ORIGINAL RESEARCH



Prediction in general relativity

C. D. McCoy¹

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Abstract Several authors have claimed that prediction is essentially impossible in the general theory of relativity, the case being particularly strong, it is said, when one fully considers the epistemic predicament of the observer. Each of these claims rests on the support of an underdetermination argument and a particular interpretation of the concept of prediction. I argue that these underdetermination arguments fail and depend on an implausible explication of prediction in the theory. The technical results adduced in these arguments can be related to certain epistemic issues, but can only be misleadingly or mistakenly characterized as related to prediction.

Keywords Prediction \cdot General theory of relativity \cdot Theory interpretation \cdot Global spacetime structure

1 Introduction

Several authors have investigated the subject of prediction in the general theory of relativity (GTR), arguing on the strength of various technical results that making predictions is essentially impossible for observers in the spacetimes permitted by the theory (Geroch 1977; Hogarth 1993, 1997; Earman 1995; Manchak 2008). On the face of it, such claims seem to be in significant tension with the important successful predictions made on the theoretical basis of GTR. The famous experimental predictions of the precession of the perihelion of Mercury and of gravitational lensing, for example, are widely understood as playing a significant role in the confirmation of the theory (Brush 1999). Well-known relativist Clifford Will observes furthermore that "today,

C. D. McCoy cdmccoy@ucsd.edu

¹ Department of Philosophy, University of California San Diego, La Jolla, CA 92093, USA

experimental gravitation is a major component of the field, characterized by continuing efforts to test the theory's predictions..." (Will 2006, p. 5). If prediction is impossible in GTR, as these several authors maintain, just what do physicists using the theory think they are testing, and why do they think that its predictions have passed their tests?

That these authors do take themselves to be providing an interpretation of the concept of prediction is evident from their approach. Their method is to consider what information about the spacetime an observer inhabits is available to her directly, and what information can then be deduced about her future on this observational basis. They use these considerations to yield two conditions that they maintain must be met in order for an event to be predictable for a given observer. Manchak (2008) adds a novel condition, motivated by "fully considering" the epistemic predicament of the observer. As he says, "the observer [must] not only have the resources to make a prediction but also the resources to *know* that she can make a prediction" (Manchak 2008, p. 320). The condition requires, roughly, that any prediction which an observer makes in her spacetime must be the same for her counterpart in any observationally indistinguishable spacetime, since the observer has no empirical basis on which to decide which of these spacetimes she actually inhabits.

The results proven on the assumption of these conditions can be understood to represent epistemic limitations on what observers can reliably predict. These limitations can moreover be traced to skeptical concerns arising from the possibility of underdetermining, observationally indistinguishable spacetimes which undermine any given prediction the observer attempts. Thus, while the famous confirmed predictions of GTR mentioned before may be called "predictions" in some sense, they simply do not meet the epistemic standard required by these authors in their definition of prediction. Accordingly, they are not to be understood as predictions in the strict sense, as they allegedly depend on epistemically unjustified assumptions about the character of the observer's spacetime (Sect. 2).

While there is certainly much to recommend this formalistic method in the philosophy of science and physics in particular, it can easily conceal crucial epistemological and physical considerations (especially in the case of an epistemological concept like prediction) which ought to be taken in account when considering the explication of the concept in a physical theory. Indeed I argue that in this particular case the concern is realized, and that the interpretation of the aforementioned technical results in terms of the concept of prediction is misleading, and in some cases outrightly mistaken.

My argument divides into two. First I dispute the main two conditions assumed by all the authors working on the topic, for these conditions rely on an overly idealized and unphysical conception of observation (Sect. 2). They assume that an observer has epistemic access to all and only events in her causal past, but I claim that no physical mechanism can account for such powers (due to the ubiquity of scattering and other physical considerations). This may sound like grist for their mill; if prediction is precluded in the idealized case as their results suggest, it must seemingly be all the more problematic in less idealized cases, where obtaining complete sets of observational data requires one to face the physical limitations I discuss (as well as ordinary practical difficulties). A realistic conception of the powers of an observer, one that recognizes the need to interpolate and "complete" observational data, suggests however that there

is not only relatively little epistemic difference between predictions and retrodictions in general relativity, but also that there is no reason to restrict the initial data from which a prediction is determined to an observer's causal past. Indeed, we often make reasonable and successful inductive inferences in cosmology about the character of spacetime beyond our causal horizon.

Second I address Manchak's distinctive contribution to the literature (Sect. 3). Manchak's arguments extend prior work on the topic by making their tacit dependence on an underdetermination argument more explicit, namely through the introduction of his novel condition. He also focuses on particular kinds of underdetermining spacetimes, namely observationally indistinguishable spacetimes that differ in specific global properties. I argue however that Manchak's "nemesis" spacetimes fail to undermine prediction, as they either rest on the unphysical assumption that predictions are of point-like events, or else are disfavored on inductive grounds. The possibility of prediction may perhaps be threatened by other underdetermining spacetimes, but not, I claim, these. I also show that his results do not in fact depend on the central assumptions of the previously criticized conditions on prediction—a crucial point for understanding their significance.

What is the significance of the various results proven in this literature, if they do not undermine or pertain to prediction, as I claim? It seems that the most positive assessment of the consequences of the first two conditions is that they demonstrate merely that ideal observational knowledge is insufficient for making practical predictions, and we must rely on somewhat less well-justified beliefs, in addition to those given by "direct" observation, to make reliable predictions. Unfortunately for those who make it, this is not a particularly novel epistemological point. It is also a lesson already well-incorporated in standard scientific practice—as witnessed by the classical tests of general relativity and by the continuing practice of experimental gravitation. I suggest that Manchak's distinctive claims are, however, of greater epistemological significance than previous ones. They are best interpreted though not in terms of prediction, since prediction applies only to global properties of spacetime in a very loose sense, but rather as pointing out the outstanding epistemological issue of justifying global inferences in relativistic spacetimes, an issue to which his later papers have especially drawn attention (Manchak 2009a, 2011).

