Chapter 11
Non-uniquely Extendible Maximal Globally Hyperbolic Spacetimes in Classical General Relativity: A Philosophical Survey

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Abstract I discuss philosophical questions raised by existence of extendible maximal globally hyperbolic spacetimes with non-unique extensions. These spacetimes represent a form of indeterminism in classical general relativity: given fixed initial data and equations of the theory, there exists more than one possible way a spacetime could be. Even though such spacetimes have been investigated in the physics literature for quite some time, a philosophical discussion of their importance is missing. Here I explore their relevance for the notion of physical equivalence, distinction between physically reasonable spacetimes and physically unreasonable ones, relation between determinism and singular spacetimes, connections between some forms of indeterminism and existence of time machines, and question whether cosmic censorship can be understood as expressing determinism of general relativity.

11.1 Introduction

I will discuss a few examples of extendible maximal globally hyperbolic spacetimes in classical general relativity (GR), and their bearing on the question whether general theory of relativity is deterministic. First, I provide overview of relevant background material (concerning notions of determinism and indeterminism, relativistic spacetimes and initial value formulation of classical general relativity). After that, I briefly present a few classess of spacetimes sharing an interesting property: in the initial value formulation, the nicely behaving maximal globally hyperbolic region of spacetime allows more than one continuation, in the sense that there exists more than one (up to isometry) inextendible spacetime into which this region can be isometrically embedded. Under some natural interpretation of what does it mean for a theory to be indeterministic, existence of such spacetimes shows that classical general relativity is indeterministic. But significance of such spacetimes

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G. Hofer-Szabó, L. Wroński (eds.), *Making it Formally Explicit*, European Studies in Philosophy of Science 6, DOI 10.1007/978-3-319-55486-0_11

is not limited to question of determinism, which I will demonstrate by connecting these spacetimes to questions of physical equivalence in classical GR, conditions for distinguishing physically reasonable models from physically unreasonable ones, time machines, and interpretation of the cosmic censorship hypothesis.

11.1.1 Defining Determinism

Various non-equivalent precisifications of determinism have been offered in the literature. Often these characterizations invoke certain features of the physical situation (such as the existence of the set of moments of time, represented by the set of real numbers $\mathbb{R}$ with its natural linear ordering), features which, in the context of classical GR, do not hold in all of the solutions. One should thus be careful in how one spells out the notion of determinism, it may or may be not applicable to classical GR.

For example, Werndl (2015) provides the following intuition:

> Determinism reigns when the state of the system at one time fixes the past and future evolution of the system.

And Wüthrich (2011), following Earman (1986), gives two definitions:

Definition 1 (Determinism for worlds) A world $W \in \mathcal{W}$ is deterministic if and only if for any $W' \in \mathcal{W}$, if W and W' agree at any time, then they agree for all times.

Definition 2 (Determinism for theories) A 'theory T is deterministic just in case, given the state description $s(t_1)$ at any time t_1 , the state description $s(t_2)$ at any other time t_2 is deducible [in principle] from T.'

Determinism for theories is sometimes called Laplacian determinism, and spelled out in terms of models: a theory is deterministic iff every two models which agree at some t have to agree at all t'.

The trouble with such concepts is that in case of GR spacetime is a dynamical entity, and (prima facie) there are models of the theory which do not have anything like moments of time (Earman (2007) discussess a few concerns related to existence of acausal spacetimes in the context of Laplacian determinism). Fortunately, concept of Laplacian determinism can be spelled out in a way which is more friendly to pecularities of classical GR. One way to do so has been provided (in the context of debates surrounding the Hole Argument) by Butterfield (1989), and seems to be the most sophisticated version of Laplacian determinism available. According to Butterfield,

> a theory with models $\langle M, O_i \rangle$ is **S**-deterministic, where **S** is a kind of region that occurs in manifolds of the kind occurring in the models, iff: given any two models $\langle M, O_i \rangle$ and $\langle M', O'_i \rangle$ containing regions S, S' of kind **S** respectively, and any diffeomorphism α from S onto S': if $\alpha^*(O_i) = O'_i$ on $\alpha(S) = S'$, then: there is an isomorphism β from M onto M' that sends S to S', i.e. $\beta^*(O_i) = O'_i$ throughout M and $\beta(S) = S'$.

Note that this definition does not assume anything like moments of time found in other explications of determinism. However, under this definition worry about existence of acausal spacetimes does not disappear entirely.

Some choices trivialize the question: if **S** is required to be a Cauchy surface of a spacetime in question, then (by Choquet-Bruhat and Geroch theorem I discuss in Sect. 11.1.4) uniqueness is ensured by fiat; but if **S** is just, say, some achronal submanifold of spacetime, uniqueness fails almost certainly. Either way question of determinism becomes trivial: choice of region **S** implies the answer to the question of determinism. Is there a choice of **S** which avoids this triviality? Such a choice would *de facto* implicitly assume that spacetimes which do not contain regions of type **S** (say, spacetimes violating some causality condition) are not physically reasonable, at least in the minimal sense that they are not relevant for the question of determinism: are neither examples of determinism nor indeterminism. One option, to which, I believe, insufficient attention has been paid by philosophers of science, is taking maximal globally hyperbolic development of an initial data set as **S** (I explain the terminology in Sect. 11.1.2). This choice has practical advantage: in a natural way it connects to the initial value problem in classical GR, and is what has been actually used by mathematical physicists. A consequence of that, however, is that spacetimes containing various kinds of holes which "mutilate" MGHD (i.e. spacetimes which have globally hyperbolic regions, but which do not contain maximal globally hyperbolic region as a subset) are excluded *by fiat*, in manner similar to Earman's take on the "dirty open secret" of Earman (1995).

Another subtle aspect is making precise what transformations are included as isomorphisms β. In most of what follows I will assume that an isometry (that is, pullback of the spacetime metric g_{ab} by a diffeomorphism) which preserves temporal orientation is the appropriate choice for β. But, as I discuss in Sect. 11.3.2, there are other possible choices one could make here.

Suppose (in case of relativistic spacetime) that spacetime is temporally orientable and that the orientation has been fixed. I will say that spacetime $\langle M, g\rangle$ is futuristically indeterministic if there are two spacetimes $\langle M', g'\rangle$ and $\langle M'', g''\rangle$ which extend $\langle M, g\rangle$ in such a way that new regions of $\langle M', g'\rangle$ and $\langle M'', g''\rangle$ are to the future of $\langle M, g\rangle$, and $\langle M', g'\rangle$ and $\langle M'', g''\rangle$ are not isometric. Similarly, spacetime could be past indeterministic, or it could be indeterministic "in both ways". Interestingly, as I will show in what follows, classical GR provides us examples of all three types of indeterminism.

