HOW LIKELY IS AN INFLATION DISASTER?

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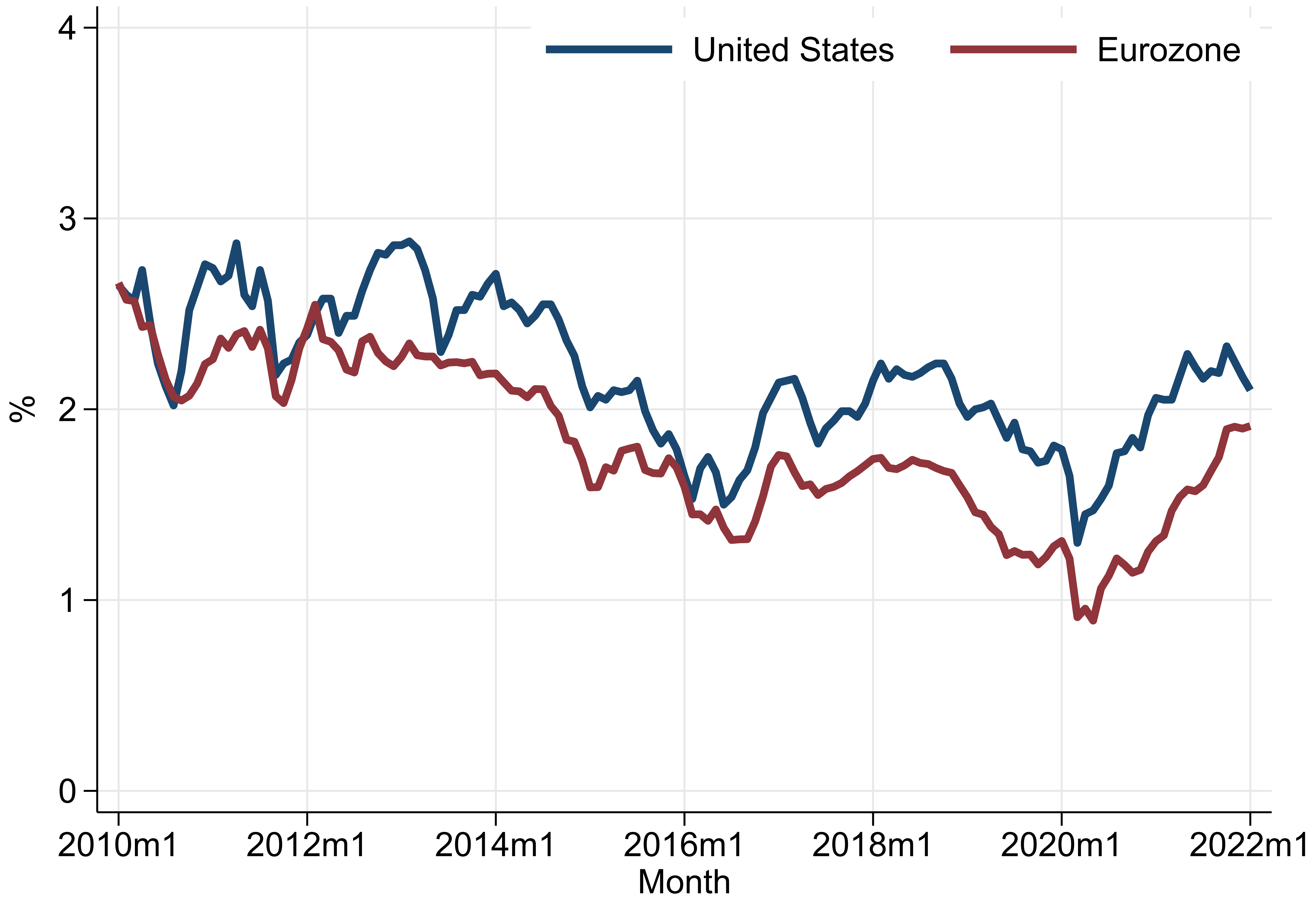
Expected long-run inflation



- 5-year, 5-year expected inflation
- Easy to build and carefully monitored
- But not about disasters
- Robust decision making cares about distribution and especially about tails. Driver of costs of inflation

Methods and options data to answer

What is the current date t market perceived probability that inflation will be persistently above or below the annual target between T and T + H?

For example, what is the current probability that average inflation will be above 4% between 5 and 10 years from now?

$$\Phi_t^{dh} = Prob[\pi_{T,T+H}/H > \bar{\pi} + d], \qquad T = 5, H = 5,$$

$$\Phi_t^{dl} = Prob[\pi_{T,T+H}/H < \bar{\pi} - d]. \qquad \bar{\pi} = 2\%, d = 2, 3\%$$

Our contribution:,

(i) use inflation swaptions data, make three important adjustments

(ii) also risk-adjusted probabilities, also probabilities over nearer (5y) horizon

(iii) re-evaluate 2010-21 monetary policy, risks of deflation and of inflation

Literature review

- Tail outcomes for inflation disasters, like literature on inflation at risk
  (Andrade, Ghysels and Idier, 2012, Lopez-Salido and Loria, 2020, Adrian, Boyarchenko and Giannone, 2019). However, we focus on markets perceptions of this risk as measured by option prices, rather than on distributions of realized inflation.

- Expectations disasters in surveys
  Reis (2022) Ryngaert (2022) but perspective of financial markets and tails.

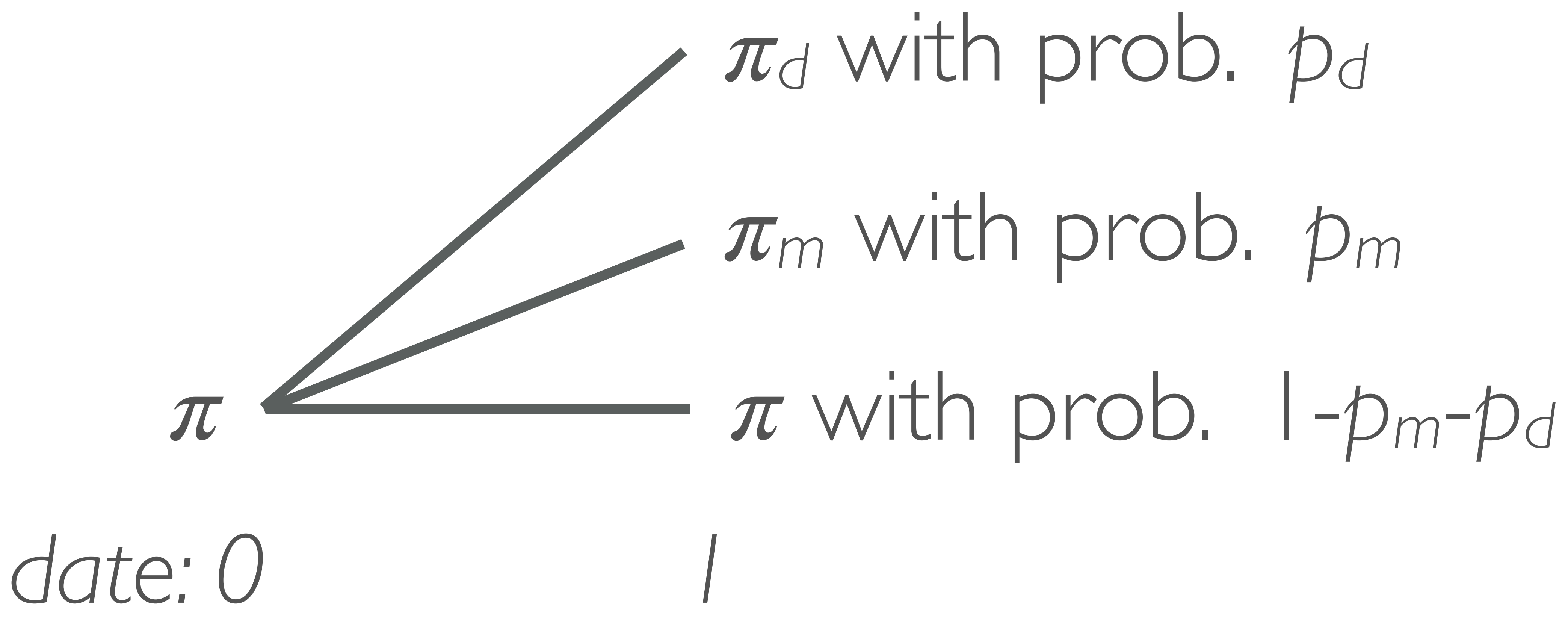
- Use inflation options data to focus on tail phenomena
  Hilscher, Raviv and Reis (2022), Mertens and Williams (2021), Kitsul and Wright (2013), Fleckenstein, Longstaff and Lustig (2017). Yet we focus on long- term forward horizons.

- Literature on equity disasters to adjust for risk
  (Barro, 2006, Gabaix, 2012, Barro and Liao, 2021), but we focus on inflation disasters.

Intuition of method

The standard reported probabilities

Panel A. Inflation event-tree



- An option that pays one unit if disaster at period 1 sells for price $a_d(1)$
- Price of that option: $a_d(1) = p_d\ m_d\ exp(-\pi_d)$
- Build probability $n_d(1) = a_d(1)\ exp(i(1))$ since positive and add to interest rate

First adjustment: risk neutral probabilities

- Arrow Debreu security pays 1 in disaster state

- Price of that security (probability):

$$q_d(1) = p_d\, m_d\, r(1) = p_d \quad \text{if risk neutral}$$

- But given definitions:

$$q_d(1) = n_d(1)\, exp(r(1) + \pi_d - i(1)\,) \approx n_d(1)\, exp(d)$$

- If horizon is short, or calculating near probabilities, adjustment is 1. But if 10-year ahead, 3% disaster, then adjustment: exp(10x1.03) = 1.35 (or 0.67)

- Intuition: when option pays, real payoffs are smaller, so option is cheaper.