2 Prediction: observation and underdetermination

As said, the contributors to the literature on prediction in GTR employ a common strategy: explicate the concept of prediction in the standard formalism of GTR by specifying certain conditions that intuitively capture the notion of making predictions in the theory, then prove various plausible conjectures on the basis of that formalism and definitions incorporating the conditions (or provide counterexamples) (Geroch 1977, p. 81). The philosophical adequacy of such a method obviously depends on whether the conditions relevantly capture the notion at issue.

So, to begin, consider the following general definition utilized in this literature, adapted slightly from Manchak (2008, p. 318):

Definition 1 Given a (relativistic) spacetime and some point q in that spacetime, a point p is in the *domain of prediction* of q if and only if (i) p is not in the causal past of q and (ii) there exists an achronal, closed, spacelike surface S in the causal past of q such that p is in the domain of dependence of the subset S.¹

The domain of prediction is the region of the given spacetime where events are in principle "predictable" by the given observer at q. Two conditions are imposed on this region in the above definition, corresponding to what are taken to be intuitive limitations on the possibility of prediction for an observer (Geroch 1977, pp. 88–89).

Condition (i) captures (in a way readily translatable to the mathematical framework of GTR) the idea that *pre*dictions are not about facts in the past. As the term itself suggests, predictions are intuitively of future events. From the point of view of those advocating it, condition (i) does not really represent an important restriction however. One may define the *domain of retrodiction* as above but with the condition (i')—that p is in the causal past of q—in place of (i). In this case, though, the domain of retrodiction is just the causal past of q (Hogarth 1997), trivially, since the observer is assumed (by all hands advocating the condition) to have in principle access to all information in her causal past.² This point of view is, for example, explicitly stated by Hogarth.³ (Others appear to take the assumption as obviously valid and do not necessarily state it.)

The same assumption about the powers of the observer motivates condition (ii). This condition captures (in a way readily translatable to the mathematical framework of GTR) the intuition that a prediction is possible only when the predicted event is fully determined by some data surface in the observer's past and an appropriate dynamical law, and when that data surface is empirically accessible to the observer as well.⁴ The initial conditions relevant for prediction are therefore restricted to the causal past of q, ostensibly on the assumption that an observer may only gather information on or from within her past light cone (Earman 1995, p. 125). The definition also makes use of the notion of a domain of dependence: the region of spacetime where events are determined from some set of given initial data (on an achronal, closed, spacelike surface S) and a given dynamical law of temporal evolution—in the case of interest here, the Einstein field equation (EFE). This paradigm of evolving initial data forward

¹ By a relativistic spacetime I mean a smooth, connected four-dimensional manifold \mathcal{M} and a smooth, non-degenerate pseudo-Riemannian metric of Lorentz signature *g* defined on \mathcal{M} .

The timelike (respectively, causal) past of a point q in a time-orientable spacetime \mathcal{M} is the set of all points p such that there exists a future-directed curve from p to q that is timelike (timelike or null). If such a curve exists, one writes $p \ll q$ (p < q). The timelike (causal) past of a point q is usually denoted $I^-(q)$ ($J^-(q)$). The timelike (causal) future of a point q, $I^+(q)$ ($J^+(q)$) are defined analogously.

A closed spacelike surface *S* in \mathcal{M} is a three-dimensional subset of *M* where every smooth curve into the subset is a spacelike curve. A subset *S* is achronal if the intersection of $I^+(S)$ and *S* is empty. The domain of dependence of a subset *S*, denoted D(S), is the set of all points *p* in \mathcal{M} such that every past or future inextendible causal curve through *p* intersects *S*.

 $^{^2}$ Of course in a relativistic spacetime not all events which fall outside an observer's past are future, since some events are spacelike related to the observer, thus one may as well allow events which are spacelike-related to the observer to count as predictable.

³ "Let it be assumed that the observer at q has full epistemic access to the influences in [the causal past of q]" (Hogarth 1993, p. 723).

⁴ "For present purposes take prediction to mean deterministic prediction from the laws of physics" (Earman 1995, p. 128).

with the appropriate dynamical law of course accounts for much of the predictive power of physics.⁵

On the basis of conditions (i) and (ii) alone—insofar as they are understood as explicating "the physical notion of 'making predictions in general relativity" (Manchak 2008, p. 319)—one can show that prediction in GTR is subject to strict limitations, as shown by, among others, Geroch (1977) and Hogarth (1993).⁶ The precise proofs and results are not especially relevant to my argument, so I do not discuss them further. It is sufficient for my purposes to note that, as Hogarth succinctly puts it (using a similar definition of prediction), "the final word must be that the prospect of predicting the future looks pretty bleak" (Hogarth 1993, p. 739).

What then of the confirmatory predictions such as of the precession of Mercury's perihelion or the bending of light? How do they fall afoul of the proofs? Logically, the answer should be evident given the definition of domain of prediction: these "predictions" are either retrodictions (violating condition (i)), or else more information than what is observationally accessible is taken as given in order to determine the predicted event (violating condition (ii)). Set aside (for the moment) any scrutiny of condition (i), i.e. whether retrodictions should in fact be thought of as predictions. Let us ask first, "is it justified to avail oneself of more initial data than what is observable?" That is to say, may an event p determined from an initial data surface S which is not (entirely) in the causal past of an observer be justifiably considered a prediction? If so, then the conceptual significance of the version of prediction adumbrated above should come into question; if not, one would presumably like some clear justification for disallowing assumptions about unobserved data for making predictions.

Although none of the various authors explicitly furnishes such a justification, suspicion naturally falls on empiricist scruples being behind the prohibition. The kind of argument one might immediately give against a more permissive notion of prediction, i.e. where S is not restricted to the causal past of the observer, is some sort of underdetermination argument. Such skeptical arguments take the familiar general form

- 1. There are some number of empirically indistinguishable *x*'s (where *x* may be scientific theories, spacetime models, dynamical laws, etc.).
- 2. The evidence cannot favor one such *x* over any other.
- 3. Therefore belief in any one of these *x*'s is insufficiently unwarranted to count as knowledge.