Another important distinction concerns indeterminism of the theories and determinism of the world. These two are very different. One can imagine that the best theory of the world is deterministic (in the sense that all models of the theory are deterministic), but the world itself is indeterministic, or the other way round—this could happen, for instance, when theory is incomplete. Indeed, we will argue that one can see extendible maximal globally hyperbolic spacetimes as showing that the theory of general relativity is indeterministic, in the sense that there are sectors of solutions in which general relativity has indeterministic solutions; but, at the same time, it seems that models most useful for the description of our world (FRLW spacetimes) are (at least in the classical theory) inextendible beyond the maximal globally hyperbolic region, which in turn strongly suggests that the world itself is deterministic, at least in the sense that it is (at the energy scales in which classical general relativity is uncontroversially applicable to our world, i.e. without taking into the account energy scales at which quantum effects are expected to play some role) best represented by a deterministic model.

11.1.2 Relativistic Spacetimes

A brief reminder on the structure of relativistic spacetimes is in order. A general relativistic spacetime is a pair $\langle M, g_{ab}\rangle$, where M is a smooth, second-countable, paracompact and Hausdorff manifold, and g_{ab} is a non-degenerate Lorentz-signature metric. In what follows I assume that spacetime satisfies Einstein's field equations, and for most of the time restrict our attention to vacuum solutions. I will follow the standard notation of Malament (2012).

Spacetime $\langle M, g_{ab}\rangle$ is globally hyperbolic iff there exists achronal subset without edge S which is intersected exactly once by every inextendible timelike curve (or, equivalently, S such that its domain of dependence is the whole spacetime, $D(S) = M$). Spacetime $\langle M, g_{ab}\rangle$ is extendible iff there exists spacetime $\langle M', g'_{ab}\rangle$ and a smooth function $f : M \mapsto M'$ which is a bijection onto its image, such that $f(M) \subsetneq M'$ and $g'_{ab} \restriction_{f(M)} = f^*(g_{ab})$; otherwise it is inextendible. Spacetime is (timelike, null) geodesically complete iff generalized affine parameter of any (timelike, null) geodesics takes arbitrary values in $\mathbb{R}$.

Spacetime $\langle M', g'_{ab}, \Lambda\rangle$ is an extension of $\langle M, g_{ab}\rangle$, if:

1. there exists a function $\Lambda : M \mapsto M'$ which is an embedding of M in $\Lambda(M)$ (which is a diffeomorphism onto its image)
2. $\Lambda^*(g'_{ab}\mid_{\Lambda(M)}) = g_{ab}$
3. $\Lambda(M) \neq M'$.

I will say that spacetime is maximal (or inextendible) iff it has no extension, and φ-maximal (or φ-inextendible) if it has no extension with the property φ. In case of spacetimes I will describe below φ will be a causality condition of global hyperbolicity. Motivation for requiring inextendibility seems to be based on metaphysics (see Earman 1995); nevertheless, inextendibility is commonly taken to be a necessary condition for a spacetime to be physically reasonable.[1]

[1] See, however, Manchak (2016b) for a dissenting view.

11.1.3 Levels of Indeterminism in Classical General Relativity

General relativistic spacetimes have many unusual and suprising features. It is appropriate to place indeterminism I am interested in here in a broader context. Depending on how exactly one conceptualizes indeterminism, some general relativistic phenomena count as examples of indeterminism or not. Here, I am interested in situations in which for an initial value problem a solution exists, but uniqueness of a solution fails. But this is by far not the only choice (even if we restrict indeterminism to an existence statement about indeterministic model, which, again, is not a neutral decision). One may distinguish few types of indeterminism in classical GR.

1. acausal solutions, i.e. spacetimes which are not even temporally orientable
2. there is no globally hyperbolic region at all: (a) because closed time-like curves exist, or (b) spacetime satisfies some causality condition which rules out closed timelike curves (strong causality), but is not globally hyperbolic (for example, due to boundary at timelike infinity);
3. globally hyperbolic region is not maximal in the sense that spacetime has been "mutilated" in some way (for example by removing some points from the spacetime manifold);
4. spacetime is singular—it abruptly "comes to an end", either (a) due to blowup of some quantities, or (b) due to some form of curve incompleteness;
5. spacetime does have the maximal globally hyperbolic region, but this region can be further extended in multiple non-equivalent ways

Even though there are many examples of type 1., 2. and 3., they are sometimes argued to be unphysical in classical GR (for example, due to presence of artificial "hole" in spacetime, or on some other grounds). Type 4. spacetimes are rampant in classical GR, as witnessed by the singularity theorems. They may be though of as indeterministic in the sense that solution cannot be continued beyond certain region. Indeterminism is by far not the only possible reading. These spacetimes could be understood as signalling breakdown of the theory (option often assumed by physicists working in quantum gravity), or as entirely new structure predicted by a theory (see Curiel 1998 for this view).

Whether any of these examples could satisfy a Butterfield-like definition depends on the choice of region **S** and the role that the region is expected to play in the analysis. For instance, if **S** is a Cauchy surface for the whole spacetime, only types 3. and 4. remain. And if the role of **S** should be something like: being a candidate for region which determines what happens in its domain of dependence, then 3. seems to be less relevant. But if—as I have argued in Sect. 11.1.1—maximal globally hyperbolic development is a promising choice of **S**, only types 4. and 5. could be of relevance.

Some authors, such as Kutach (2013), restrict their discussion of determinism in GR to singular spacetimes. I find this state of affairs unfortunate, since it ignores fascinating and subtle physical features of various general relativistic spacetimes.

In particular, spacetimes of type 5. have not, I believe, received the attention they deserve—despite being the closest example to the kind of indeterminism which motivates Laplacian concept of determinism in the first place.

There are plenty of ways of mutilating MGHDs and obtain situations in which non-maximal globally hyperbolic region of spacetime can have few extensions: the maximal globally hyperbolic one and some others. Multiple examples of such construction can be found in Manchak (2015) or Earman (1995) chapter 3.8. Such spacetimes are often thought of as artificial, in that some "unphysical" hole has been made in spacetime. It is not easy, however, to find a hole-freeness condition which gives an intuitively correct verdict in such cases (see Krasnikov (2009) and Manchak (2016a) for discussion of few such conditions). But I suppose here that one does not make recourse to any such operations, and allows the globally hyperbolic extension to be continued as far as possible, obtaining MGHD.

Philosophical analysis of GR could be carried at several levels (Malament 2012): one could look for some features at the level of an exact solution, where the right hand side of Einstein's field equation takes some particular form (say, vacuum, electrovacuum, dust, perfect fluid, etc.). Or one could not assume any particular form of the stress-energy-momentum tensor, but assume that it behaves in a certain way by postulating an energy condition. Finally, one could ignore the right hand side entirely, and work with any Lorentzian metric whatsoever.

