RESEARCH ARTICLE



Unruh versus Tolman: on the heat of acceleration Dedicated to the memory of Rudolf Haag

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Abstract It is shown that the Unruh effect, i.e. the increase in temperature indicated by a uniformly accelerated thermometer in an inertial vacuum state of a quantum field, cannot be interpreted as the result of an exchange of heat with a surrounding gas. Since the vacuum is spatially homogeneous in the accelerated system its temperature must be zero everywhere as a consequence of Tolman's law. In fact, the increase of temperature of accelerated thermometers is due to systematic quantum effects induced by the local coupling between the thermometer and the vacuum. This coupling inevitably creates excitations of the vacuum which transfer energy to the thermometer, gained by the acceleration, and thereby affect its readings. The temperature of the vacuum, however, remains to be zero for arbitrary accelerations.

Keywords Unruh effect · Tolman-Ehrenfest law · Vacuum temperature

1 Introduction

In a well-known paper, Unruh [24] considered an idealized, pointlike detector which follows a worldline of constant proper acceleration in Minkowski spacetime while its degrees of freedom are coupled to a quantum field in the inertial vacuum state. He has shown that in the limit of large measuring times and of weak couplings the detector state will be found in a Gibbs ensemble corresponding to a temperature which

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is proportional to the detector's proper acceleration; see also [6,7,19,26]. The relation is

$$T_D = \frac{a}{2\pi} \tag{1}$$

where T_D is the Gibbs ensemble temperature of the detector, *a* is the proper acceleration along its worldline, and we use units where the velocity of light, Planck's constant and Boltzmann's constant have the values $c = \hbar = k = 1$.

Constant acceleration can be described as the effect of a constant gravitational field (by the equivalence principle) which in turn can be described by a static spacetime metric. In the case at hand this is Rindler space whose metric is given in appropriate coordinates (assuming that the acceleration points into the 1–direction) by

$$ds^{2} = (ax_{1})^{2} dt^{2} - d\mathbf{x}^{2}, \quad \mathbf{x} = (x_{1}, x_{2}, x_{3}).$$
⁽²⁾

The orbit of the thermometer is given in these (Rindler) coordinates by $x_1 = 1/a$, $x_2 = x_3 = 0$ and *t* denotes its proper time.

It has been suggested to interpret the temperature T_D of the detector as the temperature of a relativistic gas which appears in the vacuum because of the acceleration, cf. [10, p. 167], [25, p. 3721], [26, p. 115]. If this picture is correct, *i.e.* if one is effectively dealing in the accelerated vacuum with an equilibrium state of a gravitating gas, one can apply a classical result of Tolman [22] and Tolman–Ehrenfest [23] who observed that the temperature in such systems is spatially varying. In the case at hand they obtain for the temperature $T(\mathbf{x})$ at point \mathbf{x} in Rindler space the relation

$$T(\mathbf{x}) a x_1 = \text{const.} \tag{3}$$

where the constant depends on the system.

If the Unruh temperature T_D at $x_1 = 1/a$ is identified with the temperature of a gravitating gas one obtains const. $= a/2\pi$, so the temperature of the vacuum depends on the position in the comoving system according to $T(\mathbf{x}) = 1/2\pi x_1$. Hence it is strongly varying with \mathbf{x} and one would expect that the gas should also exhibit corresponding pressure and density variations.

We will show in this article that this conclusion leads to contradictions, so there is no such gas. In particular, the (macroscopic) Tolman temperature and the (microscopic) Unruh temperature cannot be identified. This applies not only to the vacuum, but in fact to any accelerated equilibrium (KMS) state of a quantum field with respect to the time coordinate *t*. We will indicate the origin of this discrepancy, explain why Unruh detectors do not describe perfect local thermometers and outline how local temperatures can be determined otherwise, leading to results which consistently unify the Unruh effect and Tolman's law. Most arguments rest on results of our recent work [5].

These results appear to have consequences also for other discussions in which relation (3) is of relevance, such as considerations of black branes [16], or the idea of a fundamental link between time and temperature ("thermal time") [14]. At any rate, one has to be cautious when identifying the Tolman temperature of (3) with

temperature readings distorted by quantum effects, such as the Unruh temperature of an accelerated detector.