Due to the demonstrable formal existence of observationally indistinguishable spacetimes in GTR (Glymour 1977; Malament 1977; Manchak 2009a), it seems on

⁵ Note that the condition does remove all reference to the observer, but simply for the reason that "observer" is not part of the primitive language of GTR. One may of course easily make a suitable definition. For example, "we can associate with each *observer* his space–time trajectory or cosmic world-line which is itself, necessarily, a future-directed timelike curve" (Malament 1977, p. 63).

⁶ For example: "Given a spacetime \mathcal{M} , g and a point q of \mathcal{M} such that [the domain of prediction of q] contains a point to the future of q, then \mathcal{M} , g is a closed universe, in the sense that it admits a compact spacelike surface" (Geroch 1977, p. 92). "The domain of prediction for each point in Minkowski space–time is empty. The same is true of many of the cosmological models of general relativity, e.g., the Robertson-Walker models which have been used to describe a universe beginning or ending with a big bang" (Earman 1986, p. 193).

the face of it that an observer cannot know on the basis of observations which spacetime she inhabits. If an observer relies only on evidence from her causal past, then for all she knows her spacetime could be one of any number of various observationally indistinguishable spacetimes. It seems that her observational data by themselves cannot favor one of these spacetimes over any other, since none of them can be ruled out on these data alone. In other words, for any initial data surface *S* in her causal past, there exist some number of spacetimes with empirically indistinguishable data surfaces extending beyond her or her counterpart's causal past. The observer may then conclude that she is not justified in assuming any of these data surfaces for the purposes of prediction.

This particular underdetermination argument should hardly worry the observer, though, since the dynamical law used to determine future predictions is in the very same sense underdetermined as well, i.e. she cannot be certain that her prediction based on a particular dynamical law (like the EFE) will succeed either, based solely on observations from her causal past. Insofar as one is willing to accept that GTR and its dynamical law are adequately justified by the evidence (as conditions (i) and (ii) clearly presuppose—and as one should!), the mere existence of alternate dynamical laws is insufficient to discount genuine knowledge claims deduced on their basis. Similarly, insofar as one is willing to accept that initial data which go beyond what is observable are adequately justified by the evidence, the mere existence of observationally indistinguishable spacetimes is insufficient to discount genuine knowledge claims deduced on their basis.

There are thus two problems with this naive underdetermination argument. First, it assumes that observational indistinguishability implies that evidence cannot favor one spacetime over another. It is well-known in the underdetermination literature, at the least since Laudan and Leplin (1991), that the consequent simply does not follow—unless one advocates a particularly implausible and simple-minded empiricist epistemology. Second, its tacit acceptance of dynamical conditions (like the EFE) is in tension with the rejection of non-dynamical conditions (like an initial data surface not confined to the observer's causal past). The tension becomes more acute when one recognizes that, from an epistemological view, the exact same local induction used to project the EFE into the future for the purposes of prediction is used to project conditions.⁷

The failure of this simplistic argument suggests that experimental gravitation is not as threatened by the impossibility results as it may seem; at least its use of initial data that go beyond the restrictions of condition (ii) alone does not make it so. This suggests shifting attention to the significance of these results, since they are clearly not best understood as undermining the possibility of prediction without qualification.

⁷ Beisbart asserts that common intuitions record a difference: "In physics, we distinguish between dynamical theories and the initial conditions. The former are often supposed to hold necessarily, if they hold, whereas initial conditions are supposed to be contingent. As a consequence, induction may be on firmer grounds for laws than for initial conditions" (Beisbart 2009, p. 201). Maudlin (2007) too shares this intuition, but provides no compelling argument in support. There is, so far as I can see, nothing decisive in such metaphysical intuitions, especially given the epistemological parity of dynamical and non-dynamical conditions. Thus the burden is on those who wish to make the distinction to justify it.

The first thought one may naturally have is that the significance of condition (ii) and its consequences is primarily epistemic, namely that predictions based on observable data are "more secure"—because better justified—than predictions based on data that go beyond an observer's causal past. The underdetermination argument from above can then be re-purposed to support this modified conclusion.

I claim that this "more secure" notion of prediction is however problematic, for it rests on implausible assumptions about the powers of observers. It is in fact somewhat ironic that the problem I raise comes from a failure to portray the powers of observers correctly. The various writers on this subject invariably point to the importance of distinguishing prediction and determination, precisely because prediction is an activity of observers whose ability to gather facts about their world is limited:

Questions about scientific predictability are often posed in terms of disembodied spirits whose intelligence may range over the entire spatial extent of the universe (recall Laplace's demon) or in terms of embodied observers who are given information about the past and present state of the world. But this approach leads to a never-never land form of prediction that is unavailable to actual observers who are localized and embodied and who are not "given" any free gifts of information but must ferret it out for themselves (Earman 1986, p. 63).

Let us examine the assumed powers of observation in condition (ii). The operative intuition behind the condition is that if the putative predictable event p is indeed in the domain of dependence of the initial data surface and that surface is in the causal past of q, then "an observer at q will have at her fingertips all the information necessary to predict the state of p" (Hogarth 1993, p. 724). Note that she must have observed the *complete* surface worth of data in order to make a prediction; "missing" data correspondingly reduce the domain of dependence and therefore the domain of prediction. Obviously, observing a complete surface of data is practically impossible, so some degree of idealization is necessary. I claim, however, that there are not only practical impediments but properly physical impediments to an observer gathering all of the "in principle available" data, i.e. for any surface in an observer's causal past, she may only gather an incomplete set of data on that surface. This is indeed contrary to what is assumed by all of the authors working on the topic. Hogarth, for example, intuits the observational context as follows: "Crudely speaking, [prediction] is possible because inhabitants of appropriate relativistic worlds can gather together, by *physical* means, all the data that determine some future event" (Hogarth 1993, p. 721, emphasis added). I contend that they cannot do so, no matter how hard they tried, because well-established physics mandates that they cannot.