In what follows I will focus on exact vacuum solutions. Due to the level of technical difficulty, question of existence and uniqueness of extendible MGHD has not been yet investigated thoroughly beyond vacuum solutions, and even in that case the analysis is restricted to certain special (highly symmetric) cases.

11.1.4 Initial Value Problem in GR and Choquet-Bruhat and Geroch Theorem

Under certain conditions initial value problem in classical general relativity has a unique solution, which gives some support to the idea that classical general relativity can be Laplacian deterministic (if one has some reason to ignore spacetimes which are not amenable to initial value treatment due to violations of global hyperbolicity). Choquet-Bruhat and Geroch theorem is a fundamental result expressing this uniqueness of a solution. Informally, one tries to think of a spacelike hypersurface Σ of a 4-manifold (more generally, n-dimensional manifold) $\langle M, g\rangle$ and some data on Σ as uniquely fixing spacetime $\langle M, g\rangle$.

More precisely, initial data consist of an 3-manifold Σ (thought of as spacelike hypersurface of a sought-for M; in full generality, Σ is $n-1$-dimensional, for n being dimension of spacetime) with a Riemannian metric g_0, symmetric covariant 2-tensor k_0 (and, optionally, some scalar functions φ_0, φ_1, etc.). The task is to construct an 4-manifold M with a Lorentz metric g_{ab} (and, optionally, scalar function φ, etc.)

and an embedding $i : \Sigma \to M$ such that if k is the second fundamental form on $i(\Sigma) \subset M, \ldots$, then $i^*(g) = g_0$, $i^*(k) = k_0$ (and, optionally, $i^*(\varphi) = \varphi_0$, etc.).

For $\langle M, g, k \rangle$ developed from $\langle \Sigma, g_o, k_0 \rangle$ to satisfy Einstein's equations, constraint equations should be satisfied. For a vacuum solution, they take the form:

$$S - k_{0\,ij}k_0^{ij} + (tr\ k_0)^2 = 0$$

$$D^j k_{0\,ji} - D_i(tr\ k_0) = 0$$

where S is the scalar curvature, D is the derivative operator induced on Σ, and indices are raised/lowered by g_0. Then, $\langle \Sigma, g_o, k_0 \rangle$ that satisfies the constraints are called vacuum initial data set.

Choquet-Bruhat & Geroch theorem (Choquet-Bruhat and Geroch 1969), then, states that for an initial (vacuum) data set $\langle \Sigma, g_0, k_0 \rangle$ there exists a unique, up to isometry, maximal globally hyperbolic development (MGHD) $\langle M, g, \Lambda \rangle$ of this data set. "Vacuum" means that the MGHD is a vacuum solution, and the spacetime obtained is a "Cauchy development", that is, $\Lambda(\Sigma)$ is a Cauchy surface in M. The theorem can also be generalized to non-vacuum data sets (see Ringström 2009). This uniqueness result crucially depends on the causality condition (global hyperbolicity) and does not establish uniqueness of the maximal extension of spacetime simpliciter, which will be crucial in the following sections. The point could be stated as follows: φ-inextendible spacetime can be extendible, if its extensions do not satisfy the condition φ.

Final note: this formulation assumes that a spacelike hypersurface has been fixed. But one could just as well fix a null hypersurface and consider initial data living on it. Investigation of this formulation could shed interesting light on the question of determinism of GR, but—since the spacelike (or ADM) formulation is commonly adopted in classical and canonical quantum gravity—I will focus on the spacelike formulation.

I am now in position to inquire whether classical general relativity is deterministic according to Butterfield's definition, after **S** has been decided to be the maximal globally hyperbolic development. In other words, assuming that MGHD exists and is realized in spacetime, can one find any models of the theory which are past (or future, or in both ways) indeterministic, up to certain standard of equivalence of models? Choquet-Bruhat and Geroch theorem does assert uniqueness of MGHD, but remains silent about uniqueness of the spacetime as a whole: from the theorem it does not follow that the non-globally hyperbolic extension of MGHD is unique. If there is a spacetime which has a non-globally hyperbolic extension, it could be unique or non-unique (up to some equivalence relation). So: is there a spacetime A which is the MGHD of some initial data, for which there exist two or more non-globally hyperbolic spacetimes B_i, $i \in I$ into which A can be embedded as a proper subset?

If the answer is "yes", one has a very strong witness of indeterminism. Strength of the witness stems from the fact that even if acausal spacetimes are ignored, and singular behavior is acknowledged as physical feature of the theory, and no artificial manipulations are performed, and the natural course of events is allowed to continue as far as possible, in some cases there is an open possibility of more than one history compatible with given initial data and laws of nature (constraints given by Einstein's field equations).

And the answer, indeed, is "yes, there exist spacetimes such as A". The rest of this paper concerns few consequences of existence of such spacetimes. I discuss the ways in which non-isometric extensions of MGHD arise, to what extent such spacetimes can be considered to be physically reasonable solutions, and implications for the question of Laplacian determinism in classical general relativity.

11.2 Extendible MGHD

We will briefly summarize the current state of the art on the extendible MGHDs. These results are well known in the relevant physical literature (see Chruściel and Isenberg 1993; Ringström 2009), but for our discussion it is useful to have an overview of the situation.

The easiest known example is the Misner spacetime. Take a subregion of two-dimensional[2] Minkowski spacetime (such as a lecture room), close it in the spatial direction (i.e. identify right and left wall of the room), and start contracting it along the closed dimension at some fixed rate β. Take the reference frame (x, t) at rest with respect to the left side of spacetime, and place the clock there. On the left side: $\tau = t$, on the right side $\tau = t/\sqrt{1-\beta^2}$ (by time dilation). This will lead to creation of closed timelike curves, and chronology horizon (which separates region of spacetime with CTCs from region without CTCs). There are two classes of geodesics: leftward and rightward. Each of these classes can be extended beyond the horizon; but not both of them. In other words: spacetime is geodesically incomplete, but has "less" incompleteness. Misner spacetime is sometimes quoted as an example of spacetime which allows non-isometric extensions, but this is not quite correct: unless some additional requirements on the function which identifies extensions are specified, Misner spacetime has unique extension (a "flip" is needed to map incomplete geodesics in extension I to incomplete geodesics in extension II, and vice versa for complete geodesics).

