States vs. changes of states: a reformulation of the ontic vs. epistemic distinction in quantum mechanics

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Abstract

In this paper I challenge the distinction between "epistemic" and "ontic" states that was propounded by Harrigan and Spekkens (2010) by pointing out that because knowledge is factive, any state that represents someone's knowledge about a physical system thereby also represents something about the physical system itself, so there is no such thing as "mere knowledge". This criticism leads to the reformulation of the main question of the debate: instead of asking whether a given state is ontic or epistemic, we should instead ask whether a given change of a state is ontic or epistemic. In particular, in the context of quantum mechanics, one can ask whether the collapse of the quantum state could be understood as an epistemically successful change of the observer's beliefs about the complete state of the system that is not associated with any change in the physical reality. I argue that the answer to this question should be in the negative. This is because it is possible that in a series of measurements the collapse rule tells us to update a certain state to a different one and then back to the same state; if both of these updates are merely changes of our beliefs, then they could not both be epistemically successful.

1 Introduction

Quantum mechanics is famously difficult to interpret, mostly because in its standard version it involves two incompatible rules governing the change of quantum states: the Schrödinger evolution and the quantum state collapse (which is used only in special circumstances, namely, just after the "measurement" has been made). This has led some thinkers to the idea that at least some aspects of the formalism of quantum mechanics should be interpreted epistemically.

A contribution to this discussion was made by Harrigan and Spekkens (2010; henceforth HS), who distinguish between an "ontic" and an "epistemic" understanding of quantum states. The fact that their definitions are formulated in purely mathematical terms enabled the proof of theorems concerning this distinction. For example, Pusey, Barrett and

Rudolph (2012; henceforth PBR) have shown that (given some additional assumptions) quantum states cannot be "epistemic" in the sense of HS. If the additional assumptions they made are tenable, and the definitions of "ontic" and "epistemic" are adequate, then this result is a strong argument in favour of interpreting quantum states as representing something physically real.

However, the adequacy of HS's framework has been questioned (e.g., Oldofredi and López 2020, Hance, Rarity and Ladyman 2021). In this paper, I offer a critical assessment of the conceptual side of this debate, giving novel arguments for the philosophical inappropriateness of HS's terminology. I also propose a reformulation of the central question of the debate and provide a new argument in favour of one of the answers to the reformulated question. Before I explain this in more detail, I need to introduce some terminology.

By the "physical state of the system" I mean the state that the system is objectively in (at some given time). If a set of states is considered in an abstract way, without any reference to what it represents, its elements will be called "theoretical states". Quantum states, which are the subject of this paper, are an example of theoretical states. The crucial definition is as follows:

Definition 1 (Ontic and epistemic states)**.** *If a given theoretical state can represent a (possible) physical state of a system (perhaps in an incomplete way), then it is called an ontic state. If a given theoretical state can represent the (possible) beliefs of some observer about the physical state of a system, it is called an epistemic state.*[1]

Notice that these definitions do not exclude that the same theoretical state can be both ontic and epistemic: it is conceivable that the same theoretical state can represent a physical state of the system and the observer's knowledge about this state (cf. section 3.3 and footnote 14).

After reviewing HS's definitions of "ontic" and "epistemic"[2] states (section 2), I will spend some time introducing the epistemological concepts that are crucial for the debate (sections 3.1–3.2). Then, I will argue (sections 3.3–3.4) that if a theoretical state represents someone's knowledge about the system (and as such it is epistemic), it is thereby also ontic, that is, it also represents something about physical reality. This is because knowledge is standardly understood as factive, that is, as entailing that what is known actually holds. Therefore, the distinction made by HS cannot be conceptually adequate. Some other possible readings of this distinction are also considered (sections 3.5–3.6), with the conclusion that they are either inconsistent with some of HS's assumptions or also inadequate. I will propose that the question of whether a given state is ontic or

[1]Strictly speaking, the term "doxastic" would be more adequate than "epistemic", as we can also consider false beliefs. However, I will use the term "epistemic" to remain closer to the original terminology of the debate.

[2]To avoid any confusion, from now on I will use quotation marks whenever I mean HS's formal sense of ontic vs. epistemic (defined in terms of non-overlapping vs. overlapping probability distributions, see Def. 2) and no quotation marks whenever I mean the philosophical sense of ontic vs. epistemic (i.e., representing physical reality vs. representing beliefs, see Def. 1). The former is intended by HS to coincide with the latter, but this is what I will question, so they need to be keep separate.

epistemic should be replaced by a different one, namely, whether a given *change* of a state is ontic or epistemic, that is, whether it is a change in the physical reality or merely in our beliefs about it (section 3.7). Then, I will present a novel argument that a particular kind of change of quantum states, namely, the measurement-induced collapse, cannot be interpreted as an epistemically successful change of beliefs about the complete state of the system (section 4). In section 5, I will consider the options that remain for the defenders of the epistemic view on quantum collapse. In section 6, I will extend my analysis to some variants of quantum mechanics that do without the concept of measurement-induced collapse. Finally, section 7 will provide a short summary.

2 The standard definition of "ontic" and "epistemic" states

The distinction between "ontic" and "epistemic" states was introduced by HS (2010) within the framework of ontological models.[3] They start with an operational formulation of quantum mechanics, the primitive terms of which are preparations (denoted by P) and measurement procedures (denoted by M). Quantum states (denoted by ψ or ϕ, possibly with labels) are assumed to be in one-to-one correspondence with preparation procedures. The operational formulation of a theory gives us the probabilities of the outcomes of different measurements given different preparations (i.e., probabilities $Pr(k|M,P)$ that the measurement M will give the outcome k for the preparation P). Such operational formulation of quantum mechanics can be associated[4] with its ontological model, which postulates the set of complete[5] states the system might be in (it is denoted by Λ and its elements by λ). Complete states, by definition, capture all the information about the system (all its properties at a given time). With each preparation procedure (and, therefore, with each quantum state) there is associated a probability distribution $Pr(\lambda|P)$ over Λ, which determines what is the probability that the system created by means of the preparation P is in the state λ. The ontological model needs to agree with the predictions of quantum mechanics in the sense that $\int d\lambda\ Pr(k|M,\lambda)\ Pr(\lambda|P)$ should recover the values given by the Born rule.

Given this framework, HS claim that certain quantum states understood as probability distributions over the set of complete states Λ represent the physical reality, whereas others represent "an observer's knowledge of reality rather than reality itself" (2010:126). The former are characterized by having non-overlapping supports, whereas the supports of the

[3]Formally speaking, ontological models are the same as hidden variable models, which is a more popular term. However, the authors avoid it, because they want their framework to encompass the option that quantum states (which are not hidden) are already complete (cf. HS 2010:129, footnote 5).

[4]"Associated" in the sense that the ontological model represents the underlying ontology that gives rise to possible experimental results accounted for by the operational formulation.

[5]HS call these complete states "the ontic states". However, I will avoid this identification, as I will call "ontic" any state belonging to what they call "ψ-ontic models".

latter may overlap. Their full classification of types of ontological models is as follows (2010:129–134):

- An ontological model is ψ-complete if quantum states are complete states (see HS 2010:131 for technical details).
- An ontological model is ψ-ontic if for any pair of preparation procedures, P_ψ and P_ϕ, associated with distinct quantum states ψ and ϕ, we have $Pr(\lambda|P_\psi)\ Pr(\lambda|P_\phi) = 0$ for all λ.
- An ontological model is ψ-epistemic if it is not ψ-ontic.

An important feature of HS's proposal is that a state does not need to be complete in order to be "ontic". Concerning terminology, HS prefer to attribute the property of being "ontic" or "epistemic" to models, whereas I attribute this property to states, but these two ways of speaking are equivalent and easily inter-translatable. In ψ-complete models quantum states are complete states, in ψ-ontic models quantum states are "ontic" states and in ψ-epistemic models quantum states are "epistemic" states; quantum states considered in abstraction from what they represent are in my nomenclature called theoretical states. Therefore, I will use the following definition expressing HS's distinction:

Definition 2 ("Ontic" and "epistemic" states)**.** *Consider a set of theoretical states which are probability distributions over a certain state space (whose elements are interpreted as possible complete states of some physical systems). If a given theoretical state is such that its support does not overlap with the support of any other theoretical state in this set, it is called an "ontic" state; if its support does overlap with the support of some other theoretical state in this set, then it is called an "epistemic" state.*

Our question is whether Def. 2 captures the same distinction as Def. 1.

3 Criticism of the standard definitions of "ontic" and "epistemic" states

In this section, I will argue that the definitions of ψ-ontic and ψ-epistemic models coined by HS are conceptually inadequate.[6] I will begin by carefully introducing the epistemological concepts involved in the debate (sec-

[6] I know of two other papers that have similar aims: Oldofredi and López (2020) and Hance, Rarity and Ladyman (2021). Let me shortly discuss the differences between my approach and their approaches.

Oldofredi and López (2020) make two objections. First, they claim that complete states can be thought of as attributed to individual systems as well as to ensembles of individual systems, whereas HS take into account only the first of these options. Second, they point out that in some interpretations of quantum mechanics complete states are perspectival or relational, which again is not taken into account by HS. My criticism of HS's terminology is tangential to that of Oldofredi and López and goes much deeper than theirs—they think that *if* complete states are understood as attributed to individual systems and are regarded as intrinsic (i.e., neither perspectival nor relational), then HS's definitions are adequate, which is what I disagree with.