To argue for this contention, it is worth remarking first that to model observations in a spacetime theory some physical model of the phenomena by which an observer learns of her environment is required, since she cannot directly observe the purely geometrical metric or connection. Although the metric is sometimes referred to as an observable of GTR (Wald 1984, p. 378), this is meant in the extended sense that it is determinable by directly observable quantities (Ellis 1980). For example, the relevant observable quantities in our universe for cosmological applications include redshifts, proper motions and angular diameter distances of astronomical objects, distortion measurements (from gravitational lensing) and number counts of distant galaxies and quasars (Ellis et al. 1985). These observable data can be used to determine the spacetime metric. In other applications of GTR, the relevant observable quantities may be different, as they would be in the case of the precession of the perihelion of Mercury. GTR is a theory of gravitation, not a theory of everything, so the exact physical model may well vary from application to application. The crucial point though is that such physical models, whatever they are in a particular application, should certainly not come from the "never-never land" of physics, but should attend to how information from distant regions may be physically transported to the worldline of an observer—a point of which astronomers and cosmologists are obviously well aware in their work.

Accordingly, consider an observer in Minkowski spacetime, the prototypical vacuum solution of GTR. Obviously spacetime does not have to be completely empty to be well-described by Minkowski spacetime—it is, after all, the background spacetime used in quantum field theory. The backreaction of matter and radiation on the metric only has to be negligible for it to be a good approximation of the actual spacetime. Could the observer ever gather enough data from, say, electromagnetic radiation to make a prediction by the standards of (ii)? Evidently not, since to gather enough the observer would have to receive a complete surface *S* worth of data, and this plausibly requires photons (or whatever) communicating these data from all of *S*. But if there were a sufficient density of radiation to communicate the needed data, it would be unjustified for the observer to consider herself in Minkowski spacetime, for such a density of radiation would have a non-negligible effect on the metric. The assumption that motivates condition (ii) is therefore false in Minkowski spacetime and similar spacetimes, since the initial data the observer would use for determination are not accessible to the observer by any physical means.

The physics of information transportation also spells trouble for prediction in other spacetimes. Although other physical mechanisms may transport information to cosmological observers, e.g. neutrinos and gravitational waves, the principal source of information in our world is electromagnetic in nature. The familiar physics of electrodynamics alone provides reason to doubt that an observer could ever gather a complete surface S of data for the purposes of prediction from electromagnetic radiation—in any spacetime. But what is true about electrodynamics in this respect is true about other physical theories. From what we know about physical processes (in particular on the basis of quantum field theory), information simply cannot be losslessly transported from S in the past of q to the observer at q, free of scattering, decay, or other forms of corruption.

Accordingly, consider now a spacetime where a sufficient density of informationcarrying matter and radiation exist, such as the standard Friedman–Robertson–Walker spacetimes of cosmology. Is an observer in such a spacetime physically able to gather a sufficient amount of data to make predictions, as Hogarth and the rest assume? In a transparent universe, i.e. a universe where the optical depth is everywhere zero, the answer would be affirmative. But a universe with a sufficient density of informationcarrying electromagnetic radiation is not transparent, since any such radiation scatters (either through self-interactions or higher-order interactions). Thus the observed intensity of radiation emitted from a source is some fraction of the intensity of radiation emitted from that source, and the information transported via that radiation about the source is to some extent—however small—corrupted.

What this means is that for any surface of data *S* in the observer's causal past, only a fraction of that data is accessible to the observer by physical means. But then this fraction of data is insufficient to determine events in *S*'s domain of dependence. So the ostensibly physical assumption that motivates condition (ii) is false in *any* spacetime, since the initial data the observer would use for determination are not accessible to her by any physical means.

If one is indeed in the business of explicating "the physical notion of making predictions in general relativity", then condition (ii) fails to deliver, since it supposes that prediction is determination from initial data which are physically obtainable by an observer.⁸ No observer in any spacetime could conceivably observe a complete surface *S* of data on physical grounds. An observer simply cannot gather the data that determine some future event by physical means. It is, in short, physically impossible.

I suspect that some may not be impressed with this argument. The motivation for condition (ii) might be quickly defended by arguing that the condition only concerns idealized observations, such that the red flags I raise over information degradation are quite besides the point. But then what precisely is the point? I have argued that the notion of an observer having complete information is physically impossible—indeed, it requires the "free gift" of information maligned above in the quotation by Earman. I suppose what these authors may have in mind is that the proofs are meant to show that in the "best case" prediction is nearly impossible in GTR, from which one casually infers that in any less ideal case prediction will be correspondingly worse off. This inference, however, depends on the epistemic limitations of the ideal case carrying over to the realistic case, a point which should be resisted from a more sensible epistemological stance, one which accepts some degree of fallibility in our inferences.

Indeed, the foregoing considerations serve in part to emphasize that an acceptable interpretation of prediction should rely on the physically realistic assumption that an observer only has partial access to the observable data. Partial access to data is the case in practice as well as in principle; there are practical impediments as well to obtaining complete data sets, which lead to imprecise or incomplete observations. For sure, part of the power of physical theory is in using dynamical laws to determine unknown facts from a complete set of data, so partial data sets present a challenge to the application of dynamical theories. The means of overcoming this challenge are familiar though: we interpolate the data or we adopt a statistical approach. Some reasonable assumptions about the "missing" data have to be made, and so much is obviously nothing more than standard scientific practice.⁹

Now, if we are permitted to make ampliative inferences about data within our causal past and take the products of these inferences as knowledge, then there can

⁸ Norton (2011) too recognizes that condition (ii) rests on "an excessively optimistic assessment of our observational abilities".