Taub-NUT spacetime can be seen as a 4-dimensional version of Misner spacetime. Lets begin with $M \approx S^3 \times (t_1, t_2)$, and $g_{ab} = -U^{-1}dt^2 + (t^2 + L^2)(\sigma_x^2 + \sigma_y^2) + 4L^2U\sigma_z^2$, where $U(t) = \frac{(t_2-t)(t-t_1)}{t^2+L^2}$. Taub spacetime is maximally globally hyperbolic; it could be extended (by adding so-called NUT regions) to the past in two ways and to the future in two ways. In total there are four extensions of the

[2]Sticking to the convention followed by most presentations of Misner spacetime in the literature, I discuss two dimensional Misner spacetime. But this spacetime can be defined in higher dimensions as well.

Taub region: $M^{\downarrow+}$, $M^{\downarrow-}$ and $M^{\uparrow+}$, $M^{\uparrow-}$. In every past (future) extensions one of families of geodesics which are incomplete in the Taub region becomes complete, whereas the other family of geodesics remains incomplete (a behaviour analogous to "leftward" and "rightward" geodesics in Misner spacetime). Some of these are isometric: $M^{\downarrow+}$ is isometric with $M^{\downarrow-}$, and $M^{\uparrow+}$ is isometric with $M^{\uparrow-}$. But one can "glue together" pairs of these extensions, one to the past together with one to the future. "Gluing together" means taking a quotient by an appropriate equivalence relation. One obtains four Taub-NUT spacetimes, $M^{++} = M^{\uparrow+} \cup M^{\downarrow+}/\approx$, $M^{+-} = M^{\uparrow+} \cup M^{\downarrow-}/\approx$, $M^{-+} = M^{\uparrow-} \cup M^{\downarrow+}/\approx$, and $M^{--} = M^{\uparrow-} \cup M^{\downarrow-}/\approx$. Now, how to produce non-isometric extensions of Taub region? It turns out that M^{++} is NOT isometric to M^{+-}, even though M^{++} is isometric to M^{--} and M^{+-} is isometric to M^{-+}. These non-isometric extensions are produced out of isometric ones. But there is no trick involved. Remember that there are two families of null geodesics: in each extension one is complete whereas the other is not. Isometry "exchanges" these two types—this can be done when we have only one extension, but cannot be done (in a smooth way) when we have more of them. So non-isometric extensions of Taub-NUT have to be what I called earlier indeterminism "in both ways".

Misner and Taub-NUT spacetimes have been called counterexamples to almost anything. Are there any other example of maximal globally hyperbolic spacetimes with maximal non-equivalent extensions? Yes. These have been found in (certain) polarized Gowdy spacetimes. These are maximally globally hyperbolic, expanding, not homogenous vacuum spacetimes with two orthogonal Killing vector fields, which (historically) has been useful as simplest toy models of gravitational waves. There are four types of polarized Gowdy spacetimes, classified by topology of the spatial slice: S^3, T^3, $T^2 \times T^1$, and Lens spaces. Depending on the topology of the polarized Gowdy spacetime, it may or may not be extendible. The extendible ones have $M \approx \Re^+ \times T^3$, with a T^2 spatially acting isometry group. Spacetime metric $g_{ab} = e^{-2U(t,\Theta)}[e^{-2A(t,\Theta)}(-dt^2 + d\Theta^2) + t^2dy^2] + e^{2U(t,\Theta)}dx^2$. These spacetimes are generated by the set of triples $\langle \Pi(\Theta,t), \omega(\Theta,t), \alpha \rangle$, and the behavior near $t = 0$ is governed by $\Pi(\Theta)$. Polarized Gowdy spacetime can be extended iff there is a non-empty interval $I = (\Theta_1, \Theta_2)$ s.t. $\forall\ \Theta \in I \quad \Pi(\Theta) = 0$ (or $\forall\ \Theta \in I \quad \Pi(\Theta) = 1$). If spacetime is generated by Π such that $\forall\ \Theta \in (0, 2\pi)\ \Pi(\Theta) = 0$ (or ($\forall\ \Theta \in (0, 2\pi)\ \Pi(\Theta) = 1$) can be slightly extended ("by ϵ") below $t = 0$. Such a spacetime has two extensions, M^+_ϵ and M^-_ϵ, which are isometric. But if the spacetime is generated by Π such that Π equals 0 (or 1) in two disjoint intervals, I_1 and I_2, then it has $4 = 2 \times 2$ extensions M^{1+}_ϵ, M^{1-}_ϵ, M^{2+}_ϵ, and M^{2-}_ϵ. Consider the following pasting (by appropriate equivalence relations): $M^{++} = \{M \cup M^{1+}_\epsilon \cup M^{2+}_\epsilon\}/_\equiv$, and $\hat{M}^{++}$—the maximal extensions of M^{++}; then $\hat{M}^{++}$ and $\hat{M}^{+-}$ are NOT isometric. Recall now that polarized Gowdy spacetimes are generated by Π. If the number of disjoint intervals on which $\Pi = 0$ or 1 increases, the number of non-isometric extensions spacetimes increases as well. This implies that one can have countably infinite number of non-isometric extensions. All these non-isometric extensions are made to the past (one can think of them as extensions through the Big Bang). Of course, if one decides that polarized Gowdy spacetime should be temporally oriented in the other way, one will have extensions to the future, but the natural interpretation of polarized Gowdy spacetimes sees them as de-idealized cosmological models, whereas change of time orientation would see them as infinite to the past, contracting spacetimes ending with Big Crunch.

Other examples of similar behaviour of MGHD have been found. Ringström (2009) shows that locally rotationally symmetric Bianchi IX initial data have two non-isometric extensions which are C^2 inextendible. He also demonstrates that for other Bianchi types MGHD is inextendible. More recently, Costa et al. (2015) show that for some Einstein-Maxwell systems with positive cosmological constant similar phenomena holds.

11.3 What Do Extendible MGHDs Teach About Classical General Relativity?

If one has extendible MGHD with non-isometric extensions, then one has found indeterminism in the sense of Butterfield's definition—under precisifications concerning choices of region **S** and β I have decided for. That is, there are two models (spacetimes $\langle M, g\rangle$ and $\langle M', g'\rangle$) which contain regions of kind **S** (maximal globally hyperbolic developments) such that there exists an isometry between the MGHDs, but there is no isomorphism β (isometry preserving temporal orientation) between $\langle M, g\rangle$ and $\langle M', g'\rangle$. Therefore classical general relativity is not MGHD-deterministic. This is interesting on its own, since it shows that even if spacetime is globally hyperbolic, there could be physical phenomena (formation of Cauchy horizons) which allow for indeterminism in the sense of the existence of solutions and the failure of uniqueness. But a foundational significance of extendible MGHDs is not limited to that. In what follows I discuss common features of these spacetimes, pressure they put on the simplistic understanding of physical equivalence in GR, whether they are physically reasonable or not, their relation to questions concerning time machines, and relations between cosmic censorship and determinism.