Hance, Rarity and Ladyman (2021) make claims seemingly similar to mine, namely, that "Harrigan's and Spekkens's terms, ψ-ontic and ψ-epistemic, do not formalise the informal tions 3.1–3.2). Building on this, I will argue (sections 3.3–3.6) that "epistemic" and "ontic" states in the sense of HS (see Def. 2) can be both epistemic and ontic in the sense of representing both someone's beliefs and the physical reality (see Def. 1) at the same time. Finally, I will propose the reformulation of the debate in terms of changes of states rather than states themselves (section 3.7).

3.1 A short primer on some epistemological concepts

The aim of this subsection is to introduce certain epistemological concepts that will be used in the argumentation. The presentation will be much more detailed than in any paper I am aware of which is devoted to the quantum ontic vs. epistemic debate, but at the same time it will be very sketchy compared to what is available in the epistemological literature. I will not be assuming any particular epistemological theory and instead rely solely on theses that most contemporary epistemologists would agree with. The following common notation will be used: S will denote an epistemic subject (who may be also called "agent" or "observer") and p will denote a proposition towards which S can have various attitudes (called "propositional attitudes").

First, epistemologists distinguish between two families of propositional attitudes: full beliefs and partial beliefs; the latter are also called credences or degrees of belief (see, e.g., Jackson 2020, Genin and Huber 2021). Full beliefs are an all-or-nothing matter. For any p, S can believe p, disbelieve p or suspend judgement with respect to p; the fact that beliefs can come in various strengths is not taken into account here. In contrast, partial beliefs are graded and are often modelled by real numbers between 0 and 1. These numbers capture the strength of the subject's beliefs. If S believes that p to a degree d_S and S' believes that p to a degree $d_{S'}$ such that $d_{S'} > d_S$, then S' believes p stronger than S does.

Second, beliefs can be true or false. What this means precisely is a matter of debate, but here we do not need to delve into details. We will only assume that whether a belief is true or false depends on what the world is like (which is in accordance with the classical conception of truth). This is surely a controversial assumption in philosophy, but it seems to be implicit in the discussion this paper contributes to, so it is

ideas correctly" and that "models can be simultaneously ontic and epistemic". The latter thesis can be understood in two different ways. The first is that a state is "simultaneously ontic and epistemic" if some part or aspect of it represents the reality and another part or aspect represents the observer's knowledge. This seems to be the authors' intended reading. Therefore, they seem to agree with the common assumption of the debate that something can represent mere knowledge without representing reality, which is my main target of criticism. The second reading is that a state taken *as a whole* can represent reality and the observer's knowledge about reality at the same time. This is the reading I am sympathetic towards; I want to even strengthen this thesis, by saying that theoretical states that represent someone's knowledge not only can, but even *must* represent the physical reality as well.

Also Schlosshauer and Fine (2012) distance themselves from HS's terminology, as they rename "ψ-epistemic"/"ψ-ontic" models to "mixed"/"segregated" models (which they find to be "less charged" terms). However, their interest in PBR theorem is mathematical rather than interpretational, so they do not offer any assessment of HS's nomenclature.

not problematic in this context.[7] S's belief that p is true if it is the case that p and false otherwise. In particular, S's belief that a system Q is in a state λ_0 is true if the system Q is indeed in the state λ_0 and false otherwise.

Third, partial beliefs, as they are usually conceived, are closely related to probabilities. There are arguments that partial beliefs of a rational agent should satisfy the axioms of probability (see Genin and Huber 2021, section 3.1.3 and references therein). This connection can be used in at least two ways. On the one hand, if I want my partial beliefs to be rational, I should better ensure that they satisfy the axioms of probability—so this connection gives me a constraint that my partial beliefs should satisfy. On the other hand, given some probabilities whose nature is unknown, this connection opens the possibility of interpreting such probabilities as representing partial beliefs of some subject (instead of representing, e.g., relative frequencies or propensities). It is the latter way of exploiting the connection between partial beliefs and probabilities that will be of our interest here. If quantum states are probability distributions over Λ, then they could be understood as representing one's partial beliefs about the system being in one of the states belonging to Λ.

Fourth, various analyses of knowledge have been proposed in epistemology (see, e.g., Ichikawa and Steup 2018), but fortunately the point I am going to make depends only on the assumptions that most of these analyses share. Knowledge is usually regarded as bearing a close relation to beliefs on the one hand and truth on the other. "S knows p" means that S believes that p, p is true and some further conditions are satisfied. Contemporary epistemologists have hotly debated what these further conditions should be (e.g., justification, causal connection to the fact that makes p true, safety...), but they usually accept the first two conditions. There are exceptions to this rule, but they do not seem to be relevant in our context, so I will not discuss them here for the sake of brevity.

This way of thinking about knowledge has the following consequences. If we claim that S knows that the system Q is in the state λ_0, then this presupposes that S believes that the system Q is in the state λ_0 as

[7] One can object that for a proponent of the epistemic nature of quantum states it would be natural to use some non-classical, epistemic concept of truth (e.g., identifying truth with rational acceptability). However, we do not discuss here all possible positions that treat quantum states as epistemic, but only those that can be expressed in HS's framework of ontological models. It seems that the most popular positions that assume the epistemic nature of quantum states reject the existence of complete states altogether (cf. option (1) in section 5). However, they are just outside of the scope of this paper (and outside of the scope of the papers of HS 2010 and PBR 2012). The framework of ontological models presupposes a realistic view on physical theories because complete theoretical states are supposed to represent what the system is really like, which is in line with the classical concept of truth. What is more, choosing some epistemic concept of truth would deprive the concept of ontic states of the significance it was supposed to have, as under an epistemic view on truth, from the truth of the statement that a system is in such-and-such ontic state, nothing follows about what this system is really like—that would be a very weak notion of onticity. Of course, one can just start from accepting some non-classical concept of truth and assuming the epistemic nature of all states; however, in HS's approach, both types of states (i.e., "epistemic" and "ontic") need to be conceivable and definable in a single framework.

well as that the system Q is indeed in the state λ_0 (we have substituted "the system Q is in the state λ_0" for p in the general analysis of the previous paragraph). The latter feature of the concept of knowledge is called "factivity". If some p is known, then p must be true, that is, it must be the fact that p (this is where the name "factivity" comes from). In other words, saying that "S knows that p but p is false" or "S knows that p but p is not really the case" amounts to a contradiction.

Fifth, the relation between full and partial beliefs is a complicated issue and epistemologists do not agree how exactly it should be approached. One natural idea is to set a certain threshold such that if S has a partial belief that p with the value equal to or greater than this threshold, then S also believes[8] p in the sense of having a full belief that p. However, it is not clear, for example, what the value of this threshold should be and whether it should be the same in all contexts.

3.2 Full beliefs, partial beliefs and quantum states

The importance of the problem of the relation between full beliefs and partial beliefs for our considerations comes from the fact that we want to analyse what it means that quantum states represent knowledge, but knowledge is usually defined as a *full* belief satisfying certain constraints, whereas quantum states, being probability distributions, can only represent *partial* beliefs, not full beliefs. Therefore, it is not clear how exactly the claim that a quantum state represents knowledge should be understood (and, more generally, what could it mean to attribute the status of knowledge to partial beliefs). One minimal reading of such claims, which I think captures adequately the intentions of HS and their followers, is that a quantum state represents S's knowledge iff it represents S's partial beliefs about the state of some physical system and assigns a non-zero probability to the complete state the system is actually in (and some further conditions necessary for knowledge are satisfied).

This can be related to the standard way of understanding knowledge in the following way. Assume that Λ is the set of all states the system Q might be in and that S knows that this is the case, so that S can ascribe to the system Q only states belonging to Λ. Then, the following principle (which may be called the Full Belief-Partial Belief Link) holds:

> If $\Lambda_0 \subseteq \Lambda$ is the largest subset of Λ such that for any $\lambda \in \Lambda_0$, S's degree of belief that the system Q is actually in the state λ is non-zero, then S believes (in the sense of having a full belief) that Q is in one of the states belonging to Λ_0 and is not in any state belonging to $\Lambda \setminus \Lambda_0$.

S's partial beliefs are knowledge (in the minimal sense we are interested in here) iff the corresponding full belief given by the above Full Belief-Partial Belief Link is knowledge in the standard sense.

[8] The connection between partial beliefs and full beliefs can be understood either as descriptive (if S has certain partial beliefs, then this means that S also *has* certain full beliefs) or as normative (if S has certain partial beliefs, then S *should* also have certain full beliefs). This difference will not matter for our discussion, as we are here considering rational agents only, so we assume that S's full beliefs are as they should be given S's partial beliefs.

3.3 Quantum states and the factivity of knowledge

After this epistemological introduction, let us return to our main topic, namely, the interpretation of quantum states. Could they represent mere observer's knowledge? In the light of our observation that knowledge is standardly regarded as factive, it is difficult to make sense of such a claim. If a state represents an observer as knowing that p, then it thereby implicitly represents the reality as being such that p is the case. There is a complication here, arising from the fact that quantum states, if interpreted epistemically, do not represent full beliefs, but partial beliefs and knowledge is usually understood as a full belief satisfying certain additional conditions. This complication does not change the essence of my objection, but makes it technically more challenging to express.