Risk adjustment

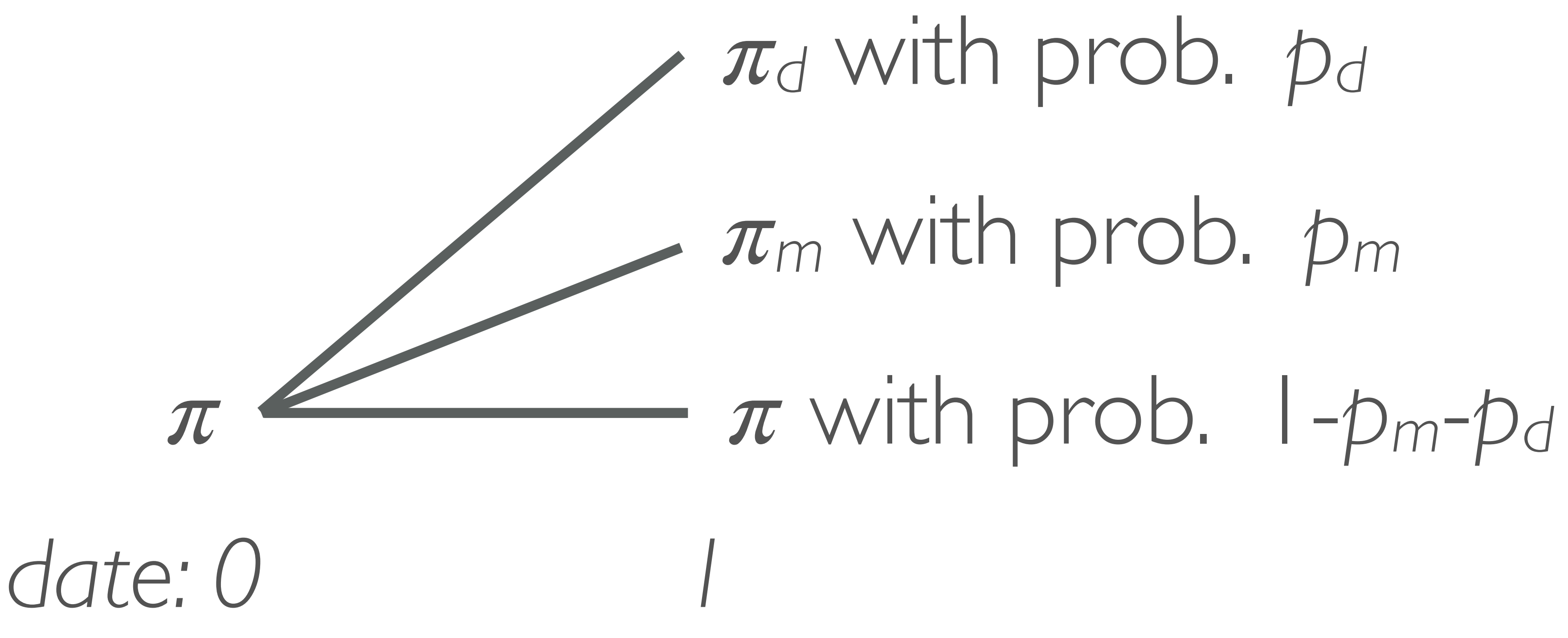
- Familiar one, go from risk neutral probabilities to actual probabilities:

$$q_d(1) = (1 + (m_d - 1)\, p)\, p_d(1)$$

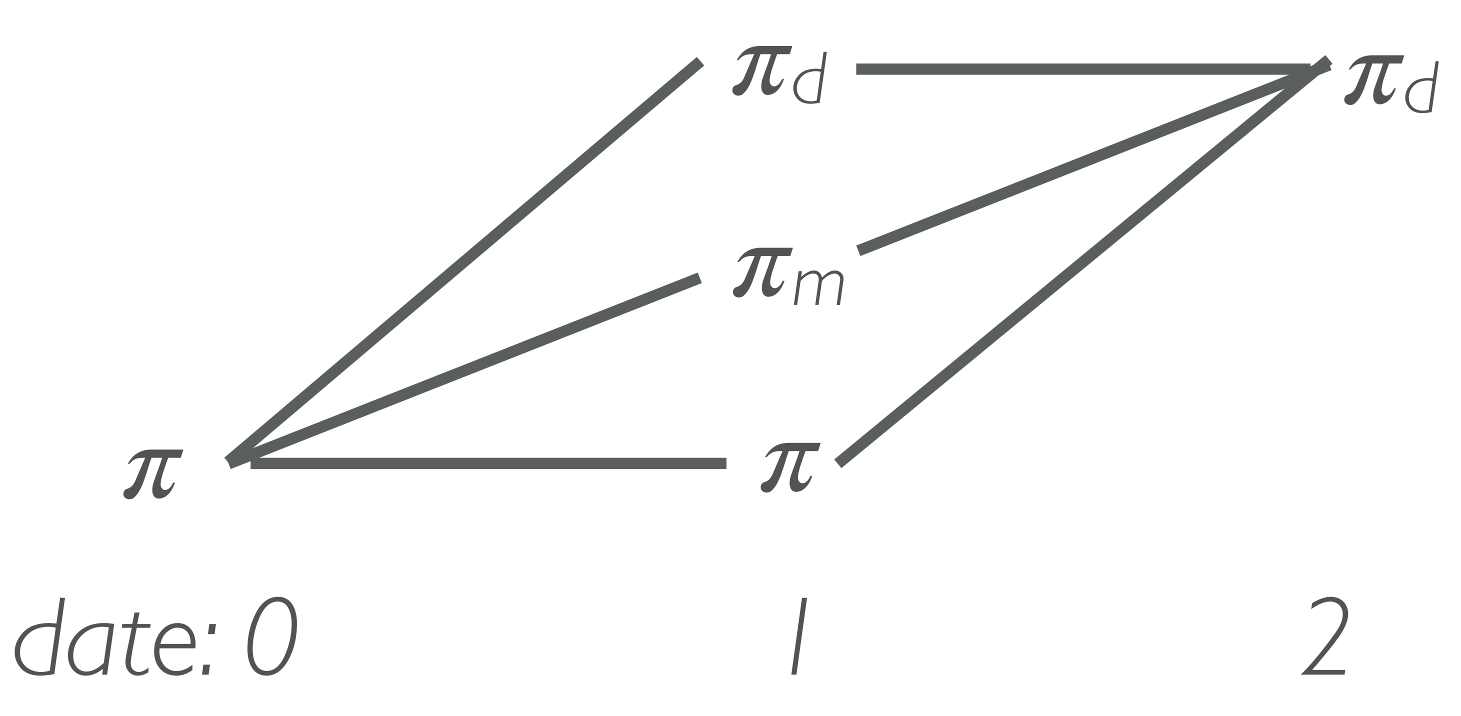
- Disaster are bad times, $m_d > 1$, so probability of disasters overstated
- But: (i) conditional distribution of output and inflation disaster p
- And (ii) average output drop in inflation disasters comes with an m_d
- Need: how often do inflation disasters and consumption disasters coincide? Less often than consumption and equity disasters, not so big adjustment…

Forward probabilities

Panel A. Inflation event-tree



Panel B. Distant inflation disaster



$$p_d(2) = p_m\, p_{md} + p_d\, p_{dd} + (1 - p_m - p_d)\, p_d$$

- First period probability: $p_d < p_d(2)$ ($p_m p_{md}/p_d$ *large enough*)
- Cumulative probability: $p_d(1\&2) = p_d\, p_{dd} < p_d(2)$
- <u>Answer:</u> $p_d(2)$ from a forward-dated option and model of persistence

Formal general analysis

The environment: focus on inflation risk

- Many booms and busts in economy with constant inflation

$$\text{States: } s \in S,\ \text{probabilities: } \tilde{p}(s) \geq 0, \sum_{s\in S} \tilde{p}(s) = 1$$

$$\text{Arrow-Debreu security price: } \tilde{b}(s) = \tilde{p}(s)\tilde{m}(s)$$

$$\text{Inflation: } \pi(s),\ \text{probabilities: } p(\pi) = \sum_{s:\pi(s)=\pi} \tilde{p}(s)$$

$$\text{Cardinality of } \Pi \leq \text{Cardinality of } S$$

$$\text{Inflation securities: } b(\pi) = \sum_{s:\pi(s)=\pi} \tilde{b}(s) = p(\pi)m(\pi)$$

$$\text{SDF: } m(\pi) = \sum_{s:\pi(s)=\pi} p(s)m(s)/p(\pi)\ \text{ only inflation risk}$$

Proposition: three adjustments to data

$$
\begin{aligned}
p_t(\pi_{T,T+H}) = &\underbrace{n_t(\pi_{T,T+H})}_{\text{Options}} \\
&\times \underbrace{\left(e^{(\pi_{T,T+H}-\pi^e_{T,T+H})H}\right)}_{\text{Real}} \\
&\times \underbrace{\left(e^{-r_{T,T+H}H}m(\pi_{T,T+H})\right)}_{\text{Risk}} \\
&\times \underbrace{\sum_{\pi_{0,T}}\left[\left(\sum_{\ldots=\pi_{T,T+H}H} q(\pi_{T,T+1},\ldots,\pi_{T+H-1,H}|\pi_{0,T})\right)\frac{q(\pi_{0,T})}{q(\pi_{0,T+H})}\right]}_{\text{Horizon}}
\end{aligned}
$$

Options

- Call option with strike price *k*, *a(k)*, knowing AD security is *b(**π**)*

$$a(k) = \sum_{\pi} \left(p(\pi) m(\pi) \max\left\{ \frac{e^{\pi} - k}{e^{\pi}}, 0 \right\} \right) \approx \int_k^{\infty} \left(\frac{e^{\pi} - k}{e^{\pi}} \right) b(\pi) d\pi$$

- Definition "nominal probabilities"

$$e^{i} a(k) = \int_k^{\infty} (e^{\pi} - k) n(\pi) d\pi \ \ \text{differentiate:} \ \ N(\log(k)) = 1 + I a'(k)$$

