The hole argument and the problem of time

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Hole argument (Earman/Norton)

- Hole diffeomorphism, $\psi : M \to M$ which:
  - acts non-trivially on a 'hole' $O \subset M$ with no matter.
  - act trivially on $M - O$.
- The pushforward of g by ψ, $\psi^* g = \tilde{g}$, is non-trivial in O.
- While $(M, g) \to (M, \tilde{g})$ is a mathematical isomorphism, a (standard) substantivalist takes different assignments of the metric at different points in O to represent ontologically distinct models.

$\Rightarrow$ representations underdetermine ontological models.

Weatherall's deflation (2015)

> *"...the default sense of 'sameness' or 'equivalence' of mathematical models in physics should be the sense of equivalence given by the mathematics used in formulating those models... mathematical models of a physical theory are only defined up to isomorphism,* **where the standard of isomorphism is given by the mathematical theory of whatever mathematical objects the theory takes as its models***...isomorphic mathematical models in physics should be taken to have the same representational capacities. By this I mean that if a particular mathematical model may be used to represent a given physical situation, then any isomorphic model may be used to represent that situation equally well." [bold added]*

Since the standard of mathematical isomorphism given by different geometry is met by $\psi^* g$, (M, g) and $(M, \tilde{g})$ should be taken to represent the 'situation' (metaphysical possibility?) equally well.[1]

1. For a critique of this deflation, see Pooley and Read 2019 (Unpublished) and Roberts 2020.

Time reparametrization versus standard gauge symmetry

- E&M gauge symmetry: $A_\mu \to A_\mu + \partial_\mu \phi$ has simple action on phase space:
$$A_i \to A_i + \partial_i \phi \tag{1}$$
Symplectic flow of Gauß constraint:$\partial_i E^i = 0$ is 'gauge orbit'.
$\Rightarrow$ symmetry acts 'at an instant'
$\Rightarrow$ symmetry can be removed by quotienting gauge orbit.
- Rep. invariance: $t \to f(t)$, $\dot{f} > 0$ acts 'on a history'.
$\Rightarrow$ Flow of Hamiltonian constraint $H = 0$ generates solutions.
$\Rightarrow$ Quotienting by flow gives space of initial data.

Formal analogy between orbits of $\partial_i E^i$ and $H = 0$:

$\Rightarrow$ time is 'gauge'

The Problem of time

- Collection of intertwined problems about the representation of temporal symmetry in canonical GR/QG.
- Distinctions: classical/quantum and local/global.
- **Classical, global** time evolution along orbits of constraints:
  $\Rightarrow$ "Time is gauge"?!
- **Classical, local:** symmetries only close 'on-shell'
  $\Rightarrow$ no natural group action on phase space. (See below.)
- **Quantum, global:** no time evolution of Ψ
  $\Rightarrow$ e.g., WdW equation $\hat{H}\Psi = 0$.
- **Quantum, local:** how to represent symmetries quantum mechanically?
  $\Rightarrow$ on-shell condition and anomalous representations

Standards of isomorphism

- On space-time: $(M, g_{\mu\nu}) \to (M, \tilde{g}_{\mu\nu})$ is an isomorphism.
  - Isomorphism at the level of mathematical objections only.
  - No input from Einstein equations.
- On phase-space: same initial data (g_{ij}, π^{ij}) lead to isomorphic space-times when evolved with Hamiltonian and difference choices of lapse.
  - Isomorphism requires knowledge of dynamics.
  - Initial data can only have the same representational capacity as a space-time if the dynamics are specified.

Weatherall's deflation revisited

- Standard of isomorphism: "is given by the mathematical theory of whatever mathematical objects the theory takes as its models".
- In canonical E&M, $U(1)$ transformations fit the standard of isomorphism set by phase space (i.e., quotient by $U(1)$ orbits).
- In space-time GR: 'hole diffeomorphisms' fit the standard of isomorphism set by differential geometry (i.e., space-time diffeomorphism)
- In canonical GR 'hole refoliations' do not fit the standard of isomorphism set by phase space
  $\Rightarrow$ they require specification of the Hamiltonian.

$\therefore$ Weaterall's deflation does not work in canonical GR.

Quantum Gravity?

This problem becomes particularly acute in quantum gravity were classical dynamics (i.e., the 'on-shell' condition) can be violated.

Moncrief 1990

"What, after all, is a **quantum space-time** *and does such an object admit a representation in terms of different* **space-like slicings?***"* [Original emphasis]

Not just a problem for canonical formalism!

⇒ How to define 'in/out'-states without space-like (?!) boundaries?

Quantum hole argument?