Consider a quantum system Q, a quantum state ψ (associated with a probability distribution over Λ) and an observer S whose partial beliefs about which state Q is in are represented by ψ. The last assumption means that for any $\lambda \in \Lambda$, ψ assigns λ probability $Pr(\lambda) = p_0$ iff S's degree of belief that Q is in the state λ is p_0.[9] Denote by Λ_0 the subset of Λ that contains all and only elements of Λ to which ψ assigns non-zero probability. By our Full Belief-Partial Belief Link, in such a case S believes (in the sense of having a full belief) that Q is in one of the states belonging to Λ_0 and is not in any state belonging to $\Lambda \setminus \Lambda_0$. If this belief is false, then we cannot attribute knowledge to S. If this belief is true, then we can say that S knows that Q is in one of the states belonging to Λ_0 and is not in any state belonging to $\Lambda \setminus \Lambda_0$ (provided that some additional conditions necessary for knowledge are satisfied; I set this issue aside as irrelevant for our discussion). Our question is the following: in this assertion about S's knowledge, do we use ψ to merely represent S's knowledge or also (some part of) the physical reality? The answer is, as already observed, that we would not be allowed to assert that S knows that Q is in one of the states belonging to Λ_0 and is not in any state belonging to $\Lambda \setminus \Lambda_0$ if it was not the case that Q is in one of the states belonging to Λ_0 and is not in any state belonging to $\Lambda \setminus \Lambda_0$.[10] Therefore, in the assertion about S's knowledge we use ψ in two ways: first, explicitly, in our statement that S knows that Q is in one of the states that are in

[9] One might be suspicious about the idea that quantum states can represent our partial beliefs about the complete state of a system, given that the space of complete states Λ is unspecified here and we do not know what it is. One can represent an agent's partial beliefs as probability distributions over options that this agent is aware of, but this move becomes dubious if the options are unknown. This might be countered by saying that we are interested in possible partial beliefs of possible agents, not in partial beliefs of actual agents, and quantum states could represent partial beliefs of hypothetical agents (perhaps future scientists) who know the space of complete states. However, under this reading, the debate becomes much less relevant for the interpretation of the practice of actual scientists (who do not know the space of complete states). In the main text, I set this problem aside and just consider agents for whom it makes sense to say that quantum states represent their partial beliefs.

[10] Of course, we could be mistaken here, that is, we can make an assertion about S's knowledge because we believe wrongly that Q is in one of the states belonging to Λ_0 and is not in any state belonging to $\Lambda \setminus \Lambda_0$. In such a case our assertion of S's knowledge would just be false.

the support of the probability distribution associated with ψ and second, implicitly, in our presupposition that Q is indeed in one of the states that are in the support of the probability distribution associated with ψ. In the statement we interpret ψ epistemically (i.e., as representing beliefs of the observer), but in the presupposition we interpret ψ ontically (i.e., as representing something about the physical reality). We cannot make this statement without making this presupposition (unless we reject the factivity of knowledge), which means that we cannot use ψ in an epistemic way without using it also in an ontic way.

Therefore, if a quantum state is epistemic in the sense of Def. 1, then it is also ontic in the sense of this definition. A similar (but weaker) connection holds in the other direction. Assume that ψ represents the state of the system Q (i.e., it is ontic) and that it is in principle possible for some observer S to know that this is the case. Then, to represent S's (possible) knowledge about the state of the system, we need to use the state ψ again. Therefore, if a quantum state is ontic in the sense of Def. 1 and is in principle knowable, then it is epistemic in the sense of this definition. The connection here is weaker because it does not hold for ontic states that cannot be known even in principle; but quantum states are assumed to be knowable, so this restriction is irrelevant for us. Therefore, any quantum state is both ontic and epistemic in the sense of Def. 1—that is, it can represent the physical state of the system and the observer's knowledge about it. This entails that Defs. 1 and 2 cannot coincide, as in the latter "ontic" and "epistemic" states are two disjoint classes of states.

Is there any way for the proponents of HS's definition of "ontic" and "epistemic" states to avoid this argument? An obvious move is to talk about mere beliefs, without presupposing anything about their truth values. However, HS's definition of "epistemic" states seems to presuppose that we have to do with true beliefs, not just any beliefs. If we allowed treating quantum states as representing both partial beliefs that assign *non-zero* probability to the actual complete state of the system and partial beliefs that assign *zero* probability to the actual state of the system, this would undermine HS's rationale behind defining "epistemic" states as having overlapping supports, because then two quantum states with disjoint supports could represent partial beliefs about the actual state of the same system, as we do not require the actual complete state to be in the supports of both of them.

3.4 A simple example

To illustrate the irrelevance of the distinction between states with overlapping and non-overlapping supports for the issue of their being ontic or epistemic, let us consider the following simple example. Suppose we are investigating the masses of objects. Consider the following three sets of possible mass states (where the subscript "M" stands for mass):

- $\Lambda_M = \mathbb{R}^+$;
- $O_M = \{(0\text{ kg}, 4\text{ kg}], (4\text{ kg}, 8\text{ kg}], (8\text{ kg}, 12\text{ kg}], \ldots\}$;
- $E_M = \{(0\text{ kg}, 4\text{ kg}], (1\text{ kg}, 5\text{ kg}], (2\text{ kg}, 6\text{ kg}], \ldots\}$.

Using HS's terminology, one should say that states belonging to Λ_M are complete (these are exact masses of objects, expressed by real numbers), whereas states belonging to O_M and E_M are incomplete. Furthermore, states belonging to O_M do not overlap, whereas some of the states belonging to E_M do overlap, which means that the former should be regarded as "ontic", whereas the latter should be regarded as "epistemic" in HS's sense.[11]

Now the question becomes: are we willing to say that there is a fundamental metaphysical difference between states belonging to O_M and states belonging to E_M? Any state belonging to either O_M or E_M can be used to represent either a mass of a physical object (albeit in a non-exact way) or an observer's knowledge about that mass. If the mass of the object is 3.5 kg, then it is objectively true that its mass is between 0 and 4 kg (so it can be described by the first state in O_M), but it is no less objectively true that it is between 1 kg and 5 kg, as well as between 2 kg and 6 kg, and so on (so it can also be described by any of the first four states in E_M). Given the exact mass 3.5 kg, it follows by pure mathematics to which intervals this mass value belongs.

What is more, irrespective of whether we use states belonging to O_M or to E_M, we will represent the mass of the object as belonging to a certain interval of length 4. Therefore, changing from E_M to O_M does not increase our precision.

Another obvious obstacle for treating the two sets as having metaphysically different status is the fact that O_M is a subset of E_M. Do the states belonging to O_M cease to represent the reality, whenever this set is extended to E_M? This sounds rather absurd.

One might object that I have used arbitrarily defined sets of states to make my point, but the sets we actually use (e.g., in quantum mechanics) are not chosen at will, and their choice is a result of a conglomerate of experimental and theoretical considerations. If such considerations (the objection might go) lead to a set of non-overlapping states, this could be only because we are "cutting nature at the joints", we are revealing some objective distinctions in the physical reality itself.

To see why this is not true, let us return to our mass example. Suppose you have a weighing scale with weights of 4 kg each. The scale has two arms. You put the object whose mass you want to measure on the left arm and one weight on the right arm. If the left arm is above the right arm, you know that the object has a mass between 0 kg and 4 kg. If the left arm is below the right arm, you put another weight on the right arm. If now the left arm is below the right arm, you know that the object has a mass between 4 kg and 8 kg. If not, then you put yet another weight on the right arm... You repeat the procedure until the left arm is above the right arm. The states you can detect in this way are precisely the elements of O_M. However, this does not mean that you have revealed some deep division in the nature of mass, namely, that masses come in

[11] There are no probabilities here, which makes the example easier to grasp intuitively, but at the same time because of this it might seem too dissimilar to the quantum mechanical case we are interested in. To avoid this objection, we can just assume that the elements of O_M and E_M are uniform probability distributions over the interval of real numbers instead of being the interval itself.

chunks of 4 kg each. Instead, this is a result of your accidental epistemic constraints—the measurement device that is available to you can detect only such states. This illustrates that the states that we actually use can be non-overlapping for rather epistemic reasons.

This does not mean (of course) that the elements of O_M should be regarded as merely epistemic states. They can reveal something about the mass of the object you are investigating—namely, its real value with a given precision. The elements of O_M can be used to represent your knowledge that a physical object has a mass between 0 kg and 4 kg or between 4 kg and 8 kg (and so on), but also to represent the object itself having such a value of mass. And the same is true for E_M.

One might object that the above example is inadequate because it involves only one property and HS's idea is that quantum states capture some properties of the system, whereas other properties (if there are any) are captured by the hidden variables, so considering multiple properties is essential. In response, observe that HS assume only that quantum states are probability distributions over Λ, properties do not enter their formal framework, even if they are important at the intuitive level. Therefore, all our analyses for the mass case carry over straightforwardly to the case with multiple properties; the only difference is that now the elements of Λ_M are n-tuples of real numbers and elements of sets O_M, E_M are sets of such n-tuples (but cf. section 3.6 for more on the topic of multiple properties).

