Which r-star, public bonds or private investment? Measurement and policy implications.*

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Abstract

Measures of the decline in r-star are (almost) exclusively based on a fall in government bond yields. Yet, measures of the return on private aggregate capital are roughly constant, or slightly increasing. This paper first shows that this aggregate fact is robust across countries, measures of capital, and measures of income, from both the perspectives of the demand and of the supply of capital. A model of monetary policy and capital misallocation concludes that the public bond r-star is relevant for the constraint on policy, but the private investment r-star is relevant for the transmission of policy. As a result, relative to the literature, monetary policy is less powerful, rises in inflation are less beneficial, aggregate demand stimulus is less important, and supply-side reforms are more central.

JEL codes: E25, E43, E52.

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1 Introduction

Just before the sudden rise in inflation of 2021-22, the major central banks had gone through extensive mission reviews. A leading motivation for these was the evidence that r^* had fallen. When Jerome Powell announced the new monetary policy framework in August of 2020, he stated that it was partly prompted by the "...fall in the equilibrium real interest rate, or "r-star" ..." Powell (2020). The ECB's July 2021 overview of its monetary policy strategy states as the first new challenge, requiring a revision, that "structural developments have lowered the equilibrium real rate of interest".[1] A vast academic literature has noted that a lower r^* implies that: (i) monetary policy will often be excessively tight because of the lower bound constraint, (ii) deflation traps are as much to fear as run-away inflation, (iii) stimulus to aggregate demand would be very effective, and (iv) improvements in aggregate supply are second order and could even backfire (Eggertsson and Egiev, 2020). These four conclusions played a part in the mission reviews, for instance in the normalization of tools other than interest rates to ease the policy stance, or in the adoption of an average inflation target to fight the prospect of deflation. They likewise played a part in monetary policy in 2021, welcoming fiscal stimulus as well as the initial rise in inflation above target.

Measures of a declining r^* rely almost exclusively on observed yields on long-term government bonds, especially for the United States.[2] Yet, r^* (or the natural interest rate, or the neutral interest rate) is defined as the long-run value of real interest rates at which investment equals savings and the economy's resources are fully employed. There is no strong theoretical case for the equilibrating margin of savings and investment (especially at the global scale) to take place in the market for U.S. Treasuries.[3] Instead, Gomme,

[1] Even earlier, in 2018, Robert Kaplan, president of the FRB Dallas, stated that: "many of us at the Federal Reserve pay close attention to the various models that seek to estimate this rate." and Ben Broadbent, deputy governor of the Bank of England reviewing the history of monetary policy over the last quarter century in March of 2022 , described how estimates of a "low 'neutral' rate of interest" had affected the stance of monetary policy in the UK since at least February of 2014.

[2] Laubach and Williams (2003), followed by Holston, Laubach and Williams (2017), is an influential article. Del Negro et al. (2017) is an exception, using prices of other financial assets, but still with the stated intention of measuring"the real return to an asset with the same safety and liquidity attributes as a 3-month U.S. Treasury bill" (page 236).

[3] Wicksell (1898) is perhaps the intellectual origin, and Woodford (2003) interpreted it as the real interest rate in an equilibrium where prices are fully flexible, while Rachel and Smith (2017) interpret it as a long-run value in observed real interest rates, and Del Negro et al. (2019) as an hypothetical return absent shocks, an actual return stripped of cyclical factors, or movements in returns at frequencies lower than the business cycle. Because capital markets are integrated, some suggest looking at interest-rate trends that are common across countries (Barro and Sala-i Martin, 1990, King and Low, 2014, Rachel and Summers, 2019), and a

Ravikumar and Rupert (2015) and Farhi and Gourio (2018) noted that an off-the-shelf measure of the return on private capital has remained stable or even increased between 2000 and 2010.

Starting from this observation, this paper makes two contributions. First, it systematically estimates the returns to private investment, dealing with many possible objections to its measurement. While levels are hard to pin down, the trend in 2000-20 is clear: the gap between the returns on private investment and the returns on government bonds has increased by 1-3%. From the perspective of the demand for capital, this conclusion is robust to trends in: (i) the relative price of investment, (ii) depreciation, (iii) self-employment, (iv) cross-country differences (v) public capital stocks, (vi) capital gains, (vii) corporate taxes, (viii) the weight of real estate, (ix) intangibles, and (x) marginal versus average returns. Next taking the perspective of the supply of capital, both the inverted Euler equation and the consumption-wealth ratio are consistent with an unchanged r^*.

Second, I modify the neoclassical model to include a misallocation of capital that generates a wedge between the returns to private capital and the returns to government bonds. In the model, while secular stagnation (a fall in the rate of productivity growth or population aging) can explain the fall in the government bond r^*, the rising gap between this r^* and the private capital r^* requires an increase in capital misallocation and a stagnation of private investment. With flexible prices, the model has the strong neutrality prediction: once the effect of the private capital r^* has been taken into account, then a change in the government bond r^* has no effect on the level of capital, labor or output. With sticky wages, in a secular stagnation steady-state with a liquidity trap, the government bond r^* regains its relevance. Its level determines whether monetary policy is constrained by an effective lower bound. The empirically observed fall of the last twenty years makes it more likely that the economy ends up in a liquidity trap, with output forever below potential, as anticipated in the central banks' mission reviews. Yet, the transmission of policy to investment and capital depends on its effect on the private capital r^*. This casts doubts on the three other policy conclusions from the literature. Now, higher inflation may come associated with lower output; more government spending lowers investment;

final view that is more tied to monetary policy is that r^* is a counterfactual policy rate at which monetary policy is neither accommodative nor restrictive within a DSGE model of business cycles (Cúrdia et al., 2015, Del Negro et al., 2017). Kiley (2020) provides a survey of these related perspectives. A large body of research has noted that Treasuries are "special" in their safety, liquidity and convenience, and that this specialness changes over time (Krishnamurthy and Vissing-Jorgensen, 2011, Jiang et al., 2019*a*).Lunsford and West (2019), Hamilton et al. (2016), Borio et al. (2017) find a weak connection between Treasury rates and the fundamental determinants of savings in investment.

increases in productivity growth may raise or lower output; and improvements in capital allocation are very effective at raising output.

Review of the literature: The more closely related papers to this one are Caballero and Farhi (2018), Caballero, Farhi and Gourinchas (2016), and Acharya and Dogra (2021). They also focus on monetary policy at the ZLB because of a fall in the interest rate on government bonds relative to the return on private capital. However, they focus on risk (as opposed to misallocation) as the main driver. I focus on riskless environments, so that my results complement theirs with no overlap. This leads to different conclusions when it comes to monetary policy.

Empirically, many have noted that returns on US capital have not fallen, very often by plotting the estimates from Gomme, Ravikumar and Rupert (2011). However, these are not strictly comparable with government bond r^* measures from the literature, as they focus on US business-cycle frequencies and tax adjustments. The focus here is instead on trends across countries and on adjustments to what is measured as the capital stock and what is measured as payments to its holders. A closely related literature, following Caselli and Feyrer (2007), compares return to capital across countries, while other work has documented a decline in the US labor share of payments partly related to the constancy of the returns to capital.[4] Both of these literatures have emphasized how facts regarding capital payments depend on crucial choices of how to treat capital, and on which country one focuses on. This paper provides a systematic analysis across countries, doing many of the adjustments from that literature and more, showing that the constancy of private returns to capital seems very robust. In contemporaneous work, Chirinko and Mallickc (2022) pursues a similar approach but focused on the dispersion of return across U.S. sectors.

I model secular stagnation following Eggertsson, Mehrotra and Robbins (2019) but now with multiple r^*'s. Like Cúrdia et al. (2015) and Del Negro et al. (2017) I try to answer which measure of r^* is relevant for monetary policy, but the focus is on financial frictions causing capital misallocation rather than price rigidities causing output misallocation. Calvo and Velasco (2021) also study monetary policy in a model with multiple r^*'s, but they focus on unconventional tools like quantitative easing or forward guidance.

The fall in the return on government bonds relative to other investments has received more attention if the study of fiscal policy. A low government bond r^* relaxes the government budget constraint and allows for higher public debt, but it can crowd out private

[4]See Neiman and Karabarbounis (2014), Rognlie (2015), Gutiérrez and Piton (2020), Koh, Santaeulàlia-Llopis and Zheng (2016), Karabarbounis and Neiman (2019).

investment. [5] This paper focuses instead on monetary policy.

Outline: The rest of the paper is organized as follows. Section 2 estimates the private capital r^* from the perspectives of the demand for capital, while section 3 takes the perspective of the supply of capital. The second contribution is split between section 4 with the neoclassical model, and section 5 with nominal rigidities leading to steady-state secular stagnation. Section 6 concludes.

2 Estimates of r^*

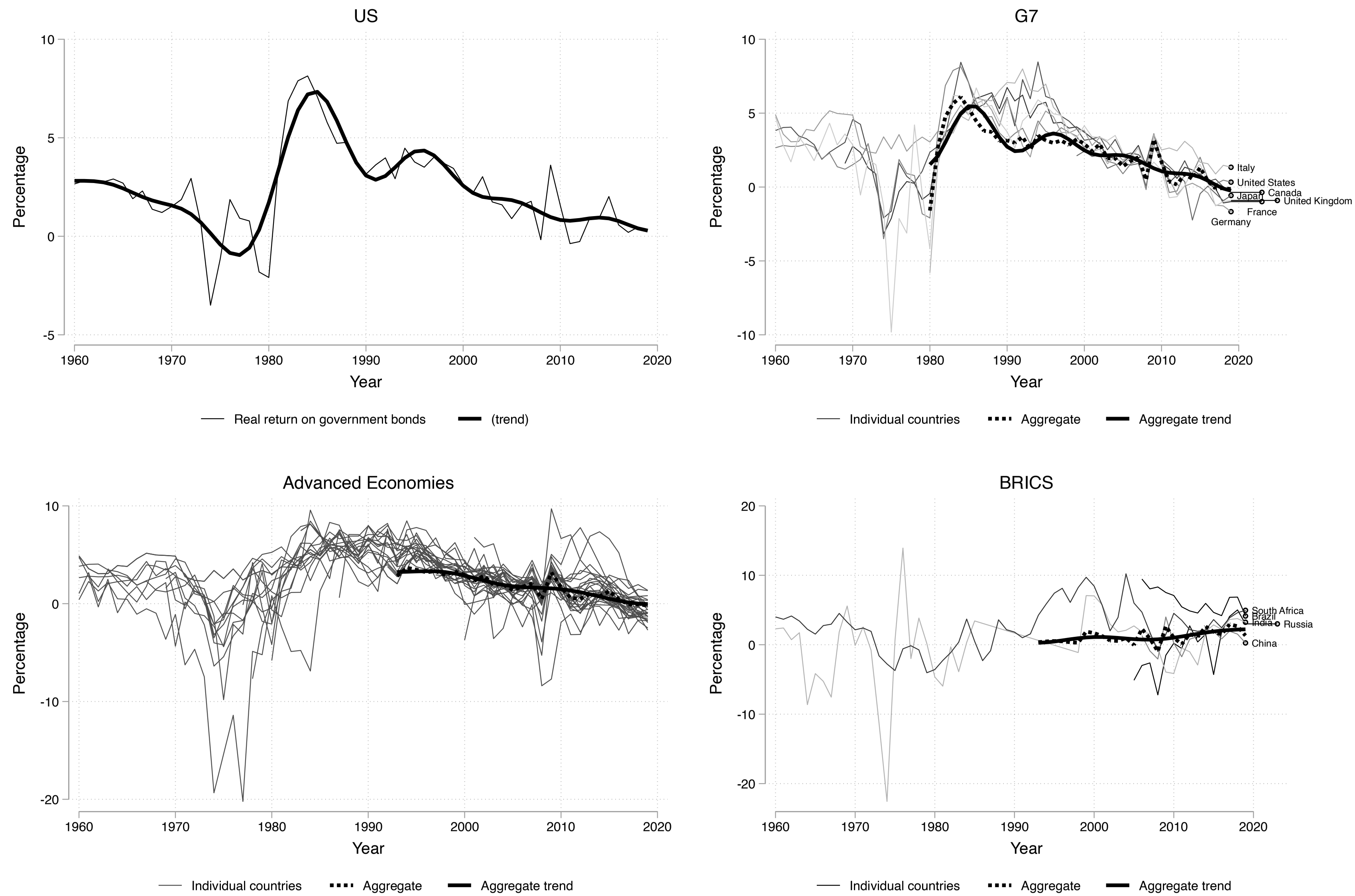
Figure 1 plots the returns from lending to the government until 2019. These r^* are calculated as the difference between a long-term government bond yield and realized CPI inflation. Here, as in all other figures, the trend is calculated using low-frequency weighted averages of cosine transforms (Müller and Watson, 2008) to extract cycles with a frequency of at least 10 years. The four panels do this for: (a) the United States. (ii) the G-7 largest advanced economies, (iii) a group of advanced economies as classified by the IMF and (d) a small set of fast- growing emerging economies. The aggregate series are GDP-weighted averages. The appendix lists the countries in each group and the series for each individual country. The government bond r^* has trended down for at least twenty years (and maybe forty) in almost all advanced economies, but not in the fast-growing emerging economies. The literature described in the introduction has shown this is robust to measuring the nominal returns on government bonds at different maturities, and expected inflation using actual inflation, predicted inflation from autoregressions or (for a few countries) inflation-indexed bonds.

If capital markets were efficient and arbitrage held across assets, then this series would be equal to the risk-adjusted return from investing in the private economy. A representative investor would be indifferent between investing in government bonds or private capital, and this unique r^* would capture the concept of the neutral interest rate at which savings meets investment. With frictions in the allocation of capital and deviations from arbitrage this is no longer the case.

Empirically, the trends in returns on different financial assets have diverged significantly in the last two decades. The return on government bonds reflect their (risky) claim

[5] See Reis (2021), Jiang et al. (2019*b*), Angeletos, Collard and Dellas (2016), Schmid, Liu and Yaron (2021) from the perspective of misallocation and Caballero, Farhi and Gourinchas (2017) and Marx, Mojon and Velde (2021) with a focus on changes in risk, and the survey in Reis (2022).