⁹ "Using observationally determined initial data, *completed* for those which are determined not precisely enough or which correspond to observationally not accessible regions on the past light cone, one may construct essentially local cosmological models inside the past light cone..." (Dautcourt 1983, p. 153, emphasis added).

be no prohibition to making the same kinds of inferences about data outside of our causal past (so long as they can be reasonably justified of course!) and to counting it as knowledge. If we, for example, infer that the cosmic microwave background (CMB) is isotropic (and has been for the last 13.5 billion years) by completing our available observational data, then we are surely within our rights to extend the spatial constraint of isotropy outside of our causal horizon (at least to some extent). Surely, in other words, we may reasonably believe that the cosmic microwave background will appear isotropic tomorrow, in a year, in 100 years, in 1000 years...after all, one must not forget that cosmological time scales are extremely long! Moreover, in this case the degree of justification should permit us to claim knowledge of any of these predictions of isotropy...fallible knowledge to be sure, but knowledge all the same.

With these remarks in mind, the seemingly innocuous condition (i) also appears in a different light than that with which it was introduced previously. When observation was taken as ideal, retrodiction was trivial, so one could focus on prediction in the "strict" sense, i.e. toward the future. But partial epistemic access makes retrodiction non-trivial, and no different in kind from this strict sense of prediction (modulo concerns over entropy being relevant to the distinction between prediction and retrodiction). It therefore seems sensible to widen the scope of prediction to allow predicted events to be anywhere in the spacetime manifold at all, removing any physically or epistemically significant role for condition (i). Indeed this is the way the term "prediction" is commonly used in theoretical physics presently, and especially in cosmology.¹⁰

In conclusion, I have argued that physical and epistemological considerations related to prediction militate against conditions (i) and (ii) as adequate or even relevant to the explication of the concept of prediction within the context of GTR. The naive underdetermination argument which one might take them as gesturing at fails in all generality, although that does not of course mean that there are not stronger underdetermination arguments that could be made. Indeed I turn to stronger such arguments in the following section, where I evaluate Manchak's contribution to the literature. Furthermore, because the impossibility results mentioned in this section are essentially based on these two conditions and these conditions do not adequately explicate prediction, the results proved on their basis should not be interpreted in terms of prediction. Instead it seems they are perhaps better re-interpreted as something like "no-go" results on the empiricist, infallibalist epistemology that would lead to positing conditions (i) and (ii).¹¹ They indicate that an overly idealized conception of observation is inadequate to our predictive practices, and therefore that a realistic conception thereof, one that acknowledges the epistemic validity of ampliative inferences, is essential to making sense of these practices.

¹⁰ Examples of this contemporary understanding of prediction abound. Here is one: "The cosmological singularity (in all examples where its character is not known to be unstable) involves infinite curvature and infinite density. One's abhorence of such a theoretical prediction is particularly heightened by the correlative prediction that these infinities occurred at a finite proper time in the past,..." (Misner et al. 1973, p. 813).

¹¹ To be sure this is an epistemology that was easier to maintain in pre-relativistic days. I am inclined in fact to interpret this as one of the central points of Geroch (1977).

3 Prediction: global spacetime structure

Manchak (2008) explicitly accepts conditions (i) and (ii) as partial explications of prediction in GTR, but argues that they are by themselves insufficient.¹² His position is that an observer must not only have access to the information needed to make a desired prediction, but must also know that those facts would indeed determine the predicted state of affairs in any spacetime that is observationally indistinguishable from her own. According to him, conditions (i) and (ii) are insufficient to guarantee this. He argues this on the grounds that the observer's epistemic situation is such that she cannot know in which of these spacetimes she is an observer, at least given the data she has available (Manchak 2009a). The epistemic threat to predictions arises if, for any prediction which the observer would like to make, there always exists a spacetime in this collection that can undermine that prediction.

These considerations lead Manchak to define the "domain of *genuine* prediction" (Manchak 2008, p. 320):

Definition 2 Given a spacetime and some point q in that spacetime, a point p is in the *domain of genuine prediction* of q if and only if (i–ii) p is in the domain of prediction of q, and (iii), for all inextendible spacetimes, if there is an isometric embedding of q's causal past into that inextendible spacetime, then there is an isometric embedding of the union of q's causal past and p's causal past into that same spacetime such that the embeddings are equivalent when the latter is restricted to the causal past of q.^{13,14}

Manchak motivates the adoption of condition (iii) by claiming that it is necessary to give "a definition of the domain of prediction which requires that the observer not only have the resources to make a prediction but also the resources to *know* that she can make a prediction" (Manchak 2008, p. 320). He therefore finds the definition of prediction based on only (i) and (ii) wanting, since "it seems that knowledge about one's domain of prediction requires knowledge not only of one's causal past but also of the spacetime in which one's causal past is embedded" (Manchak 2008, p. 320). This is because the domain of prediction (including only (i) and (ii)) depends not only on the restriction of initial data to the observer's causal past but also on the structure of the given spacetime, which as has been stressed is something the observer

¹² "One may wonder if the definition of the domain of prediction given above accurately reflects the physical notion of 'making predictions in general relativity.' It is [my] position that it does not" (Manchak 2008, p. 319).

¹³ An inextendible spacetime is a spacetime for which all isometric embeddings into another spacetime are surjective. Every extendible spacetime has an (not necessarily unique) inextendible extension.

¹⁴ As written, this definition is too weak. There may be multiple isometric embeddings of q's causal past into another spacetime, only one of which has the required isometric embedding of the union of q's causal past and p's causal past. In short, the observer may not only inhabit one of many observationally indistinguishable spacetimes, but may also suffer from self-locating uncertainty within a spacetime. Requiring that the first embedding is unique avoids this issue, as does requiring that the second embedding exists for each of the first.

As written, this definition is also too strong. The predicted event p's causal past need not be shared in all observationally indistinguishable spacetimes, since the entire causal past of p is unnecessary to determine p. A suitable requirement would only have that the intersection of the future domain of dependence of the surface S that determines p and p's causal past is shared in all spacetimes.

cannot know by observations alone. In this way the definition of domain of prediction treats the structure of spacetime somewhat infelicitously as both unknown and known. The underdetermination argument which motivates adopting (ii) concludes that the observer cannot know which spacetime she inhabits, but the definition itself assumes the observer's spacetime is given.