- The rhs is how sensitive the price of the option is to the strike price, easy to measure. But only actual probabilities if probabilities are all zero but one

$$n(\pi) = p(\pi) \ \ \text{only if} \ \ m(\pi) e^{r} e^{\pi - \pi^{e}} = 1$$

Risk neutral probabilities

- Obtain them instead from:

$$q(\log k) = e^r k a''(k)$$

- Which equals actual probabilities "only" requires risk neutrality

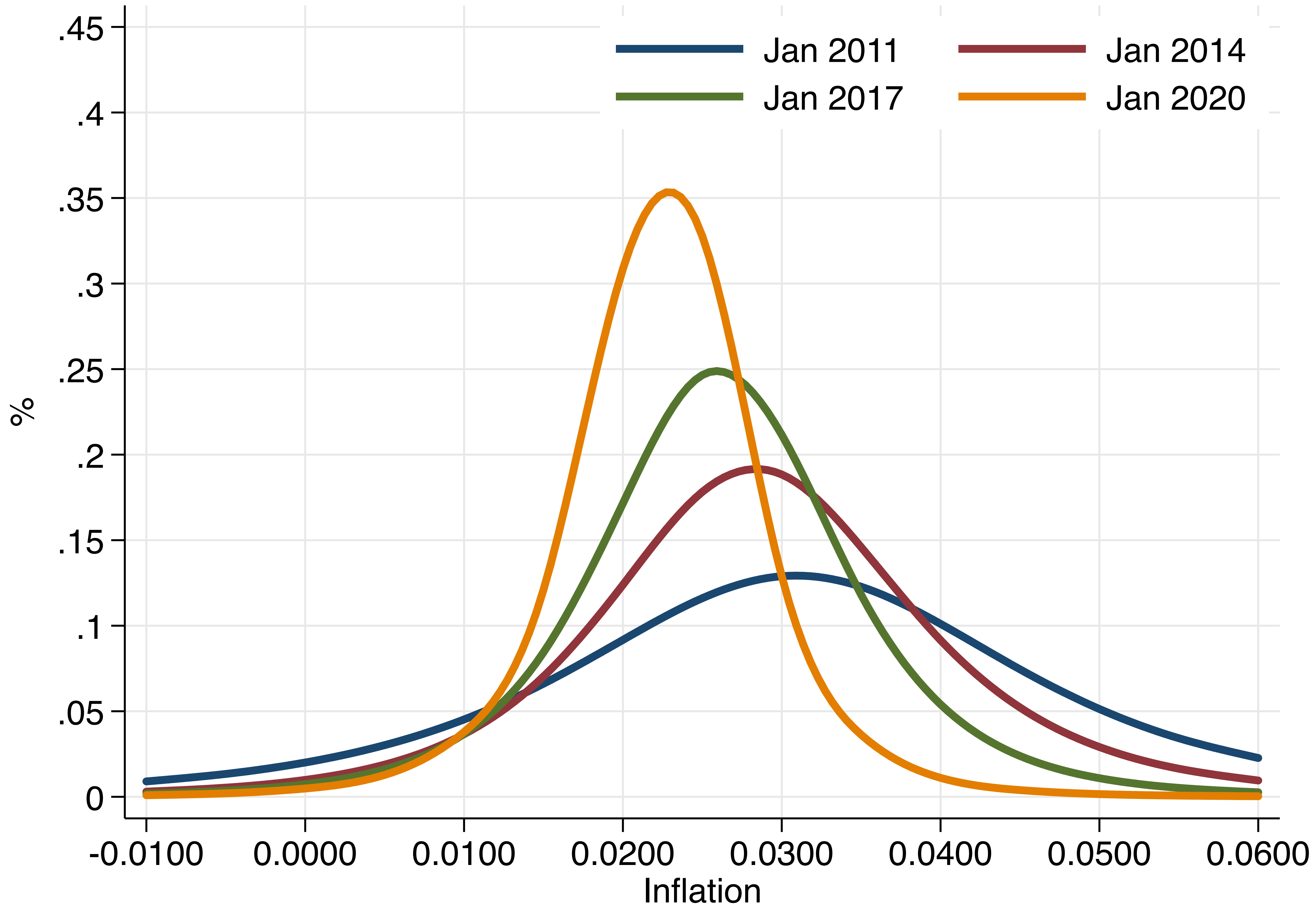
- Data on options to get these distributions:

  (i) Bloomberg, November 2009 to March 2022, will be updating monthly

  (ii) "cleaning:" in enforcing no arbitrage, using all quotes

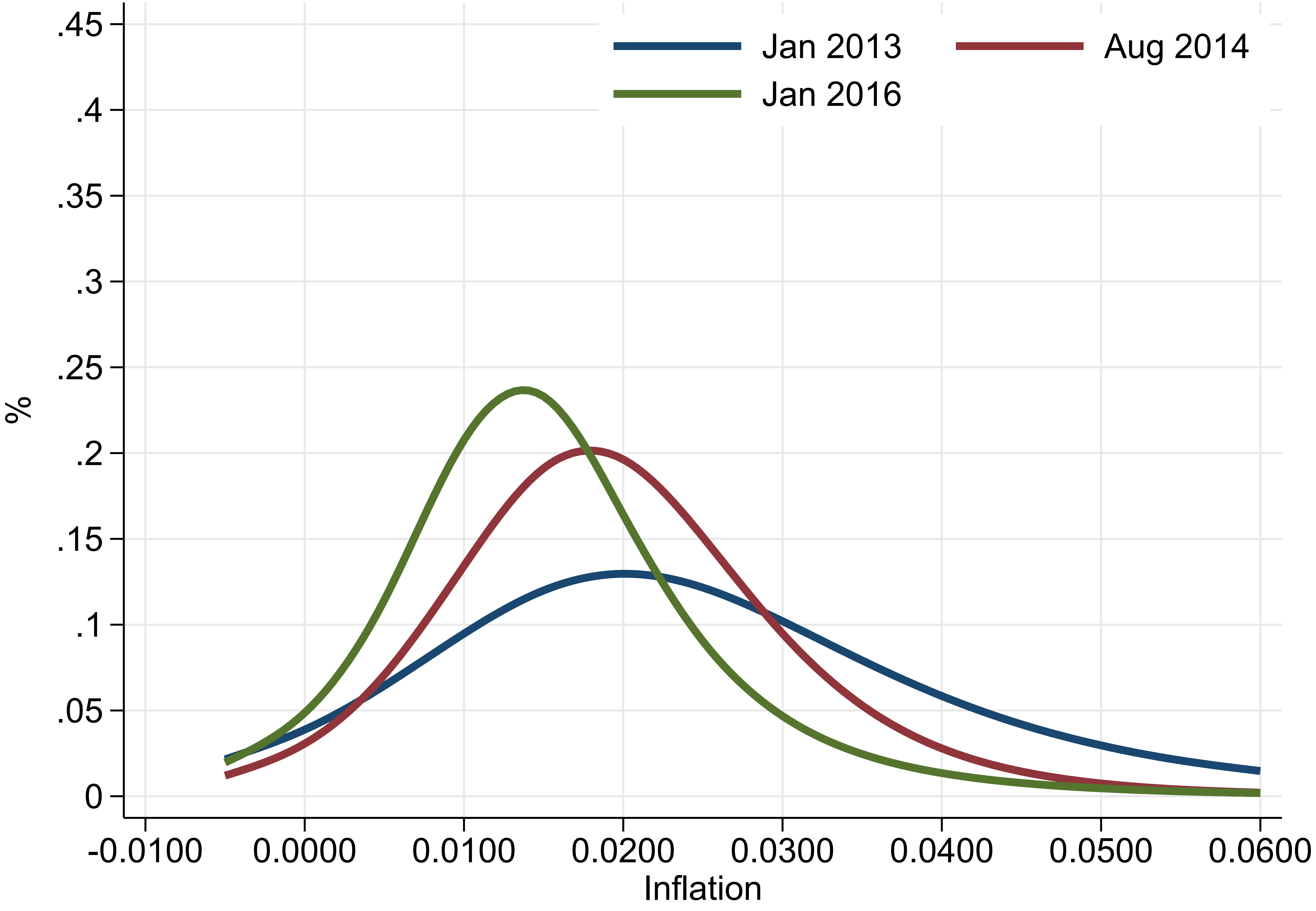
  (iii) while data exists daily, monthly is more conservative

US 10y distributions: 2011-20 re-anchoring



- Started with uncertainty on both sides
- Remarkably successful re-anchoring after the great financial crisis