Figure 1: Trends in real interest rates



on the tax revenue and spending of the government, which has gone through large trend changes. Financial assets give different claims on the private economy that reflect different risks, the structure of corporate financing, or the split between private and public businesses, all of which have gone through large long-run changes over the last twenty years. Which return should one look at to measure r^*? The celebrated Modigliani-Miller theorem provides an answer. The returns earned by any project, whether an individual firm or the aggregate economy, can be split into different financial claims and rewards to different holders leading to returns across multiple assets in an infinite number of ways. But the payoffs of the project remain the same, and ultimately they makes their way to reward the providers of capital on aggregate. These aggregate payoffs in an economy are comprehensively measured by the national accounts.

2.1 Estimates from the demand for capital

Firms (formal or informal) demand capital for production whether through issuing shares, selling bonds, raising private equity, or myriad other forms of financing. Let k be the capital stock, p^k its price, and δ its depreciation rate. Combined with labor, which earns wage w, capital produces output y that sells for price p. The aggregate operating surplus, net of depreciation, across all firms is:

$$NOS = py - wl - \delta p^k k \tag{1}$$

The net average return to capital is simply:[6]

$$\frac{NOS}{p^k k} = \frac{1-\theta}{p^k k / py} \tag{2}$$

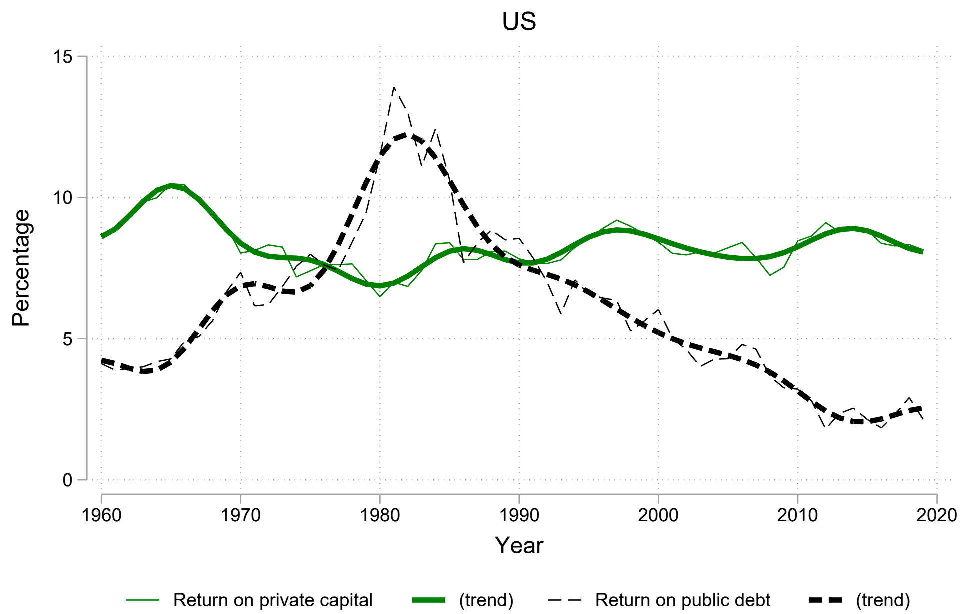
where θ is the labor share of income net of depreciation. This formula, derived from the demand for capital, can be applied directly to the national accounts. At the same time, in its simplicity, it reveals three concerns with using the data.

2.1.1 Three first concerns and baseline estimates

First, one must use the nominal capital-to-output ratio. There have been large trends in the relative price of investment goods. Using the real ratio would confuse those trends

[6] Under constant returns to scale, this is also the marginal return.

Figure 2: US returns on private capital versus government bonds



for trends in the return to capital.

Second, the net operating surplus or labor share are net of measured depreciation. There have been large trends in the composition of the capital stock, between equipment and structures, and between intangible and tangible capital, which not only have quite different depreciation rates as these have changed overtime. Using a fixed depreciation rate across time to cumulate gross investment flows with the perpetual inventory method, as is often done in the literature, could lead to spurious trends.

Third, the national accounts assume that all payments to the self-employed are returns to capital, when likely these include a remuneration to labor. Trends in self-employment, or in the relative weight of the informal economy, could create trends in the labor share that map to trends in measured returns. It is important to adjust for self-employment. An extensive literature, from Caselli and Feyrer (2007) to Gutiérrez and Piton (2020), has discussed how to adjust for self-employment, and I follow it by adding 2/3 of mixed income to the labor share (and so subtracting it from net operating surplus).[7]

[7] A related issue is the rise in the profit share of income (Barkai, 2020). Yet, whether the returns to the capitalists come through the profits of the firm or rental payments, these are still the returns to owning the private capital stock.

Figure 2 shows the US estimates using data from the BEA. The pattern is clear: the return to private capital has been roughly constant over the least 50 years. There has been a large and growing gap between the private capital r^* and the government bond r^*.

2.1.2 Fourth concern: is this true across countries?

There have been important trends in the return that the US earns abroad as it mostly invests in private capital, while foreigners invest in the government bonds that the US issues. This role as the supplier of the world safe asset could explain why r^* for government bonds has fallen, as the foreign savings glut is directed to them, while the r^* for private capital does not. The constancy of the US private returns to capital may therefore be special to that country.

Figure 3 shows the private return to capital for other countries keeping the same scale for the US and the G-7. For the G-7, I use the national statistics from each country, complemented with IMF data when necessary, while for the advanced economies I use the AMECO database and the IMF infrastructure governance dataset, which suffer from the second concern of not measuring disaggregated actual depreciation across different types of capital for most counties. The returns in most advanced countries are more volatile than in the US. Yet, for very few is there a clear trend in the last twenty years. Aggregating across the group, they are as stable as in the United States.

For the fast-growing emerging countries, not only is depreciation poorly measured, but also the adjustment for self-employment is not available. Therefore these estimates must be taken with greater care. Still, they show a declining trend in capital returns the last twenty years, consistent with converging to a Solow-model steady state from below.

2.1.3 Fifth concern: include public capital stock?

By accounting convention, the value added of the public sector is zero, so the net operating surplus of the government is likely understated. The baseline estimates excluded the public capital stock from the denominator since. This is consistent with a view of public investment being divorced from the private returns in the economy but devoted to providing some public goods. Moreover, the measured net operating surplus is paid to the owners of the private capital stock, so the baseline estimates capture private returns. However, it is as defensible to think of the public stock as an essential input to production generating the net operating surplus that we observe. Some countries have more

Figure 3: The private return to capital: baseline estimates

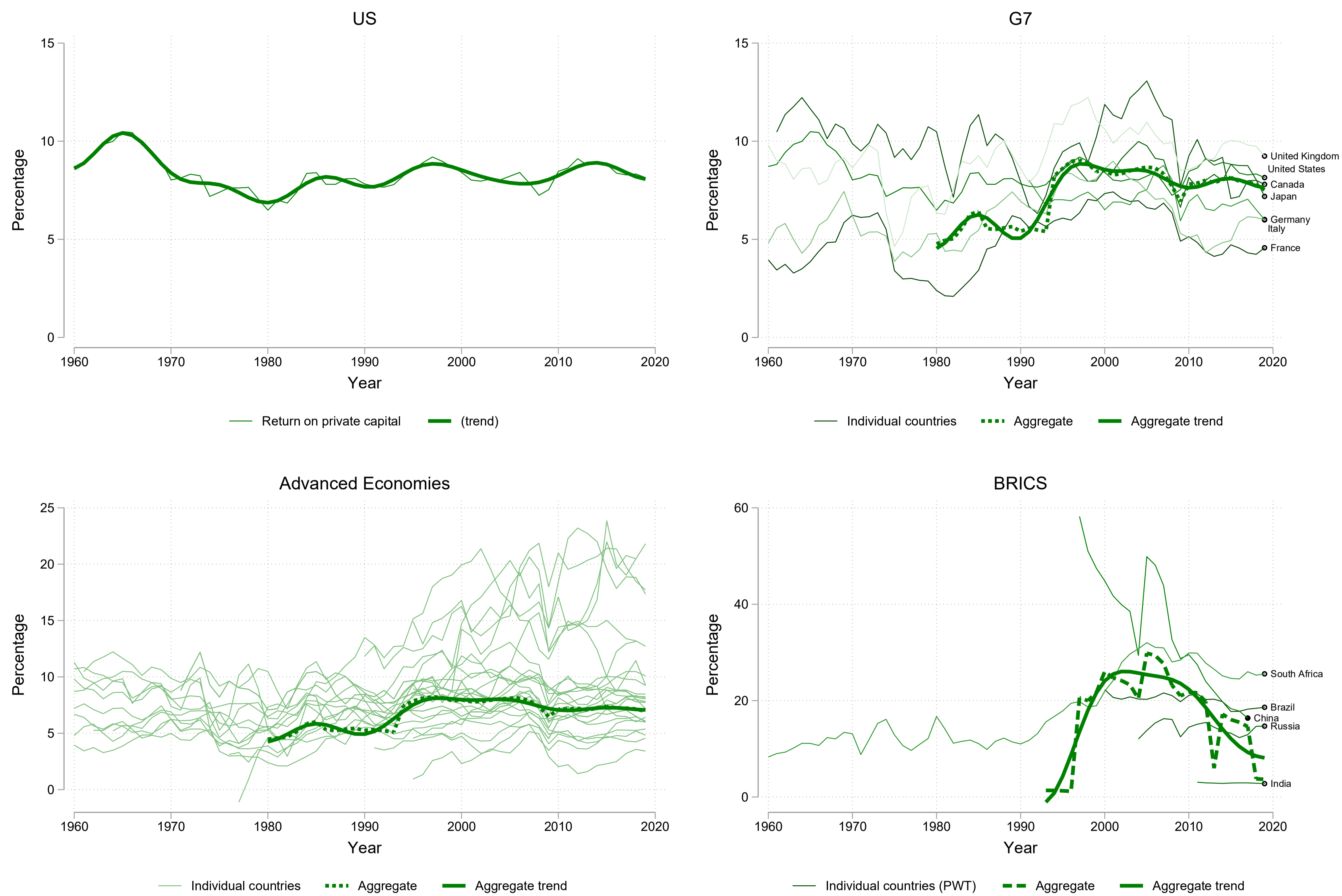
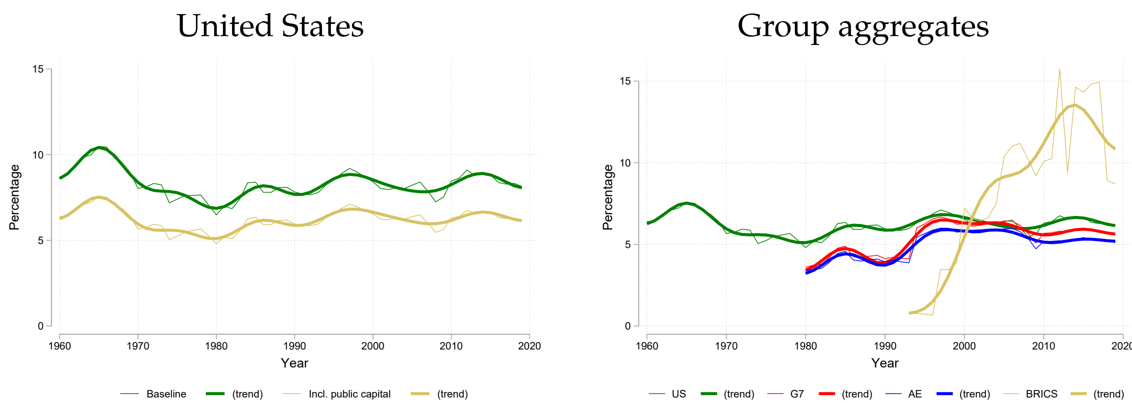


Figure 4: Returns to total, private and public, capital



infrastructure in the public sector, while others have the same train tracks or roads in the private sector. Furthermore, these patterns of ownership have changed over time. Including the public capital stock in the denominator of the calculations provides a measure of total, private and public, returns.

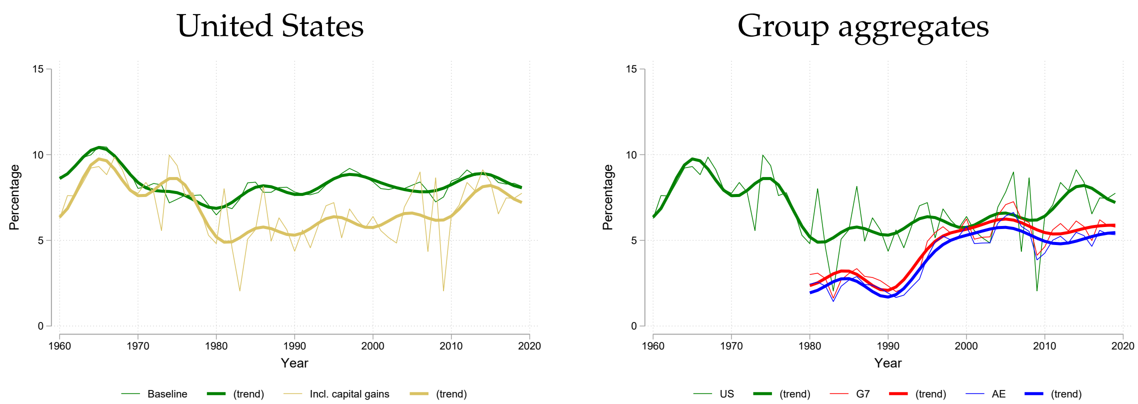
Figure 4 shows estimates using the total capital stock. Of course they have to be lower. But within the advanced economies, the conclusion that the trend is unchanged in the last twenty years remains. For the fast-rising economies though, the trend reverts from declining to sharply increasing.

2.1.4 Sixth concern: include capital gains?

The formula in equation (2) took the perspective of the aggregate economy, and implicitly assumed that capital was not reversible into consumption goods. Instead, if the holders of capital can sell them for consumption, there should be an extra term in the returns: $(p^k_{t+1}/p^k_t)(p_t/p_{t+1})$, capturing capital gains. Because of the large trend in the price of investment goods, if this price appreciation had accelerated in the last twenty years this could generate the downward trend in returns.

Figure 5 shows this is not the case. If anything, it is the opposite. The decline in investment prices has slightly decelerated so including it makes the returns to private capital now slightly rise since the turn of the century.