Condition (iii) is meant to capture the intuition that genuine prediction, i.e. prediction where the observer knows without recourse to knowledge beyond what is observationally available that the prediction is determined, requires empirically indistinguishable spacetimes to have identical observable consequences for the prediction in question. If the causal past of an observer is replicated (isometrically embedded) in another spacetime, then one can say that those spacetimes are empirically indistinguishable to that observer.¹⁵ To make a genuine prediction, the observer's actual spacetime and any spacetime observationally indistinguishable from it must share the causal past of the predicted event in addition (since the causal past of the predicted event determines that event). Then she can know that her prediction is genuine, and that it cannot be foiled by her lack of knowledge about her actual spacetime.

Manchak (2008, pp. 320–321) uses this novel definition to prove a theorem that depends on constructing a particular nemesis spacetime that undermines any given prediction by an observer in any given spacetime. The specific nemesis spacetime is constructed by cutting a hole in the given spacetime to yield another, then finding a maximal extension that preserves the hole (Manchak 2008, p. 321).

Theorem 1 Given a spacetime and a point q, the domain of genuine prediction of q is a subset of the boundary of q's causal past.

Since points that are connected to q by null curves are on the boundary of q's causal past and in q's causal past, these boundary points are not in the domain of genuine prediction. Only those points that are on the boundary of q's causal past and not in the causal past are genuinely predictable according to Manchak's theorem. Thus the consequence of this theorem is that "the only possible predictions are those 'on the verge' of being retrodictions." On the basis of this theorem, he concludes that "if the epistemological predicament of the observer is fully considered, there seems to be an interesting and robust sense in which genuine prediction is not possible in general relativity" (Manchak 2008, p. 321).

Manchak is careful not to run afoul of at least one of the criticisms of the previous section. He observes (in a different context, but applicably here as well) that "even under the assumption of an inductive principle—that the physical laws we determine locally are applicable throughout the universe—these general epistemological difficulties remain" (Manchak 2011, p. 410). Although the quotation may suggest that he has only dynamical laws in mind, it is clear from his definition (Manchak 2011, p. 413) that any local condition, such as local spatial homogeneity and isotropy, is an admissible assumption as well.¹⁶

¹⁵ A given spacetime is completely observationally indistinguishable from another if for every point in the given spacetime, there is a point in the other where the causal pasts of the points are isometric (Malament 1977, p. 68). See also Glymour (1977).

Before turning to evaluate Manchak's theorem, it is important to point out that the criticisms of condition (ii) are in fact irrelevant to the theorem he proves. Although Manchak incorporates condition (ii) into his definition of genuine prediction, his theorem does not in fact depend at all on its presence. The definition of genuine prediction is easy to modify to avoid its assumption:

Definition 3 Given a spacetime and some point q in that spacetime, a point p is in the *domain of genuine prediction* of q if and only if (ii') there exists an achronal, closed, spacelike surface S such that p is in the domain of dependence of the subset S, and (iii), for all inextendible spacetimes, if there is an isometric embedding of the union of q's causal past and S into that inextendible spacetime, then there is an isometric embedding of the union of q's causal past and p's causal past and p's causal past into that same spacetime such that the embeddings are equivalent when the latter is restricted to the union of the causal past of q and S.

Manchak's theorem is provable on the basis of this definition without any significant modification of the details of the proof.

In any case, observe that based on the definition—strictly speaking—even a single observationally indistinguishable spacetime in which the putative prediction fails is enough to preclude genuine prediction. This seeming demand for certainty in predictions is, as noted previously, a high standard for knowledge. It is therefore an easy target for underdetermination arguments. If the required warrant for knowledge is that the evidence picks out x with certainty, then the mere exhibition of empirically indistinguishable x's is sufficient to infer the conclusion of the general underdetermination argument sketched in the previous section. But if one allows that knowledge is possible even when the warrant for belief falls short of certainty, then it is insufficient. As noted previously, one must in that case show that the evidence cannot favor one x over to some appropriate degree (Laudan and Leplin 1991). Therefore, if the theorem is to be interpreted as demonstrating a failure of predictability, it must be because this nemesis spacetime is not disfavored by evidence compared to the actual spacetime.

The theorem and its proof make essential use of global features of spacetime (a spacetime hole is a global property of spacetime), and are motivated by the thought that there is a significant epistemic difference between extending conditions locally and extending them to all of spacetime. Manchak suggests that "induction on such large scales would seem to be suspect, given that we are able to observe only a 'negligibly small region' of the universe" (Manchak 2011, p. 416). Even if we were able to observe a large fraction of the universe, though, one might argue that such local observations could still never sufficiently justify belief in a global property of spacetime. Indeed, in keeping with this latter sentiment, Manchak (2009a) argues that there is a "robust sense in which the global structure of every cosmological model is underdetermined" by observationally indistinguishable spacetimes.

¹⁶ As Norton comments, "Our observable spacetime is four-dimensional and has a Lorentz signature metrical structure. We are allowed the inductive inference that this will persist in the unobserved part. Footnote 16 continued

More generally, we are allowed to infer inductively to the persistence of any local condition, such as the obtaining of the Einstein gravitational field equations, in both the observer's and the [observationally indistinguishable] spacetimes" (Norton 2011, p. 170).

It is important to recognize, however, that Manchak first proposes a different nemesis spacetime than the one constructed in the proof of his theorem, one in which the predicted event's spacetime point is simply removed from the observer's manifold. Yet he does not dwell long on the simple failure of predictability due to the existence of the "plucked point" nemesis spacetime. The mutilated spacetime is extendible (just add the point back), and "one could require that spacetime be inextendible" (Manchak 2008, p. 320). For precisely the reasons given in the previous paragraph one may wonder whether the global property of inextendibility is in fact epistemically justified. Manchak (2011) himself argues that it is sometimes problematic to insist on inextendibility, since not every well-behaved spacetime has a well-behaved maximal extension and not every spacetime has a unique extension.