Eurozone 10y distributions: the birth of QE

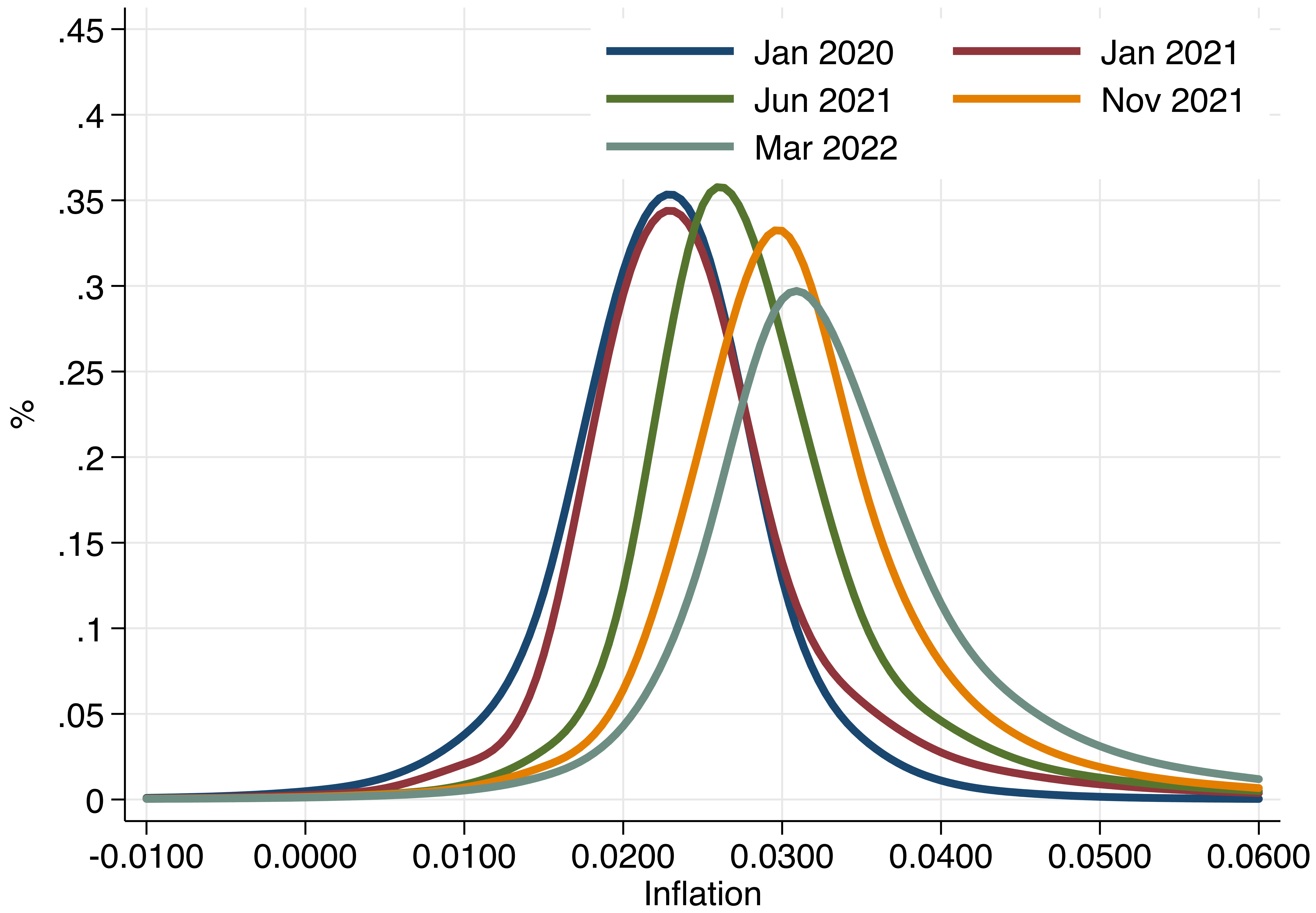


The start of the "anchored too low" period clearly there but:

(i) very concentrated in 0-1%

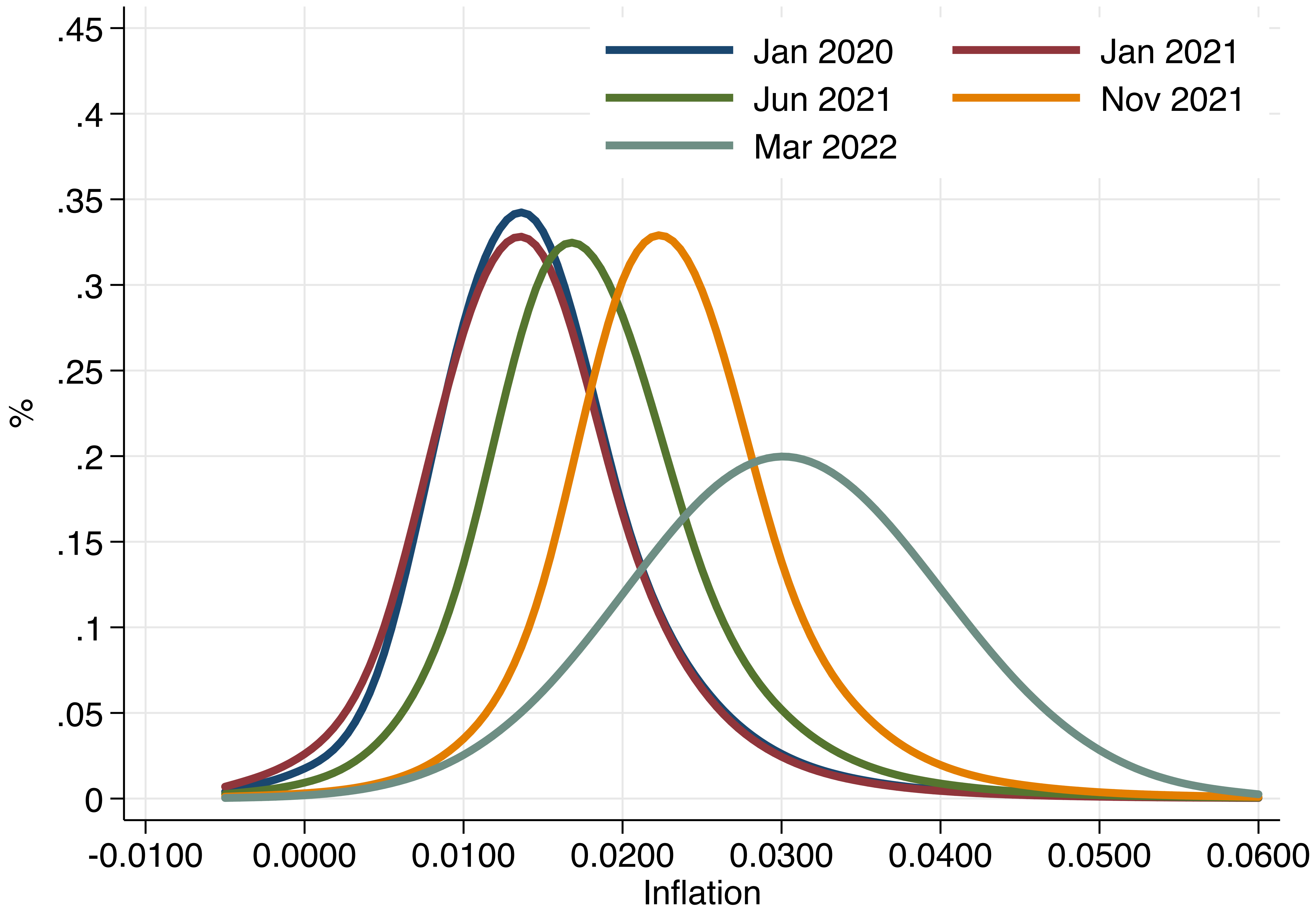
(ii) doubling of deflation disaster probability, but still only 7%

US 10y distributions: the pandemic



- During first half of 2020 had seen more mass on left tail, but by en do fear, all gone.
- But 2021 seeing steady shifts to the right
- Fed's pivot did not halt it

Eurozone 10y distributions: the pandemic



- Qualitatively similar, quantitatively different until 2022
- Huge change since start of year

Risk adjustment

- Consumption and inflation follow processes with *one* common disaster, since inflation disasters alone lead to no risk adjustment, and consumption disasters alone do not trigger the option:

$$\pi_{t+\Delta} = \bar{\pi} + u^{\pi}_{t+\Delta} + \varepsilon_{t+\Delta} + d^h_{t+\Delta} - d^l_{t+\Delta},$$

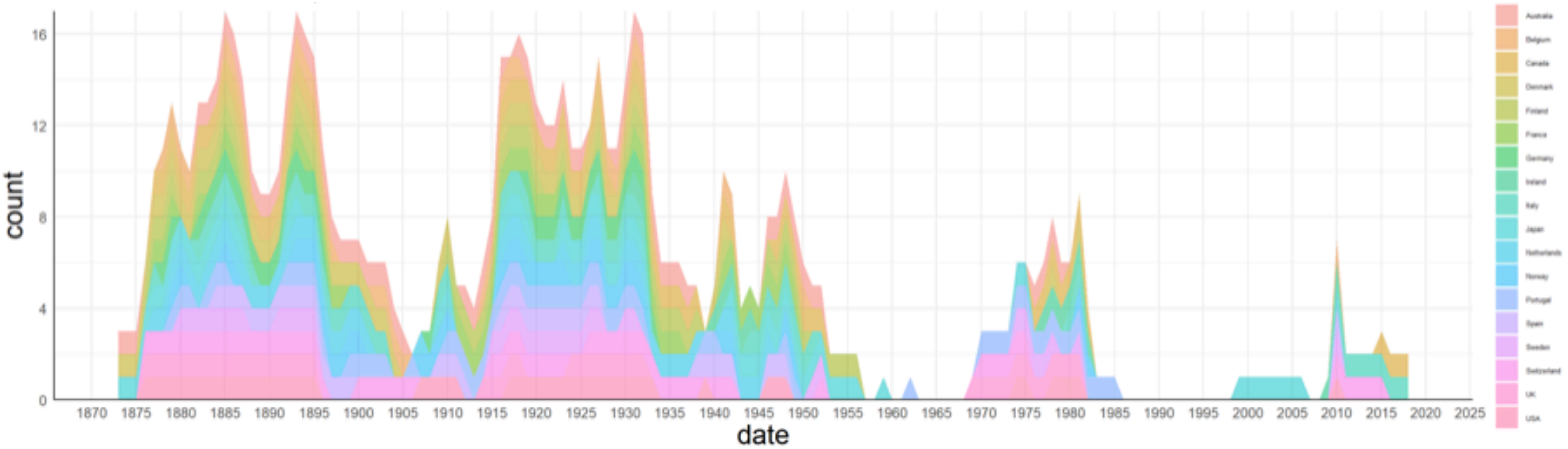
$$\log(c_{t+\Delta}) = \log(c_t) + g + u^c_{t+\Delta} + \beta_0 \varepsilon_{t+\Delta} - \beta^h d^h_{t+\Delta} - \beta^l d^l_{t+\Delta}.$$

- With probability p, disaster $\beta^h d = 1 - 1/z$, where z has a Pareto distribution:

$$F(z^h) = 1 - \left(\frac{z^h}{z^h_0}\right)^{-\alpha^h} \quad \text{with } z^h \geq z^h_0 > 1, \alpha^h > 0$$

