - x_{t+j} \right] + (1+\phi)^{-T+t}\, \mathbb{E}_t \left(\pi_{T+1} - \pi^*_{T+1} \right) . \tag{4}$$

If the last term on the right-hand side is uniquely pinned down by the switch in regime, then inflation on the left-hand side is uniquely pinned down as well. If the switch leads to an inflation close to target, then the last term will be close to zero. Therefore, the differences between actual and expected inflation are still approximately given by the formula that we derived earlier for the Taylor rule.

A.2 Steering inflation using the money supply

Money growth rules: The classical monetarist rule proposes that the supply of currency grows at a constant rate over time: $h_t = \bar{x}t$, where $\bar{x}$ is a constant. The price level is thus determinate and given by:

$$p_t = \bar{x}t + \eta\bar{x} + \frac{1}{1+\eta^h} \sum_{j=0}^{\infty} \left(\frac{\eta^h}{1+\eta^h} \right)^j \mathbb{E}_t[\eta^h r_{t+j} - c_{t+j} - \eta^h u_{t+j}] . \tag{5}$$

Without currency shocks, in a long-run balance growth path where consumption grows at a constant rate, inflation is equal to the money growth rate $\bar{x}$ minus the growth rate of consumption. Thus, choosing $\bar{x}$ to be the long-run inflation target of the central bank plus the real growth rate of the economy provides an effective way to achieve the target.

Seignorage policy rules: A seignorage policy rule will print banknotes to ensure that seignorage equals an exogenous amount: $s^h_t = x_t$. This gives a difference equation between inflation and seignorage:

$$\pi_t + \eta^h \mathbb{E}_{t-1}(\pi_t) = \eta^h \bar{\Pi}\, \mathbb{E}_t(\pi_{t+1}) + (\bar{\Pi} - 1)x_t - \bar{\Pi}(c_t - \eta^h r_t + \eta^h u_t^h) + (c_{t-1} - \eta^h r_{t-1} + \eta^h u_{t-1}^h)\,. \tag{6}$$

Iterating this equation forward, it is enough that $\eta^h \bar{\Pi} > 1$. Since p_{-1} is given, the price level is determinate at all dates, including 0.

A.3 Steering inflation using net shortfalls

Shortfall policy rules The solution for the price level under a feedback rule for the central bank's shortfall is given by:

$$p_t = v_t - \left(\frac{\bar{A}}{\bar{V}/\bar{P}}\right) a_t + \left(\frac{\bar{W}}{\bar{V}/\bar{P}}\right)\left(\frac{1}{1-\phi}\right)\sum_{j=0}^{\infty}\left(\frac{\beta}{1-\phi}\right)^j (x_{t+1+j} - r_{t+j})\,. \tag{7}$$

B Microfoundation

B.1 Households

The representative household derives utility from leisure, the consumption of a set of $I+1$ differentiated goods, $C_t(i)$, and holding financial assets indexed by j, $B_t(j) \geq 0$ in nominal units. The lifetime utility is given by:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \log C_t + \sum_j \left(\frac{B_t(j)}{P_t Y_t}\right)^{1-1/\eta^j} U_t(j) - \frac{L_t^{1-1/\varphi} - 1}{1 - 1/\varphi}\right], \tag{8}$$

where $\mathbb{E}_t$ is the expectation operator conditional on the information set available at time t and β is the discount factor.

There is a consumption aggregator with constant elasticity of substitution $\sigma > 1$ across the $I+1$ varieties:

$$C_t = \left[\sum_{i=0}^{I} C_t(i)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{9}$$

If the consumer can buy each good at the dollar price $P_t(i)$, and can buy J assets with return $I_{t+1}(j)$, then the optimality conditions with respect to the purchase of each good will give:

- The optimal demand across goods:

$$\left(\frac{C_t(i)}{C_t(0)}\right)^{-\sigma} \equiv \mathbb{R}_t(i) = \frac{P_t(i)}{P_t(0)} \quad \text{for } i = 1, ..., I\,. \tag{10}$$

- The cost-of-living price index:

$$P_t = \mathbb{P}\left(\{P_t(i)\}_{i=0,...I}\right) \equiv \left(\sum_{i=0}^{I} P_t(i)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{11}$$

such that $\sum_{i=0}^{I} P_t(i)C_t(i) = P_t C_t$.

- The saving condition for each asset (including currency) is

$$\mathbb{E}_t\left[\mathbb{M}_{t+1}\left[\frac{P_t(1+I_{t+1}(j))}{P_{t+1}}\right] + \mathbb{Q}_t(j)\right] = 1 \quad \text{for all } j \tag{12}$$

where:

$$\mathbb{M}_{t+1} = \beta\left(\frac{C_t}{C_{t+1}}\right) \quad \text{and} \quad \mathbb{Q}_t(j) \equiv \frac{\eta^j - 1}{\eta^j}\left(\frac{B_t(j)}{P_t Y_t}\right)^{-1/\eta^j} U_t(j) \tag{13}$$

- The optimal supply of labor

$$L_t^{\varphi} C_t = Q_t \tag{14}$$

where Q_t is the real wage.

B.2 Firms

Each good is produced by a monopolist firm that faces a consumer demand function with a constant price elasticity and uses only labor under constant returns to scale and pays a sales tax τ_t:

$$Y_t(i) = A_t L_t(i)\,, \tag{15}$$

The desired price the firm would set equals a constant markup over the marginal cost.:

$$\tilde{P}_t(i) = Z_t P_t \mathbb{C}_t(i) \quad \text{where} \quad \mathbb{Z}_t \equiv \frac{(1-\tau_t)\sigma}{\sigma - 1} \quad \text{and} \quad \mathbb{C}_t(i) = Q_t \tag{16}$$

B.3 Market clearing

There is no investment, so the consumption of each good equals output:

$$C_t(i) = Y_t(i) \tag{17}$$

Labor supplied is equal to labor demanded so:

$$Q_t = \mathbb{C}_t(i) = (Y_t/A_t)^{1+\frac{1}{\varphi}} \tag{18}$$

And both productivity A_t and the markups $\mathbb{Z}_t$ are exogenous.