11.3.1 Common Features

Known cases of non-unique extensions share few common features. These spacetimes

1. are exact solutions with non-trivial symmetries,
2. are spatially compact (i.e. topology of the spacelike section is compact),
3. are vacuum solutions (i.e. right-hand side of the Einstein's field equations vanishes),

4. do not satisfy strong causality in the non-isometric regions (a topic I return to in Sect. 11.3.5),
5. are singular
6. and these which have non-unique up to time preserving isometry extensions have disconnected Cauchy horizons (i.e. the Cauchy horizon has more than one connected component: one to the past and one to the future in Taub-NUT case, multiple components separated by curvature singularities in Gowdy case)

There is no proof that any of these needs to hold. It is just a fact that all MGHDs with non-unique extensions found so far share these properties.

Costa et al. (2015) found electrovacuum solutions with positive cosmological constant which admit non-unique extensions. This shows that feature 3. is not necessary for the MGHD to have non-unique extensions. Similarly, one may expect that features 1. and 2. could be relaxed – these features, then, arise merely due to our lack of control over the set of relativistic spacetimes (in contrast with being a property that MGDHs with non-unique extensions need to satisfy).

Note, moreover, that the fourth feature holds only if some form of hole freeness condition is assumed – otherwise, one can always remove closed regions from the extension and use conformal transformations or pass to universal covering space to ensure inextendibility, while simultaneously removing closed time-like curves from the extension (graphically speaking, by cutting them into half in the process of removal of the closed regions). I expand on this theme in Sect. 11.3.3, linking this question to some difficulties encountered in investigations concerning the existence of time machines in classical general relativity.

Fifth feature raises a question about relation between spacetime singularities and indeterminism. If, as I assume here, one conceptualizes indeterminism as situation in which solutions exist but their uniqueness fails, then singular spacetime does not have to demonstrate indeterminism of the theory. In case of "singular" spacetimes, there is **no** evolution of system for some parameter; in case of extendible MGHDs, there is **more than one** evolution of the system compatible with the given initial data. But all cases of extendible MGHDs are singular in the sense of being timelike geodesically incomplete (and it is difficult to imagine that this particular feature could be relaxed, at least in case of vacuum solutions): any of the inextendible non-unique extensions of Misner, Taub-NUT, polarized Gowdy or Bianchi IX spacetime is timelike geodesically incomplete (even if one allows non-Hausdorff extensions; see Hawking and Ellis (1973, p. 174) for the argument in Misner case). But they do not have to be singular in the sense of curvature blow-up: Misner and Taub-NUT spacetimes are blow-up free.

A topologically disconnected Cauchy horizon is present in all known cases of extensions which are non-unique up to isometry which preserves temporal orientation. Going back to the classification of indeterministic solutions with respect to temporal orientation, one could conjecture that non-uniqueness of a solution requires either that the solution is "in both ways" indeterministic, or that it needs to have curvature singularity in some (but not all) regions close to the Cauchy horizon, effectively splitting it into at least two components. And one could also try to play some forms of singular behaviour against some other forms: for example, if it could be demonstrated that Cauchy horizons can be replaced with blow-up singularities (a theme explored by Misner and Taub in the context of classical instability in Misner and Taub (1969), and, in the context of semi-classical gravity, by Hiscock and Konkowski (1982) for Taub-NUT horizons and Thorne (1993) for Misner spacetime), then one type of singularity (more bening, so to speak) could be used to protect us from more malign phenomena.

11.3.2 Physical Equivalence

One often hears that diffeomorphic spacetimes are equivalent, or other statement to that effect (these are made in the context of debates concerning the Hole Argument). An interesting concern raised by existence of extendible MGHDs is that mere existence of an isometry between spacetimes may not be sufficient condition for physical equivalence. Consider following conditions:

1. Isometry preserves time orientation
2. Isometry belongs to the connected component of the identity in the diffeomorphism group (with the intuition being that only isometries which can be obtained by continuous deformation from an identity represent physically equivalent situations)
3. Isometry preserves location of a Cauchy surface[3]

These conditions are sensitive to whether $\langle M, g\rangle$ has non-isometric extensions: a spacetime can have unique extension if one uses weaker (i.e. broader) condition, but non-unique if one uses stronger (more strict) condition. For instance (Chruściel and Isenberg 1993) if one considers 2-dimensional Misner spacetime and demands that isometry is in the connected component of identity, then maximal extension is non-unique. Thus, whether a given spacetime counts as an example of indeterminism or not can be changed by demanding more fine-grained equivalence condition. Similarly, extensions to the future (or to the past) of the Taub region are isometric by a time orientation preserving isometry, but not by an isometry belonging to the identity component of the diffeomorphism group. And in all these cases extensions are not Cauchy equivalent. But is there a single correct condition for physical equivalence which should be used in the context of initial value problem? Equivalently, do these more strict conditions capture some physical structure which should be preserved by mappings which identify physically equivalent spacetimes? In particular, are there theoretical contexts in which these strict conditions are appropriate?

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[3]More precisely: two maximal extensions $\langle M_1, g_1, \Lambda_1\rangle$ and $\langle M_2, g_2, \Lambda_2\rangle$ of $\langle M, g\rangle$ are Cauchy equivalent wrt Σ, i iff $\psi \circ \Lambda_1 \circ i = \Lambda_2 \circ i$

Note also that choice of a differentiability class does not seem to influence whether spacetime has non-isometric extensions. In case of Taub-NUT and polarised Gowdy, extensions are analytic; in case of Bianchi IX extensions are C^2. This makes the extensions slightly different from Norton's dome, where differentiability conditions (singularity at the summit) play crucial role.

11.3.3 Distinguishing Physically Reasonable Spacetimes from Physically Unreasonable Ones

In the context of indeterminism of Newtonian mechanics (as demonstrated by the Norton's dome) Malament (2008) suggested that interesting question raised by Norton's dome is not a yes-or-no question whether Newtonian mechanics is indeterministic; rather, the interesting question is to whether and in what sense examples discussed by Norton can be thought of as examples of Newtonian systems. Similar question could be asked about extendible MGHDs: to what extent are they examples of physically reasonable general relativistic spacetimes?

This leads to a more general point: analysis of determinism by asking about existence and properties of extendible spacetimes is closely related to questions concerning physical possibilities in classical general relativity. Consider some Problematic Feature φ of the theory; in our case the feature is indeterminism, but in other cases it may be something else, like singular structure, observational indistinguishability, or possibility of time travel. Questions concerning Problematic Feature φ can be thought of as investigated in two stages. In the first stage, one questions whether φ is possible by looking at all spacetimes $\langle M, g \rangle$. It turns out that it is, in general, very easy to find some examples of spacetimes with property φ, for most properties which are of some philosophical interest. Thus, in the second stage, one considers only those spacetimes $\langle M, g_{ab} \rangle$ which satisfy some Nicety Condition, which distinguishes physically reasonable spacetimes from physically unreasonable ones. Whether it is still easy to find spacetime with property φ depends on the form of the additional condition.