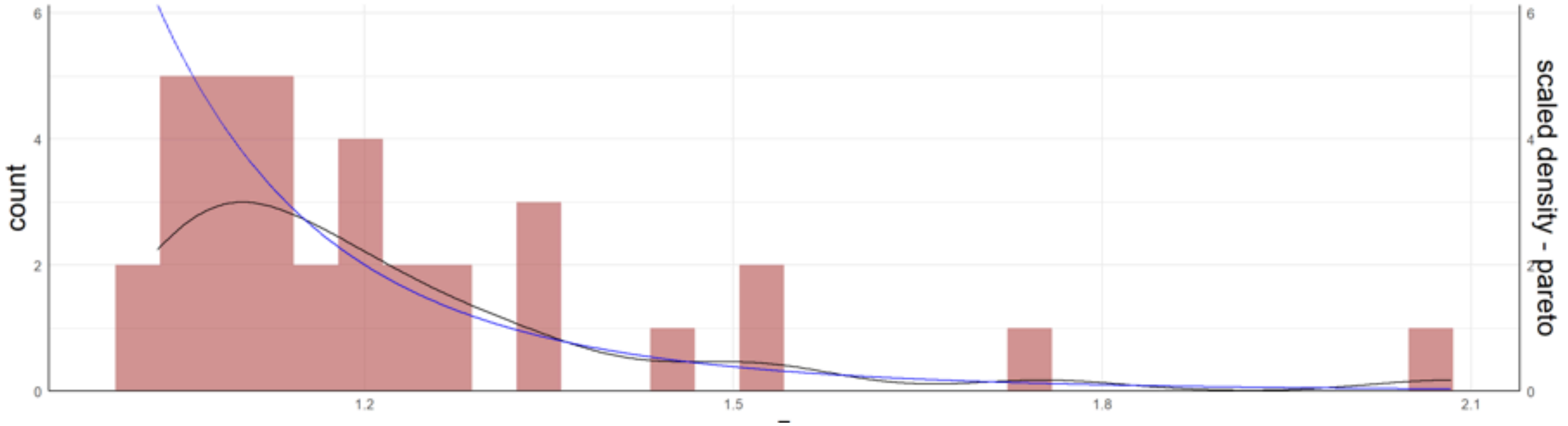
- Risk aversion 3 (E-Z utility), key parameters are: z, α

Distributions in the data



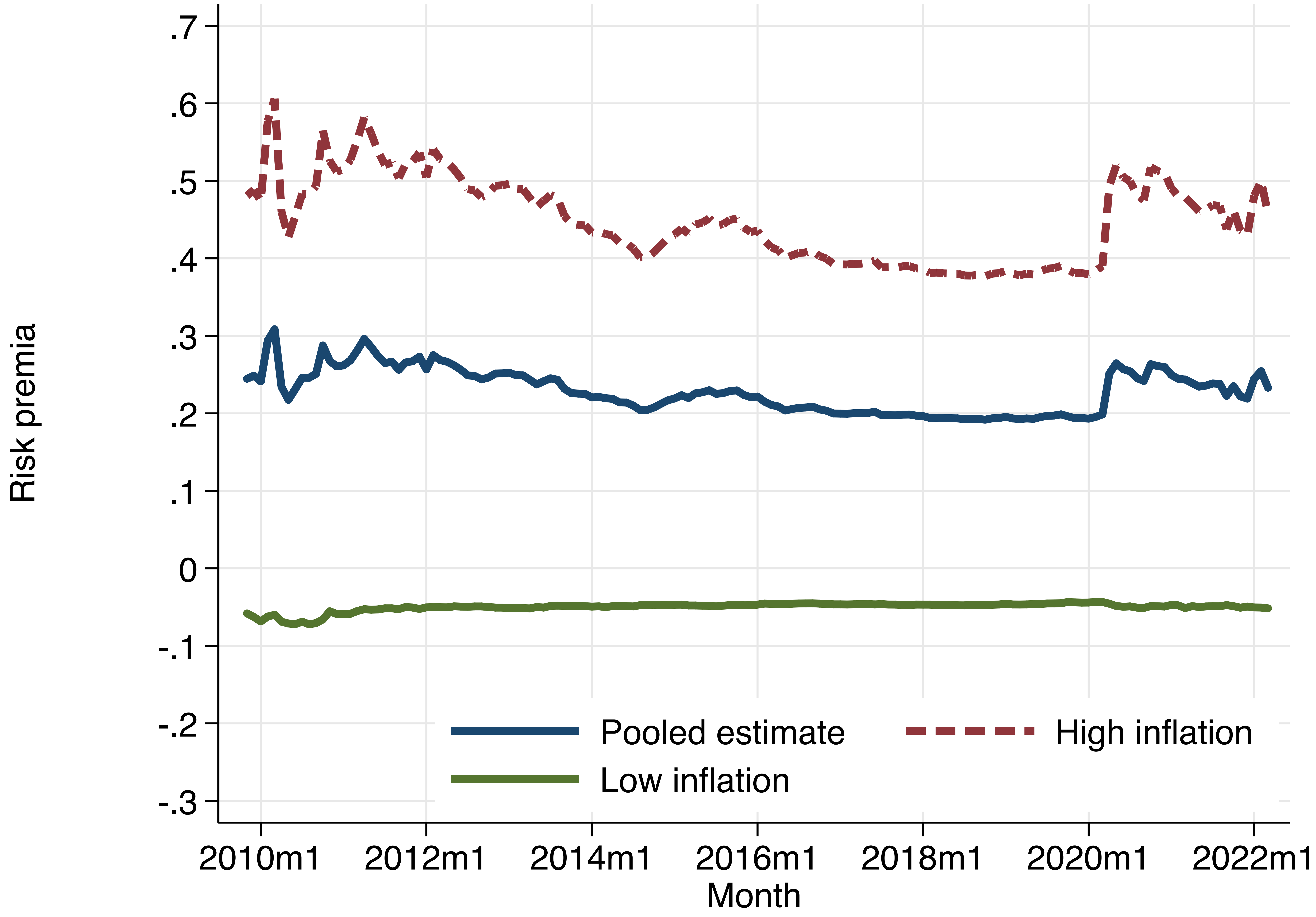
- Data: Barro (2006) consumption, Jorda Schularick Taylor (2019) inflation
- Find peaks/troughs in 5-year windows; relative to band pass filter freq>20 years
- Probability of output disaster conditional inflation disaster: 20.3%

Pareto distribution



- Estimates: $\alpha = 6.38$, $z_0 = 1.03$.
- Or $\alpha^h = 5.45, z^h{}_0 = 1.03$ and $\alpha^l = 15.18, z^h{}_0 = 1.06$
- (Barro-Liao (6-8 , 1.03))

Risk premia then



- Not all inflation disasters were output disasters
- Size of those disasters very asymmetric

Model of inflation dynamics

- Inflation = normal inf. (continuous-time process) + disaster inf. (Poisson jump)
- Assumption: jump size is large enough and variance of normal inflation is low
- Estimation (given discrete time, coarse options data):
  - Inflation into 8 bins: $\pi(i) = \{ \leq -1 , (-1,0] , (0,1] , (1,2] , (2,3] , (3,4] , (4,5] , >5 \}$
  - Markov chain approximation of continuous time: $A=\{a_{i,j}\}$ is an 8x8 matrix with probability of going from $\pi(i)$ to $\pi(j)$ (so rows add to 1)
  - Use only options data so model of $q(.)$
  - 24 moments from three distributions at every date:

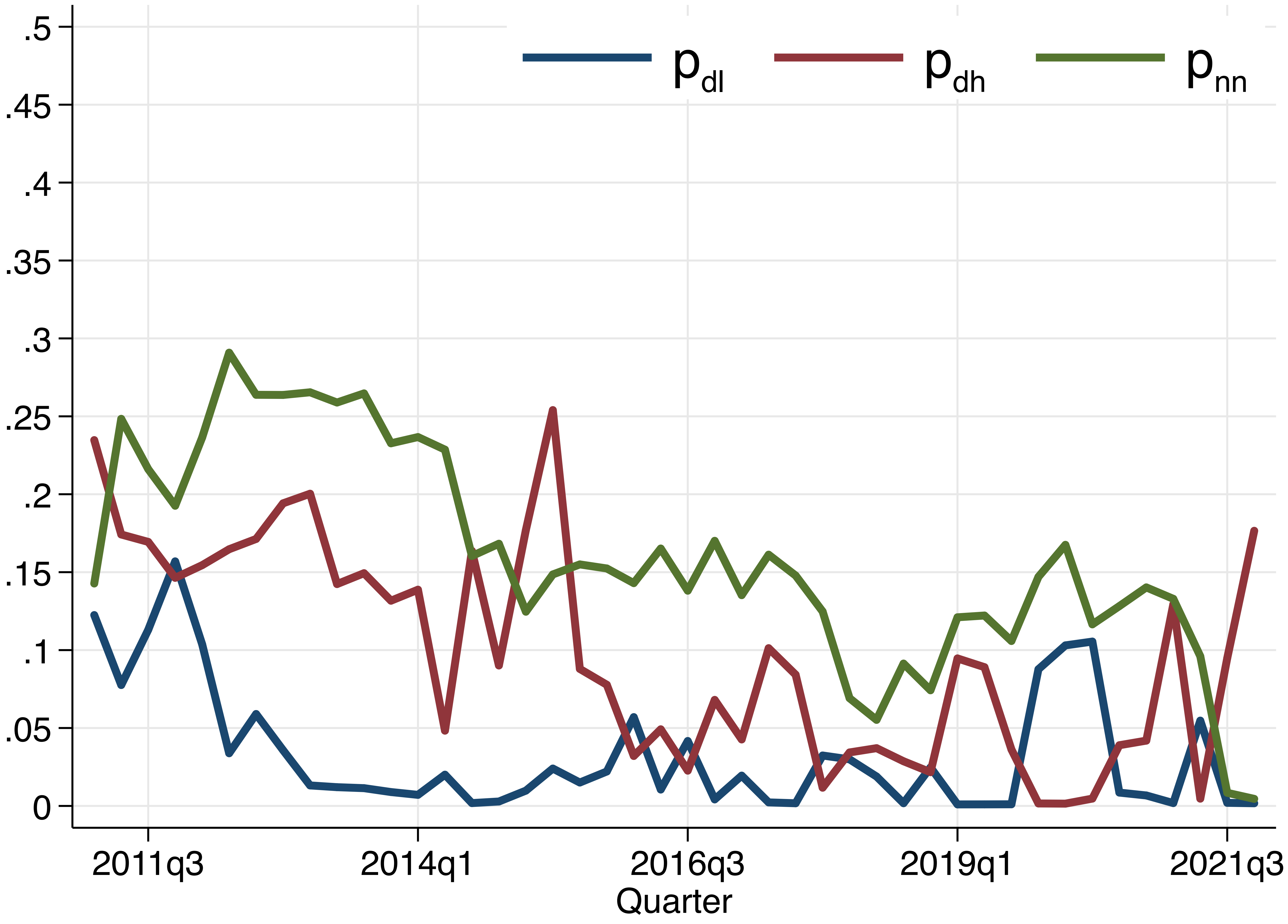
$$q(\pi_{0,5}) \quad , \quad q(\pi_{0,10}) \quad , \quad q(\pi_{5,6}) \approx q(\pi_{6,7}) \approx q(\pi_{7,8}) \approx q(\pi_{8,9}) \approx q(\pi_{9,10})$$