3.5 Epistemic "informational holes" in states and epistemic changes of states

We have seen that statements such as "the quantum state represents mere observer's knowledge" or "the quantum state is a representation of an observer's knowledge of reality rather than reality itself" are problematic in the light of the factivity of knowledge. Are there any claims in the vicinity of these that are more reasonable? I think there are at least two. Even though one cannot say that some state is merely epistemic, one can reasonably say that some lack of information (or lack of knowledge) is merely epistemic or that some change of the observer's beliefs is merely epistemic. Let us look at these in turn.

Whenever non-trivial (i.e., different than 0 and 1) probabilities are used in the representation of the physical state of an individual system, there is some indeterminateness involved and one can reasonably ask whether the physical reality itself is indeterminate in a given respect or this is only our lack of knowledge. It is commonly believed that in the case of classical statistical mechanics the latter holds: the classical particles have precise positions and momenta, but we do not know them. However, this does not mean that the states of classical statistical mechanics represent "mere knowledge" about individual systems—rather, they represent the states of physical particles in the world, albeit in an incomplete way.[12] What is merely epistemic here is the "informational

[12]This claim might seem to be in conflict with the ensemble interpretation of classical statistical mechanics, but it is reconcilable. If the probabilities represent relative frequencies

hole" in the theoretical state, not the positive informational content of it. This "informational hole" (e.g., the missing information about the exact positions and momenta in the case of classical statistical mechanics) does not have any counterpart in reality—the real physical state has this hole "filled in", it is only a hole in our knowledge. However, the positive informational content of the state (e.g., the known information about positions and momenta in the case of classical statistical mechanics) is not merely our knowledge, it also captures (incompletely) the real physical state.[13] It should be clear that this sense of "merely epistemic" does not underlie the distinction between "epistemic" and "ontic" made by HS, as all incomplete states involve such informational holes, not only "epistemic" ones. If quantum states have overlapping supports, then they cannot be complete, so overlapping supports are indicators of epistemicity in the sense of the presence of informational holes, but this is not HS's intended meaning of epistemicity. States with non-overlapping supports can also have informational holes (cf. O_M in section 3.4).

The second type of claims, concerning the nature of changes of states, can be illustrated by the following example. Assume that I know for sure that my keys are somewhere in my house and that this house can be divided into a number of places. The set of such places (closed under the union and intersection) is the set of possible states of my keys. Are these states ontic or epistemic? As we should expect, they are both. On the one hand, these states can represent the actual location of the keys—a fact about the physical reality. On the other hand, they can represent my beliefs concerning the location of my keys.[14] This is a synchronic

of states in an ensemble, then what is primarily represented by the probability distribution is that ensemble, but individual systems are also represented, even if in a derivative way—namely, as being in one of the states that belong to the support of this probability distribution. As long as not the whole state space of an individual system is in the support of the probability distribution, this is non-trivial information about the individual systems belonging to the ensemble.

[13] Sometimes in the literature one can encounter statements that some states represent only our ignorance. I think that such phrases cannot be read literally as ignorance is something purely negative (what we do not know), so such a state should be only the list of things that we do not know. However, in fact a state specifies the things that we know about a system (unless we made a mistake and attributed the wrong state to the system)—just our knowledge happens to be incomplete. Therefore, phrases such as "this state represents our ignorance" are acceptable only if regarded as abbreviations for something like "the informational hole in this state represents our ignorance and not an objective indeterminateness".

[14] It should be stressed that this is true even for complete states. Assume that λ_{key} represents the actual (complete) state in which my keys are. Therefore, λ_{key} is surely ontic, but it is also epistemic: if I know that my keys are in the state λ_{key}, then to represent my (complete) knowledge about the position of my keys, one should also use the same state, λ_{key}. This is a consequence of how ontic and epistemic states were defined in section 1 (Def. 1): a theoretical state is ontic if it can represent a (possible) physical state of a system, whereas it is epistemic if it can represent the (possible) beliefs of some observer about the physical state of a system. This is why even a complete state can also be epistemic—how else could we represent the perfect knowledge of an observer if not by means of a complete state? If I had defined an epistemic state as representing "merely knowledge" (i.e., representing knowledge and not being able to represent anything else), then a complete state certainly could not count as epistemic (because it is able to represent something that is not knowledge—the state of the physical system); but I doubt that anything could count as an epistemic state defined in this way (because of the factivity of knowledge) unless we use some non-standard concepts of

level: at any given time, the states can represent both the location of my keys and my beliefs concerning their location. However, there might be a difference at the diachronic level, that is, in how the states evolve in time. Suppose I wake up in the morning and start looking for my keys. Initially, I have no idea where they are located, so my beliefs are represented by the state that is the union of all places in my house. But when I start checking place by place, then I exclude more and more places as possible locations of my keys. My beliefs change—they are no longer represented by the most encompassing state, the consecutive states representing my beliefs correspond to smaller and smaller places. Therefore, the epistemic evolution is here non-trivial. Meanwhile, during this whole process of my looking for my keys, the keys themselves stay wherever they have been initially. Therefore, at least up to the point when I find them and pick them up, the ontic evolution is trivial.

In the example with keys there is a change in my beliefs without any change in the world.[15] However, this should not be conceived as a difference between two types of states because the same set of states is used to represent possible locations of my keys and my possible beliefs about this location. It is the change of states that can be said to be merely epistemic here, not the states themselves. The same intuition can be applied to the case of quantum mechanics (section 3.7).

3.6 Exact and inexact values of properties

One can object to the argument from the factivity of knowledge (section 3.3) and to my simple example (section 3.4) that "getting things 'more or less right' (λ, in this case) is not enough for onticity, since onticity depends on ψ univocally capturing λ".[16] In response, let us observe that for ψ to univocally capture λ, there must be a one-to-one correspondence between wave functions and complete states, which amounts to the claim that the wave function is itself complete. Therefore, under the most natural reading of this objection, it identifies the concept of an ontic state with the concept of a complete state, contrary to HS's intention. In HS's definition of "onticity", it is λ that univocally determines the "ontic state", not the other way around (because there might be more than one λ in the support of ψ), so the intuition invoked in this objection is not satisfied here.

A way of modifying this objection is to say that "getting things 'more or less right' is not enough for onticity as it requires getting *something* exactly right, even though not necessarily the entire λ". One can continue this train of thought by assuming that λ consists of a set of exact values of certain properties (more than one), and a state is ontic if it captures

knowledge and/or truth. If I have defined an epistemic state as representing "merely beliefs" (i.e., representing beliefs and not being able to represent anything else), then a complete state would also not count as epistemic (for the same reason as before), although there presumably are some epistemic states in this sense (e.g., being a circular square is perhaps a state such that someone might believe that something is in this state, but nothing actually could be in this state because it is self-contradictory).

[15] Of course, the opposite situation is possible as well, namely a change in the world without any change in my beliefs.

[16] This objection was suggested to me by...

the exact value of at least one of these properties. This is in fact how HS seem to think about λ's—as consisting of ψ and (perhaps) several hidden variables (HS 2010:129–130). A similar idea is expressed by PBR (2012:475–476). Let us grant this understanding of complete states as consisting of exact values of several properties and call one of them f with values belonging to $\mathbb{R}$. Then, for each λ, the value of f is unique (so it can be written as $f(\lambda)$). Consider two different values of this property, denoted by f_1 and f_2. Each of them determines the set of all complete states whose value of f is f_i (for $i = 1, 2$)—that is, the set $F_i := \{\lambda \in \Lambda | f(\lambda) = f_i\}$. What is more, F_1 and F_2 are disjoint sets because every λ corresponds to exactly one value of f. Therefore, if every wave function uniquely determines the exact value of f, then the supports of such wave functions must be disjoint, so in this case they are "ontic" states in HS's sense.

This relation might be used to motivate HS's definition of "ontic" states in the following way: a state can be said to be "ontic" only if it captures exactly the value of at least one of the properties that constitute complete states. However, for this way of motivating the definition to have a chance to work, the relation would need to hold in both ways, that is, it should be the case not only that capturing the exact value of some property implies disjoint supports, but also that disjoint supports imply capturing the exact value of some property. But this is not the case: the implication in the latter direction is in general false, as is shown by the following example. Assume that every complete state consists in the specification of the values of two properties, f and g. Consider sets of complete states $X_1 := \{\lambda_1, \lambda_2\}$ and $X_2 := \{\lambda_3, \lambda_4\}$, where $\lambda_1 = \langle f_1, g_1\rangle, \lambda_2 = \langle f_2, g_2\rangle, \lambda_3 = \langle f_1, g_2\rangle$ and $\lambda_4 = \langle f_2, g_1\rangle$. These sets provide a sought-for counterexample, as X_1 and X_2 are disjoint, but they do not determine an exact value of either of the properties f or g.