Nevertheless, he recognizes the common demand in the relativity literature that spacetimes be inextendible, and so subsequently includes the requirement in his definition of genuine prediction, since even under the assumption of inextendibility he is able to prove that any observer's spacetime is underdetermined in a way that undermines any given prediction she wishes to make.

Some will likely believe that the contentious point is whether "cut-and-paste" or "holed" spacetimes are permissible. There has indeed been some skepticism of the technique of creating new spacetimes by the method of cutting holes, a method which is featured in the construction of Manchak's nemesis spacetimes. Manchak (2011) argues that a compelling case for a categorical prohibition of the kind of spacetime holes used to construct the nemesis spacetime has not been made. In support of the physical reasonableness of spacetime holes, he cites Geroch (1971b, p. 78): "The space–times obtained by cutting and patching are not normally considered as serious models of our universe. However, the mere existence of a space–time having certain global features suggests that there are many models—some perhaps quite reasonable physically—with similar properties." While the global property of hole-freeness is often assumed to hold by fiat in practice, Earman (1995, p. 98) diagnoses the motivation for holding it as a mere desire to preserve determinism, an attitude he argues is impermissible and question begging when determinism is at issue.¹⁷

I certainly have no argument as to whether hole-freeness is a physically motivated condition or not. I am therefore happy to agree with Manchak that prohibiting holes by assuming the hole-freeness condition is unjustified. Instead I only wish to make the narrow claim that the specific nemesis spacetime in Manchak's theorem is not a threat to making predictions in GTR. In particular, I claim that an observer in that spacetime would consider it predictively indistinguishable with respect to the spacetime from which it is constructed, and thus the (suitably modified) theorem cannot be proved on its basis.

Recall that a fundamental assumption of relativistic spacetime theory is that spacetime is a continuum and appropriately modeled by a manifold. It is usually assumed that every point in the spacetime manifold represents an event. These assumptions, however, are on somewhat dubious empirical grounds, since no measurement could possibly ever register any such pointlike event. Observable events are in point of fact

¹⁷ Manchak (2009b) agrees, and for this reason adopts the view that it may be possible to demonstrate hole-freeness in GTR by assuming suitably physically motivated conditions.

always larger than a point. (It is of course usually convenient to treat them as points in most formal models.) Since predictions are of measurable events, we should therefore demand that any theorem that treats predicted events as pointlike must show that that prediction is sufficiently "robust" in order to have physical significance.¹⁸

The construction of the nemesis spacetime in Manchak's theorem begins by cutting out the boundary of a closed spacelike surface of the actual spacetime (the boundary including the intended pointlike prediction), this surface itself in some (open) neighborhood of the pointlike prediction. Cutting such a hole, however, does not affect the properties of the remaining points in this neighborhood. (One stitches two of these "holed" spacetimes together to make a new inextendible spacetime-Manchak does not formally demonstrate how they are patched together, but it can be shown.) Since the properties of points in the hole's neighborhood remain unaffected, one cannot claim that the original spacetime and the nemesis spacetime differ in their authentic predictions (for the given observer and her counterpart in the nemesis spacetime), since the actual prediction an observer makes is more robust than a single missing point, line, or surface—the predicted event is intuitively an open region. The actual observer and her counterpart in the nemesis spacetime would therefore make effectively identical authentic predictions, up to an arbitrary degree of measurement accuracy. Thus, with a suitably modified condition (iii') which incorporates the robustness condition, I conclude that Manchak's theorem cannot be proven for physically significant predictions using this particular nemesis spacetime.

One might think that the modification to recover Manchak's result is trivial, since it seems that one could easily cut out larger holes. More precisely, for any open region O one requires in order to insure robustness of predictions, one could remove a closed region including the open region O such that one can then follow Manchak's recipe for patching together copies of that spacetime. Such a procedure will not work, however, insofar as one maintains the requirement of inextendibility. The spacetime constructed in this way can always be extended to the nemesis spacetime constructed in Manchak's original proof (just replace all the points save one, keeping the existence of a hole intact), which spacetime, as argued above, fails to invalidate the observer's prediction.

Perhaps some different inextendible nemesis spacetime can be constructed that meets the "robustness of predictions" requirement and also invalidates any observer's predictions. It may even be of some interest to investigate this possibility, but in my view a serious threat to prediction remains so long as one does not produce grounds to justify inextendibility. Manchak's initial nemesis spacetime, where one takes the actual spacetime and removes the predicted point, can easily be adapted to invalidate predictions no matter how robust one demands them to be. That is, without the inextendibility requirement, one can cut out as big of a hole as one requires to create a nemesis spacetime that invalidates predictions. Thus the real issue that needs resolution is whether such specific "holed" spacetimes are evidentially disfavored over their "unholed" counterparts.

¹⁸ "But if a prediction depends crucially upon the precise data—if it undergoes a drastic change under even arbitrarily small perturbations of that data—then our prediction, while perhaps suggestive and useful, has little physical significance" (Geroch 1971a).

Once again, I have no desire to argue for or against hole-freeness as a physical constraint on spacetime, since I am willing to accept that spacetime holes may have empirical consequences. In order to rebut Manchak's claim about prediction in GTR I need only claim that the "holed" nemesis spacetime which undermines a given prediction should be disfavored compared to the unmutilated spacetime. I argue for this claim inductively. The relevant basic fact is the following: In every case where we have or could have made a prediction on the theoretical basis of GTR, that prediction has not been or would not have been undermined by our spacetime in fact having a spacetime hole at the spacetime location of the predicted event. Given this evidence, we have good reason to expect that our predictions will not be prevented by such arbitrary holes in our future, i.e. these nemesis spacetimes are inductively disfavored in our world and worlds relevantly similar. This argument should incline us to think that such spacetime holes are in fact mere skeptical artifice and underdetermination arguments that depend on them are not to be taken seriously.