Markov chain approximate model

$$\mathbf{P} = \begin{bmatrix} 1-5p_l & p_l & p_l & p_l & p_l & p_l & 0 & 0 \\ p_{dl}+p_{nn} & p_{ml} & p_{mr} & 0 & 0 & 0 & 0 & 0 \\ p_{dl} & p_{nn} & p_m & p_{mr} & 0 & 0 & 0 & p_{dh} \\ p_{dl} & 0 & p_{nn} & p_n & p_{nn} & 0 & 0 & p_{dh} \\ p_{dl} & 0 & 0 & p_{nn} & p_n & p_{nn} & 0 & p_{dh} \\ p_{dl} & 0 & 0 & 0 & p_{mr} & p_m & p_{nn} & p_{dh} \\ 0 & 0 & 0 & 0 & 0 & p_{mr} & p_{mh} & p_{dh}+p_{nn} \\ 0 & 0 & p_h & p_h & p_h & p_h & p_h & 1-5p_h \end{bmatrix}.$$

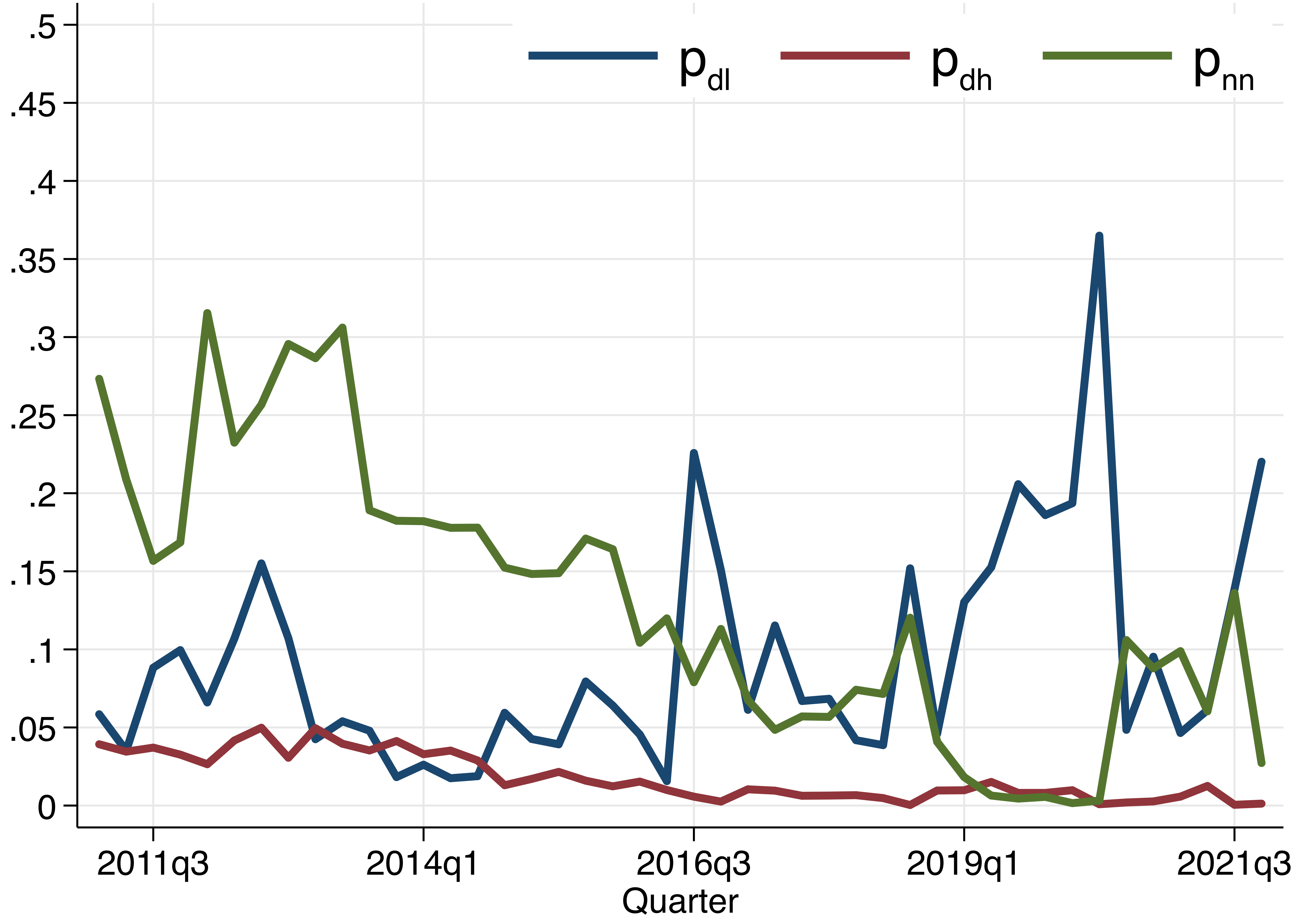
- In normal states, can move one up or down (normal) or jump to disaster
- Allow time variation in perceived disaster-entering probabilities (credibility of central bank) and in volatility of normal inflation (stochastic vol in markets)

US model parameter estimates



- If rise, 50% chance will revert, strong mean reversion
- probability of leaving disaster after one year is high (5x19% = 90%) and symmetric, transitory.
- Strong fall in stochastic volatility
- Recent rise in H jumps.

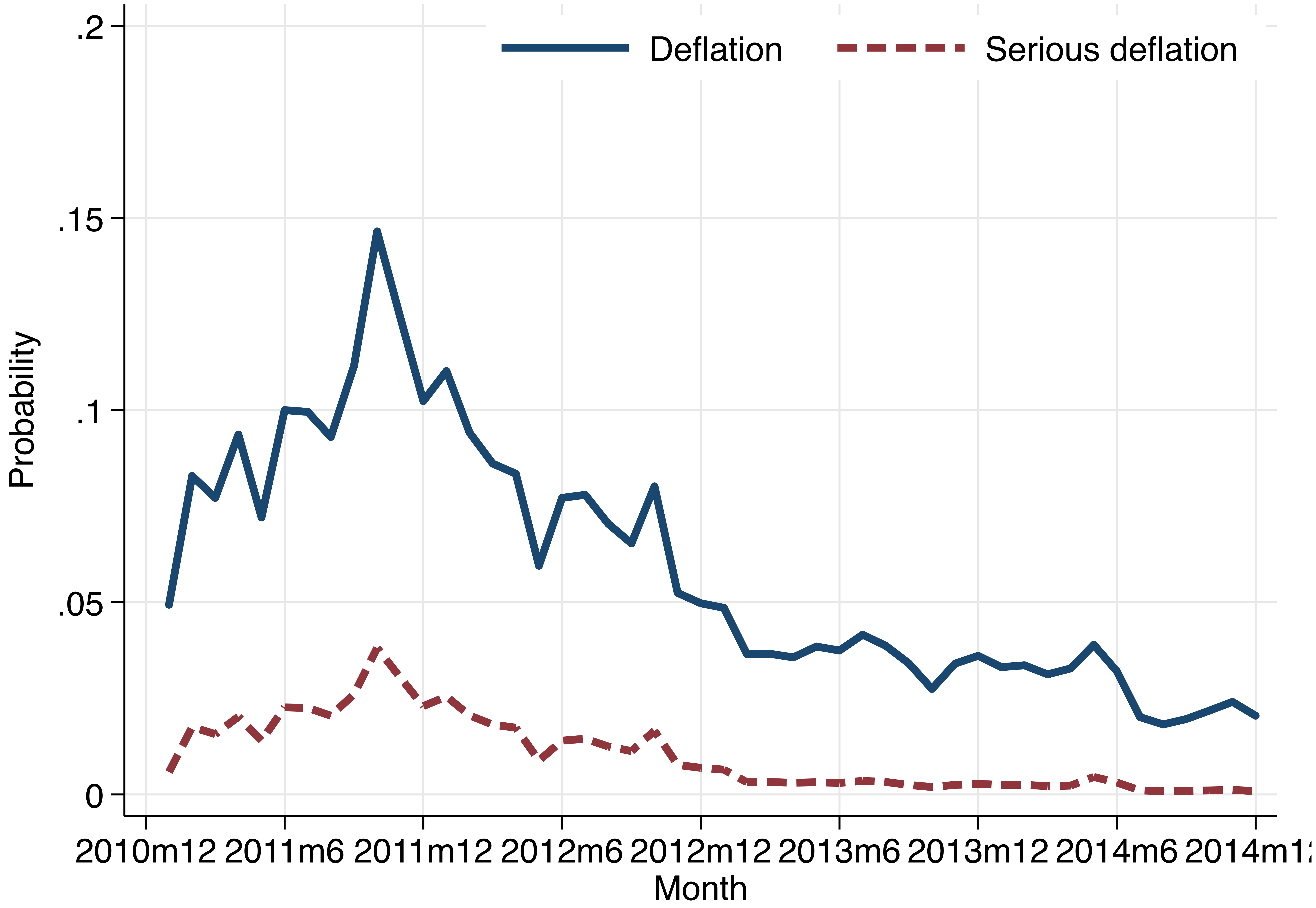
EZ model parameter estimates



- Again strong mean reversion and fall in stochastic volatility
- But post 2016, fluctuation in risk of being driven to deflation state.
- Much less movement in H-disaster in 2021