One can try to prevent situations of this kind by defining properties in a way that makes such counterexamples impossible. This in fact seems to be done by PBR (2012:476, description of figure 1, notation changed), who define a physical property in the following way: they consider a collection of probability distributions over the set of complete states labelled by $l \in L$ (i.e., $\{\mu_l(\lambda)\}_{l \in L}$) and say that if in such a collection every pair of distributions have disjoint supports, then the label $l \in L$ is uniquely determined by λ and therefore is called "a physical property". It is not clear whether a probability distribution over complete states is an object of the right category to be called a property (especially in the light of the intuition mentioned earlier that complete states are specifications of the values of physical properties), but at least it looks reasonable to say that a given value of a given physical property might *correspond* to a certain probability distribution (namely, the one that is non-zero for all and only complete states that for this particular property determine this particular value). However, there is a more serious problem with this approach to defining properties: being a physical property depends here on what the collection of probability distributions we started with is. The same probability distribution will then correspond to a physical property when "immersed" in some collections of probability distributions, but not when "immersed" in others. This seems to be too high a level of arbitrariness in specifying what a physical property is.

Another way of objecting to my counterexample to the implication from disjoint supports (i.e., "onticity" in HS's sense) to capturing the exact value of some property is to say that even though such counterexamples cannot be excluded in general, they do not hold in the particular case we are interested in. What is needed here is the assumption that quantum mechanics is such that either (i) probability distributions over complete states associated with wave functions are not disjoint, or (ii) they are disjoint and each wave function captures an exact value of some property. Using our former notation and assuming again for simplicity that there are only two properties, case (ii) might be realized as follows: the wave function captures the exact value of f, whereas the value of g is a hidden variable. Then, every wave function ψ_{f_i}[17] would correspond to the set $\{\lambda \in \Lambda | f(\lambda) = f_i\}$, which is equal to $\{\langle f_i, g_j \rangle | \exists_{\lambda \in \Lambda} g(\lambda) = g_j\}$. Wave functions ψ_{f_i} correspond to disjoint probability distributions and capture the value of one property (i.e., the value of f). Of course, both f and g can be replaced by any number of properties. Currently, I believe this is the closest to HS's (2010) understanding of this issue. However, some additional argument is needed to support the hypothesis that either (i) or (ii) holds. In other words, we need to exclude the remaining option that (iii) probability distributions over complete states associated with wave functions are disjoint but wave functions do not capture the exact value of any property. That is, we need to show that the supports of probability distributions associated with wave functions are not similar to sets X_1 and X_2 above. I am not aware of any argument for this hypothesis.

The remaining question is whether providing such an argument would be sufficient to justify HS's definitions of "ontic" and "epistemic" states.[18] Associating the concept of onticity with exact values of properties seems to be supported by the following intuition: what is real in the strict sense are the exact values of properties, and their inexact specifications do not form a part of our ontology, but statements about them might be true because they are made true by these exact values obtaining in the world.[19] However, in this way we do not gain any support for calling states with partially overlapping supports "epistemic" (unless we assume that whatever is not ontic is thereby epistemic, which is rather dubious). In particular, the fallaciousness of saying that something represents "merely knowledge" is not dismissed. What is more, the equivalence between disjoint supports of states and the capturing of the exact value of some property by these states is at best accidental: if it holds in quantum mechanics, this is because of particular features of this theory, not because this equivalence is

[17]Wave functions under these assumptions can be labelled by the values of f because each of them is associated with a different value of f.

[18]Of course, one can use any terminological conventions one wants (in particular, one can define "ontic", "epistemic", "real", "knowledge" etc. in whatever way one wants), but for the results obtained using these definitions to have philosophical importance, they must be sufficiently close to how these notions are typically understood in philosophy.

[19]For example, the value f_i of f is a part of our ontology, but the value of f between f_j and f_k (where $f_j < f_k$) is not because adding it would be superfluous: the ontology consisting of exact values is sufficient to provide facts that make true statements concerning both exact and inexact values. For the latter this works as follows: the statement "system's S value of f is between f_j and f_k" is made true by the (physical) fact that system's S value of f is f_i and the (mathematical) fact that f_i is a number between f_j and f_k.

analytically true for any collection of probability distributions (we have seen that it is easy to construct counterexamples to it).

A more appropriate distinction here seems to be between what is ontologically fundamental vs. everything that supervenes on it[20] (instead of "ontic" vs. "epistemic"). The exact values of physical quantities might then be said to ontologically fundamental, whereas the inexact values of these physical quantities might be said to supervene on them (i.e., they are still in some sense real properties, albeit non-fundamental ones and fully determined by fundamental ones).

3.7 A reformulation of the problem

In the light of the above arguments, I would like to propose a reformulation of the problem of onticity vs. epistemicity with regard to quantum mechanics. Instead of asking "is a given state ontic or epistemic?" (a synchronic question), one can ask "is a given *change* of a state ontic or epistemic?" (a diachronic question); cf. my toy example with the location of my keys in section 3.5.

Perhaps, at least to some extent, this diachronic thinking was a guiding intuition for the HS's definitions of ψ-ontic and ψ-epistemic models. This is suggested, for example, by the following quote: "By our definitions, ψ has an ontic character if and only if a variation of ψ implies a variation of reality and an epistemic character if and only if a variation of ψ does not necessarily imply a variation of reality" (HS 2010:132). However, it is unlikely that "variation" here should be understood in a dynamical way (as a change of a state), as the predicates "ontic" and "epistemic" are attributed by the authors to states and not to the changes of states. Instead, "variation" here seems to mean only "difference". Therefore, even if the underlying intuition was somewhat similar, the difference between the two formulations (i.e., the synchronic one and the diachronic one) is conceptually important and deserves a strong emphasis.

One can object here that whether a change of a state is ontic or epistemic should depend (at least to some extent) on whether the state itself is ontic or epistemic. However, my question is posed for states that are both ontic and epistemic (in the sense of Def. 1). The case under consideration is precisely of this kind: both "ontic" and "epistemic" states in HS's sense (defined in terms of non-overlapping vs. overlapping supports, see Def. 2) are both ontic and epistemic in the philosophical sense (i.e., any of them can be used to represent the state of the system and the knowledge of an observer that the system is in that state, see Def. 1). Depending on the way in which they are used in a particular context, their change is either ontic or epistemic. The question is now whether some particular ways of changing these states (such as measurement-induced collapse or unitary evolution) are an instance of the former or the latter type of change.

[20] A collection of entities A is said to supervene on the collection of entities B (where "entities" might be of any ontological category—objects, properties, relations, facts and so on) iff there could be no difference on A without any difference on B. Another way of expressing this condition is to say that the specification of entities of type B uniquely determines the specification of entities of type A.

4 Can the collapse of the quantum state be interpreted as a successful change of partial beliefs?

In section 3, I argued that as long as we do not consider the evolution of states, any state that represents knowledge (and as such is epistemic) by the factivity of knowledge is also ontic. The appropriate question is then not whether a given state represents the reality or our knowledge about reality, but whether a given change of a state is a change in reality or only a change in our beliefs about reality. This leads to a question whether by looking merely at the formal features of the evolution of states, we can tell what kind of change we are dealing with. I believe that, in general, the answer is negative, in the sense that there is no universal criterion distinguishing ontic change from merely epistemic change (especially if we allow partial beliefs that do not have the actual state in their support). However, I believe that we can conclude something about particular cases, including the quantum mechanical case we are interested in.

First, the PBR theorem (Pusey, Barrett and Rudolph 2012) is relevant for our modified question about changes of states, despite the fact that it was formulated as an answer to the original question about states. Assume that at time t_i an observer S ascribes a quantum state ψ_i to a system Q and at a later time t_f the same observer S ascribes to Q a different state ψ_f. Could the change from ψ_i to ψ_f be merely epistemic, assuming that PBR theorem is true, that is, assuming that ψ_i and ψ_f have non-overlapping supports? The answer depends on whether we allow quantum states to represent partial beliefs that are entirely false in the sense of assigning zero probability to the actual complete state of the system. If we allow this, then the answer is "yes". For example, if Q was in the same complete state λ throughout the whole interval $[t_i, t_f]$, ψ_i assigns zero probability to λ and ψ_f assigns non-zero probability to λ, then S ascribed wrongly ψ_i to Q at t_i and ascribed rightly ψ_f to Q at t_f. In this case, the change from ψ_i to ψ_f is merely a change of S's beliefs about the complete state of the system. However, if we use quantum states to represent only partial beliefs that assign non-zero probability to the actual complete state of the system, then the change from ψ_i to ψ_f cannot be merely epistemic. If the system was at t_i in λ, then it cannot be at t_f still in λ because in t_f its quantum state is ψ_f, which, as PBR theorem tells us, assigns zero probabilities to all complete states that are assigned non-zero probabilities by ψ_i, including λ.

Second, below I will provide a new and independent argument that the measurement-induced collapse of the quantum state cannot be regarded as a successful change of a partial belief state. On the one hand, my argument is more limited than that of PBR in the sense that it concerns only the quantum state collapse, whereas PBR theorem is relevant for any change of the quantum state. However, it is usually the collapse, not the Schrödinger evolution, that is conjectured to be a merely epistemic change.[21] On the other hand, my argument does not assume anything

[21] An exception to this might be Bartlett, Rudolph and Spekkens (2012).

about the structure of the prepared state, whereas the PBR theorem relies on the Preparation Independence Postulate (or some weakened version of it, see Myrvold 2018), which makes my argument more general in this respect. Additionally, it is interesting on its own because the plot of the argumentation is entirely different than in the PBR theorem.

How can our belief states change? First, consider full belief states. I will make here two assumptions (hopefully not very controversial, at least in the context of the debate to which this paper contributes). Every belief has a certain logical value and I will assume that there are only two such values, truth and falsity. Every belief also has some degree of specificity or informativeness: for example, a belief that the mass of the object is between 1 kg and 4 kg is less specific (less informative) than the belief that the mass of the object is between 1 kg and 2 kg.