Figure 5: Returns to private capital including capital gains



2.1.5 Seventh concern: adjust for taxes, and if so which ones?

The returns on government bonds and on private capital are both subject to personal income taxes. Accounting for taxes does not change the conclusions on the gap between them.[8] However, corporate and business taxes only fall on the private capital, but not on government bonds. There has been a large trend in corporate taxation since the late 1980s in advanced economies that could impart a tend in this wedge.

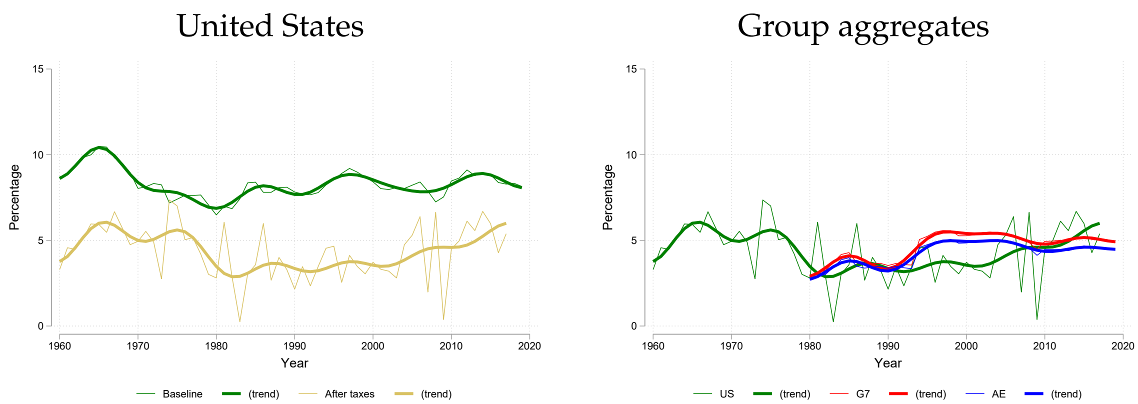
Figure 6 shows that it is indeed so. The trend downwards in corporate income taxes leads to a trend upwards in the returns to private capital. The trend in the gap between the two r^* is even higher.

2.1.6 Eighth concern: exclude real estate?

Real estate is a significant part of the capital stock and of investment, so it makes sense to include it as in the baseline estimates. However, a part of the income from real estate is a remuneration for land, which cannot be productively accumulated. The literature on measuring trends in the labor share has noted this is important because of the large changes in house prices (Rognlie, 2015). However, excluding payments to land from NOS should also come with excluding it from the measures of the reproducible capital stock.

[8]Gomme, Ravikumar and Rupert (2011, 2015) produce US estimates that exclude taxes on corporate income, business, and on household income at the local, state, and federal level. Strictly speaking their estimates are then not comparable with the returns on government bond in figure 1 since those did not take out the household income taxes, even though this is often done.

Figure 6: Returns to private capital after corporate taxes



Here I borrow form the state of the art in that literature, using the estimates of Gutiérrez and Piton (2020) that exclude real estate. Sadly these are gross, not net of depreciation, and they end in 2015. Figure 7 shows that returns are much higher, since real estate is such a large component of the capital stock. In the United States there is now a very slight trend up in the returns to capital, while in the other advanced economies, there was a sharp fall after the financial crisis.

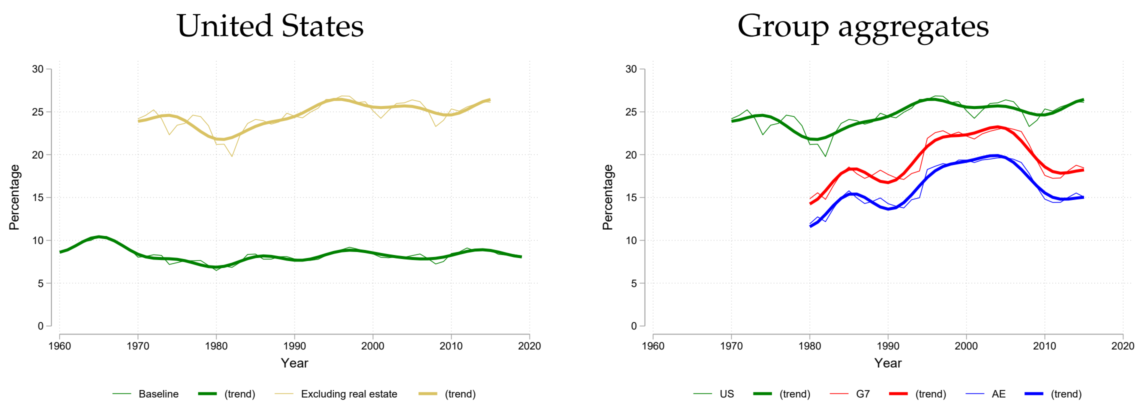
2.1.7 Ninth concern: include more or fewer intangibles?

The share of intangibles in investment has been growing at a fast pace since the 1980s. Including intangibles in the national accounts consists of no longer subtracting expenses in intangibles from value added as an intermediate good, but instead including them as part of investment. This requires also measuring their depreciation, and accumulating net investment to raise estimates of the capital stock. Using superscript I to refer to intangibles, the returns to private capital including intangible are now:

$$\frac{NOS + Inv^I - Dep^I}{K + K^I} \tag{3}$$

At first, when investment in intangibles starts, this unambiguously raises the estimates, since $Inv^I > 0, Dep^I = K^I = 0$. By the steady state, when $Inv^I = Dep^I$, the return to capital is now lower, driven down solely by the higher measured capital stock $K^I > 0$. Therefore, in the transition to an economy with more intangibles, the returns to private

Figure 7: (Gross) returns to private capital excluding real estate



capital should trend down.

Fortunately, the baseline estimates already included intangibles, following the BEA's 2013 revision of the national accounts. Figure 8 shows two alternatives for the United States. The first follows Koh, Santaeulàlia-Llopis and Zheng (2016) in using the BEA procedures pre-revision, and therefore including few to no intangibles. The second uses the estimates of Corrado et al. (2020) which take a broader view than the BEA on what to include in intangibles.

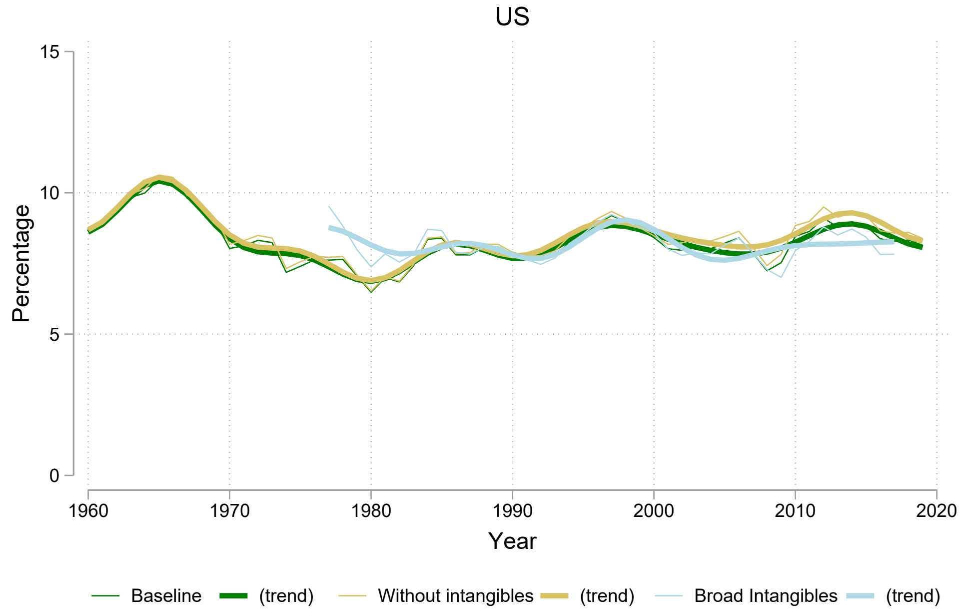
Confirming the discussion above, the broader the measure of intangibles, the more there is a relative downward trend. However, his effect is quantitatively small. Why is this the case if intangibles are such a large share of investment? Because the depreciation of intangibles is also high, so that even by 2019, the share of the total capital which is intangible was only 7%.

2.1.8 Tenth concern: marginal or average

All of the calculations so far have plotted average returns to private investment. Of equal interest is whether marginal returns have changed. Of course, if the production function is linearly homogeneous, then average and marginal are the same, but otherwise the two can significantly differ. Unfortunately, measuring marginal returns is challenging.

Figure 9 plots the results of two strategies to approach this problem. The first in the left panel is more conventional. It comes from estimating a trans-log function with net operating surplus on the left hand side, and both the capital stock as well as a measure of

Figure 8: Returns to private capital including more or fewer intangibles



labor input (hours) on the right-hand side with jump to second order terms. The estimates start by construction at zero, since only changes can be identified. Notably, these returns change even less, quantitatively.

The second strategy, in the panel on the right, starts by building measures of the return on private capital at the sectoral level, following the same strategy as in our aggregate baseline (a few outlier measures above 100% are dropped but results are broadly similar with them). Then, for each country, it regresses this on time fixed effects, using weighted lest squares with weights determined by the change in the share of each sector capital stock. Insofar as the marginal sectors are those that are growing faster, then the estimates of the time-series fixed effects provide an approximation of the marginal return in the economy. These estimates are understandably quite volatile, since sectoral data is noisy, so only the trends are shown. While for some countries, returns rise and others fall, on aggregate again there is no discernible trend.

The estimated sectoral fixed effects of this regression are plotted in figure 10, but now weighted by their share in the capital stock, as opposed to its change. The goal is to illustrate the heterogeneity across sectors, even as this paper has emphasized the difference

Figure 9: Returns to private capital: marginal estimates

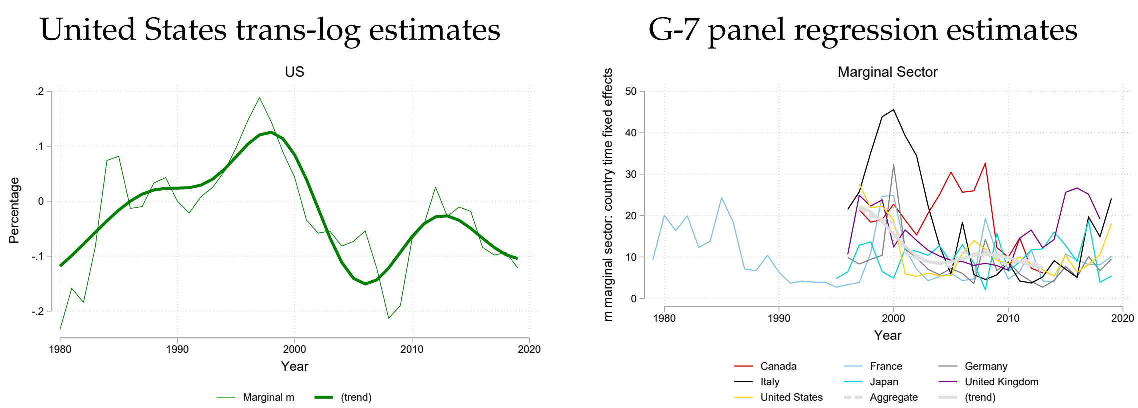
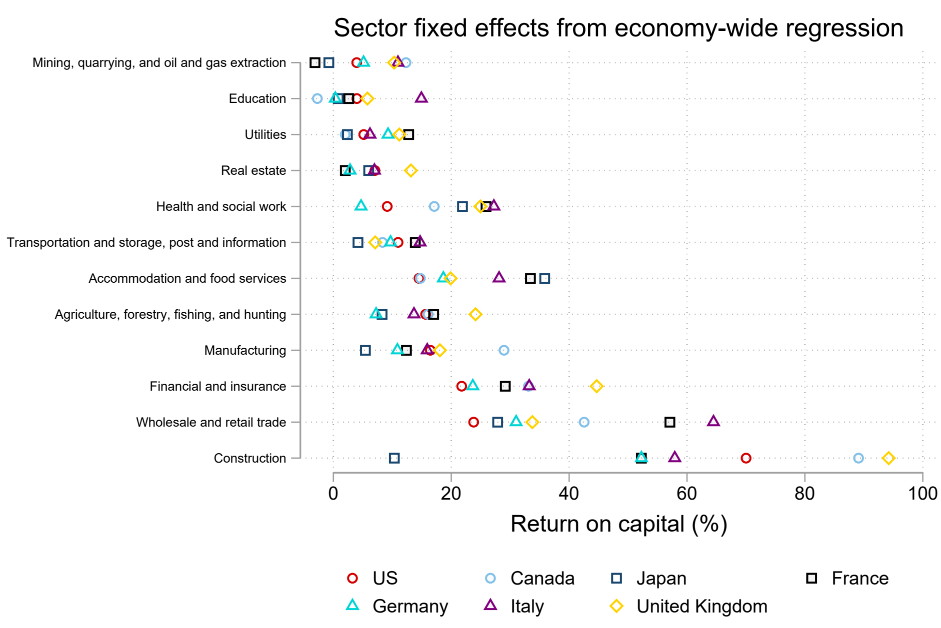


Figure 10: Returns to private capital across sectors



between private investment and government bonds. Sectors are ordered by the estimates return for the United States. Estimates within each country are similarly downward sloping just as in the United States. There is an important sectoral component in returns that is similar across advanced economies. Likewise the dispersion within each sector across countries is large.

3 Estimates from the supply of capital

Capital is supplied by household savings. While the previous section used the demand of capital from firms to back out the return, this section uses the supply of capital from households to measure it. The two should be consistent in equilibrium, so this provides an alternative approach. The theory of consumption dictates that the supply of savings comes from an Euler equation and from a budget constraint. I explore estimates of the inverse supply of capital from either of them.

3.1 Euler equation estimates: classical

The classic Euler-Ramey formula states that the difference between r^* and the time-preference rate is equal to the growth rate of consumption divided by the intertemporal elasticity of substitution. Under the (very strong) assumptions that consumption equal output and that there is a single r^* in the economy, then a supply based estimate of its trend is:

$$r^* = g(y)^{1/\nu}/\beta \tag{4}$$

where $g(.)$ is the growth function (so $g(y) = y_{t+1}/y_t$), β is discount factor, and ν is the intertemporal elasticity of substitution.