2 The Unruh detector, encore

Turning to the details, let $\phi(\mathbf{x})$ be a real, scalar quantum field on Minkowski space \mathbb{R}^4 , where we use inertial coordinates $\mathbf{x} = (\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$ (sans-serif letters). The action of spacetime translations $\mathbf{y} \in \mathbb{R}^4$ and Lorentz transformations $\Lambda \in \mathscr{L}_+^{\uparrow}$ on the field is given by the map $\phi(\mathbf{x}) \mapsto \phi(\Lambda \mathbf{x} + \mathbf{y})$ and the field is assumed to be local, *i.e.*[$\phi(\mathbf{x}), \phi(\mathbf{y})$] = 0 if \mathbf{x}, \mathbf{y} are spacelike separated.

We will consider different Hilbert space realizations (representations [11]) of this field which correspond to globally differing states. The basic reference state is the inertial vacuum, simply called vacuum in the following, which is described by a unit vector Ω_0 in the vacuum Hilbert space \mathcal{H}_0 . On \mathcal{H}_0 there exists a continuous unitary representation U_0 of the Poincaré group $\mathcal{P}_+^{\uparrow} = \mathbb{R}^4 \rtimes \mathcal{L}_+^{\uparrow}$ such that (1) Ω_0 is invariant under its action, (2) the generator (Hamiltonian) P_0 of the inertial time translations is positive and (3) $U_0(\mathbf{y}, \Lambda)\phi(\mathbf{x})U_0(\mathbf{y}, \Lambda)^{-1} = \phi(\Lambda \mathbf{x} + \mathbf{y})$. For the sake of concreteness, we take as a simple example fitting into this setting the theory of a free field of mass m = 0, acting on Fock space. But our arguments are, to a large extent, model independent.

Since we are interested in the spatial dependence of temperature in accelerated systems we consider a Minkowski space based observer who enters with his clock a laboratory which, at some instant of time, is at rest and then undergoes constant acceleration a > 0 into the 1-direction. The laboratory is assumed to have rigid walls which can withstand the tidal forces caused by the acceleration, cf. [5]. Using Rindler coordinates, the laboratory occupies at proper time $t \ge 0$ of the observer some region in the half-space $L_t = \{x \in \mathbb{R}^3 : x_1 > 0\}$ where the observer stays at $x_o = (1/a, 0, 0)$. Proceeding to Minkowski coordinates, this half-space corresponds to the half-hyperplane $L_t = \{x \in \mathbb{R}^4 : x_0 = th(at)x_1, x_1 > 0\} = \Lambda_1(at)L_0$, where Λ_1 denotes the one-parameter group of Lorentz boosts into the 1-direction, parametrized by *at*. Thus the time evolution of all points in the laboratory is determined by this action.

The observables carried along by the observer, testing the properties of the field are, at time t = 0, described by polynomials $A = \sum c_n \phi(f_1) \dots \pi(f_n)$ of the field operators ϕ and their canonical conjugates π which are integrated with test functions f having support in the region L_0 . Taking into account the preceding remarks about the evolution of points in L_0 and the transformation properties of the field under Lorentz transformations, it follows that the observables at time t are given by (Heisenberg picture) $A(t) = U_a(t)AU_a(t)^{-1}$, where we have put $U_a(t) \doteq U_0(0, A_1(at))$.

Since the vacuum vector Ω_0 is invariant under Lorentz transformations, the accelerated observer finds with his observables A, B that the vacuum state ω_0 is stationary, $i.e.\omega_0(A(t)) = \langle \Omega_0, U_a(t)AU_a(t)^{-1}\Omega_0 \rangle = \omega_0(A)$. Moreover, as observed by Unruh [24] and independently by Bisognano and Wichmann [2] (see also [9]), the correlation functions $t \mapsto \omega_0(BA(t))$ satisfy the KMS condition which is a distinctive feature of equilibrium states [12,15]. This condition can be presented in the form $\omega_0(B\widetilde{A}(k)) = e^{k/T_D}\omega_0(\widetilde{A}(k)B), k \in \mathbb{R}$, where the tilde denotes the Fourier transform

(in the sense of distributions) of the operator functions $t \mapsto A(t)$ and T_D is the Unruh temperature given above. Thus there arises the question of the physical significance of the parameter T_D .