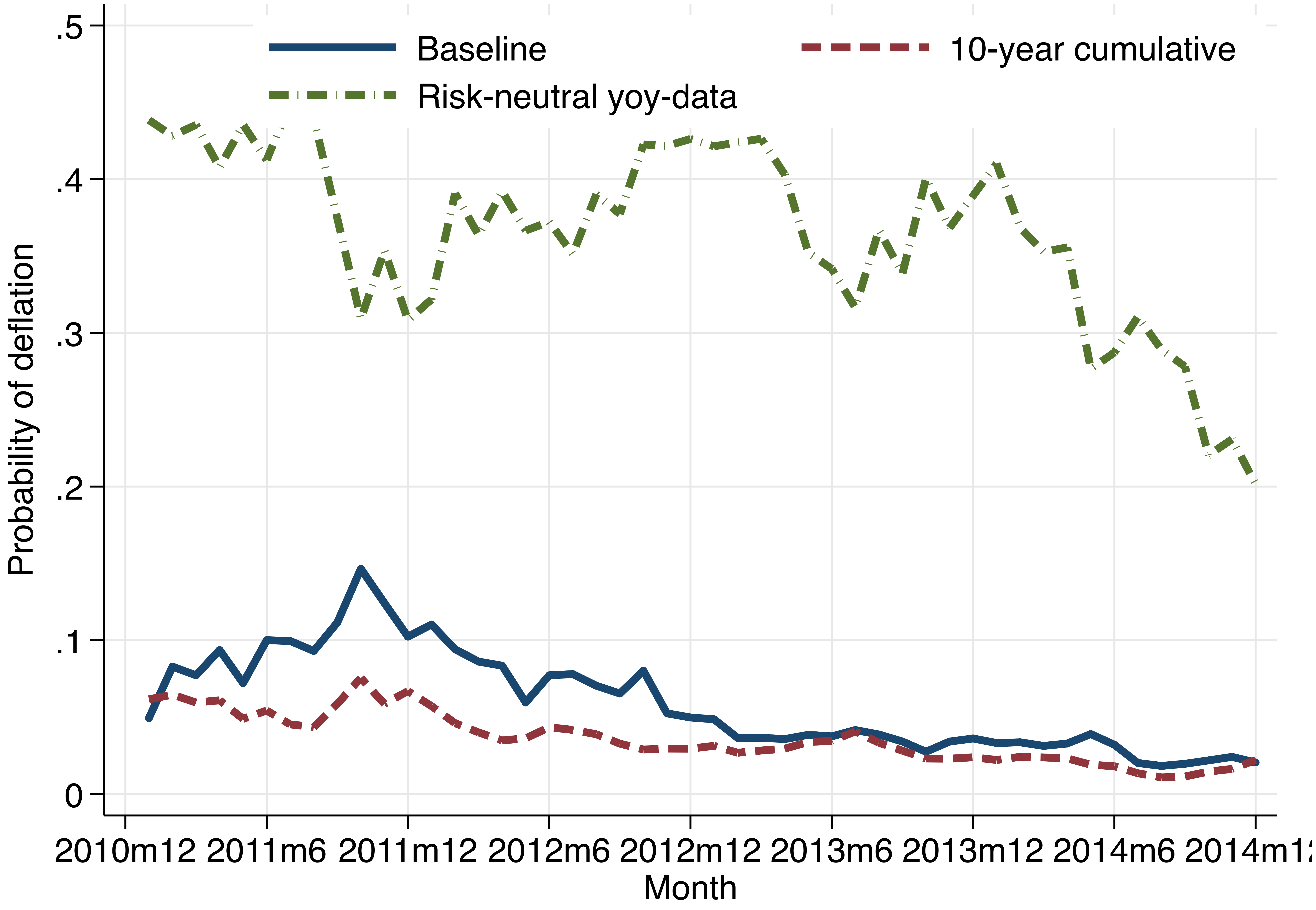
Inflation disaster estimates

US deflation fears 2011-14



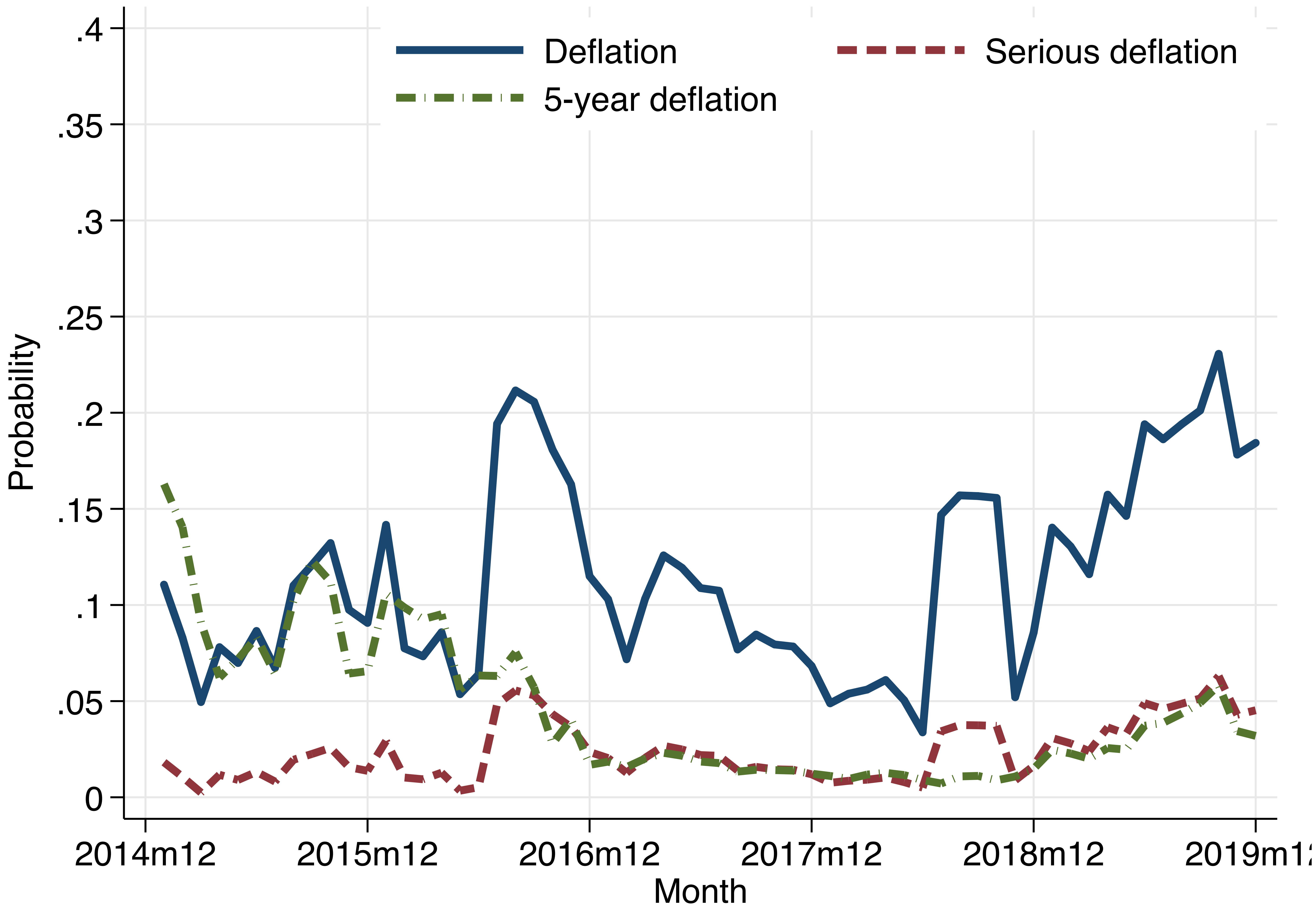
- Not all that large
- Unlike previous estimate
- Unclear justified such aggressive policy and lingering Japanization fears

The conquest of US deflation risk



- 10-year probability lower but not much so
- Big real adjustment, some horizon adjustment, small risk adjustment
- one-year forwards larger and more volatile