The slight fall in output growth over the last twenty years requires an intertemporal elasticity of substitution for the representative agent that is lower than 0.1 to suggest a large decline in r^* like what we observed from the demand for government bonds. More conventional values for this elasticity, like 1 or 1/2, instead produce estimates of an almost unchanged r^* consistent with what we measured using the demand for private capital.[9]

[9]Laubach and Williams (2003) combine the return on government bonds with an IS equation, as opposed to an Euler equation, relating output to interest rates to measure r^* using both supply and demand. However, their estimated fall comes entirely form the demand side; replacing the return on bonds with a return on private capital in their estimates leads to an estimates r^* that rises throughout their sample.

3.2 Euler equation estimates: modern

The modern literature on consumption deviates from this benchmark by noting that a fraction χ of households live hand to mouth consuming their labor income, and that among the other fraction returns on savings can differ substantially. For simplicity say that a fraction α of savers earn a leveraged return over the return on private capital, while the remainder earn the return on government bonds, the supply of capital now is:

$$g(c) = (1-\chi)\beta[\alpha \times lev \times m + (1-\alpha)r]^{\nu} + \chi g(wl) \tag{5}$$

I used m to denote the return on private capital, and r the return on government bonds, while lev is leverage.

Two assumptions turn this into a measurement equation for r^*. First, assume that leverage through private credit pays approximately the same return as government bonds. Second, assume that the economy is approximately closed, so that total outside assets are equal to $k+b$ where b are government bonds. A few steps of algebra then show that:

$$m = r + \left(1+\frac{b}{k}\right)\left(\frac{x}{\nu} - r\right) \tag{6}$$

$$\text{with } x \equiv \log\left(\frac{g(1-sav) - \chi g(1-\theta)}{1-\chi}\right) + \log(g(y)) - \log(\beta) \tag{7}$$

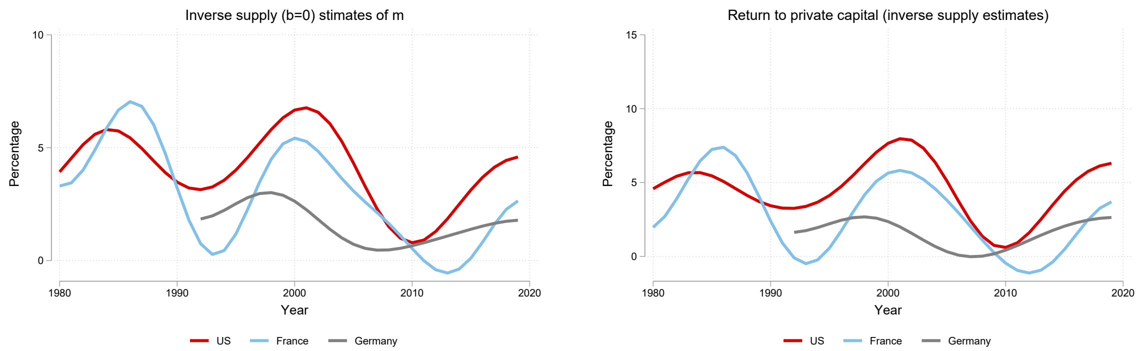
If $b = 0$ and all savings go to the private economy then $m = \frac{x}{\nu}$. The estimate of r^* from private capital differs from the classic one only because x is an adjusted growth rate. Starting from the growth rate of output, it accounts for changes in the savings rate sav, and changes in the labor share. This matters because both have trended in the last twenty years.[10] With public debt, $b > 0$, then the equation uses the fact that the adjusted growth rate from the Euler equation is a weighted average between the two r^*'s, private and public, to back out the wedge between them.

Figure 11 shows the supply-based estimates of m with an intertemporal elasticity of substitution of 0.5 and $\chi = 0.4$, a common value in savers-spenders and TANK models. The left panel takes the $b = 0$ case, and the right panel measures b using IMF data on public debt. The estimates of m are significantly more volatile, partly because the great recession came with such a large fall in output that it appears to be a change in trend that

[10] At the other extreme, if the only form of savings is the public debt, and the capital stock is zero, then $r = \frac{x}{\nu}$. Once the growth rate of output is adjusted for savings and hand-to-mouth households, the standard argument holds.

Figure 11: Estimates of r^* from the inverse supply of savings

With $b = 0$ | With $b > 0$



quickly reverses. Still, it is hard to argue that there is a steady decline in the last twenty years. The trend down in the savings rate and the labor share of income offset the decline in income growth, consistent with a constant return on investments. In turn, the rise in b offset the fall in r to lead as well to the unchanged m. In words, the rise in the public debt absorbed a larger share of savings, but since the growth rate did not fall, then it must be that returns on private capital rose relative to the returns on government bonds.

3.3 Budget constraint estimates

Following Lettau and Ludvigson (2001) and Gourinchas, Rey and Sauzet (2022) start with the resource constraint for the economy:

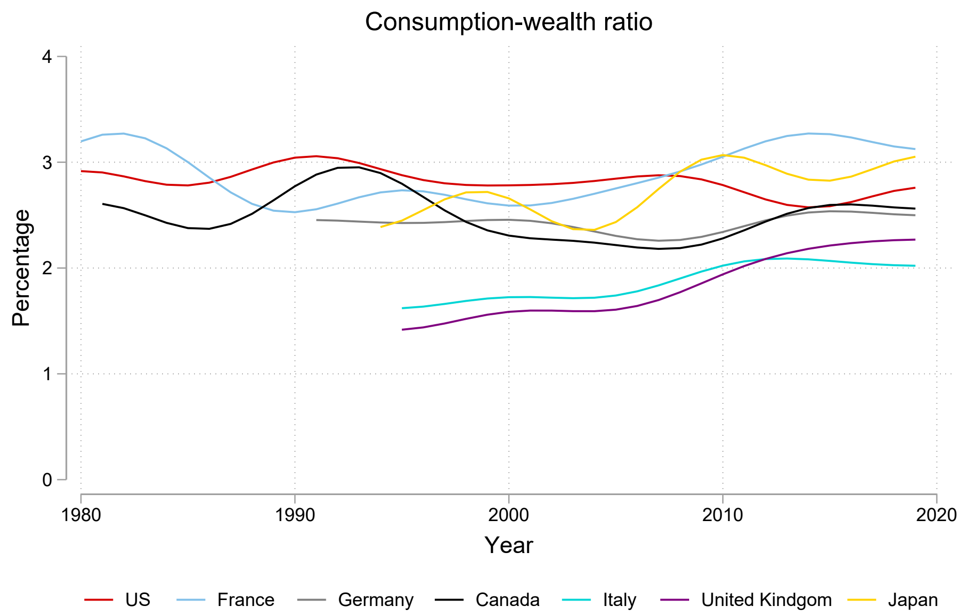
$$p_t^k k_{t+1} + p_t^c c_t = r_{k,t} p_t^k k_t + w_t l_t \tag{8}$$

Defining the consumption-wealth ratio as $CW = p^c c / p^k r^k k$ this is equivalent to the non-linear difference equation:

$$\frac{1}{CW_{t+1}} = r_{t+1}^k \left(\frac{p_t^c c_t}{p_{t+1}^c c_{t+1}} \right) \left(\frac{1}{CW_t} - 1 + \frac{w_t l_t}{p_t^c c_t} \right) \tag{9}$$

Now, log-linearizing this around a constant consumption-wealth ratio $C/W = \rho$ and a constant labor share, using hats to denote log-deviations, taking expectations, and iterat-

Figure 12: Consumption-wealth ratio



ing forward, one gets:

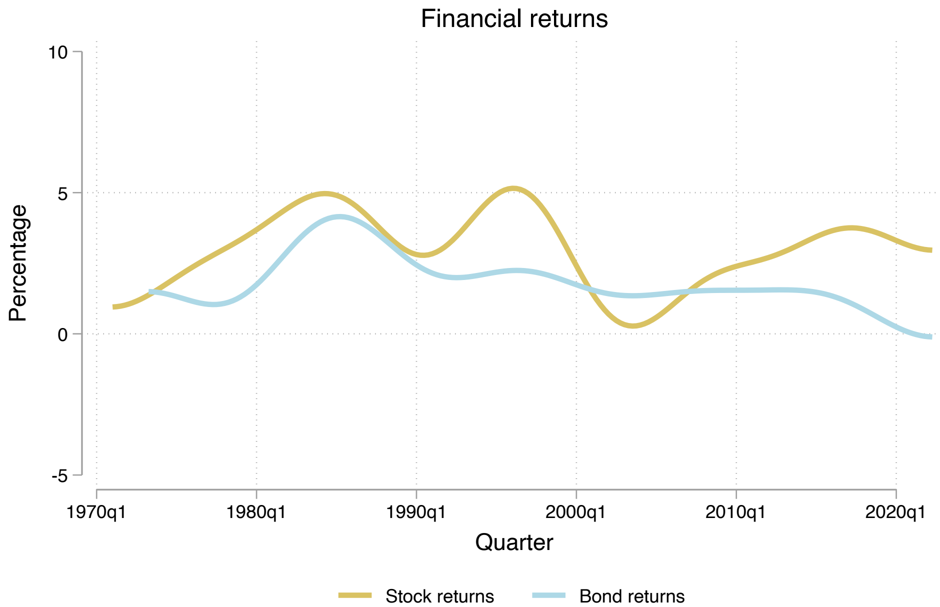
$$C\hat{W}_t = \mathbb{E}_t \left[\sum_{s=1}^{\infty} \rho^s \left(\hat{m}_{t+s} - \Delta(\hat{p}^c_{t+s} + \hat{c}_{t+s}) \right) \right] \quad (10)$$

The null hypothesis is that there is no change in the forward-looking returns to private capital. Under this hull hypothesis, there should be no discernible trend in the consumption-wealth ratio. The data plotted in 12 confirms it: there is no discernible trend in the ratio of consumption to capital payments.

3.4 Financial returns estimates

Financial returns provide another approach to inspect the returns to private capital form the perspective fo the investors and the return they get from the firm. It is important to look at broad indices, since financial innovation moves risk across different asset classes. Figure 13 plots the return on a broad stock index (Wilshire 5000) and a broad set of corporate bonds (Bloomberg). Neither has trended down since 2000. Looking to the compo-

Figure 13: Financial asset returns



nents of these indices, the returns of the largest, most liquid firms (SP500) with the highest credit-worthiness (AAA bonds) have declined, albeit not as much as the returns on government bonds. But looking at smaller, yet still listed firms, or at BBB bonds, where there has been much growth in the last two decades, returns have stayed unchanged or risen. The same is true of returns on venture capital or private equity.

4 A neoclassical model

This section presents a simple modification of the classic neoclassical model that is at the heart of almost all modern DSGEs in which there is a gap between the two different r^*'s.

4.1 Firms and workers

Firms operate a Cobb-Douglas production function:

$$y = k^{\theta}(Al)^{1-\theta}. \tag{11}$$

Relative to the measurements in section 2, this assumes that the capital share is now constant. Firms are able to charge a markup μ over marginal costs, so:

$$\mu = \frac{(1-\theta)y/l}{\omega}, \tag{12}$$

where $\omega = w/p$ is the real wage. It follows that the return to the owners of this technology matches the formula used in section 2. I use m to denote the return on private capital, and r to denote the return on government bonds, so as to more clearly distinguish between these two r^*.

Labor is supplied by hand-to-mouth households that have an endowment of one unit of labor and supply it inelastically. Therefore, when the economy is at potential, all labor is employed and $l_t = 1$. But, this is not always the case.

4.2 Households and financial frictions

There is a continuum of households, that may enter each period with different assets but have identical log preferences. Relative to the measurement model in section 3, I assume the elasticity of intertemporal substation is 1 for simplicity ($\nu = 1$).

At the start of the period, households learn their type for that period. With probability α, they will be capitalists with access to the production technology for that period, and so they will be able to earn the returns on capital. They can leverage their investment using private credit, but are subject to a leverage constraint such that they can only commit to repay on their loans z up to a fraction γ of what their returns will be.[11] The capitalists can also save in government bonds and, since there is no uncertainty, the return on credit is r. Therefore, the debt repayments to revenue ratio of the private sector is $rz/mk \leq \gamma$. It follows that relative to the measurement in section 3, the leverage ratio γ is now a constant parameter, and the ratio of capitalists α is constant as well.

With probability $1-\alpha$, instead, households are savers, who only have access to the private credit market and to government bonds in order to save or borrow. The Bellman

[11] This can be micro-founded by assuming that the entrepreneur in charge of the firm can only credibly pledge to pay back a fraction γ of her final output, since she could run away with the remaining output.

equations characterizing each type's behavior are:

$$V(a, cap) = \max \{\log(a + z - k) + \beta\alpha V(mk - rz, cap) + \beta(1 - \alpha)V(mk - rz, sav)\}$$
$$\text{subject to} \quad rz \leq \gamma mk \tag{13}$$
$$V(a, sav) = \max \{\log(c) + \beta\alpha V(r(a - c), cap) + \beta(1 - \alpha)V(r(a - c), sav)\} \tag{14}$$

This model therefore has two financial frictions. First, that only $\alpha < 1$ of the wealth entering a period can be used as equity of firms. There is a limit on the net worth of entrepreneurs or in the amount of equity capital available. Second, that only a fraction $\gamma < 1$ of final net output can be funded through private credit. There is a limit on borrowing and credit.

The solution of this problem leads to the modern aggregate Euler equation that I used for measurement.

4.3 Market clearing and government

Letting g be the ratio of net government spending to the capital stock, the government budget constraint is:

$$b_{t+1} = r_t b_t + g k_t \tag{15}$$

I take g as an exogenous policy choice.

The market clearing condition for assets is:

$$k_t = \alpha(b_t + k_t) + z_t, \tag{16}$$

since total capital employed is equal to that funded through equity, which is the share α of the private wealth in the economy (capital plus government bonds), and that funded through private borrowing.

4.4 The neoclassical benchmark

It is easy to show that if $\alpha + \gamma > 1$, the financial constraints do not bind. There is enough equity and credit for all the capital to be employed in production. In that case, returns are equated and, through the Euler equation, they equal the rate of impatience:

$$m = r = \frac{1}{\beta}. \tag{17}$$

Using these lenses, the fall in r^* that the literature has measured has then to be associated with a higher β. It is easy to extend the model to have a balanced growth path driven by a constant growth rate in the productivity and population growth. In that case, a fall in the growth rate of productivity or in the rate of population growth, would be equivalent to a rise in β. This has led to the standard close association between low r^* and secular stagnation.