In order to answer this question, Unruh studied in [24] the effect of the coupling of the accelerated vacuum state with a small system (probe). The simplest such example is a two level system with Hilbert space \mathbb{C}^2 and internal Hamiltonian $H_o = E_o \sigma_3$, where $\sigma_0, \sigma_1, \sigma_2, \sigma_3$ are the Pauli matrices and E_o is the internal energy relative to the time scale of the observer. The generator of the time translations of the field in the accelerated laboratory is aK_1 , where K_1 is the generator on \mathcal{H}_0 of the boosts in the 1-direction. In order to describe the coupling between the vacuum and the probe on the product Hilbert space $\mathcal{H}_0 \otimes \mathbb{C}^2$ we choose some smooth non-negative function $\mathbf{x} \mapsto p(\mathbf{x})$ that integrates to 1 and has support in L_0 about the chosen position of the redshift factor ax_1 , which scales energies at the points \mathbf{x} in the accelerated laboratory, the generator of time translations of the coupled system is modeled by

$$G_{g,p} \doteq aK_1 \otimes 1 + 1 \otimes E_p \,\sigma_3 + g \,\phi(p) \otimes \sigma_1,\tag{4}$$

where $E_p = \int d\mathbf{x} p(\mathbf{x}) a x_1 E_o$ is the internal energy of the probe at its respective position, g is a coupling constant and $\phi(p)$ is the field integrated with the function p. Thus the time translations of the coupled system relative to the proper time of the observer are described by the unitaries $V_{g,p}(t) = e^{itG_{g,p}}$.

Now let $\Omega_{\otimes} \doteq \Omega_0 \otimes \eta \in \mathcal{H}_0 \otimes \mathbb{C}^2$ be the product of the vacuum vector and any given state vector η of the probe and let $A_{\otimes} = \sum_i A_i \otimes \sigma_i$ be the observables of the coupled system. It has been shown by de Biévre and Merkli [7] that for arbitrary coupling functions p the expectation values of observables in the coupled state exist at large times,

$$\lim_{t \to \infty} \langle \Omega_{\otimes}, V_{g,p}(t) A_{\otimes} V_{g,p}(t)^{-1} \Omega_{\otimes} \rangle \doteq \omega_{g,p}(A_{\otimes}).$$
(5)

These limits define stationary KMS states $\omega_{g,p}$ for the coupled dynamics corresponding to the same KMS parameter T_D as for the uncoupled vacuum. Moreover, if one proceeds to small couplings one obtains

$$\lim_{g \to 0} \omega_{g,p}(A_{\otimes}) = \sum_{i} \omega_0(A_i) \operatorname{Tr}((1/Z)e^{-E_p \sigma_3/T_D} \sigma_i).$$
(6)

Even though the proof is rather involved, this result is physically not so surprising. For it says that a microscopic probe cannot disturb an infinite equilibrium state, whilst it is itself driven to equilibrium, described by a Gibbs ensemble.

In a similar manner one can treat the case of several probes, placed at different positions in the laboratory. There the internal Hamiltonian of the probe, appearing in the resulting Gibbs ensemble (6), has to be replaced by the sum of the respective internal probe Hamiltonians.

This observation has been taken as justification to interpret probes as thermometers and to relate the KMS parameter T_D of their ensembles to the temperature of the vacuum in the accelerated system, cf. [7, 19, 24]. As is apparent from the preceding relation, T_D does not depend on the position of the probes within the laboratory, fixed by the support of p. This is in accordance with the known fact that the KMS parameter of infinite equilibrium states is a global, superselected quantity [21]. But it shows that probes cannot be used offhandedly to determine the temperature at their respective position. In fact, as has been explained, the local temperature of equilibrium states in the accelerated laboratory varies according to Tolman's law (3).

Commonly, one copes with this problem by a reinterpretation of the readings of probes. Thinking of some fixed hardware, one compares the properties of the probe in the accelerated system with those in an inertial equilibrium state [24]. By a rearrangement of the redshift factors, appearing in (6) in the energy E_p , one argues that each probe behaves at its respective position in the laboratory as if it were exposed to an inertial equilibrium state of temperature $T_D / \int d\mathbf{x} \ p(\mathbf{x}) \ ax_1$. Thus one *defines* as "true local temperature" of the vacuum at point \mathbf{x} the quantity $T_D(\mathbf{x}) = T_D/ax_1$, seemingly in accordance with Tolman's law. However, there then appears another conceptual problem.