I conclude therefore that the prediction of events in spacetime has not at all been shown to be essentially impossible—at least not impossible for the reasons that Manchak produces. Introducing spacetime holes in the way of the nemesis spacetimes merely represents a kind of restricted skepticism about prediction that can be resisted on straightforward inductive and physical grounds as I have argued. If one extends the scope of prediction to global properties of spacetime, then it can perhaps be said that our ability to predict is limited (Manchak 2009a), but this usage only loosely makes contact with the practically relevant notion of making and confirming predictions in GTR. We cannot observe global properties of spacetime, only their consequences, so it makes little sense to describe such properties as predictable. Nevertheless, I emphasize that the underdetermination of global spacetime structure does represent a peculiar and interesting epistemological challenge that has not yet been satisfactorily decided; it is, however, a challenge which is only misleadingly described as a challenge for prediction.

4 Concluding remarks

Geroch (1977) remarks that a discussion of predictions in physics only makes sense in the context of a particular physical theory. He suggests therefore that "one might divide a discussion of prediction in physics into two parts: (1) the choice of a physical theory and (2) the establishment and interpretation of certain theorems within the mathematical formalism of that theory" (Geroch 1977, p. 81). Such a method is fine as far as it goes¹⁹; if one's aim is merely to gain some insight into the structure of a theory or to seek modifications to a theory that has certain undesirable pathologies, then there is much to recommend such a strategy in the philosophy and foundations of physics.

The physical and philosophical significance of the results of an investigation conducted in this manner can be easily distorted however—precisely because the

¹⁹ Geroch, at least, acknowledges that to follow this method is to operate in a "rather narrow framework" (Geroch 1977, p. 83).

explication of a concept in a theory depends, sometimes sensitively, on physical details and philosophical motivations. Indeed, when discussing the prospects of prediction in Newtonian mechanics, where the possibility of "space invaders" (particles entering or leaving spacetime from "infinity") ruins the uniqueness of solutions to an initial value problem, Geroch considers fixing this theoretical flaw by requiring that no information comes into spacetime from infinity. He remarks, however, that "this result would not, at least to me, suggest prediction, for it constrains both the initial state of the system and its future behavior" (Geroch 1977, p. 82). Dynamical laws obviously constrain a system's future behavior as well, but in their case Geroch takes them on the contrary as essential to prediction. The apparently diverging standards he thus holds for dynamical constraints and non-dynamical constraints is, it seems, a philosophical predilection of some kind. Rejecting the significance of this distinction to prediction (the diverging standards of which strike me in any case as epistemologically ill-founded)²⁰ would at least straightforwardly obviate the suppositious claim that "there seems to be no theorem in ordinary Newtonian mechanics that suggest possibilities for prediction" (Geroch 1977, p. 82).

I have argued that claims of the impossibility of prediction in the general theory of relativity are similarly suppositious, and unsoundly so. As they are interpretations of technical results in terms of the concept of prediction, they depend on the availability of precise and relevant explications of this concept. I have argued that the conditions which constitute these explications are ill-motivated and defective. Conditions (i) and (ii) are unphysical and over-idealized in a way that greatly limits their significance; condition (iii) is problematic as well, as it fails to recognize the physical import of robustness in predictions. Insofar as prediction is an activity of "actual observers who are localized and embodied and who are not 'given' any free gifts of information," I conclude that one should not be led to motivate these conditions nor maintain that prediction is in any sense impossible because of results proved on their basis.²¹

I also argued that Manchak's nemesis spacetimes constitute insufficient grounds for a compelling underdetermination argument for undermining the possibility of predictions. My arguments parallel standard moves for disputing underdetermination arguments: a putative case of underdetermination is argued either to fail to be distinguished in the relevant way or it is disfavored on epistemological grounds. In the one case this is because the spacetime does not yield a different prediction, and in the other inductive arguments favor the unmutilated spacetime, at least in worlds like our own.

²⁰ As I observe in Sect. 2, there is no epistemologically significant difference between the way we come to learn about dynamical and non-dynamical constraints. Indeed, some dynamical laws—the EFE being a particularly salient example in this case—can be understood as incorporating dynamical and non-dynamical constraints, such as when GTR is treated as a constrained Hamiltonian system. Tradition and metaphysical intuitions may incline one toward inflating the distinction's importance in interpretational contexts, but nobody has made it clear why one should. There are, in any case, reasonably good accounts of laws which do not presuppose that laws are necessarily dynamical. Cf. the relevant parts of Callender (2004, 2007).

²¹ Although I do not propose a positive definition of prediction in this paper, I think the two conditions I suggest in the paper, namely conditions (ii') and (iii'), are important and relevant to having a clear conception of prediction in GTR. Nevertheless, I do not think they necessarily constitute an adequate definition of prediction.

If the definitions and results do not have the interpretive import that they claim, they are still definitions and results in the mathematical framework of GTR. What significance can be claimed for them? I urged that Manchak's argument is best understood as focusing attention on an important and under-investigated epistemic challenge, namely the justification of assuming global properties of spacetime. Only in a loose sense of prediction can these properties be thought of as predictable; nonetheless, the arguments justifying such assumptions leave much to be desired, a contention which Manchak has emphasized elsewhere (Manchak 2009a, 2011). The arguments based on consequences of conditions (i) and (ii) however appear to serve at best as pointing out that idealized observations and naive deduction are insufficient to predict anything in GTR. Earman at least appears to take this to be a significant epistemological point, remarking that "observation combined with inductive reasoning may recommend one hypothesis over all others, but rarely, if ever, does the combination yield a confidence that approaches the certainty with which Laplace's demon went about its prediction tasks" (Earman 1986, p. 66). Perhaps some still cling to the fancy of certitude in our epistemological endeavors; for the most part philosophy and science have long since come to terms with the fallibility of knowledge, and indeed without any apparent forfeiture of confidence in the viability of successful prediction.

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