The problem with this account is that it would have predicted a fall in the private investment r^*, or at least no change in the wedge between it and the government bond r^*. This is not what we see in the data.

The neoclassical model also has strong neutrality results. Either a change in markups or a change in TFP have no effect on the level of interest rates, just on the level of output. In turn, an increase in government net spending would lower private consumption, but leave the capital stock and output unchanged. By Ricardian equivalence, the level of the public debt is irrelevant for interest rates.

4.5 The economy with financing constraints leading to misallocation: solution

Now assume that $\alpha + \gamma < 1$. The financial frictions in the economy are binding and there is misallocation of capital.

Combining the modern aggregate Euler equation in equation (6) with total returns evaluated at a steady state gives:

$$\frac{1}{\beta} = \alpha \left(\frac{1 - \frac{rz}{mk}}{1 - \frac{z}{k}} \right) m + (1 - \alpha) r. \tag{18}$$

Moreover, the market clearing condition for assets in equation (16) delivers an equation for the public debt to capital ratio:

$$\frac{b}{k} = \frac{1 - \alpha}{\alpha} - \frac{\gamma m}{\alpha r}. \tag{19}$$

Combining the last two equations with the government budget constraint in equation (15) leads to:

$$m = r + \left(\frac{1}{\beta} - r \right) \left(1 + \frac{g}{1 - r} \right). \tag{20}$$

This equation shows that there are two jointly necessary conditions for $m > r$. First, that r is low, below the discount rate that would prevail under a perfect allocation of capital. Second, that there is a positive amount of public debt outstanding absorbing some of the national savings, in spite of the low returns that the debt offers. The equation also shows that, all else equal, a lower r does not necessarily result in a larger $m - r$ gap. It depends.

Substituting out for m in equations (19) and (20) gives the solution for r:

$$\left(\frac{1}{\beta r} - 1\right)\left(1 + \frac{g}{1-r}\right) = \frac{1}{\gamma}\left(1 - \alpha - \frac{\alpha g}{1-r}\right) - 1. \tag{21}$$

This is one equation in one unknown and shows that the level of the government-bond r^* will depend not just on the conditions for secular stagnation (β), but also on how many resources are absorbed by government spending (g) and the two financial friction parameters (α and γ), that is, the extent of misallocation in the economy. Generally, we will get $r < 1/\beta$ and easily one can get government bond real interest rates net of growth that are negative.

Finally, given the solution for m, the level of output is connected to the private return to capital in steady state by:

$$y = \frac{A}{(m - 1 + \delta)^{\frac{\theta}{1-\theta}}}\left(1 - \frac{1-\theta}{\mu}\right)^{\frac{\theta}{1-\theta}}. \tag{22}$$

Intuitively, because of diminishing returns, a higher return on private savings is a sign that there is less capital in the economy, and so less output.

This concludes the description of the neoclassical, neutral, or long-run equilibrium. Solving it is straightforward and can be done sequentially: equation (21) gives the solution for r, then equation (20) delivers m, after which equation (22) gives the solution of output. The exogenous variables are impatience β, deficits g, the degree of financial frictions misallocating capital α and γ, and productivity A and markups μ.

4.6 Explaining the trends in the data

In the model with misallocation, an increase in β (or a fall in the growth rate of productivity) lowers r just as it did before. However, it also lowers m because a less productive economy provides a lower return to investment. Worse, the gap between the two rates of return shrinks. Therefore, the observed decline in productivity growth, by itself, would have led to the opposite patterns in the rates of return relative to what we observed.

Novel to the model of misallocation are the parameters α and γ. In different ways, both proxy for financial development in the sense of the existing capital making its way to those who can put it to productive use earning the return to private capital. A higher α means that a larger share of assets is in the hands of the capitalists. A higher γ means that more of the assets are in the hands of the investors that can take it into the firms through leverage. A fall in either increases the gap between m and r. That is, less financial development hampers investment and capital so, by diminishing returns, it will tend to raise m. Public debt competes with the private capital stock as a means of saving, so the government is now absorbing more of the private savings. Therefore, r will rise, even if not as much thus closing the gap between the two returns.

Similar, an increase in the primary deficit as a ratio of the capital stock, g, (consistent with the data) is only consistent with the government budget constraint if r is lower. In this model, more spending means crowding out of the capital stock, and more misallocation. The increase in the public deficit draws resources to the government debt market away from private investment. By diminishing returns, this means a rise in m. Just as we observe in the data, the gap between the two returns rises. Therefore, because g is another measure of misallocation, it also can explain the facts.

Therefore, a combination of secular stagnation and worsening capital allocation can explain the patterns in the data. Note that changes in the level of technology (A) or in market power (μ) affect the potential amount of output in the economy, both in the standard model, as well as when there is misallocation of capital. However, they do not affect equilibrium returns. They can explain permanent declines in well-being, and may well have been present in the last two decades, but they do not explain the trends in returns that this paper documented.

4.7 A strong neutrality result

Which measure of r^* should we focus on? With flexible prices, there is a a stark and strong (surely too strong) answer to this question: only the returns on private investment matter. From equation (22), the partial derivative of y with respect to r keeping m fixed is zero. In words, *once the effect of m is taken into account, then r is irrelevant for the level of real activity*. In the model, policymakers should ultimately not care about the r^* on government bonds; they should focus on the r^* in private investment instead.

The reason for this can be inferred from the standard neoclassical model, as it is usually taught. In that model, the level of government bonds is irrelevant for the equilibrium allocation. What pins down real activity is the marginal product of capital, and how it is affected by productivity, patience, and public policies. This is also true in the canonical DSGE model that is used for monetary policy, built on Christiano, Eichenbaum and Evans (2005). Frictions introduce wedges between the marginal product of capital and the return on private investment, but it is still the concept of returns to private investment that is relevant for equilibrium. Within this class of models, the return on government bonds is only useful insofar as it provides an imperfect measure of the return on private capital.

In the canonical new Keynesian model of Woodford (2003), there are neither government bonds nor capital. The Wicksellian interest rate is a a shadow rate. Its relevance comes from its role in the Euler equation and its link to the growth rate of consumption. There is no theoretical prior for why either r or m is to be preferred in the Euler equation in the new Keynesian model. Section 3 noted that modern consumption theory suggests instead a weighted average of both. That model does not provide a defense for the focus on the empirical estimates of r.

One argument for focussing on r is the presumption that it provides an estimate of m adjusted for risk. Investing in the private economy is riskier than saving in government bonds in advanced economies, but with efficient capital markets risk-adjusted returns are still equated by arbitrage. A long literature has explored this argument (e.g., Barro, 2020). The model above purposefully left any risk out to show that r and m can move in opposite directions in response to shocks and policies. This showed that this presumption is not justified and is potentially dangerous to evaluate fiscal policy.

5 A new Keynesian model

The model in the previous section had no nominal rigidities, and focussed on the steady state. The object of attention was the neutral, potential, or long-run equilibrium in the economy, and in particular in r and m. This section solves for equilibria with nominal rigidities while still focussing on long-run steady states, so these are still, in some interpretations, versions of r^*.

5.1 Introducing nominal rigidities and monetary policy

In the neoclassical equilibrium where labor is fully employed, so $l = 1$, the real wage ω is the ratio of the marginal product of labor to the markup :

$$\omega_t = \frac{(1-\theta)k_{t-1}^{\theta}A_t^{1-\theta}}{\mu}. \tag{23}$$

Following Eggertsson, Mehrotra and Robbins (2019), I assume downward nominal wage stickiness. Namely, the nominal wage is set according to:

$$w_t = \max\{\omega_t p_t \;\; ; \;\; (w_{t-1}\pi_t^e)^{\eta}(\omega_t p_t)^{1-\eta}\} \tag{24}$$

If the neoclassical real wage has grown enough, then the actual wage set is equal to it. However, if this dictates a too low increase in wages, workers and firms agree instead to not let wages fall too much relative to a reference value. This reference is the value of nominal wages yesterday, updated to reflect the inflation that workers expected π_t^e. If $\eta = 0$, then there are no nominal rigidities, and we are in the neoclassical equilibrium. For η closer to 1, expected inflation becomes critical. If expected inflation is well above actual inflation, then real wages will be well above their neoclassical market-clearing level.

The other side of high real wages is low employment. Instead of employment equalling its unit supply, now employment comes from combining the wage rule with the demand for labor from firms to get, in steady state:

$$l = \left(\frac{\pi}{\pi^e}\right)^{\frac{\eta}{\theta(1-\eta)}} \tag{25}$$

If actual inflation $\pi_t = p_t/p_{t-1}$ is below expected inflation, then this reduces employment since the wage demands are larger in real terms.

Monetary policy chooses a nominal interest rate, and has an inflation target π^*. I ignore the practical implementation of monetary policy.[12] Instead, to focus on the effective lower bound (ZLB), I assume that nominal interest rates cannot be below some exogenous parameter $\zeta \leq 1$. Monetary policy can then be in one of two states. Either it delivers $\pi_t = \pi^*$, which is possible as long as $r_t\pi^* > \zeta$. Or, it is bound to set nominal interest rates to zero, so that $\pi_{t+1} = \zeta/r_t$ in equilibrium, which is likely away from target.

[12] See Castillo-Martinez and Reis (2019) for a review of how a central bank that controls interest rates, the money supply, or its portfolio can achieve its desired inflation.

5.2 Policy: constraints and transmission

The economy can be stuck in a stagnation trap in which the zero lower bound binds forever. This is the case if productivity growth is persistently disappointing and people systematically expect higher inflation than actual inflation.

Start with the case where markets efficiently allocate capital. In the model, this corresponds to $\alpha + \gamma > 1$ and, in this case $r_t = m_t$ at all dates. Now, if the ZLB binds, then r is too high in steady state relative to the neoclassical r, so the capital stock is lower. Inflation is below target, and it will be lower the higher is the real interest rate. Since wages are driven by the reference norm, they are above their neoclassical counterpart ω. Employment is low. The combination of low employment and low capital stock means that output is persistently below potential.

Consider now the same scenario—low productivity growth and expected inflation exceeding higher inflation—with a binding zero lower bound in steady state, but now when there is misallocation of capital due to financial frictions. The pair of equilibrium (m, r) is still the solution of equations (20)-(21). Inflation is then pinned down by $\pi_{t+1} = 1/r_t$, and employment is given by equation (25). Note that having inflation below expected is a necessary condition for under-employment.

Either way, note that whether we are in this equilibrium, or in the neoclassical one of the previous section, the equilibrium depends on the value of r. In other words, the government bond r^* now plays a key role, unlike in the previous section. It is this government bond r^* that determines whether policy is constrained, and so whether the economy is in this secular stagnation equilibrium

However, turn to output. The level of output in equilibrium at this steady-state secular stagnation equilibrium is:

$$y = A \left(1 - \frac{1-\theta}{\mu}\right)^{\frac{\theta}{1-\theta}} \underbrace{\left(\frac{\zeta}{r\pi^e}\right)^{\frac{\eta}{\theta(1-\eta)}}}_{\propto\, l} \underbrace{\left(\frac{1}{m-1+\delta}\right)^{\frac{\theta}{1-\theta}}}_{\propto\, k}. \tag{26}$$

The first two multiplicative terms on the right-hand side are the standard effect of markups and productivity on output. The third term is proportional to employment and appears because of equation (25). We see that r matters for the level of output because as it falls, this indicates the economy becoming closer to what it would have been without the ZLB, thus boosting aggregate demand and employment. These three effects are present with and without misallocation

However, with misallocation there is a fourth multiplicative term showing that m plays a role. This term is proportional to the capital stock. The transmission to investment of either monetary policy or relaxing the ZLB, comes through m. As in the neoclassical model, it is m, not r, which determines investment. Therefore, for the important transmission of monetary policy to investment, it is m that should be the focus of monetary policy.

5.3 Monetary policy when capital markets are inefficient

The classic answer to the problem of the ZLB is for monetary policy to get the economy out of this rut by raising inflation or relaxing the ZLB. In the model this corresponds to raising π^* relative to π^e, or to lowering ζ. Either would have the real interest rate fall, and possibly succeed in removing the constraint on monetary policy. Moreover, as inflation is brought back in line with its expected value, real wages can fall to their neoclassical level, and the economy would return to full employment. The priority for monetary policy is clear: generate inflation, or "commit to being irresponsible" (Krugman, 1998).

With this equilibrium in mind, central bankers would fear inflation becoming too low. Undershooting the inflation target becomes a measure of failure. Because of this, there is a strong case for raising the inflation target, in case the original source of the problem was having a too low target relative to what workers expected in their norm. Analyses that focused on r^* in the 2010s subverted the 1980s dominant view that high inflation was the danger.

However, relative to the case where $m = r$, note that the m on the right-hand side of equation (26) no longer falls one-to-one with r and therefore one-to-one with π. With financial frictions, the benefits of higher inflation in stimulating real activity are lower.

Another common policy prescription during a secular stagnation is to increase government spending g in the model. Whether with efficient or inefficient capital markets, this will raise inflation and boost employment. However, with $m = r$, this also raises capital, crowding in investment because of the resulting fall in real returns. With inefficient capital markets, instead r falls, but the $m - r$ wedge rises, and investment is now crowded out. The government spending multiplier is smaller.

With efficient capital markets, increases in productivity growth that raise β will backfire and lower output because they lower inflation, and employment. Again, this effect is present with inefficient capital markets, but now there is a countervailing effect, because investment rises. Output may raise or fall.

The model in this paper suggests a new policy that can be very effective at the ZLB: reforming capital markets. Raising α and/or γ will lower m as more savings makes its way to investment in the capital stock. This raises output directly. Moreover, the fall in m also indirectly lowers the equilibrium r. Therefore, inflation rises, employment increases, and there is a second round effect further boosting capital. Overall, output increases through both channels. A sufficiently ambitious and effective improvement in the allocation of capital can close the gap between m and r so much that it brings r higher and the economy away from the zero lower bound.

In the other direction, policies meant to raise r focussed exclusively on exiting the zero lower bound, can easily backfire if they raise m. This may be the case with monetary policies that interfere so actively in financial markets with the goal of stimulating demand, that they end up hurting supply by worsening the allocation of capital across the private economy.