3 The vacuum seen from an accelerated laboratory

The observer can determine, besides the temperature, other intensive properties of the vacuum, such as densities and pressures at different points in the laboratory. These observables are described by operators A of the form given above. Note that the interpretation of these observables does not depend on the motion of the observer, he can rely on their readings independently of the dynamics. A spatial shift y and evolution in time t of the observable A results in the corresponding operator $A(t, y) = U_a(t)U_0(y, 1)A U_0(y, 1)^{-1}U_a(t)^{-1}$, where we have identified Rindler and Minkowski coordinates at time t = 0. Because of the invariance of the vacuum under Poincaré transformations, all expectation values of observables (hence also their variances *etc*) satisfy

$$\omega_0(A(t, \mathbf{y})) = \langle \Omega_0, U_a(t)U_0(\mathbf{y}, 1)A U_0(\mathbf{y}, 1)^{-1}U_a(t)^{-1}\Omega_0 \rangle = \omega_0(A).$$
(7)

Thus the vacuum is homogeneous also in the accelerated system, there appear no nonzero density or pressure gradients. This fact is incompatible with a locally varying temperature of the vacuum state, in conflict with the above *ad hoc* definition. Thus there is no hot gas appearing in accelerated vacuum states.

This raises the question as to why the probe gains energy from the vacuum in the accelerated system. As has been pointed out long ago, cf. [1, pp. 54–57] and [18], the answer rests upon the quantum nature of the coupling between the probe and the field, given by the last term in relation (4). The operator $\phi(p)$ appearing there describes a local operation in the region fixed by p whose quantum effects inevitably change the energy content of the underlying ensemble, excitations are randomly created.

Upon maintaining the acceleration of the laboratory, these excitations gain energy from the external forces, and they deliver parts of this energy to the probe in the course of time. Thus, due to these quantum effects, the probe also exchanges mechanical energy with its environment, not only heat as is expected from a perfect thermometer. The increase of energy of the probe due to this effect leads to an increase of temperature in its readings. In inertial systems or for small accelerations this effect causes errors in the temperature readings which lie beyond any measuring accuracy, so they do not matter. But for large accelerations this systematic effect becomes prominent and can no longer be neglected.

The replacement of $\phi(p)$ by another operator, coupling the probe with the vacuum, does not cure this problem. In fact, there does not exist any non-trivial operator A that is localized in the region L_0 and does not change the energy content of the vacuum; this is a consequence of the Reeh–Schlieder theorem [20] according to which the only local operators preserving the vacuum are multiples of the identity. Moreover, if one replaces in relation (4) the field $\phi(p)$ by another local operator A one obtains in the limit of large measuring times and small couplings always the same final state of the probe, given in (6), cf. [8]. So the KMS parameter T_D does not depend on the specific nature of the coupling. All probes exhibit the same systematic error and indicate at asymptotic times their own temperature, induced by the measuring process, instead of the temperature of the vacuum.

So what is the local temperature of the accelerated vacuum state? In order to answer this question it has been proposed [4] to exhibit sufficiently many local observables *A* which are appropriate to determine intensive properties of equilibrium states and to rely on the zeroth law of thermodynamics and the Gibbs phase rule according to which the temperature of equilibrium states is uniquely fixed by these data. Quantum fluctuations can be suppressed by proceeding to large time limits, respectively averages, of these observables.

This idea has been applied in [5] to, both, inertial and accelerated observers. Denoting by $U_0(t)$ the time translations on \mathcal{H}_0 in the inertial system, the expectation values of all observables A in states of \mathcal{H}_0 attain sharp values at asymptotic times. Their (weak) limits are given by

$$\lim_{t \to \infty} U_0(t) A U_0(t)^{-1} = \omega_0(A) \, 1, \tag{8}$$

fixing all intensive parameters in this case. Performing the analogous limits in the accelerated laboratory with the corresponding time translations $U_a(t)$ one obtains

$$\lim_{t \to \infty} U_a(t) A U_a(t)^{-1} = \omega_0(A) \, 1, \tag{9}$$

i.e. the asymptotic expectation values of the intensive observables are not affected by the acceleration. Since all intensive parameters of the accelerated vacuum coincide with those in the inertial system one may conclude that the temperatures coincide as well, *i.e.* the accelerated vacuum has temperature zero everywhere.

