‘Notes on Symmetries’: in the footsteps of the great Belot ...

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1 Symmetry

Belot defines the notions of: structure, automorphism (hence: group of automorphisms); and hence: symmetry ‘in the first sense’; and (given a definition of ‘nomenclature’) symmetry ‘in the second sense’. Cf ‘active’ and ‘passive’.

JNB: a recent elegant short preprint about how and why symmetries tell us about structure is T Barrett’s ‘What do symmetries tell us about structure?’: which is at: http://philsci-archive.pitt.edu/13487/

2 Symmetry arguments

Belot proposes a general form of a symmetry argument:–

Namely: that a proposed extension to a given structure (in his notation: a new relation R defined on the domain D of objects of the given structure) is evaluated by whether or not it is preserved by the symmetries of the original structure.

The argument is of course only as good as one’s confidence that the symmetries of the original structure are the “correct” symmetries for the domain of investigation.

3 Examples

3.1: Platonic cosmology.

3.2: Simultaneity in special relativity.
We expect any proposed notion of simultaneity in special relativity to involve an equivalence relation ‘at the same time as’, on Minkowski spacetime, whose equivalence classes are each of them spacelike, 3-dimensional and connected. But there are no such equivalence relations that are invariant under the symmetries of Minkowski spacetime.

Cf. Malament 1977, Giulini 2001: the theorem was influential in (apparently resolving) the historical debate (Reichenbach, Grunbaum) about whether simultaneity in special relativity was a matter of convention, or fact.

3.3: Time in dust cosmology.
The verdict of 3.2 falters if one adds non-invariant structure/ information to Minkowski spacetime, so that the resulting symmetry group of automorphisms is smaller (or at least: different!); so that a proposed notion of simultaneity *can* be invariant under the smaller (or at least: different!) group of symmetries.

This is exactly what happens when you add e.g. a single timelike geodesic representing an inertial observer. (Or a congruence of timelike geodesics, all parallel to one another.) Then the hypersurfaces everywhere Minkowski-orthogonal to that geodesic will be “good” equivalence classes, i.e. “good” simultaneity surfaces, in the sense of respecting the smaller set of symmetries. (The set of symmetries is now just the euclidean group (spatial translations and rotations), plus time-translations: 'boosts' are absent.). NB: "good" though (famously!) frame-dependent. And they correspond to how Einstein 1905 defined simultaneity by what he there called a *convention*. (Thus also: Allen Janis' 'reply' to Malament.)

This also happens if you consider slightly realistic cosmologies in general relativity, e.g. dust cosmologies like FRW. The comoving 'dust' particles, representing galaxies, form a congruence of timelike lines that 'break the symmetry of Minkowski spacetime' (indeed a congruence of timelike geodesics, relative to the FRW metric).

Thus NB: the role of Gödel spacetime: and Gödel's sympathy for *idealism*.

3.4: Huygens on collisions.

3.5: Symmetries in classical mechanics.
Belot briefly introduces 'phase space + symplectic form + Hamiltonian'. Then he points out that for forces (like Newtonian gravitation) that depend only on inter-particle distances, not on location in space: the set of symmetries will include the euclidean group (spatial translations and rotations). So again: 'boosts' are absent.

JNB: Be warned: symmetry in classical mechanics is a large subject! So I make bold to recommend my expositions, in increasing order of sophistication ...
Between Laws and Models: Some Philosophical Morals of Lagrangian Mechanics: at http://philsci-archive.pitt.edu/1937/
On Symmetry and Conserved Quantities in Classical Mechanics: http://philsci-archive.pitt.edu/2362/
On Symplectic Reduction in Classical Mechanics: http://philsci-archive.pitt.edu/2373/
These eternal papers, especially the last, build on the topics in Belot's Section 5.2 (A), below.

4 Symmetries of solutions and symmetries of laws

4.1: Structuring the space of solutions.
NB: Belot's website preamble says: 'The discussion of Section 4.1 of the paper is very misleading: from knowledge of the differential equations alone one can of course determine the physically relevant symmetries ... But there is a reasonable point behind this misleading discussion.'
I, JNB, agree with this: and I will not try to report, or mend, his Sec 4.1: but instead reproduce Belot's wise words with which the website preamble continues...

> Consider the following two observations (i) Perfectly generally, a symmetry of a structure is a map from the structure to itself that preserves the relations between the elements of the structures. (ii) The Lorentz (or Galilei) group is a symmetry of a physical theory if under the obvious action of this group on the space of kinematic possibilities, the group maps solutions of equations of motion to solutions of equations of motion.
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> These are both sound. But taken together they provide some temptation to think that we can characterize a symmetry of a theory as a (smooth) map on the space of kinematic possibilities that maps solutions to solutions. This characterization is in fact not uncommon among philosophers. But the resulting notion of a symmetry is useless, since under it, e.g., any two solutions of the n-body problem are related by a symmetry of Newtons theory.

What has gone wrong? Clearly we need to think of the relevant structure encoding the physics of a theory as more than just a (smooth) space of solutions sitting inside of a space of kinematic possibilities.

There are two ways we can go here, each of which is completely standard.

The first, emphasized in the paper, is that we can take the space of solutions to be equipped with a symplectic form and a Hamiltonian. Then the physical symmetries that we know and love can be characterized as maps from the space of solutions to itself that preserve these structures.