5.4 Implications for fiscal policy

Reis (2021) argues for rewriting the government budget constraint in equation (15) as follows:

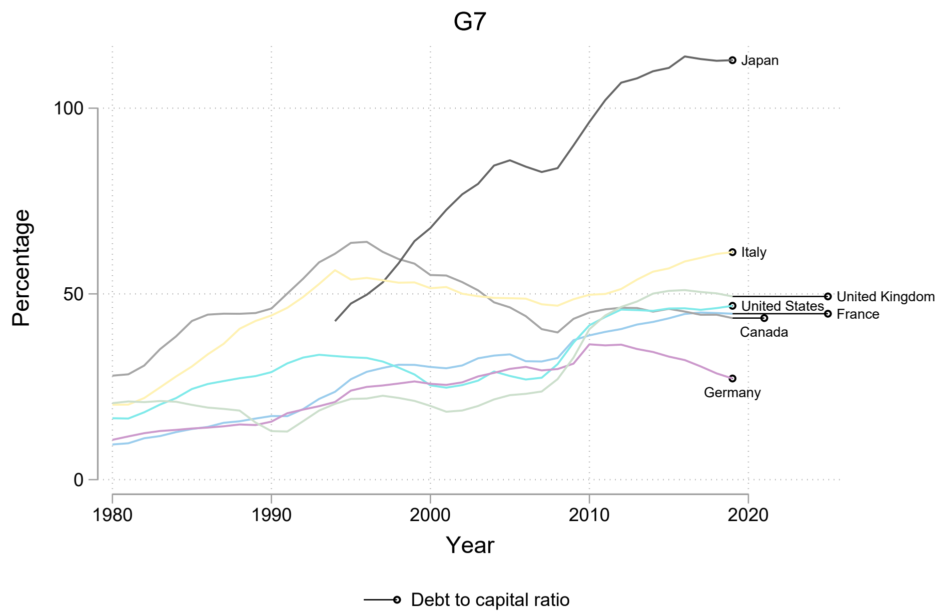
$$\frac{b}{k} = -\frac{g}{m-1} + \frac{(m-r)b}{m^*-1} \tag{27}$$

The first-term on the right-hand side is the present value of primary surpluses, using the private return on assets to discount the future. The second-term is the present value of debt revenues: the implicit revenue for the government of being able to borrow at rate r when the opportunity cost of assets in the economy is the higher m. When $r < m$, the government can run a deficit forever ($g > 0$), while sustaining a positive level of debt.

As Blanchard (2019) emphasized, low interest rates on the public debt are consistent with permanently higher spending. A fall in r with a commensurate fall in m, leaving $m - r$ unchanged, raises the sustainable amount of debt. This happens because both the flow debt revenue and the primary deficit are unchanged, but its future values are discounted less. Figure 14 shows that many countries used this extra fiscal capacity.

However, when r falls, but m stays unchanged, a rough characterization of the data, the present value of primary surpluses remains unchanged, as Jiang et al. (2019*a*) found to be the case. The debt can rise because the debt revenue is now higher.

Figure 14: Debt to capital ratio



6 Conclusion

This paper makes one empirical observation and one theoretical point.

Empirically, it measures r^* as the return on private capital. Taking either the side of investment or the side of savings, different views on what is capital, or different advanced economies, the robust finding is that r^* has been constant or slightly risen. At the same time, because the return on government bonds has fallen, there has emerged a large wedge between these two rates of return in the economy.

Theoretically, this paper explored one hypothesis for this increasing wedge: a rise in the misallocation of investment because of financial frictions. I focus on what this implies for monetary and fiscal policy, and reach five conclusions. First, that while the return on government bonds is the right focus to understand if monetary policy will be constrained, the return on private capital is the right object to focus on to measure the transmissions of monetary policy to investment. Second, the benefits of higher inflation at the ZLB are smaller if there are financial frictions breaking the tight link between the two rates of return. Third, boosting aggregate demand is less powerful, because while it will lower the return on government bonds, it will either raise the return on private capital, or leave it unchanged, crowding out investment. At the same time, raising productivity growth may boost the economy, even at the zero lower bound. Fourth, the more urgent and effective policies are to improve the mechanisms by which the economy allocates capital to different agents, whether that is achieved by better regulation, lowering market power in the financial industry, promoting financial innovation, or others. Closing the gap between the two return is a stronger priority, and policies that focus on improving the efficiency of the allocation of capital hold more promise. Fifth, and last, for fiscal policy, it is the gap between these two returns that creates the fiscal space that has allowed the public debt to grow.

All together, focusing exclusively on the return on government bonds as the measure of r^*, while neglecting the return on private capital, leads to the wrong policy advice. Was this a cause of the rise in inflation in 2021-22? Perhaps, since this framework contributed to a tolerance towards higher inflation and elevated inflation expectations as well as a focus on aggregate demand. The arguments and theory in this paper point instead to being more worried about high inflation and more focused on aggregate supply reforms.

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Appendix – For Online Publication

This appendix has three sections. Section A describes which country are in which group. Section B describes the sources of the data. Section C displays the estimates of m for each country individually, both baseline, trend, and alternatives.

A Countries

The list of countries included in the graphs, and which groups they were placed in, is listed in the following table.

Table A1: Country groups

G7	BRICS	Advanced Economies			
Canada	Brazil	Australia	Finland (*)	Latvia	Singapore
France	China	Austria (*)	France (*)	Lithuania	Slovakia (*)
Germany	India	Belgium	Germany (*)	Luxembourg (*)	Slovenia (*)
Italy	Russia	Canada (*)	Greece (*)	Malta	Spain
Japan	South Africa	Cyprus	Iceland	Netherlands (*)	Sweden (*)
United Kingdom		Czechia (*)	Ireland	New Zealand	Switzerland
United States		Denmark (*)	Italy (*)	Norway	United Kingdom (*)
		Estonia (*)	Japan (*)	Portugal	United States (*)

(*) included in measure without real estate.

B Data Sources

For the baseline estimates of m, the source of data for the net operating surplus are the national statistics: the BEA for the US, StatCan for Canada, the Cabinet Office for Japan, SingStat for Singapore, IBGE for Brazil, Rosstat for russia, MoSPI for India, NBStat for China, SARB for South Africa, AMECO for European countries. From the net operating surplus, 2/3 of mixed income was always subtracted when that series was available. Regarding European countries, the series adjusted for self-employment was obtained directly from AMECO. For the private capital stock series, the same sources are used, but when a series was not found, I used the IMF's Investment and Capital Stock website. Two adjustments needed were: (i) for Canada series, for private capital subtracted the government capital stock series from the capital stock for all industries and added all residential capital (ii) for Brazil net operating surplus was gross, so depreciation was taken out using estimates of it from the World Bank.

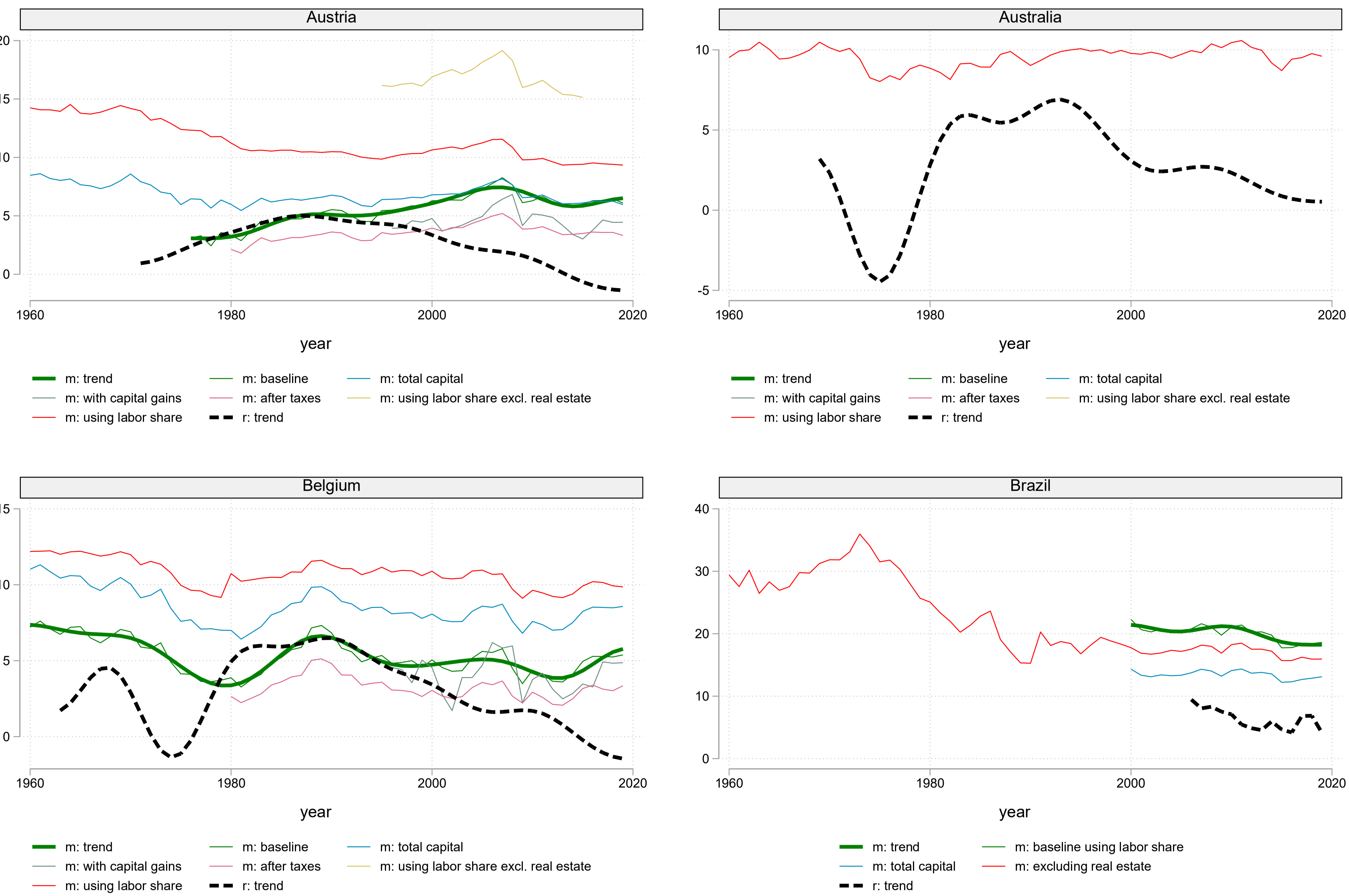
For the alternatives shown: (a) the series with total capital replaces private capital with the total capital stock from the same sources, (b) the series with capital gains adds to the baseline series, the expression $(p^k_{t+1}/p^k_t)(p_t/p_{t+1})$ using the investment deflator and the GDP deflator from AMECO and IMF, (c) the series after taxes comes from subtracting corporate taxes available from the OECD, and from Gomme, Ravikumar and Rupert (2015) for the US available in Paul Gomme's website.

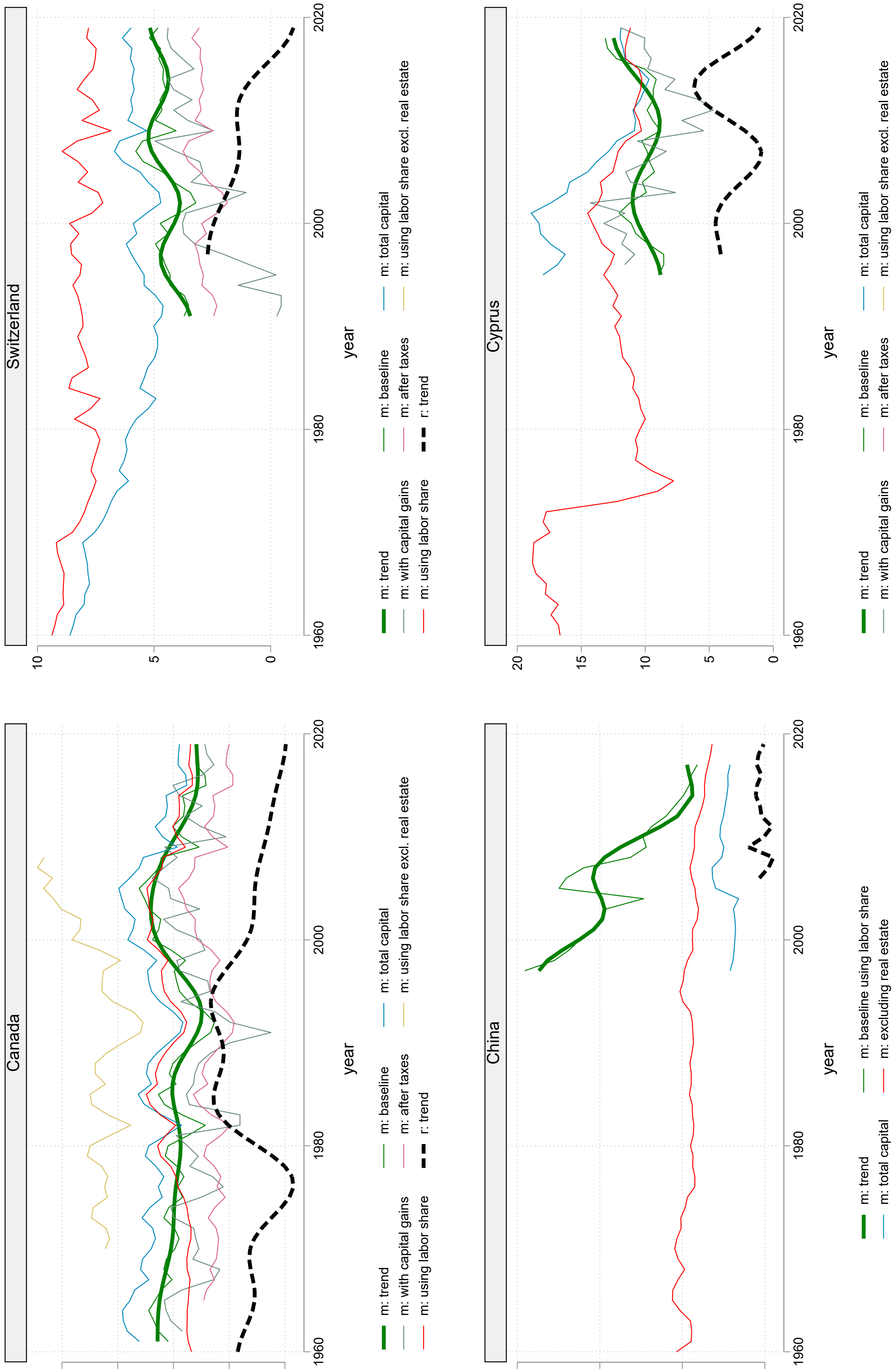
The labor share estimates come from the Penn World Tables, and I use the series for: the labor share, the PPP price, real GDP, the GDP deflator and the investment deflator.