Since there are many senses in which a spacetime can be unphysical, there is no single Nicety Condition appropriate for all contexts. For example, in cosmological context the Nice Feature often boils down to the demand of isotropy and homogeneity, whereas for some other contexts, such as possible distributions of matter, this demand would be much too strong.

Various conditions have been used as a Nicety Condition: being a solution of Einstein's equations is an obvious one; closely related are energy conditions; less obvious ones are various inextendibility and hole freeness-type conditions; and stability conditions (since stability is related to an important issue of cosmic censorship, I discuss it separately in Sect. 11.3.4). One thing is clear: known examples of extendible MGHDs are not empirically adequate as a representation of the large scale structure of the universe we live in. In this sense classical general relativity is indeterministic, even though the world as described by the theory could be deterministic.

Einstein's equations are satisfied in the case of extensions I discussed earlier. But there is a sense in which Einstein's equations are a vacuous constraint anyway: whatever the metric g_{ab} is, it can always be used to define some stress-energy-momentum tensor on the right hand side. Very often the right hand side obtained in this way will be "unphysical", for example because energy density will be negative. Energy conditions are then postulated to distinguish between stress-energy-momentum which are reasonable and those which are not. But (see Curiel 2014) the status of energy conditions seems to be highly problematic, as it is very easy to produce examples of classical and quantum scalar fields which violate any known energy condition. In addition, when a distinction between reasonable and unreasonable spacetime is drawn using an energy condition, even though it is assumed the former ones satisfy some energy condition, an explicit choice of a particular condition is very rarely made.

Another group of conditions are those which require that spacetime is "as large as it could be"—geodesic completeness, hole freeness, inextendibility, and similar. Should extendible MGHD be counted as physically possible according to these conditions? All extensions of extendible MGHDs have incomplete geodesics (under the assumption that spacetime is a smooth Hausdorff manifold; see Müller and Placek (2015) for a dissenting view). But most solutions of GR have incomplete geodesics, due to singularity theorems. Whether hole freeness is satisfied or not depends on the details of the definition. But since most available definitions are variations on the theme of the domain of dependence being as large as possible, and since one always has MGHD realized in the case of spacetimes I described above, these hole freeness conditions cannot be used to dismiss indeterminism present in case of extendible MGHDs as physically unreasonable. Finally, inextendiblity condition actually motivates going beyond MGHD (since in these cases MGHD is as large as it could be, but the spacetime as a whole is not as large as it could be). Insofar as inextendibility is motivated by something like a principle of sufficient reason, this principle forces us to leave the calm waters of MGHD and accept wild indeterminism of non-unique regions.

Of course, any non-unique, inextendible non-globally hyperbolic extensions of maximally globally hyperbolic spacetime violates some causality condition (for instance, if they are non-unique, they cannot be globally hyperbolic). But dismissing these extensions on the grounds of violating causality condition *by fiat* seems question-begging: by doing so, one would effectively assume that physically reasonable spacetimes need to be deterministic.

And if one would hold the view[4] that a spacetime is physically reasonable iff it could be used to model actual physical phenomena, then—since Gowdy spacetimes have been developed as toy models for studying gravitational waves—some extendible MGHDs would count as physically reasonable according to such a criteria.

[4]I am not aware of anyone actually subscribing to this in writing.

To wrap up: from most commonly used conditions for distinguishing physically reasonable spacetimes from unreasonable ones, all those which are not question-begging are satisfied by non-isometric extensions of MGHDs. Thus, there is no easy dismissal of the form of indeterminism brought by extendible MGHDs as physically unrealistic.

11.3.4 Rarity, Stability, Cosmic Censorship and Determinism of General Relativity

Cosmic censorship (Penrose 1979) is the statement that physically reasonable spacetime is globally hyperbolic. And if, as Penrose suggest, spacetime is physically unreasonable iff it is unstable against some time of perturbations, there are prospects for a rigorous proof of the cosmic censorship. More recent statement takes the following form: generic initial data have inextendible MGHDs. This version is more precise, but still not precise enough, since there are multiple ways of specifying what is generic. Ringström (2010) attributes this statement of cosmic censorship to Chrusciel, hence I will call it Chrusciel's Strong Cosmic Censorship, CSCC (even though similar statements appear in literature before, for example in Moncrief 1981).

In a similar vein, Hawking (1971) argued that only properties stable against some type of perturbations are physically relevant. But being generic depends on topology one chooses, and Fletcher (2015) shows that there are different choices of topologies available, each coming with its own shortcomings. So if one takes the property of "having MGHD extendible in non-unique way", identifies being generic with being stable against perturbations, and finds a physical justification for the choice of the particular topology (all of which are non-trivial conceptual choices), and in that topology statement expressing CSCC holds (see Ringström (2010) for a summary of known results), then spacetimes I have been discussing are not physically reasonable witnesses of indeterminism. Note that known theorems do not assert a typicality result in a given measure, but mere a typicality in given topology.

There seem to be few types of rarity present in extendible MGHDs. First, these spacetimes have non-trivial global symmetries, and it is widely accepted that highly symmetric spacetimes are non-generic. Second, in case of Misner spacetime a peculiar geometric setup (with contraction along the spacelike axis) is present, which presumably could be used to argue that this spacetime is non-generic. Third, initial data whose MGHDs are extendible polarized Gowdy spacetimes need to contain analytic functions Π which take the value 0 (or 1) on two or more disjoint intervals, which are topologically atypical. Under these particular understandings of "generic", CSCC holds for known extendible MGHDs.

Lets now go back to the notion of Laplacian determinism introduced at the beginning. This notion does not care about size or relative placement of indeterministic solutions in the space of all solutions. Indeterminism is merely an existence statement: if there are at least two solutions which have isometric region **S** but are not isometric overall, the theory is indeterministic. CSCC, on the other hand, does not care about existence statements like this, unless they are being followed by a typicality statement. This leads to a disconnect: even though the examples I discussed show that GR is indeterministic in the sense of Butterfield's definition, they are compatible with CSCC. Since Laplacian determinism fails and CSCC holds, CSCC cannot express Laplacian determinism.