When is a change of a full belief state successful? In terms of our two parameters characterizing beliefs (i.e., the logical value and the degree of informativeness), one can distinguish two such cases. The most obvious one is if the initial belief is false and the final belief is true. In this case, the informativeness of these beliefs does not matter: even if the initial belief was more informative, we surely want to replace it by a true belief, even if the latter is less informative. The subtler case is when the initial belief is true and the final belief is also true but more informative. It seems that at least if we are concerned only with these two parameters (i.e., logical value and informativeness), these are the only cases of a successful change of full beliefs.

In the case of partial beliefs, the issue becomes much more complex. Instead of asking whether a partial belief is true or false, we should be asking how close it gets to the truth. Let me use an example to explain what difficulties we are encountering here. Assume that there are only three possible complete states, that is, $\Lambda = \{\lambda_1, \lambda_2, \lambda_3\}$, and that the actual state of the system is λ_2. S's partial beliefs about the state of the system are represented by $Pr(\lambda_i) = p_i$, where $0 \leq p_i \leq 1$ and $\sum_{i=1}^{3} p_i = 1$. If $p_2 \neq 0$ and $p_j \neq 0$ for at least one $j \neq 2$, then we cannot say that S's partial beliefs concerning the state of the system are simply true or false, because a non-zero number is assigned to the actual state and a non-zero number is assigned to one of the non-actual states. However, if $p_2 \gg p_1 + p_3$, then it seems reasonable to assert that S is closer to truth than to falsity and the reverse for $p_2 \ll p_1 + p_3$. Can this proximity to the truth be determined in a systematic way or even measured? It turns out that this issue is investigated in formal epistemology and various measures (called inaccuracy measures or epistemic utility measures) have been proposed.[22]

However, it is debatable which of the proposed measures (if any) is adequate. Furthermore, one might question whether the closeness to the truth of our partial beliefs admits a linear order at all, that is, whether we indeed can say for any two partial belief states whether one of them is closer to truth than the other (and if there is no such linear order, then

[22] The most popular such measure is the Brier score, which for our example would be $(1-p_2)^2 + p_1^2 + p_3^2$ (the lower the value, the closer to truth is the partial belief state; see, e.g., Fallis and Lewis 2016:578–579, Wroński 2018, ch. 6).

a fortiori there could be no measure). To see this, assume the same Λ as before and consider two subjects, S and S', with partial beliefs given by $(\frac{1}{8}, \frac{3}{4}, \frac{1}{8})$ and $(\frac{2}{5}, \frac{3}{5}, 0)$ (cf. Fallis and Lewis 2016:577 for a similar example). The former probability distribution is more "peaked" over the actual complete state, but it does not exclude any of the non-actual complete states, whereas the latter is less "peaked" but excludes entirely one of the non-actual complete states. Therefore, S is closer to truth in the sense that his credence in the true hypothesis (that the system is in the state λ_2) is higher than that of S', whereas S' is closer to truth in the sense that he eliminated one of the false hypotheses altogether, which S did not do.

For our purposes, the safest move is to assume that closeness to the truth can be captured by a parameter that is at least partially ordered and stay silent on whether this order is also linear (and whether this parameter is a measure, as many formal epistemologists want it to be). Are there any other parameters that we should take into account in assessing which of two partial belief states is better than the other? In the analysis of the full belief case, we have mentioned informativeness and perhaps there are also some other parameters. Fortunately, we do not need to decide what and how many such parameters there are. We can afford to proceed in an entirely general way and assume that there are $n \geq 1$ parameters of epistemic goodness of partial belief states (denoted by G^j, $j = 1, \ldots, n$), each of which is partially ordered. One can define a function $g^j : B \to G^j$ that assigns one of the elements of G^j to all partial belief states belonging to the set B; for short we will write $g_i^j := g^j(b_i)$. Given two belief states, b_i and $b_{i'}$, there are four possible ways in which their j-th goodness parameters may be related: (1) $g_i^j > g_{i'}^j$, (2) $g_i^j < g_{i'}^j$, (3) $g_i^j = g_{i'}^j$, (4) $g_i^j \not\sim g_{i'}^j$ (incomparable).

What amounts to a successful change of partial belief states? We need to take into account all our parameters G^j and the fact that each of them might be only partially ordered. A transition from a partial belief state b_i to b_{i+1} is a *clear* epistemic success when at least one of these parameters increases (in the sense of going up in the partial order) and the rest of them either increase or do not change, that is: $g_i^j < g_{i+1}^j$ for some $j \in \{1, \ldots, n\}$ and $g_i^k \leq g_{i+1}^k$ for all $k \in \{1, \ldots, n\}$ such that $k \neq j$.

Arguably, however, the cases of clear epistemic success are not the only cases of epistemic success. If some of the parameters increase, some stay the same and some others change into incomparable ones, then this still seems to be a case of epistemic success, albeit not a clear one. An even more subtle case would be that some of the parameters decrease, but the increase of some others compensates this. For this compensation, we need some numerical measure that tells us how large a given increase or decrease was. If each of our parameters has a numerical value, then the measure of increase/decrease could just be the difference $g_{i+1}^j - g_i^j$. However, in general it could be the case that even though our parameters are only partially ordered, a measure of increase/decrease (call it v) is locally defined, so that it enables the mentioned compensation.

What should v look like? It can be defined for some pairs of the form $\langle g_i^j, g_{i'}^j \rangle$ but not necessarily for all of them. Formally, v should be a partial function on $(G^1 \times G^1) \cup \ldots \cup (G^n \times G^n)$ and its values should be real numbers. Additionally, v should satisfy the following conditions:

- if $g_i^j < g_{i'}^j$, then $v(g_i^j \to g_{i'}^j) > 0$ or $v(g_i^j \to g_{i'}^j)$ is undefined[23] (i.e., v is larger than zero for the change from a parameter that is lower in the partial order to the parameter that is higher);
- if $g_i^j > g_{i'}^j$, then $v(g_i^j \to g_{i'}^j) < 0$ or $v(g_i^j \to g_{i'}^j)$ is undefined (i.e., v is smaller than zero for the change from a parameter that is higher in the partial order to the parameter that is lower);
- $v(g_i^j \to g_{i'}^j) = -v(g_{i'}^j \to g_i^j)$ for any g_i^j and $g_{i'}^j$ for which $v(g_i^j \to g_{i'}^j)$ is defined (i.e., v should have the same absolute value and the opposite sign for two transitions that are reversals of each other).

To sum up, in the most general case of an epistemically successful transition of partial belief states $b_i \to b_{i+1}$ (for a given i) we require that some parameters increase and allow that some parameters decrease (as long as this is compensated by the increase of other parameters), some do not change and some change into incomparable ones. Therefore, a transition $b_i \to b_{i+1}$ is epistemically successful iff one can renumber the parameters G^j so that the following five conditions hold:

- $g_i^j < g_{i+1}^j$ for $1 \leq j \leq m_1$ (where $0 < m_1 \leq n$) and
- $g_i^j > g_{i+1}^j$ for $m_1 < j \leq m_2$ (where $m_1 \leq m_2 \leq n$) and
- $g_i^j = g_{i+1}^j$ for $m_2 < j \leq m_3$ (where $m_2 \leq m_3 \leq n$) and
- $g_i^j \not\sim g_{i+1}^j$ for $m_3 < j \leq n$ and
- if $m_2 > m_1$, then $\sum_{j=1}^{m_1} \left| v(g_i^j \to g_{i+1}^j) \right| > \sum_{j=m_1+1}^{m_2} \left| v(g_i^j \to g_{i+1}^j) \right|$ (the increase of parameters in the first group compensates the decrease of the parameters in the second group).

There are m_1 increasing parameters, $m_2 - m_1$ decreasing, $m_2 - m_3$ unchanging and $n - m_3$ changing into incomparable ones; out of these numbers, only m_1 is required to be greater than zero (as otherwise there would be no improvement in our partial beliefs and so the transition would not be successful).

Having developed this abstract account of what a successful change of partial belief states might amount to (which is not a particular theory, but a scheme that encompasses many possible theories, hopefully all reasonable ones), we have tools to address the main question of this section: can the measurement-induced collapse of the quantum state be interpreted as a successful change of a partial belief state? To show that this is not the case, we will consider a particular experiment.