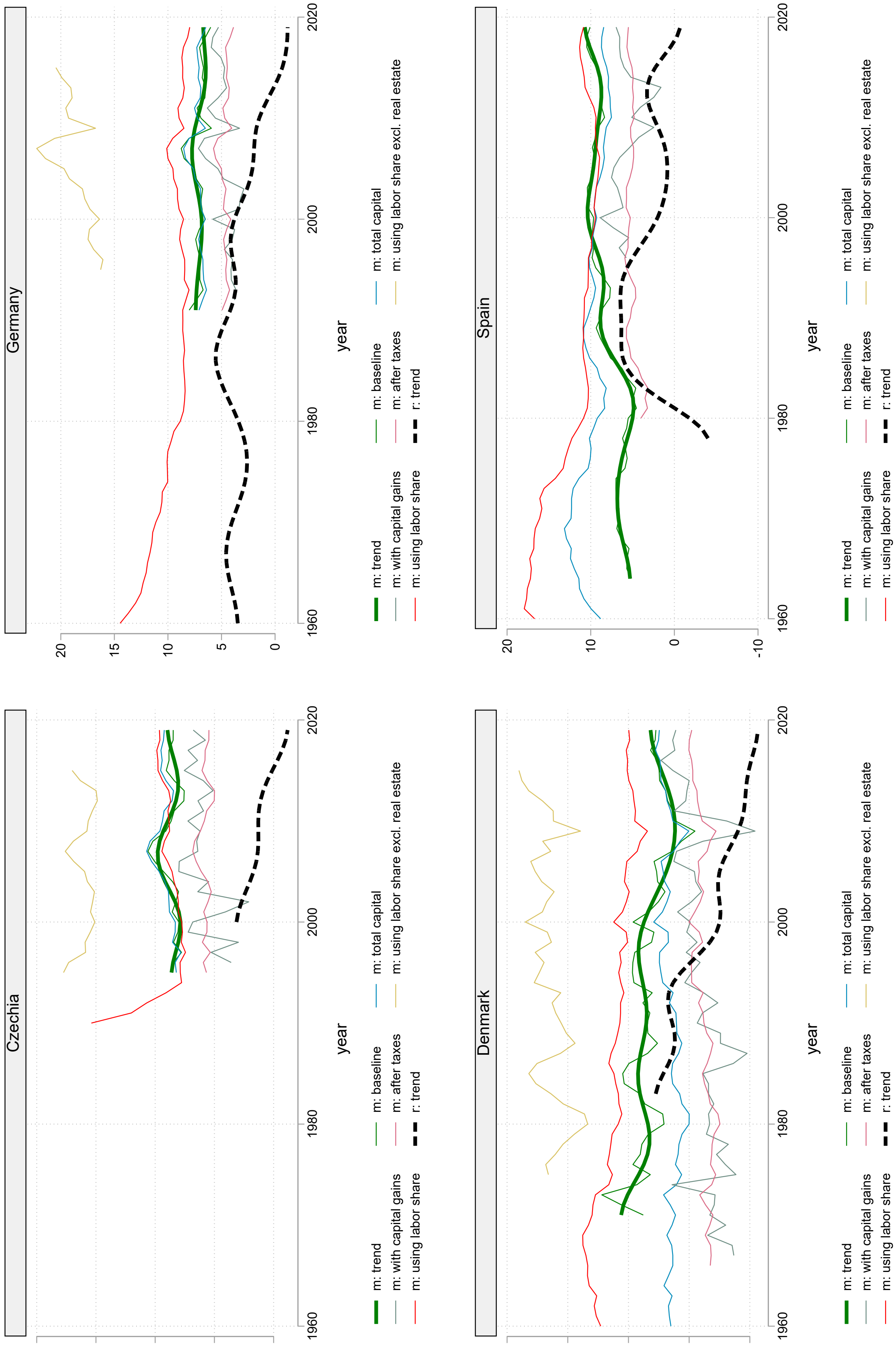
The labor share estimates excluding real estate come from: the labor share in Gutiérrez and Piton (2020), the gross value added in KLEMS (wither EU or World), and the nominal private capital in KLEMS as well. The data for the sectorial decomposition comes from: the BEA for the US, StatCan for Canada, the Cabinet Office for Japan, and Eurostat for European countries, for both net operating surplus and capital stocks.

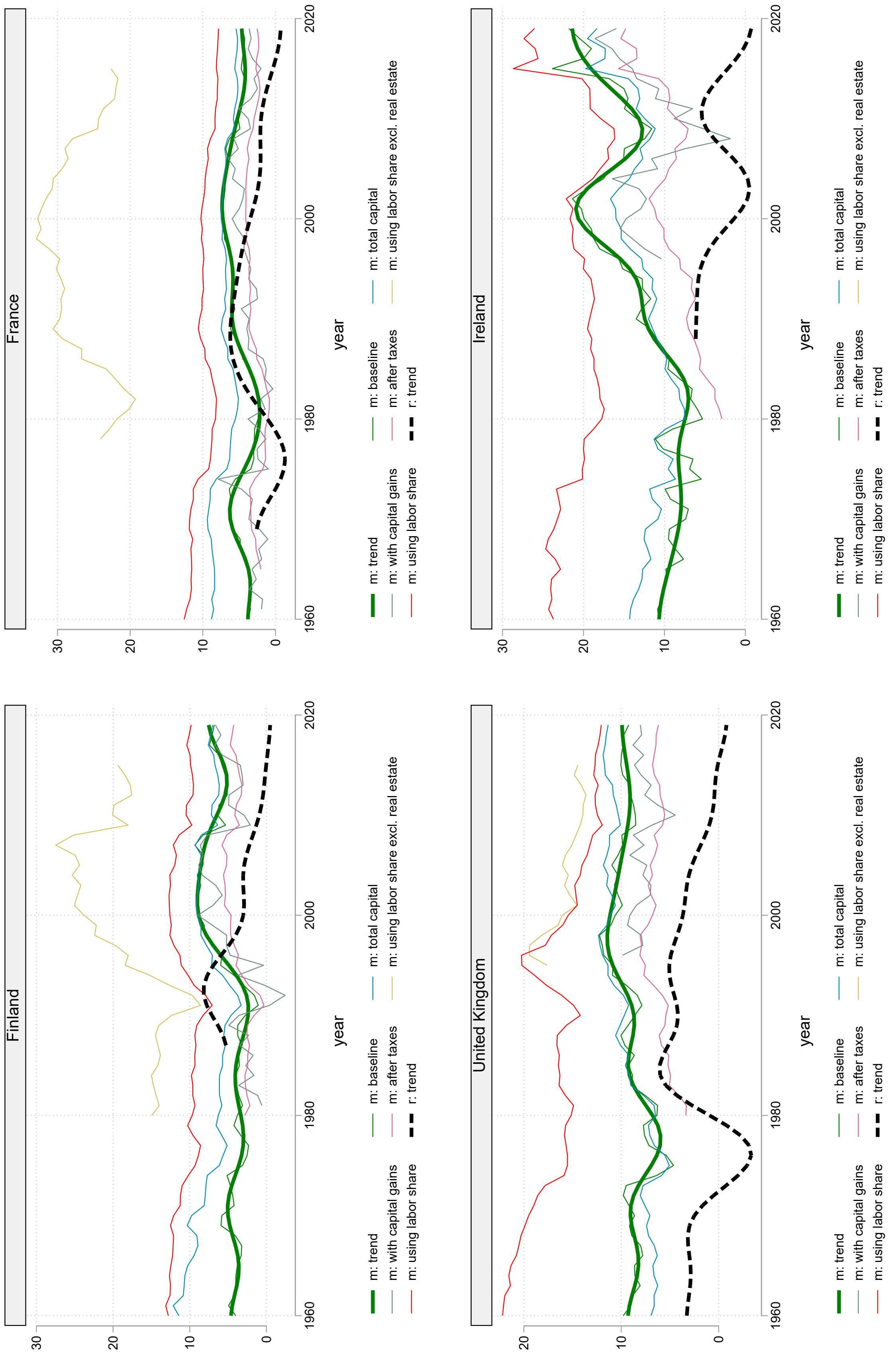
The data on intangible capital comes from INTAN-invest produced by Corrado et al. (2020) following the methods in Corrado and Hulten (2010)

C Estimates of m and r per country

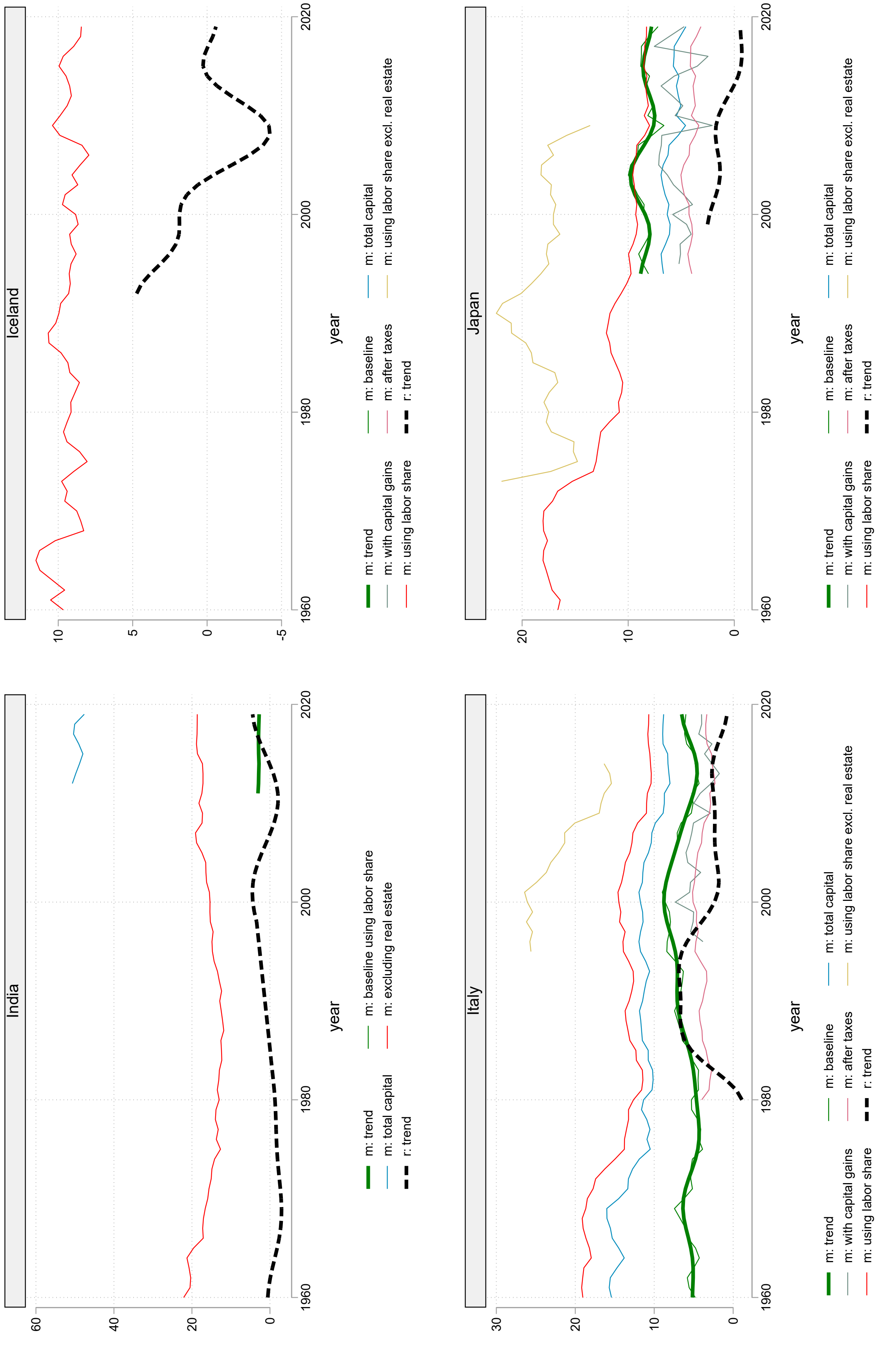

Austria
Australia
Belgium
Brazil
year
m: trend
m: with capital gains
m: using labor share
m: baseline
m: after taxes
r: trend
m: total capital
m: using labor share excl. real estate
m: baseline using labor share
m: excluding real estate