Is there, then, some other notion of determinism which CSCC expressess? One could try to understand it as expressing a form of well-posedness in the Hadamard sense. Well-posedness in the Hadamard requires that solutions exist, are unique, and moreover that solutions continuously depend on the initial data, meaning that small changes to the initial data result in small overall changes in the solution. Note, however, that here uniqueness fails in "rare" cases, and so well-posedness expressed by CSCC must be somewhat non-standard. It remains to be seen whether there is a robust, conceptually well-motivated notion of determinism which would find formal expression in such a non-standard notion of well-posedness. Certainly notion of determinism discussed in the philosophy of science literature has nothing of it.

So far, I have focused on vacuum solutions with cosmological constant Λ set to zero. What happens if Λ is non-zero? Earman (1995)(p. 82) noted that "it remains to be seen whether or not, on balance, the cosmological constant helps or hinders the quest for cosmic censorship". I will that the tentative answer is: it hinders the quest. In an interesting recent development, Costa et al. (2015) argue that positive cosmological constant makes things worse, in the sense that they are able to find spherically symmetric characteristic initial data for the Einstein-Maxwell-scalar field, with positive Λ, such that there are non-isometric and generic C^1 extensions through the horizon of the Reissner-Nordström black hole. Since negative cosmological constant often makes CSCC inapplicable (due to non-existence of MGHD in spacetimes with asymptotically anti-de Sitter behaviour at infinity), on balance it seems that cosmological constant leads to more indeterminism (although, in the negative case, it is not the Laplacian determinism in the sense I explicated in Sect. 11.1.1.

11.3.5 Connections Between Some Forms of Indeterminism and Existence of Time Machines

At this point careful reader may ask the following: aren't all known extendible spacetimes instances of Thornian time machines, that is, spacetimes which do not contain closed time-like curves, but in which the arrangement of matter and geometric features brings about closed time-like curves?[5,6] And if so, is it not

[5]Similar questions could be asked about Malament-Hogarth spacetimes.

[6]For difference between Thornian and Wellsian time machines, see Earman et al. (2016).

the case that question of determinism is subsumed by question whether time machines are genuine physical possibilities?

There are various ways of making the notion of a time machine precise (see Earman et al. (2009) for discussion of these issues). A precisification suggested by Earman, Smenk and Wüthrich (and often called ESW time machines in subsequented literature, cf. Manchak (2009) or Manchak (2014)) demands, intuitively, that region in which time machine operates is free from closed time-like curves (and MGHD, of course, is CTC-free), and every extension to the future of that region contains closed time-like curves.

Extensions of Misner and polarized Gowdy spacetime with reversed temporal orientation are extensions of the sort one is interested when investigating existence of ESW time machines. But extensions of Taub-NUT spacetime (which are "in both ways") and extensions of polarized Gowdy spacetime with natural temporal orientation (which are made to the past) resemble a reversed time machine: there are CTCs before the Big Bang, but once an observers passes through the horizon to MGHD, there is no way back to the region which allows time travel.

Moreover, since one cannot presently exclude existence of extendible MGHD such that all of the non-isometric extensions satisfy strong causality condition (which prevents CTCs), it does not follow that every spacetime admitting non-isometric extensions to the future is an example of spacetime allowing for operation of a time machine. Additionally, one can perform various mathematical operations in the non-globally hyperbolic region: for instance, remove various subsets of the manifold in such a way that there will be no CTCs in the resulting spacetime. If such extensions are allowed, the number of non-isometric extensions available increases dramatically. Resulting spacetime will have unique MGHD with various extensions: some with CTCs, some without. Most of such extensions would, intuitively speaking, have artificial "holes"; but, again, it is rather difficult to find a formal hole freeness condition which gives correct verdict in these cases.

11.3.6 Open Question

All of this leads to an interesting open question. Is there any spacetime which is maximal globally hyperbolic, satisfies some energy condition and admits at least two non-isometric extensions which are (a) hole free (in a yet to be made precise sense, if that is possible), (b) not Malament-Hogarth, (c) satisfy at least strong causality, (d) extensions are to the future only, (e) Cauchy horizon has single connected component, (f) satisfies some energy condition? Is there a robust or stable (in some well-defined sense) class of such spacetimes?

Of course, even if such a robust class of spacetimes were found, it would not mean that our world is indeterministic. Classical general relativity is often conceptualized as having "sectors" (that is, space of solutions is divided into various subsets, where spacetimes lying in a given subset share some relevant features – for example, symmetry group, form of the stress-energy-momentum tensor, or a value of the cosmological constant), and maybe the most useful representations of our world would be found in the deterministic sector. So finding a class of extendible spacetimes satisfying above constraints which could serve as a useful model for the universe we actually seem to live in would be the ultimate triumph of indeterminism in classical GR.

11.4 Summary: Laplacian Indeterminism in Classical General Relativity

Examples of extendible MGHDs can be interpreted in two ways:

(a) classical GR is indeterministic
(b) classical GR is deterministic: extendible MGHD are not physically relevant

I have argued that option (b) is untenable: there is no question-begging condition for being physically reasonable spacetime which is violated by extendible MGHDs; some of these spacetimes (polarized Gowdy solutions) have been used as useful toy models, and we have no reason to expect that phenomena of extendible MGHD with non-unique extension is limited to toy models only; and foundational significance of stability condition (not to mention difficulties with finding a formal expression of such a condition) is murky at best.

Since option (b) seems untenable, one is left with indeterminism of the theory (again, in the sense of having some indeterministic models). This assessment agrees with sentiments (sometimes) expressed both by philosophers – Belot (2011):

> Instances in which globally hyperbolic solutions admit non-isometric extensions are instances of genuine indeterminism, not gauge equivalence

and physicists – Ringström (2009):

> The fact that there are inequivalent maximal extensions [of maximal globally hyperbolic developments] means that the initial data do not uniquely determine a maximal development. In this sense, the general theory of relativity is not deterministic.

or Ringström (2010), where he comments that extendible MGHDs:

> demonstrate that Einstein's general theory of relativity is not deterministic; given initial data, there is not necessarily a unique corresponding universe

(and goes on to discuss the consolation brought by rarity of such spacetimes, i.e. Chrusciel's version of the cosmic censorship).

I have argued that it is not straightforward to understand the strong cosmic censorship in Chrusciel's sense as expressing determinism. But if it does not express determinism, what does it say?

I suggest that it merely expresses the hypothesis that GR is not radically indeterministic: there may be solutions which are indeterministic, but there are few and far between. Similar theme can be found in Geroch (1970) metaphor of black and white patches on the paper (where paper represents the set of all spacetimes, black patches are singular spacetimes and white patches are spacetimes which are no singular; the question, then, is whether from a large distance paper looks rather white, black, or kind of greyish; and Penrose and Hawking's singularity theorems are taken to imply that it is at least dark greyish). I propose that existence of extendible MGHDs be interpreted in the very same way; in particular, cosmic censorship in Chrusciel's formulation does not express determinism of classical GR, but rather a well-formed mathematical hypothesis that classical general relativity (under auxiliary assumptions, such as lack of holes and restriction to spacetimes which are amenable to initial value formulation at all) is not radically indeterministic. Using Geroch's metaphor concerning patches of paper, cosmic censorship hypothesis states that extendible MGHDs are rare enough that the set of solutions is light grey or white when looked at from the distance. Lack of radical indeterminism may be the next best thing, but is not the same as determinism *simpliciter*.