Assume we have three spin-measuring devices and a beam of electrons in a superposition state

$$\psi_1 = \alpha \left| z \uparrow \right\rangle + \beta \left| z \downarrow \right\rangle . \tag{1}$$

We set the devices so that the first one measures spin in the z-direction, the second one measures spin in the x-direction and the third one again

[23]Our function v can be undefined for certain pairs of parameters because we allow it to be a partial function only.

measures spin in the z-direction. We assume that the measurements are very fast one after another, so that the Schrödinger evolution between them can be ignored. One of the courses of events allowed by quantum mechanics for this experimental setup is the following: we perform the measurement on some electron and get the answers "up", "right" and again "up". Therefore, the state of the electron changes first from ψ_1 to

$$\psi_2 = |z \uparrow\rangle = \frac{1}{\sqrt{2}} |x \rightarrow\rangle + \frac{1}{\sqrt{2}} |x \leftarrow\rangle \, ; \tag{2}$$

then to

$$\psi_3 = |x \rightarrow\rangle = \frac{1}{\sqrt{2}} |z \uparrow\rangle + \frac{1}{\sqrt{2}} |z \downarrow\rangle \, ; \tag{3}$$

and finally to

$$\psi_4 = |z \uparrow\rangle \, . \tag{4}$$

Importantly, $\psi_4 = \psi_2$, that is, in this series of measurements we attribute to the system the same state twice, although not in consecutive measurements, but with some measurement leading to a different collapsed state in between. If we interpret the quantum states ψ_i as representing S's partial belief states b_i, then this amounts to the following series of changes of partial beliefs being allowed by quantum mechanics: $b_1 \rightarrow b_2 \rightarrow b_3 \rightarrow b_4$ with $b_2 = b_4$.[24] However, one can show that no possible series of successful changes of partial belief states can be like this. The intuition is as follows: if we improve in the change from b_2 to b_3, then we cannot improve in the reverse change from b_3 to b_2, but the latter is exactly the same as the change from b_3 to b_4, because $b_4 = b_2$. The intuition behind the argument is as simple as this, but because our conditions defining a successful change of a partial belief state are quite complex, showing this in detail requires more effort.

Let us start by observing that $b_2 = b_4$ entails that $g_2^j = g_4^j$ for all $j = 1, \ldots, n$. This will be used in the following lemma:

[24] One can ask why it is claimed that ψ_4 represents a partial belief and not a full belief, given that in this case we know with certainty that the system has spin up. There are two responses to this. First, even though this state is not a non-trivial superposition in one basis, it is a non-trivial superposition in another basis. Therefore, the fact that it is not a non-trivial superposition in the basis $\{|z \uparrow\rangle, |z \downarrow\rangle\}$ is not enough to claim that it is associated with exactly one complete state λ. Second, even if one (or more) of the states we are analysing was associated with exactly one λ, in which case the corresponding probability distribution would attribute probability 1 to this λ, such a state could still be conceived of as representing a partial belief in our sense (albeit a trivial one). Of course, this trivial partial belief would strictly correspond to some full belief, but this is not important for us here. The framework used in this section can be applied equally well to partial beliefs that are non-trivial (i.e., all probabilities are smaller than 1) and to those that are trivial (i.e., one of the probabilities is equal to 1). This is because partial beliefs belonging to both classes can be assessed with respect to their proximity to the truth and degree of informativeness (as well as other parameters evaluating the goodness of partial beliefs, if there are any)—and this is their only feature that is relevant for my framework. In the case of trivial partial beliefs, some of these parameters would perhaps obtain only extreme values (e.g., a trivial partial belief might be either true *simpliciter* or false *simpliciter*, it cannot have any of the intermediate degrees of closeness to truth). However, this does not prevent in any way the applicability of my framework to them.

Lemma 1. *For every $j \in \{1, \dots, n\}$, if $b_2 = b_4$, then a parameter G^j increases in one of the transitions $b_2 \to b_3$ or $b_3 \to b_4$ iff it decreases in the other.*

Proof. Assume that G^j increases in the first transition, so that $g_2^j < g_3^j$. Our assumption that $b_2 = b_4$ leads to $g_2^j = g_4^j$, so we have $g_4^j < g_3^j$, which means that in the second transition this parameter decreases (i.e., the other two options, $g_3^j = g_4^j$ and $g_3^j \not\sim g_4^j$ are excluded). Analogously, if G^j decreases in the first transition, it needs to increase in the second transition. Now, assume that G^j increases in the second transition, so that $g_3^j < g_4^j$. Our assumption that $b_2 = b_4$ leads to $g_2^j = g_4^j$, so we have $g_3^j < g_2^j$, which means that in the first transition this parameter decreases. Analogously, if G^j decreases in the second transition, it needs to increase in the first transition. □

The above lemma will help us in proving the theorem that excludes the scenario we are investigating:

Theorem 1. *If in a series of changes of partial belief states $b_2 \to b_3 \to b_4$ the first and the last belief state is the same (i.e., $b_2 = b_4$), then it cannot be the case that each transition $b_i \to b_{i+1}$ is an instance of clear epistemic success or unclear epistemic success.*

Proof. Consider a series of of changes of partial belief states $b_2 \to b_3 \to b_4$ such that $b_2 = b_4$. For reductio, assume that each transition $b_i \to b_{i+1}$ in this series is an instance of epistemic success (clear or unclear). Assume without loss of generality that in the first transition the parameters G^j with $j \leq m_1$ increase, parameters G^j with $m_1 < j \leq m_2$ decrease, parameters G^j with $m_2 < j \leq m_3$ do not change and parameters G^j with $m_3 < j \leq n$ change into incomparable ones. We know that $m_1 \neq 0$ (because for any kind of success at least one parameter must increase) and that $m_2 > m_1$ (the lemma together with the fact that in the second transition at least one parameter increases entail that in the first transition at least one parameter decreases). To compensate for the decrease of the parameters G^j with $m_1 < j \leq m_2$, we need to have

$$\sum_{j=m_1+1}^{m_2} \left| v(g_2^j \to g_3^j) \right| < \sum_{j=1}^{m_1} \left| v(g_2^j \to g_3^j) \right|. \tag{5}$$

However,

$$\sum_{j=1}^{m_1} \left| v(g_2^j \to g_3^j) \right| = \sum_{j=1}^{m_1} \left| v(g_4^j \to g_3^j) \right| = \sum_{j=1}^{m_1} \left| v(g_3^j \to g_4^j) \right| \tag{6}$$

(the first equality comes from $g_2^j = g_4^j$ and the second from the properties of function v). Similarly,

$$\sum_{j=m_1+1}^{m_2} \left| v(g_2^j \to g_3^j) \right| = \sum_{j=m_1+1}^{m_2} \left| v(g_4^j \to g_3^j) \right| = \sum_{j=m_1+1}^{m_2} \left| v(g_3^j \to g_4^j) \right|. \tag{7}$$

This implies that

$$\sum_{j=m_1+1}^{m_2} \left| v(g_3^j \to g_4^j) \right| < \sum_{j=1}^{m_1} \left| v(g_3^j \to g_4^j) \right|, \tag{8}$$

that is, in the second transition the sum of the values of the function v for the first m_1 parameters is larger than the sum of the values of the function v for parameters G^j with $m_1 < j \leq m_2$. However, this is precisely the opposite of what we wanted, because in the second transition the first m_1 parameters increase, whereas parameters G^j with $m_1 < j \leq m_2$ decrease (and there are no other increasing or decreasing parameters, which follows from the lemma and from the fact that these are the only increasing or decreasing parameters in the first transition). Therefore, equation (8) means that in the second transition the increasing parameters do not compensate for the decreasing ones, which contradicts the assumption that both transitions are cases of epistemic success. □

5 The remaining options for the epistemic view

I have argued that because knowledge is factive, states that represent someone's knowledge about the physical reality thereby also represent something about the physical reality itself. This led me to the conclusion that instead of asking whether a given state is ontic or epistemic, we should rather ask whether a given change of state is ontic or epistemic. In particular, the question worth asking in the context of quantum mechanics is whether the measurement-induced collapse of the quantum state can be interpreted as an epistemically successful change of our partial beliefs about the complete state of the system. I have argued for the negative answer to this question. However, this does not mean that no options are left for the defenders of the epistemic view on the change of quantum states in the measurement-induced collapse.

I think that they can take one of the following positions:

(1) Deny the existence of complete states.

(2) Accept the existence of complete states but treat quantum states as only associated with probability distributions over measurement results, not over the space of complete states.

(3) Assume that the changes of our partial beliefs in accordance with the collapse rule are not always successful, that is, they are sometimes changes from better partial beliefs to worse or incomparable partial beliefs.

(4) Deny the assumption that the change of quantum state between consecutive measurements due to the Schrödinger equation can be ignored.

(5) Interpret the collapse as partially epistemic and partially ontic.

Let us look more closely at each of these positions in turn.

The first option amounts to rejecting the whole framework of ontological models, as put forward by HS (2010) and reviewed in section 2. This seems to be the most popular position among the defenders of the epistemic view on quantum mechanics (cf. Leifer 2014:72, where the list of proponents of this option, called by him "neo-Copenhagen" is much longer than the list of defenders of the epistemic view who would accept the framework of ontological models).[25] This is a significant limitation for both my argument and for PBR theorem (as well as HS's framework of ontological models in general), as they rely crucially on the assumption of the existence of complete states, which means that the most popular variety of epistemic positions is entirely immune to these arguments.

Similarly, the approach of PBR and my own assume that a quantum state can be understood as a probability distribution over the set of complete states, which means that a defender of the second option on our list can also ignore these arguments. However, this position seems to be unstable, because if there is a complete state the system is in, then this state should constrain what the possible results of measurements conducted on this system are and do this at least as precisely as the quantum state does; but then, if both the quantum state and the complete state constrain the possible measurement results, it is difficult to imagine that the quantum state is not related in any way to the complete state.