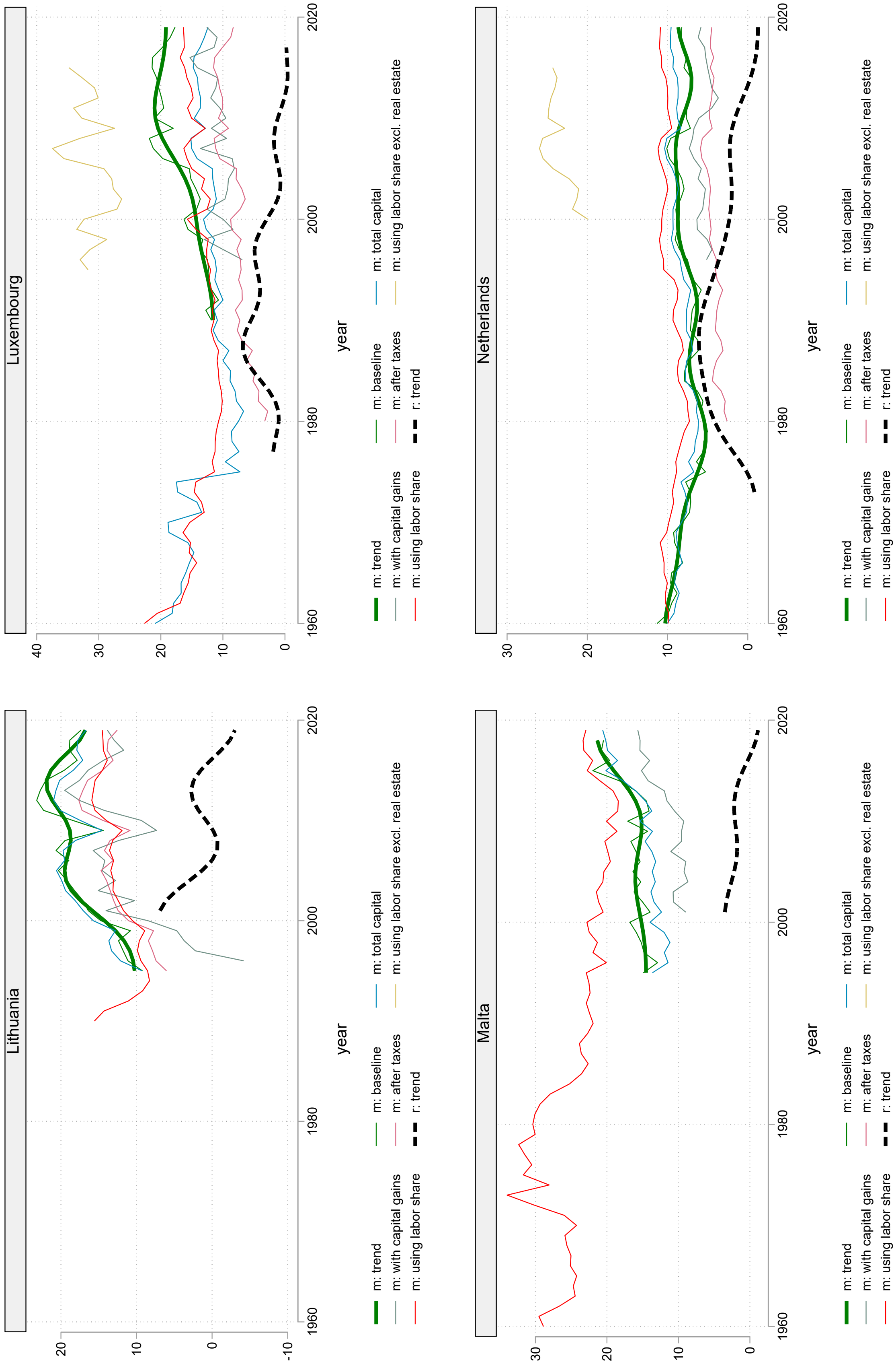

Canada
Switzerland
China
Cyprus
year
m: trend
m: baseline
m: total capital
m: with capital gains
m: after taxes
m: using labor share excl. real estate
m: using labor share
r: trend
m: baseline using labor share
m: excluding real estate

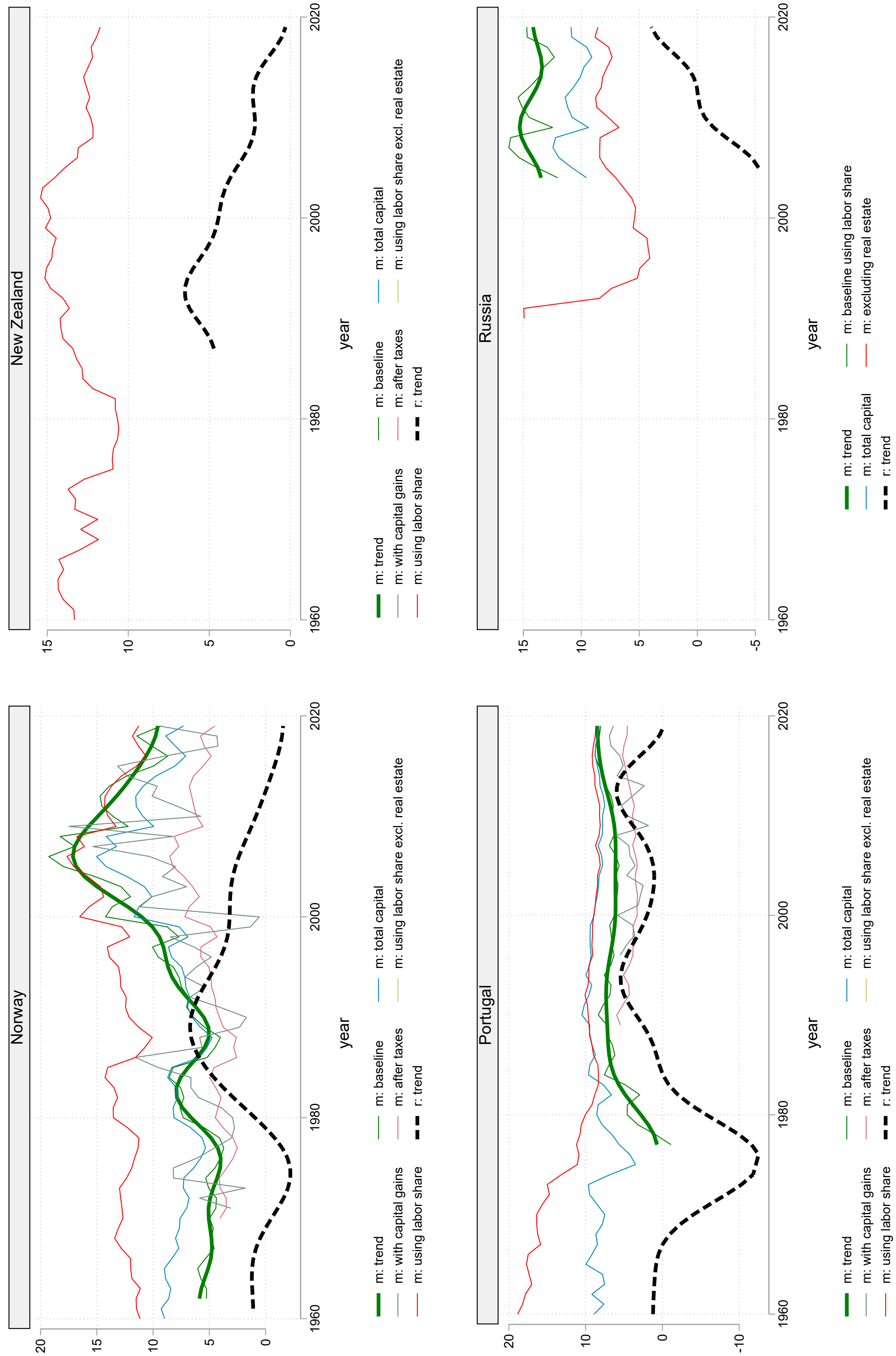

Czechia
Germany
Denmark
Spain
year
m: trend
m: baseline
m: total capital
m: with capital gains
m: after taxes
m: using labor share excl. real estate
m: using labor share
r: trend

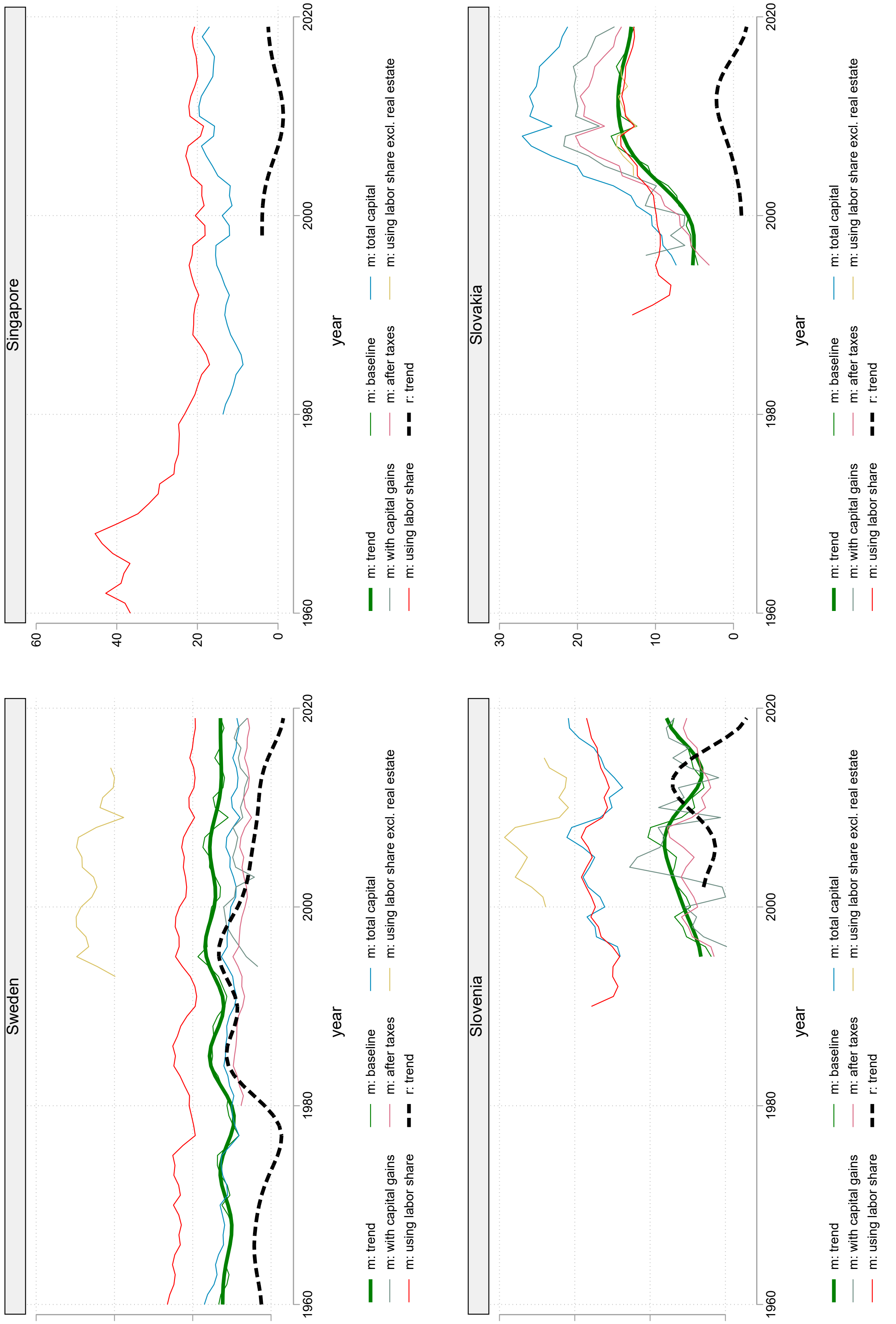


Finland
France
United Kingdom
Ireland
year
m: trend
m: with capital gains
m: using labor share
m: baseline
m: after taxes
r: trend
m: total capital
m: using labor share excl. real estate

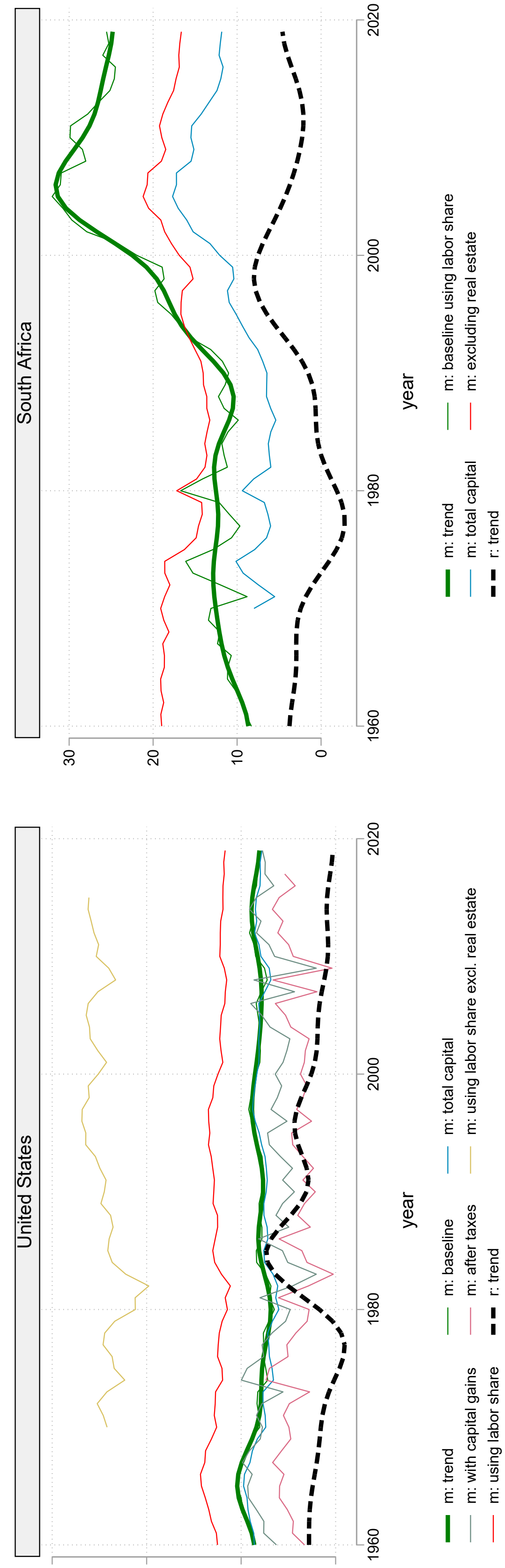

India
Iceland
Italy
Japan
year
m: trend
m: total capital
r: trend
m: baseline using labor share
m: excluding real estate
m: with capital gains
m: using labor share
m: baseline
m: after taxes
m: using labor share excl. real estate


Lithuania
Luxembourg
Malta
Netherlands
year
m: trend
m: with capital gains
m: using labor share
m: baseline
m: after taxes
r: trend
m: total capital
m: using labor share excl. real estate


Norway
New Zealand
Portugal
Russia
year
m: trend
m: with capital gains
m: using labor share
m: baseline
m: after taxes
r: trend
m: total capital
m: using labor share excl. real estate
m: baseline using labor share
m: excluding real estate
1960
1980
2000
2020



Sweden
Singapore
Slovenia
Slovakia
year
m: trend
m: with capital gains
m: using labor share
m: baseline
m: after taxes
r: trend
m: total capital
m: using labor share excl. real estate


United States
South Africa
year
year
m: trend
m: with capital gains
m: using labor share
m: baseline
m: after taxes
r: trend
m: total capital
m: using labor share excl. real estate
m: trend
m: total capital
r: trend
m: baseline using labor share
m: excluding real estate
1960
1980
2000
2020
0
10
20
30