The second option is to look ... This second approach is developed in detail in, e.g., Olver, *Applications of Lie Groups to Differential Equations*. It is sketched in Section 2 of Giulini, *Algebraic and geometric structures of Special Relativity* [http://xxx.lanl.gov/abs/math-ph/0602018] (where it is observed that if one drops the locality condition then Maxwells theory counts as Galilei-covariant).

N.B. Each of the two approaches just described can lead to surprising results: it is not plausible, for instance, that being related by a symmetry of one or the other sort is sufficient for two solutions of a theory to be physically equivalent. See my *Symmetry and Equivalence* [http://philsci-archive.pitt.edu/9275/] for further discussion.

4.2: Symmetries of equations vs. symmetries of solutions.
(1): NB: generic solutions have less symmetry than do the equations that determine them
(2): For heuristic reasons, the question (i): 'What symmetry does this law/the law we seek to formulate have/lack?' is nowadays taken more seriously than the question (ii): 'What symmetry does this solution/the solutions we in fact find in nature have/lack?'. (EG: parity violation by (i) laws of physics as vs (ii) biomolecules (Pasteur!!)).
(3): *Curie's principle*: In a sense, it is a trivial theorem: provided the dynamics is deterministic, and the symmetry of the initial state is indeed a symmetry of the dynamics in the usual sense. (*JNB: cf. my whiteboard last week!*). BUT...one can violate it with some artificial examples...discussion of example: a triangle of point particles moving under gravity, including idea of *relational mechanics ... cf. the articles by Bryan Roberts we hope to read in future weeks.*

5 Quotienting out symmetries

The idea: $\mathcal{S} = (D, \{R_i\}_{i \in I})$ is quotiented by taking the equivalence classes under any symmetry. So the equivalence classes are the *orbits* of the symmetry group acting on D.
Each n-ary relation R_i on $\mathcal{S}$ defines, on the set $[D] := \{[x] : x \in D\}$ of equivalence classes, a relation in the obvious way: namely, $[R_i] := \{([x_1], ..., [x_n]) : (x_1, ..., x_n) \in R_i\}$.
One can also discuss quotienting out in terms of 'nomenclatures' and symmetry 'in the second sense'. This approach is used in Belot's brief discussion of ...

5.1: Quotienting solutions.

5.2: Quotienting the space of solutions.
Taking the quotient, in the above sense, of a space of solutions often leads to an interesting result—with interpretative implications. Besides, there is a centuries-old mathematical theme here: and often, a considerable historical, as well as philosophical, story. Belot takes two examples.

(A): *Euclidean symmetries of n point-particles under Newtonian gravitation.*
Suppose we take the orbits of the natural action of the euclidean group on phase space. That means, roughly speaking, we:

(i) identify configurations related by Euclidean transformations composed of translations, rotations and reflections; then we

(ii) use (i) to identify states in phase space (i.e. we make momentum information 'carry along' in the natural way); and we also need to ...

(iii) take care about excising states with (a) symmetric configurations or (b) configurations representing collisions.

Roughly speaking: this gives a *relational mechanics*: i.e. a realization of Leibniz's vision in his dispute with Clarke/Newton, or indeed of Mach's! A bit more precisely:

(1): If we also excise, before quotienting, all states in which the non-zero linear momentum and-or non-zero angular momentum (with respect to 'absolute space'): then the resulting quotient is a very close cousin of a relational mechanics (JNB: that has been repeatedly re-discovered since Mach's time, and which is well represented) by Barbour and Bertotti 1982.
(JNB: A fine, easy, partly historical, read is: J. Barbour, 'The development of Machian themes in the twentieth century', in JNB ed. *The Arguments of Time*, OUP 1999.)

(2): If we also *keep*, before quotienting, states with non-zero linear and-or angular momentum, the story is a bit more complicated.

JNB: For more details about (A), cf. e.g. Section 2.3 ('Appetizer: Belot on relationist mechanics') of: On Symplectic Reduction in Classical Mechanics: http://philsci-archive.pitt.edu/2373/

(B): *Permutation symmetries of n point-particles, of equal mass, under Newtonian gravitation.*
Suppose we take the orbits of the natural action of the symmetric group S_n on phase space. That means, roughly speaking, we:

(i) identify states (comprising both configurations and momentum assignments) related by permutations ('delete the *haecceitistic* information');

(ii) again take care about excising states with (a) symmetric configurations or (b) configurations representing collisions.

This *anti-haecceitistic* formulation of (part of) classical mechanics:

(a) supports a classical statistical mechanics (cf Belot);

(b) contrasts with quantum mechanics where, in effect, anti-haecceitism is 'built in': more precisely, all states are required to be fixed by all permutations

(c) is explored further in the eternal works by Caulton and JNB: On Kinds of Indiscernibility in Logic and Metaphysics, at http://philsci-archive.pitt.edu/8450/; and: Symmetries and Paraparticles as a Motivation for Structuralism, http://philsci-archive.pitt.edu/5168/

Belot ends with wise words about two topics.

(i): Examples (A) and (B) are both about rival ways of counting possibilities, one haecceitistic and the other anti-haecceitistic; the main difference being that in (A) there is a sense thatthe quotienting purges the ontology of spacetime points.

(ii): (A) uses a continuous group of symmetries, (B) a discrete group. This means that while (A) involves reducing the dimension of the manifold of states—leading in to the world of *gauge*—(B) does not.