One could object that study of philosophical consequences of extendible MGHDs is a waste of time: after all, even if they are indeterministic models of the theory, they are far from being a most fitting to the data representation of the observable universe. So why bother? I would say that there are at least two reasons. First, that we are in philosophically interesting situation in which the observable universe is best represented as a deterministic model of an otherwise indeterministic theory. Second, that we can and do learn a lot about the theory by studying its various solutions, and extendible MGHDs are useful for that purpose; in particular, they demonstrate subtle ways in which various forms of indeterminism and singular behaviour of spacetime can mingle with each other.

Acknowledgements The author acknowledges the funding of the research grant 'Mistrz 2011' of the Foundation for Polish Science, contract nr 5/2011 for financial support. The work on this paper was partically carried out as a part of the "Probability, Causality and Determinism" Bilateral Mobility Grant of the Hungarian and Polish Academies of Sciences, NM-104/2014. I would also like to thank Dennis Dieks, Erik Curiel, Sam Fletcher, F. A. Muller, Christian Wüthrich, and the audience at BSPS 2014 in Cambridge for helpful feedback. Most of this material has been developed through numerous conversations with Tomasz Placek, to whom I am very grateful.

References

Belot, G. 2011. Background-independence. *General Relativity and Gravitation* 43: 2865–2884.

Butterfield, J. 1989. The hole truth. *British Journal for the Philosophy of Science* 40(1): 1–28.

Choquet-Bruhat, Y., and R. Geroch. 1969. Global aspects of the cauchy problem in general relativity. *Communications in Mathematical Physics* 14(4): 329–335.

Chruściel, P.T., and J. Isenberg. 1993. Nonisometric vacuum extensions of vacuum maximal globally hyperbolic spacetimes. *Physical Review D* 48(4): 1616.

Costa, J.L., P.M. Girão, J. Natário, and J.D. Silva. 2015. On the global uniqueness for the Einstein–Maxwell-scalar field system with a cosmological constant. *Communications in Mathematical Physics* 339(3): 903–947.

Curiel, E. 1998. The analysis of singular spacetimes. *Philosophy of Science* 66: S119–S145.

Curiel, E. 2014. A primer on energy conditions. In *Towards a theory of spacetime theories*, 43–104. Springer. arXiv preprint:1405.0403.

Earman, J. 1986. *A primer on determinism*, vol. 37. Dordrecht: D. Reidel.
Earman, J. 1995. *Bangs, crunches, whimpers, and shrieks: Singularities and acausalities in relativistic spacetimes*. New York: Oxford University Press.
Earman, J. 2007. Aspects of determinism in modern physics. *The Philosophy of Physics* 2: 1369–1434.
Earman, J., C. Smeenk, and C. Wüthrich 2009. Do the laws of physics forbid the operation of time machines? *Synthese* 169(1): 91–124.
Earman, J., C. Wüthrich, and J. Manchak. 2016. Time machines. In *The Stanford encyclopedia of philosophy* (Winter 2016 ed.), ed. E.N. Zalta. Stanford: Metaphysics Research Lab, Stanford University.
Fletcher, S.C. 2015. Similarity, topology, and physical significance in relativity theory. *The British Journal for the Philosophy of Science* 67(2): 365–389.
Geroch, R. 1970. Singularities. In *Relativity*, 259–291. New York: Plenum.
Hawking, S., and G. Ellis. 1973. *The large scale structure of space-time*. Cambridge monographs on mathematical physics. Cambridge: Cambridge University Press.
Hawking, S.W. 1971. Stable and generic properties in general relativity. *General Relativity and Gravitation* 1(4): 393–400.
Hiscock, W.A., and D.A. Konkowski. 1982. Quantum vacuum energy in taub-nut (Newman-Unti-Tamburino)-type cosmologies. *Physical Review D* 26: 1225–1230.
Krasnikov, S. 2009. Even the Minkowski space is holed. *Physical Review D* 79(12): 124041.
Kutach, D. 2013. *Causation and its basis in fundamental physics*. Oxford: Oxford University Press.
Malament, D. 2012. *Topics in the foundations of general relativity and Newtonian gravitation theory*. Chicago lectures in physics. Chicago: University of Chicago Press.
Malament, D.B. 2008. Norton's slippery slope. *Philosophy of Science* 75(5): 799–816.
Manchak, J.B. 2009. On the existence of time machines in general relativity. *Philosophy of Science* 76(5): 1020–1026.
Manchak, J.B. 2014. Time (hole?) machines. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 48: 124–127.
Manchak, J.B. 2015. On Gödel and the ideality of time. *Philosophy of Science* 83(5): 1050–1058. Chicago: University of Chicago Press.
Manchak, J.B. 2016a. Epistemic "holes" in space-time. *Philosophy of Science* 83(2): 265–276.
Manchak, J.B. 2016b. Is the universe as large as it can be? *Erkenntnis* 81(6): 1341–1344. Springer.
Misner, C., and A. Taub. 1969. A singularity-free empty universe. *Soviet Physics – JETP* 28: 122.
Moncrief, V. 1981. Infinite-dimensional family of vacuum cosmological models with taub-nut (Newman-Unti-Tamburino)-type extensions. *Physical Review D* 23(2): 312.
Müller, T., and T. Placek. 2015. Defining determinism. *The British Journal for the Philosophy of Science*: axv049. The British Society for the Philosophy of Science.
Penrose, R. 1979. Singularities and time-asymmetry. In *General relativity: An Einstein centenary survey*, 581–638.
Ringström, H. 2009. *The Cauchy problem in general relativity*. Zürich: European Mathematical Society.
Ringström, H. 2010. Cosmic censorship for Gowdy spacetimes. *Living Reviews in Relativity* 13(1): 2.
Thorne, K.S. 1993. Misner space as a prototype for almost any pathology. In *Directions in general relativity: Papers in honor of Charles Misner*, vol. 1. Cambridge University Press.
Werndl, C. 2016. Determinism and indeterminism. In *The Oxford handbook of philosophy of science*, ed. P. Humphreys. Oxford University Press.
Wüthrich, C. 2011. Can the world be shown to be indeterministic after all? In: *Probabilities in physics*, 365–390. Oxford: Oxford University Press.