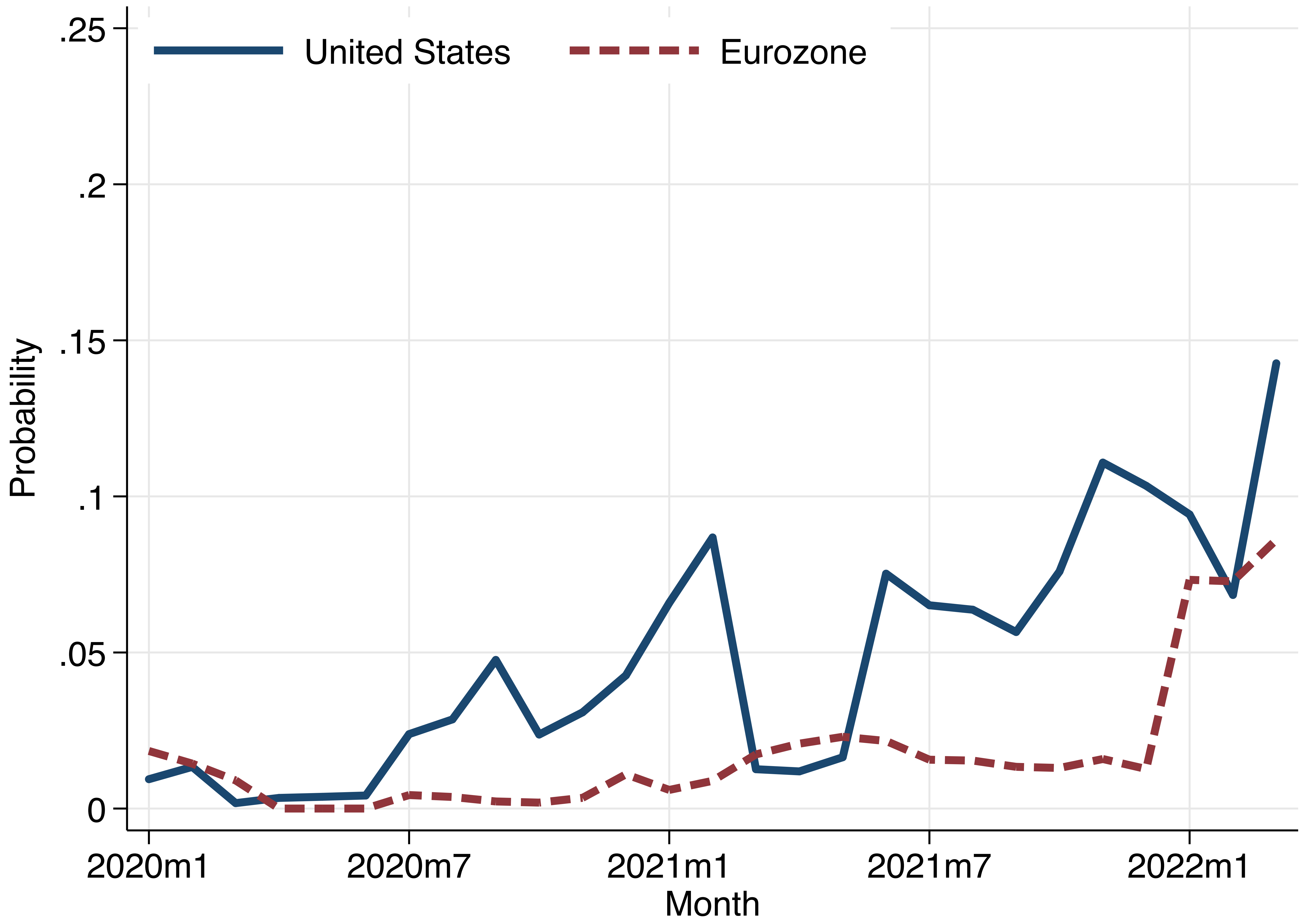
Resilient EZ deflation risk



- Lost battle by the ECB
- QE and other policies did not improve
- After 2016 became a trap scenario
- Did not disappear with 2021

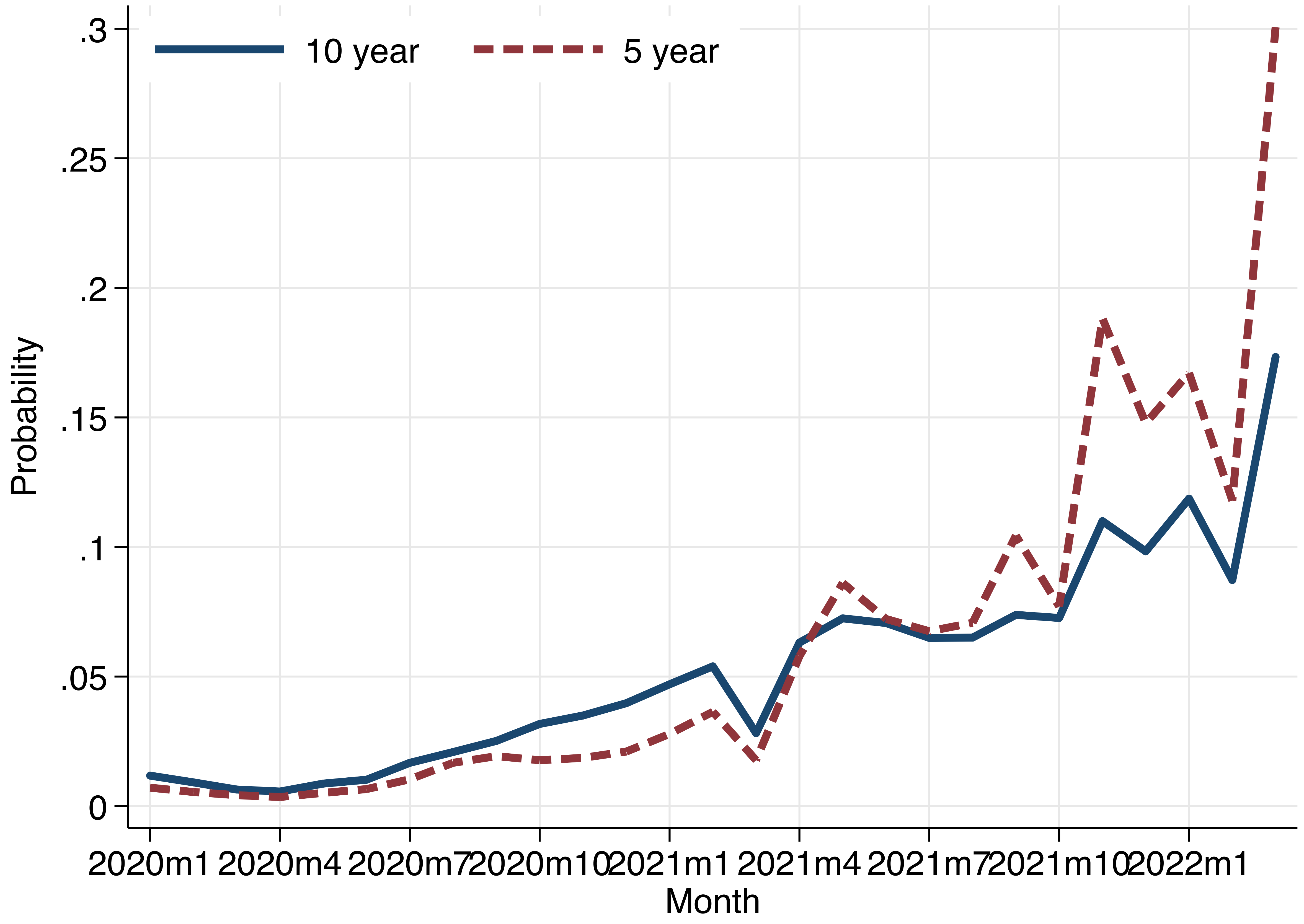
~~Pandemic~~ and 2021 inflation fears

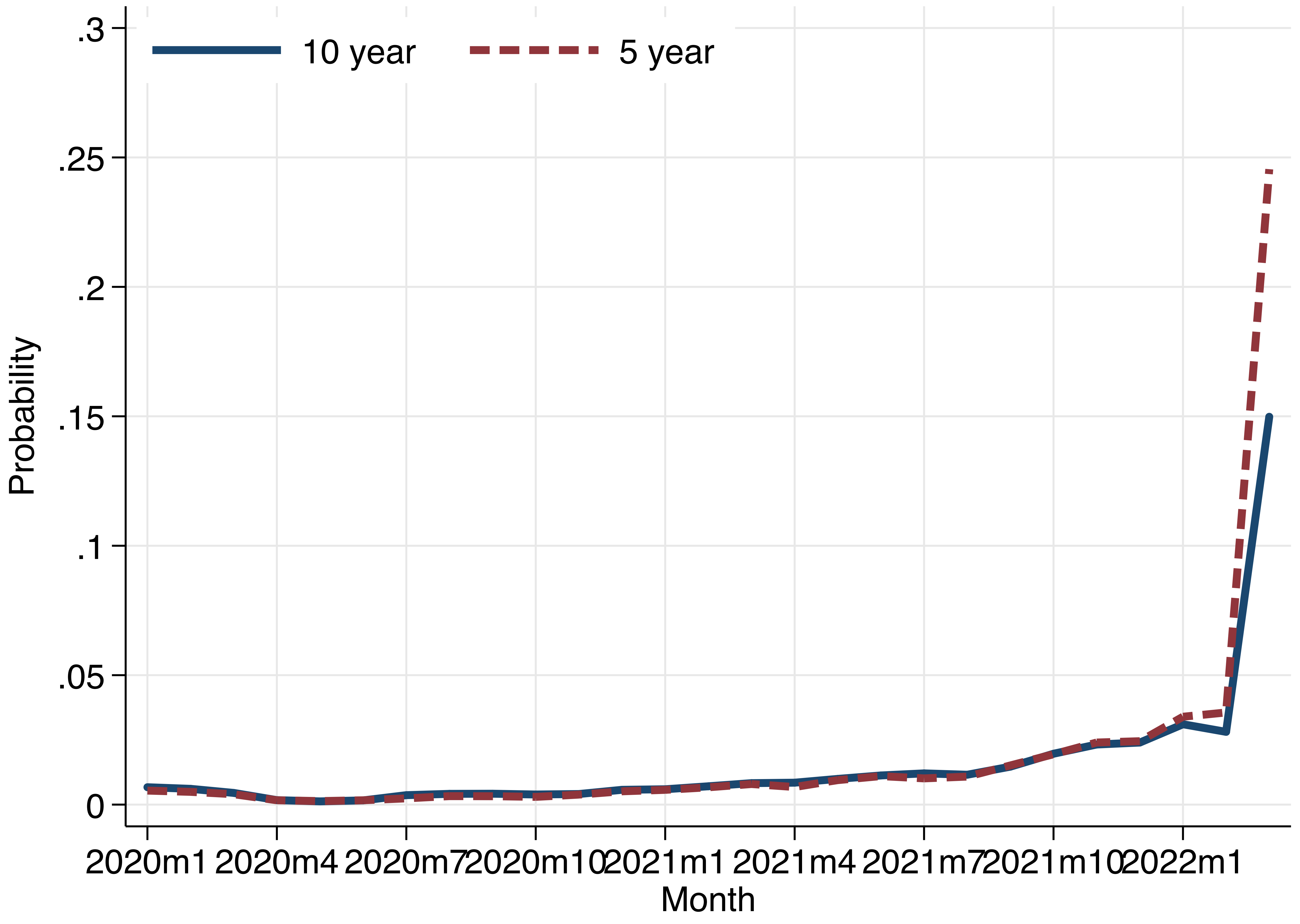
Inflation 5y5y > 4%



- Last data point: March 2022
- In a sense shockingly high
- Risk tolerance of Federal Reserve
- And EZ since start of the year

Pandemic and 2021 inflation fears





- Even higher in shorter horizon, consistent across both

Conclusion

Conclusion

- How to calculate counterpart to 5y5y figure that focusses on tails?
- Natural to use options, but needed to develop machinery to use the data
- Applications results (noting that these are market perceptions):

1. Fed deflation fears 2011-14 were exaggerated
2. ECB's still stuck in deflation-risk trap, surprisingly little improvement in spite of different policies and regimes
3. Fed in 2021: quite significant drift in risk of high inflation that persists
4. ECB in 2022: quick jump