4 Local temperature observables

The consistency of this approach has been tested in [5] for arbitrary inertial and accelerated equilibrium states. Assuming for simplicity that for each temperature (KMS parameter) T > 0 there exists only a single equilibrium state ω_T in the inertial system, one finds that on the corresponding thermal Hilbert spaces \mathscr{H}_T there holds the analogue of relation (8), where on the right hand side the vacuum state ω_0 has to be replaced by ω_T . The functions $T \mapsto \omega_T(A)$ are the macroscopic equations of state for the intensive observables A in the inertial system. (If, for given T, there exist several equilibrium states, these functions also depend on chemical potentials, the phase structure, *etc.*) It is evident that the value of T can be recovered from the collection of these data, *i.e.* temperatures can be determined with the help of the localized observables.

In the uniformly accelerated laboratory there likewise exist for all KMS parameters $T_a > 0$ equilibrium states ω_{T_a} with regard to the accelerated dynamics, given by the adjoint action of the unitaries $U_a(t)$ on the observables [13]. In order to simplify the notation we adopt here the convention that all quantities with an index *a* refer to this dynamics. In particular, the vacuum corresponds to the Unruh parameter $T_{a,0} = a/2\pi$, i.e. $\omega_{T_{a,0}} = \omega_0$ on all observables in the accelerated laboratory. Again, there holds an analogue of relation (9) on the Hilbert spaces \mathscr{H}_{T_a} attached to the accelerated equilibrium states, where one now has to replace $\omega_0 = \omega_{T_{a,0}}$ by ω_{T_a} for given KMS parameter T_a .

In order to determine the thermal interpretation of the KMS parameters T_a in the accelerated system one compares the expectation values of local observables A in the accelerated equilibrium states with those in the inertial system. This approach is analogous to that used for probes, where the temperature scale is likewise calibrated in inertial equilibrium states.

In the present simple free field model one may take as a "local thermometer" [3,4] the normal ordered square of the field, $\Theta \doteq 12 : \phi^2 :$ (or any other of its even powers). The numerical factor is suggested by calibration in the inertial equilibrium states ω_T , giving $\omega_T(\Theta(\mathbf{x})) = T^2$ for any $T \ge 0$. So the readings of Θ indicate the square of the temperature which is equal at all points \mathbf{x} in the inertial system. Plugging this observable into the accelerated equilibrium states one obtains [4]

$$\omega_{T_a}(\Theta(\mathbf{x})) = \left(T_a^2 - (a/2\pi)^2\right)/(ax_1)^2.$$
 (10)

Hence, for given KMS parameter $T_a \ge a/2\pi$, the thermometer indicates at any given point x in the accelerated laboratory the temperature

$$T_a(\mathbf{x}) = \sqrt{(T_a^2 - (a/2\pi)^2)}/ax_1.$$
 (11)

This relation is consistent with Tolman's law (3) with const. = $\sqrt{(T_a^2 - (a/2\pi)^2)}$. Notably, the systematic error in the readings of probes is corrected and the result is in accord with the statement that the temperature is 0 everywhere for $T_a = a/2\pi$, *i.e.* in the vacuum state. Note that Tolman's law in this concrete form is obtained here as a result, it is not put in by hand.

It also follows from relation (11) that the temperature tends to zero in all accelerated equilibrium states at sufficiently large distances from the boundary of L_0 . As a matter of fact, in these remote regions the expectation values of all local observables in the

accelerated KMS states coincide with those in the vacuum [5]. Thus the observer can calibrate his observables up there according to inertial standards.

For KMS parameters $T_a < a/2\pi$, the expectation values of Θ in the corresponding KMS states are negative; hence one cannot assign to them a meaningful temperature. This can be understood if one notices that also all densities and pressures are negative in these states (taking the remote vacuum as a reference). In the presence of acceleration, the excitations created by local measurements effectively equilibrate these states, but from an inertial point of view they are to be regarded as ensembles which are far from (local) equilibrium. Note that the restriction of any accelerated KMS state to the observables in any given compact region can be represented by density matrices in Fock space [17]. Hence an accelerated observer, launched in Minkowski space where he has calibrated his observables, can interpret in these terms the properties of the states and has no reason to rely on elusive Rindler quanta.

So we conclude that the vacuum does not exhibit any thermal properties in accelerated laboratories, its temperature remains to be zero. The increase in temperature indicated by microscopic probes is due to the quantum induced creation of excitations caused by the interaction; they transmit energy to the probes, gained from the accelerating forces. This energetic quantum effect can be understood in rough analogy to the production of heat by friction, but it is not the result of an exchange of thermal energy between probes and a heat bath (Rindler gas). As we have shown here, the latter interpretation is not tenable on several theoretical grounds.

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