To comment on the third option, it might seem very dubious at first glance: if using the collapse rule renders our partial beliefs worse than they were before (or neither better nor worse), why would we use it at all? It seems that in such a case it would be more epistemically profitable to abandon this rule. However, if we did not "update" the state after our first measurement from ψ_1 to ψ_2, then our predictions would be empirically less adequate, so the "updating" clearly contributes to the predictive success of quantum mechanics. Despite its initial appeal, this argumentation does not entirely exclude the position with number 3 on my list. Even if using the collapse rule always improves our predictions concerning the measurement results, this does not automatically mean that it always improves our partial beliefs about the complete state of the system. Perhaps in this latter regard, the collapse rule sometimes leads to changes of our partial beliefs that are not epistemically successful, but in a way that is impossible for us to recognize, so that despite this drawback quantum mechanics is still the best theory of quantum phenomena available to us.

The fourth option is based on the observation that our decision to ignore the Schrödinger evolution between measurements was an idealisation. However, I find it implausible that it is this idealisation that was responsible for the contradiction at which we have arrived. If the Schrödinger evolution was non-negligible in this scenario, then we should get an improvement of our predictions of measurement results by using it, but this is not the case—if the temporal distance between the measurements is very small, then it can be safely ignored.

The last option might seem to be similar to what is proposed by Hance,

[25]For example, QBism is based on the idea that probabilities represent partial beliefs; however, these are not probabilities assigned to complete states but probabilities of measurement outcomes calculated via the Born rule (see, e.g., Caves, Fuchs and Schack 2002:3). Fuchs, Mermin and Schack (2014) identify measurement outcomes with an agent's personal experiences and understand quantum mechanics as "a tool anyone can use to evaluate, on the basis of one's past experience, one's probabilistic expectations for one's subsequent experience" (2014:749); they explicitly deny the existence of complete states (2014:752) and justify this denial by appealing to the fact that λ's do not correspond to anything in quantum theory or in our experience.

Rarity and Ladyman (2021), who claim that a wave function can have in some aspects an epistemic nature and in some aspects an ontic nature. However, recall that what we are considering here is not the ontic or epistemic character of a wave function (which I argued is a rather misleading terminology) but of the changes of a wave function. The approach to epistemic change presented here (as well as the standard Bayesian epistemology) is not suitable to deal with such a mixed view. This is because for a change to be a change of someone's beliefs about a certain state of affairs, this state of affairs itself must remain the same—otherwise, what would these beliefs be about and how could we compare them with respect to how accurate they are in capturing this state of affairs? Changes of beliefs are implicitly understood as changes of beliefs on the same subject. Of course, the improvement of our knowledge about some state of affairs does not require that this state will still obtain in the world: this might be an improvement of our knowledge about the state of affairs that obtained in the past. What is required to not change is only the specification of the subject of our beliefs. However, the mixture of epistemic and ontic change seems to amount to a change of beliefs together with what these beliefs are about; therefore, it is precisely this combination that is problematic. I do not want to claim that one cannot build an account to deal with epistemic change of this kind, but for this some new formal tools would be needed.[26]

Summing up, my argumentation surely does not rule out all epistemic views on the nature of quantum collapse, but puts restrictive constraints on the class of available options. This argumentation is relevant only to approaches assuming that there are complete states of quantum systems, so one might say that it threatens less extreme variants of the epistemic view and does not have any bearing on more extreme ones, such as QBism.

6 Remarks concerning theories without measurement-induced collapse

One can object to my analysis that because quantum theories based on measurement-induced collapse are not the most philosophically interesting quantum theories currently available, proving a statement whose application is limited only to such theories is not philosophically interesting

[26] In analogy to our former toy example in which the location of my keys was sought, one might illustrate the mixed epistemic-ontic change by means of a similar example with a running hamster. I want my beliefs about the location of the hamster to coincide with its actual location at any moment. Because the hamster is running, it might happen that my beliefs improve or worsen not as a result of considering some new evidence but merely as a result of the change of the state of the hamster. This is in stark contrast with the way of thinking in the standard Bayesian epistemology, where the improvement or worsening of my beliefs might happen only due to the updating of my belief state in the light of new evidence. The reason for this is that in the standard case the subject of my beliefs is held fixed, whereas here we want our beliefs at any time to track the state of the system at that time, so the subject of my beliefs is constantly changing.

as well.[27] I do not agree that it is uninteresting, as the concept of a measurement-induced collapse is central, not marginal, in the discussion about quantum mechanics throughout its history, and it is still commonly used in the introductory expositions of quantum mechanics nowadays. What is more, the defenders of interpretations of quantum mechanics that do not refer to measurement-induced collapse usually regard changes of a wave function as ontic.

In any case, one could ask whether my ideas can be extended in some way to cover other interpretations (or versions) of quantum mechanics. My argument in section 4 could not be straightforwardly carried forward to this case as it crucially relies on the assumption that the collapse is associated with a measurement because, without that, we would not get the effect of coming back to the same state $|\psi_2\rangle$. However, I believe that the epistemic understanding of the change of a wave function is unattractive for many interpretations (or versions) of quantum mechanics that do not postulate measurement-induced collapse, which I will argue for below using some elements of the framework developed earlier.

Consider the following three classes of interpretations (or versions) of quantum mechanics without measurement-induced collapse (i.e., assuming that there is a single evolution rule for the wave function): class 1, the wave function is complete and evolves deterministically; class 2, the wave function is incomplete and evolves deterministically; class 3, the wave function is complete and evolves indeterministically. Taken together, these three classes cover many of the known interpretations (or versions) of quantum mechanics. For example, classes 1 and 3 encompass all interpretations according to which the ontology of the physical world is exhausted by the wave function, such as the Everettian interpretation, wave function realism[28] and some versions of GRW. Bohmian mechanics, another widely discussed approach, belongs to class 2. Below I will formulate three arguments that it is better not to combine interpretations (or versions) of quantum mechanics belonging to these three classes with the epistemic view on the change of a quantum state as this leads to some very implausible consequences.

Argument 1 (for classes 1 and 3): the wave function is complete and evolves either deterministically or indeterministically. Assume that this evolution is epistemic. This leads us to an absurd conclusion that nothing changes in the physical world: from the completeness assumption, all conceivable changes in the physical world could only be changes of the wave function, but its changes are, by assumption, epistemic, so no place is left for any ontic changes whatsoever.

Argument 2 (for classes 1 and 2): the wave function is either complete or incomplete and evolves deterministically. Assume that this evolution is epistemic. Consider an isolated system and assume that we attribute to it the wave function ψ_1 at t_1 (e.g., as a result of performing some mea-

[27] This objection was suggested to me by... For criticism of the concept of measurement-induced collapse see, for example, Wallace (2012:11–45) and Maudlin (2019:xi).

[28] The name of this position might be misleading, as there are many other ways of being a realist with respect to a wave function in the sense of assuming that it represents something real. Wave functional realism is understood as a conjunction of two theses: that the fundamental space is a high-dimensional space and the wave function is a field in that space.

surements on it or because we know in which way it has been prepared). Then, we can use the equations of evolution to compute that at t_2 the wave function of the system will be ψ_2, at t_3 it will be ψ_3, and so on for k different times t_i, $i = 1, \dots, k$. Now, looking back, which wave function should we attribute to our system at t_1? A natural response here is ψ_1 as it was supposed to be the state of the system at t_1. However, recall that we assumed that the change due to our deterministic equations is only epistemic. If this change is an instance of clear epistemic success, then we should attribute to the system at t_1 some state ψ_i with $i > 1$ (and if it is an instance of unclear epistemic success, we at least do not lose anything by doing so). This is because our computations of the wave function for consecutive t_i's are not supposed to track any changes in the physical system that we investigate, but only how our beliefs about it should change. The most reasonable thing to do (under the assumptions of this argument) seems to be to attribute to our system at t_1 the wave function ψ_∞ (if it is well defined). However, this is clearly in disagreement with how the formalism of quantum mechanics is actually used.

Argument 3 (for class 3): the wave function is complete and evolves indeterministically, so that given the wave function at t_1, the probability distribution over the space of wave functions at $t_2 > t_1$ is uniquely determined (where these probabilities might be non-trivial, that is, different from 0 or 1). Assume that this evolution is epistemic. Consider an isolated system and assume that we attribute to it the wave function ψ_1 at t_1, and then we compute its evolution according to the appropriate equations, concluding that at t_2 it will be ψ_2 with probability p_2, in ψ_3 with probability p_3, and so on. If our description at t_1 was adequate at all, then it was fully adequate because of the completeness of the wave function. However, if probabilities calculated for t_2 are non-trivial, then our description at t_2 is not fully adequate (because unless we attribute probability 1 to exactly one state, our closeness to truth cannot be perfect). But recall that the change is assumed to be epistemic. Therefore, it was either a change to a less adequate description, or our initial attribution of the state to the system was inadequate.

7 Summary

I have argued that HS's terminology of "epistemic" vs. "ontic" models/states is conceptually inadequate and that the debate could be reformulated in terms of the ontic/epistemic character of the changes of states rather than states themselves. I have also shown that the epistemic understanding of the change of a quantum state under the measurement-induced collapse has certain consequences that are undesirable for those who adopt the framework of ontological models. For interpretations of quantum mechanics that do not use the concept of measurement-induced collapse, the situation is less clear as they are varied, but it has been argued that—at least for some classes of them—the epistemic nature of the change of a quantum state is also implausible.

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